An economic and actuarial analysis of *death bonds*

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1. Introduction

Life insurance is a product conceived under the premise of altruism from the contractor. The amount of the benefit is related to the financial security and comfort that the payment of the benefit can provide to the insured dead person's dependent. Traditionally, life insurance has been treated as an illiquid asset, as there is a secondary market in which it is possible to sell policies. For this reason, the only options of the insured party were to maintain the policy until the end of his life or return it to the insurance company by a pre-determined value. In this second situation, the goal of keeping the proportional value to the time of the contract could not be fully complied, once the insurance company would excessively penalize the carrier, due to various costs such as administration fees, underwriting, reinsurance premiums or commissions, imposing a high discount for the early discharge of accumulated resources.

However, due to many reasons, such as health problems, need for resources or changes in preferences, might be of interest of the insured to make liquid, still in life, the resources that would only be paid after his death. If this alternative were feasible, the insured would offer his policy in the financial market, so any agent could offer a higher value than the proposed by the insurer, obtaining, therefore, a higher rate of return than the obtained in the transaction with the insurance company. In recent years, the understanding of the financial possibilities of life insurance has changed considerably. In countries such as Germany and the United Kingdom, some policyholders have traded their policies on the secondary market. With the execution of the contract between the policyholder and the financial market institution, the bond is named of *death bond*. Although it is, from this moment, a financial bond, the issue of pricing the policy in the secondary market has actuarial characteristics, since the benefit payment is assured, but the runtime is unsure, being the mortality table the main decreasing factor for the rate of return of the life bond holder.

For the insured, the gain on this transaction is related to the early payment of the resources, with a lower discount than the insurance company would pay. This new market can be an alternative to the rigidity imposed by the insurer in its contractual relation with the insured, increasing his welfare. On the other hand, in this new market there may be a particular set of imperfections. There is a possibility of adverse selection, attracting to negotiate their policies less desired individuals to the claimant.[1] To the purchaser of the policy, the gain is inversely proportional to the survival time of the seller. This characteristic can create perverse incentives for the desire of the early death of the former insured, which would increase the profit of purchaser firms of life insurance. These incentives are the main reason why the subject has become polemic.[2] Therefore, given the existing flaws, for the good functioning of this market, a proper regulation and the existence of mechanisms to reduce any unwanted behaviour of the involved parties are needed.

Based on the described framework, the objective of the present article consists in analyzing the viability of this market by means of simulations from the insurer's pricing by contract until the pricing of such securities by the secondary market. Furthermore, when evaluating the impacts of possible failures in this market, the article identifies not only the target audience of the new product that makes the highest gains for its investors, but also quantifies the potential financial returns for various scenarios.

The article is divided into five sections, in addition to this introduction. In the following section is made a review of the literature on the subject. In the third section are presented the characteristics of operation of the secondary market. In the fourth section are defined the notations, terminologies and methodologies employed. Section 5 presents the obtained results by the simulations. Section 6 presents the results considering a market failure and, finally, in Section 7 are made the final remarks.

2. LITERATURE REVIEW

There are several journalistic publications, but few articles about *death bonds*. One of these academic articles is Quinn (2008) that, addressing important moral implications, does a survey of news and interviews to describe three conceptions of this market: *sacred revulsion, consumerist consolation, and rationalized reconciliation*. The first concept, quite original, sanctifies the life insurance policies, lives and deaths that the contracts represent, so those are appropriate to be protected through the market. The second term, according to the author, mentions the high costs associated with the death of a person, justifying the negotiations as the financing of a dignified death. Finally, the last one explains that the anonymity and distance of the buyers and the former insured eliminate any moral dilemma: the sale of contracted insurance policies is classified as moral, as a purely rational, impersonal and liable to pricing business.

Gatzert (2009) identifies characteristics and economic implications of the secondary market for life insurance, comparing the United Kingdom, Germany (in both, the asset is treated as a fixed-income security) and USA markets (whose predominance is to negotiate life insurance for elderly people with reduced life expectancy). The author argues that the major factor for the success of the secondary market is the proportion of the primary market: with many policies issued, there would be more possibilities for negotiation. Doherty and Singer

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[1] It should be noted that in this case, the less desired client can be the one with greater life expectancy.

[2] Some national and international press publications gave special attention to the issue: the July 30, 2007 edition of *Business Week* magazine, the March 10, 2010 edition of *The Wall Street Journal* and, in Brazil, an article in edition 271 (nov/2009) of *Superinteressante* magazine. On the internet, there were several demonstrations rejecting the existence of this market, the more expressive is the www.nodeathbonds.com site.

(2002) estimate that the development of the secondary market could cause an increase in demand for life insurance, bringing benefits for the providers of the primary insurance market as such an increase would reduce the liquidity risk of insurers. On the other hand, there could be a significant increase in the valuation of the premiums for the insured, since an efficient secondary market increases the liquidity of the bond, making life insurance an important asset to consumers. Such increase in value, assuming everything else constant, would increase the demand for life insurance, resulting in a higher market value.

The demand side is also remembered for Gatzert (2009), because investors will require adequate compensation by investing in this product, besides the development of this market due to improvements in pricing methods. In this context, lower interest rates will lead to a diversification of their portfolios, creating a higher demand for securities backed by life insurance.

Focusing its attention on the insured, Doherty and Singer (2002) show that the emergence of a secondary market causes welfare gains, in particular for those who need liquidity for any reason. They warn to the fact that the insured, for having an active role in the decision to sell its bond, needs to be well informed about all possibilities of changes of the original policy regime, as, for example, the option of *paid-off insurance* (in case of possible non-payment, as will be discussed in subsection 4.1), among other options, so that there are not unexpected losses.

The contractual relation between the parties has legal implications for a secondary market, discussed by Gatzert (2009), which focuses on the issue of insurable interest[3] between the insured, the insurer and the beneficiary. The author cites a British Court understanding dated from 1854 that determined that the insurable interest must be present at the time of issuance, but not in subsequent periods. In the United States, the understanding that the bonds come from unknown people of investors is considered problematic by insurers and popular associations, due to the fact that the contracts are acquired without legitimate insurable interest, initiated and controlled by investors, whose only intention is to resell them on the secondary market.

In the same way, Bozanic (2008) justifies legally the reasons for the non-existence of this market. For the author, a person has legitimate insurable interest in their own life and in the lives of others with whom he has personal relations. This principle was clearly being violated by obtaining return with the death of someone, whose name is unknown by the investor and there is no relation of intimacy. Gasner (2008), in a quite similar legal argument, describes the mechanisms of life insurance securitization and argues that, among the various types of possible agreements, there is only one specific type that, according to American laws, is illegal. It is the trading of life insurance based on lives of strangers (STOLI).

Kohli (2006) argues that the American legislation has provided an appropriate framework to protect policyholders in a *death bond* transaction, even requiring better price regulations, publicity for the market and having the possibility of any eventual conflicts of interest. The author goes further: with due consideration of the legislation, the secondary market of life can become a great source of value for the policyholders.

Differently than in more developed markets, the culture of the Brazilian insurance market is to treat life insurance as a temporary, short-term insurance (typically one year),

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[3] According to the Art. 757 of the Brazilian Civil Code (2002), "by the insurance contract, the insurer is obliged, upon payment of the premium, to ensure legitimate interest of the insured, relative to a person or a thing, against predetermined risks". So, it is understood by *insurable interest* an object of the insurance contract, subject of claim.

treatment that does not allow accumulation of capital. In this sense, the creation of the secondary market in Brazil is quite impaired, although there are no legal restrictions to the accumulation of reserves in life insurance. In addition, the Brazilian legislation, when compared to the countries above mentioned, is also dubious about the possibility of the existence of this market. The Art.760 of the Civil Code of 2002 (which regulates the insurance contracts in Brazil) has the following wording:

> "Art. 760. The policy or the insurance ticket will be nominative, to the order or to the bearer, and shall state the assumed risks, the beginning and the end of its validity, the limits of the guarantee and the due premium, and, if applicable, the name of the insured and of the beneficiary.
> Sole paragraph. In people insurance, the policy or the ticket may not be to the bearer".

It can be concluded that the policy should not be to the bearer. However, the Art. 791 of the same code has the following wording:

> "Art. 791. If the insured does not renounce to the faculty, or if the insurance does not have as a declared cause to guarantee any obligation, is lawful the replacement of the beneficiary, through act between living parties or of last will.
> Sole paragraph. The insurer that is not properly made aware of the replacement will be disobliged, paying the insured capital to the former beneficiary".

The Art. 791 allows the exchange of life insurance beneficiaries at any time, including after the death of the insured. Through the joint reading of Articles 760 and 791, it is possible to infer that the Brazilian legislation admits the existence of this market, as long as it has an explicit agreement between the former insured and the purchaser of the policy and that the insurance company must be notified about such an agreement. The only barrier to the establishment of the new market in Brazil appears to be the lack of treatment of life insurance as the constitution of accumulation of long-term financial reserve.

Kohli (2006) argues that this market can benefit insurance brokers in five ways. The first is by the reception of a commission paid by investors to facilitate the trading on the secondary market. The second reason is that, in the case of not-paid insurance, brokers can receive commissions for renewals of policies that would have gone into default and are in situations where the value of the benefit would have been reduced. The third is the possibility of reinvesting the resources from the operation, generating commissions. The fourth is that the broker can earn commissions for new life insurance policies issued on the primary market. Finally, the fifth mode is the participation of the brokers in amendments to the clauses of the policy. Given these possibilities, the market may be attractive to brokers: with the financial incentive to recommend operations that may not be the most appropriate to the situation of their customers, there is a need of additional attention on the part of legislators.

Specifically about the pricing of life insurance securities, Menoncin (2009) presents a closed form for the pricing of a *death bond*, when adopted a family of survival functions, in continuous time, called *Gompertz–Makeham* using stochastic calculus. In addition, the author presents the way *death bond* should enter the buyer agent's portfolio of life security from a former insured in order to maximize the expected utility of inter-temporal consumption and their final wealth when the time horizon coincides with the moment of receiving the benefit. This modeling contributes to the pricing of the rates of return for the agents on the market, since the author finds closed forms of pricing when the above-mentioned premises are valid. However, despite of the academic contribution, the author's approach focuses on algebraic statements for the pricing of the bond, not concluding about the viability of this market.

The expected operating contribution of this work is to give a numerical and actuarial treatment for pricing of *death bonds*, using a mortality table widely used by insurance market for the primary pricing. Based on the obtained results, it is made an extension through which it is attempted to identify the most interesting insured profiles from the point of view of the investor, as well as it is presented a discussion of the impact of possible market failures in expected financial returns.

3. The financial product *death bond*

In this section will be detailed the operation of the market of the bond, also its definition. Suppose that an insured holder of a life insurance, for any reasons, decides to sell the right of his beneficiaries of receiving an amount stipulated by the contract; thus, the new holder of this right, after paying an agreed value, becomes the new beneficiary, and will receive the benefit after the death of the insured. This bond corresponds, therefore, to a change of beneficiary of the insurance for death by payment to the insured during his lifetime, being configured as a derivative of life insurance. Therefore, a *death bond* is a bond backed in life insurance contracts, in which the reception of benefit by the carrier (in general, a financial institution) occurs at the end of the period of death of the insured. The random nature of this bond comes from the fact that despite receiving the benefit is certain, since all the insured will die, there is uncertainty about the time to be elapsed until it happens.

It is possible to describe the operation of the market of *death bonds*, as done by Menoncin (2009) as follows:

1. The insured is the underwriter of life insurance. When this agent has no more interest in life insurance, he might want to discard his policy;

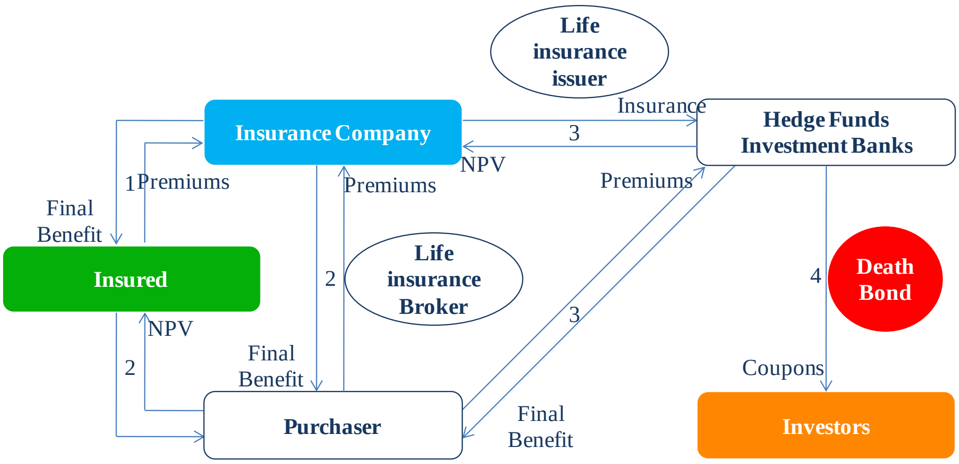
2. The insured hires a life insurance broker (compulsory figure in insurance negotiations in several countries, including Brazil) that must find a buyer for his policy. The buyer pays the net present value of accumulated reserve for the payments of the insured and receives the insurance policy. Thus, the buyer must make the remaining payments to the insurer in order to keep the initial value, or adjust its present value. The buyer will receive, from the insurer, the benefit when the insured dies. The insured pays the insurance broker a pre-established commission;

3. Another agent is the provider of life insurance. Through this agent, an investment fund or a bank buys a set of life insurances from one or more insurers. In this case, the hedge fund or investment bank will receive the buyer's premiums and will pay the final benefit and;

4. In the last step, after a sufficient amount of insurance policies are collected, such policies can fund the issuance of a *death bond*. Thus, the policies have the same role as the assets in an asset-backed security or the mortgages in mortgage securities. The new *death bond* is called a transferred asset (*pass through*) if the hedge fund premiums are paid directly to the investors.

This operating mechanism can be summarized as shown in Figure 1.

Figure 1 – Flowchart of the issuance of a *death bond*



Source: adapted from Menoncin (2009).

After the issue of a bond, it is interesting to the institution to group bonds of several individuals in a portfolio to mitigate the risk of decrease of financial return. The most interesting profiles for the creation of the portfolio will be identified further.

4. Methodology

As discussed before, the rate of return of a bond backed in a life insurance will depend basically on the financial amount that an individual has managed to accumulate in the entity in capitalization financial system insurer, which technical name is *annuity reserve (AR)*. It is an obligation of the insurer to provide future benefits to the insured, upon payment of a premium, whether it is fractionated or single.

The premium calculus is set in a way it equalizes the amount to be paid by the insured from the moment of the contracting of the product until the *expected present value (EPV)* of the flow of payments in the future made by the insurer. The EPV depends on two fundamental premises, which are the adopted survival function, which provides the probability of an individual to survive to a given period of time, and the interest rate to incur in the accumulation period of *AR*, in order to represent the financial enhancement over time.

Because it is a life insurance, it has as a premise that the benefit will be paid at once at the end of the year of the death of the insured to his dependent. Jordan (1991) argues that this premise is convenient, since the probability of death can be obtained immediately from a mortality table, not being necessary to adopt additional assumptions about the instantaneous rate of mortality. In addition, to this work, will be calculated only whole life insurance (that is, with coverage throughout the individual's life, from the hiring date), to ensure that there will be an assured payment to the beneficiary when the insured's dead.

In order to get financial protection for his family against his untimely death, the option offered by the market is the so-called *life insurance*, in which an amount is paid to the beneficiaries when the insured has died. This insurance can be for a specified period of time (*temporary*) or cover the entire life of the insured, being called *whole life insurance*.

The focus of the work is in the secondary pricing of *death bonds*, which depends on the primary contracting of life insurance, which costing form will be the capitalization financial system, normally used in the insurance market. There are several ways an insured can pay for this product: whether single-mode or in installments. There is also the possibility of irregular payment, but as it is not usual and its modeling is difficult, it will be disregarded in this work.

4.1. The model

This subsection has the main objective to present the model to be adopted for the realization of the simulations. Is also presented in summary the form the way a whole life insurance is priced, as is done by insurance companies, applying the concepts of actuarial mathematics. More details about the actuarial theory can be obtained in Jordan (1991) and Bowers et al. (1997).

The discount rate v is defined as the future flow financial reducer factor, given the interest rate (i) incurred in the period between the contracting of the insurance and the death of the insured. Financial inter-temporal discount factor, discrete-time, between t periods is set as $v = \frac{1}{(1+i)^t}$. However, the cash flow from the receipt of benefits is probabilistic, given the uncertainty of survival of the insured, being needed to multiply the flow of payments by the probability of occurrence of the event of interest; in this case, the death of the insured.

Let l_x and d_x be the quantities of living individuals aged x (measured in years) and individuals deceased before completing x+1 years respectively, given by a mortality table. It is possible to determine the probability of an individual die at age $x+t$, which notation is ${}_tp_x$, from the classical interpretation of probability, that is, by the ratio between the living aged $x+t$ and aged x. Because it is a probability, its complement ${}_tq_x$ = 1- ${}_tp_x$ is the probability of an individual aged x dies between x and $x+t$.

The payment at once (no installments) of a whole life insurance in the amount of one monetary unit (A_x), also called *pure single premium (PSP)*, is defined as the present value of the probability of death of the individual, and can be obtained using the following formula:

$$A_x = \sum_{t=0}^{\infty} v^{t+1} {}_t q_x = \frac{1}{l_x} \sum_{t=0}^{\infty} v^{t+1} d_{x+t} \,. \tag{1}$$

Simplifying with some commutation functions,[4] $D_x = v^x l_x$, $C_x = v^{x+1} d_x$ and $M_x = \sum_{t=0}^{\infty} C_{x+t}$, the value of the whole life insurance is obtained, a unit of capital paid to the beneficiary at the end of the year of the death of the insured brought to present value:

$$A_x = \sum_{t=0}^{\infty} v^{t+1} {}_t q_x = \frac{1}{v^x l_x} \sum_{t=0}^{\infty} v^{x+t+1} d_{x+t} = \frac{1}{D_x} \sum_{t=0}^{\infty} C_{x+t} = \frac{M_x}{D_x} \tag{2}$$

As already pointed out, the individual could make the insurance payment in tuition fees or monthly fees. One option is to pay for a lifetime or only for a predetermined period. However, the individual could die during the payment period (trivial case when the option is to pay throughout life) and, therefore, it is necessary to multiply the flow of payments by the probabilities of death.

For the determination of the value of the premium paid in installments, it is necessary to determine the lifetime annuity formula contracted at age x ($\ddot{a}_x$). The umlaut denotes that the premium payment will be anticipated, that is, at the beginning of each period, to ensure that

[4] In this work will be assumed the notation from *Actuarial Notation International (IAN)* for the commutation functions, which use of symbols is designed to simplify the calculation of insurance premiums and it is a standard in actuarial literature. More details at INSTITUTE OF ACTUARIES (1949).

there will be resources to make the payment, if death occurs. By defining the commutation $N_x = \sum_{t=0}^{\infty} D_{x+t}$, it is obtained:

$$\ddot{a}_x = \sum_{t=0}^{\infty} v^t \, {}_t p_x = \sum_{t=0}^{\infty} \frac{D_{x+t}}{D_x} = \frac{1}{D_x} \sum_{t=0}^{\infty} D_{x+t} = \frac{N_x}{D_x} \tag{3}$$

If the installment option is temporary, it is possible to obtain, from the equation 3, the so-called *temporary k years annuity*, which notation is $\ddot{a}_{x:\bar{k}|}$, from the subtraction between $\ddot{a}_x$ and $\ddot{a}_{x+k}$.

Therefore, the sum to be paid annually, if the insured wants to spread the value of whole life insurance, should be determined by the principle of equivalence between the expected present value at age *x* of annual income payments for life and the expected present value at age *x* of the contracted insurance. In this case, the premium is called *net level annual premium (NLAP)*, and thus:

$$\ddot{a}_x P_x = A_x \Leftrightarrow P_x = \frac{A_x}{\ddot{a}_x} = \frac{M_x}{N_x} \,. \tag{4}$$

However, the insured can opt to pay the insurance parceled, but in a temporary period of *k* years, which technical term is *net level annual premium paid in k years ($NLAP_k$)*, and which commuting formula is given by:

$${}_k P_x \ddot{a}_x = A \Leftrightarrow {}_k P_x = \frac{A_x}{\ddot{a}_{x:\bar{k}|}} = \frac{M_x}{(N_x - N_{x+k})} \,. \tag{5}$$

The accumulation of reserves depends on the number of payment of installments of premiums by the insured to the insurer. Naturally, the reserve of an individual who paid the *PSP* yields financial interest annually from the hiring date. For *NLAP* and $NLAP_k$, there are two ways to assess the *AR*: *prospective* method, which is a discounted function of the benefits and premiums to be made in the *future*, and *retrospective*,[5] which deals with the recognition of cumulated values in the *past*. For the present study, was made the option for the *prospective* method.

For a whole life insurance, in installments as a $NLAP_k$, hired at age *x*, the accumulated reserve until age *x+t*, ${}_tV_x$, can be written as:

$${}_tV_x = A_{x+t} - P_x . \ddot{a}_{x+t} = \frac{M_{x+t} - P_x N_{x+t}}{D_{x+t}} \,. \tag{6}$$

For the case of whole life insurance, which installment option follows a $NLAP_k$, the accumulated reserve to age *x+t*, which notation is ${}^k_tV_x$, can be written as:

$${}^k_tV_x = A_{x+t} - {}_k P_x . \ddot{a}_{x+t:\overline{k-t}|} = \frac{M_{x+t} - {}_k P_x (N_{x+t} - N_{x+k})}{D_{x+t}} \,. \tag{7}$$

Another possibility of insurance is the called *paid insurance (paid-up insurance)*. This life insurance modality allows that, to avoid a possible default on the part of the insured, it is possible to renegotiate the value of the benefit, from a reserve already accumulated. The

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[5] Jordan (1991) discusses and presents both methods, in addition to showing that the results are always the same, under maintenance of premises.

revaluation of the reserve of an insurance formula, denoted by ${}_tW_x$, and which payment option was *NLAP*, is given by:

$$ {}_tW_x = \frac{{}_tV_x}{A_{x+t}} = \frac{A_{x+t} - P_x.\ddot{a}_{x+t}}{A_{x+t}} = 1 - \frac{P_x}{P_{x+t}} . \quad (8) $$

Similarly, it is possible to get the value of the coverage in case it has the option to $NLAP_k$ premium installment:

$$ {}_t^kW_x = \frac{{}_t^kV_x}{A_{x+t}} = 1 - \frac{{}_kP_x}{{}_{k-t}P_{x+t}} . \quad (9) $$

4.2. Profitability of the *death bond*

In order to analyze the expected rate of return for those who get the *death bond*, it is necessary to assess the amount to be paid by the buyer. It is reasonable to assume that the buyer has the expenditure of a proportional amount to the accumulated reserve by the insured and, in case of death of the insured, the benefit would be paid to the bearer of the *death bond*.

In this way, it is defined in Equation 10, the rate of return ($RR_{x,t,n}$) of *death bond*, analogously to Fabozzi (2000), described below:

$$ RR_{x,t,n} = \frac{\left(\frac{K.{}_tW_x}{(1+j)^n}\right) - {}_tV_x}{{}_tV_x} \text{, where:} \quad (10) $$

x denotes the age at which the insured has contracted the insurance;

t is the period between the contracting of the insurance until the negotiation with the institution that acquires, i.e., the number of installments of the *NLAP* paid by the insured to the insurer;

n is the period from the acquisition of insurance by the institution until the receipt of the benefit, when the death of the insured happens, with the restriction $t \leq n$;

${}_tV_x$ is the reserve accumulated by the insured (the amount paid by the bond) between ages x and $x+t$;

j is the interest rate, denoting the opportunity cost of the institution that acquired the insurance;

K_tW_x is the amount of the benefit to be received by the institution, when the death of the insured happens.

To evaluate the expected return in *death bond* transaction for the buyer of the policy, it is necessary to assess an expected rate of return ($ERR_{x,t,n}$) at the moment of negotiation. This rate of return should be pondered by the probabilities of payment of benefit, that depends solely and exclusively on the probability of death of the insured, as evidenced by the equation 11.

$$ ERR_{x,t,n} = \frac{\sum_{n=t}^{\infty} \left\{ q_{x+n} \prod_{i=t}^{n-1} (p_{x+i}) \left[\frac{\left(\frac{K.{}_tW_x}{(1+j)^n}\right) - {}_tV_x}{{}_tV_x} \right] \right\}}{\sum_{n=t}^{\infty} \left[q_{x+n} \prod_{i=t}^{n-1} (p_{x+i}) \right]} \text{, where:} \quad (11) $$

x denotes the age at which the insured has contracted the insurance;

t is the period between the contracting the insurance until negotiations with the institution that acquires, i.e., the number of installments of the *NLAP* paid by the insured to the insurer;

n is the period from the acquisition of insurance by the institution until the receipt of the benefit, when the death of the insured happens, with the restriction $t \leq n$;

${}_tV_x$ is the reserve accumulated by the insured (the amount paid by the bond) between ages x and $x+t$;

j is the interest rate, denoting the opportunity cost of the institution that acquired the insurance;

K_tW_x is the amount of the benefit to be received by the institution, when the death of the insured happens;

q_n is the probability of the individual's death between the ages n and n+1, and;

p_i is the probability of the individual's survival at ages i and i+1. If $n = t$, the expression $\prod_{i=t}^{n-1}(p_{x+i})$ will not be calculated, given that the insured is alive when the *death bond* transaction is made.

The calculation will be presented in the next section in order to illustrate the magnitude of the returns that can be obtained by the investor in the negotiation of a *death bond*.

5. SIMULATIONS

In the following subsections are presented the calculations of rates of return that will serve as a subsidy to verify the possibility of the existence of the market for *death bonds*. The initial calculation is done for a particular profile of insured. Then will be performed a sensitivity analysis to identify which profiles of insured can be more interesting from the point of view of the *death bond* investor. Finally, it is analyzed how a market failure – the adverse selection-may affect investor return rates.

For the insured's mortality modeling will be used the most possibly conservative table, the AT-2000[6] (male and female), when it comes to survival[7]. This interest in survival is due to the unwanted character, from the point of view of *death bond* carrier, that the former insured dies after what would be expected for it.

5.1. Calculation of the Rate of Return for the investor

The calculation of the rate of return $RR_{x,t\,n}$ for the investor is made by equation 10. In the process of calculation are adopted the following hypothesis: suppose a man who hired at age x equal to 35 years a whole life insurance with face value K equal to R$ 100,000.00,

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[6] An actuarial *Annuity Table (AT)* consists of a structured method to determine the present value of a series of future payments, on the value of one monetary unity, composed of a financial discount factor pondered by the probability of occurrence of death. The AT-2000 table, standard usage in the insurance markets of Brazil and USA, was calculated based on the mortality standard of 2000 of the mass of policyholders of life insurance in the USA.

[7] Chan et al. (2006) present a comparison between various mortality tables and verify that the AT-2000 presents, for almost all ages, lower death probabilities when compared to other commonly used in the market, evidencing that the expectation of survival of the population is increasing. For this reason, it was decided to employ the most used table in the market for pricing and not to make any simulations of the impact of changing tables for it is an expected result.

whose payment option was for a *NLAP* and was compliant until he was 50 years old (therefore, t = 15) when he decided to negotiate his insurance with a financial institution. The benefit to be received in case of death of the insured is the amount corresponding to the *paid-up insurance*.

The above-mentioned values were used to calculate the *accumulated reserve* (${}_{15}V_{35}$) until the date of negotiation, from the equation 6 and the revision of the *amount to be paid as benefit* ($K_{15}W_{35}$) from a paid-up insurance (calculated with the Equation 8), assuming the carrier will no longer continue the payment of annuities of the life insurance after the sale to the investor. The results are presented in Table 1.

Table 1 – Pricing of a life insurance to an hypothetical individual

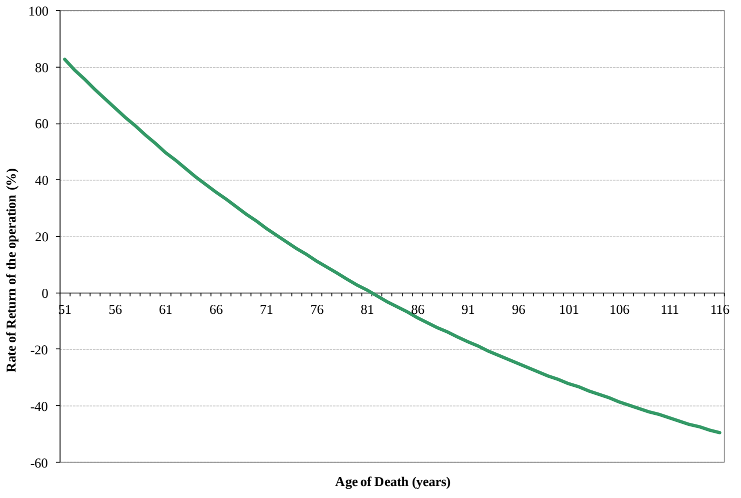
Gender (M/F)	M
Age at hiring (x, in years)	35
Number of *NLAP* paid (t, in years)	15
Age at negotiation ($x + t$, in years)	50
Financial discount (per year)	2%
Accumulated Reserve (${}_{t}V_{x}$)	R$ 21,654.58
Capital (K)	R$ 100,000.00
Paid-up insurance ($K.{}_{t}W_{x}$)	R$ 40.352,28

Source: authors' calculations.

The amounts shown in Table 1 show that the investor would be willing to pay R$ 21,654.58 (value of the accumulated reserve until age x+t = 50 years old), by an insurance that will pay a benefit of R$ 40,352.28 when the former insured dies. In case of death of the insured in the first year after the negotiation of his life insurance (n= 1, between the ages 50 and 51 years old), it is estimated, by the Equation 10, a $RR_{35,15,1}$ of 82.69%. In other words, this is the yield obtained by the investor if the insured dies within the first year after the sale of the insurance.

Once computed the investor's rate of return for the first year, were calculated the rates of return ($RR_{35,15,n}$) of the investor for each possible age ($x + t + n$, with $n \geq 1$) of death of the insured. These are the rates of return, by Equation 10, for all possible values of n, until the insured person reaches the age limit from the adopted AT-2000 table, which is 115 years. The results are shown in Chart 1. As it is possible to observe, the return function of the entity, new bond carrier is monotonically decreasing, in such a way that the moment of time when the profit would be maximized would be immediately after the conclusion of the contract. This fact can evidence the buyers' interest in acquiring bonds of individuals with higher probability of premature death, indicating the possibility of adverse selection in this market.

Graphic 1 – Rate of Return to the investor, according to the age of death of the insured

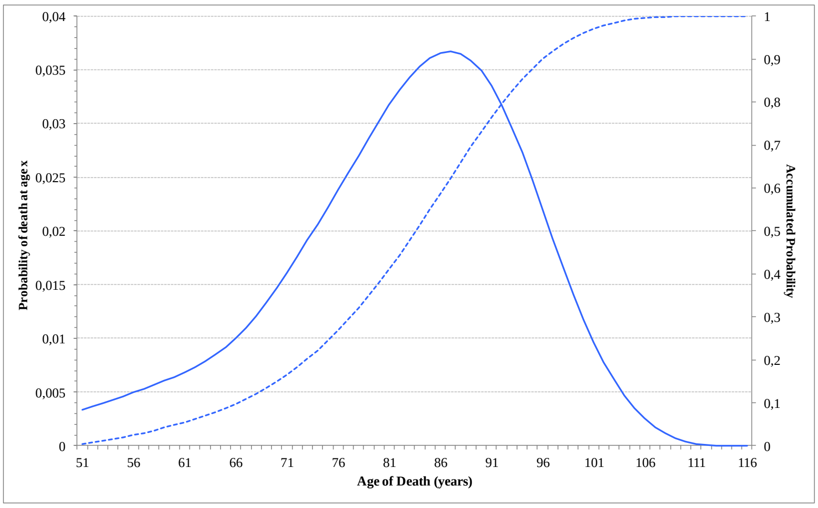


Source: authors' calculations.

It is also possible to realize, from the Graphic 1, that there is change in the sign of the return of the bearer of the bond exactly in the age in which it is expected the death of the former insured. According to the probabilities of the AT-2000 male table, the expectation of survival of a 50-year-old man is between 32 and 33 years, as you can see in Graphic 2, by the average of the mortality curve. This reversal demonstrates an intrinsic feature of the bond: given the age of the insured at the time of negotiation, the returns start to be negative exactly when the expected age of person's survival is achieved.

Once computed the investor returns, depending on the age of the insured, it is necessary to evaluate the probabilities of the insured to reach each one of the ages, in such a way that it is possible to ponder the returns according to their survival in order to get the expected returns by investors, as well as the associated dispersion with the calculated values.

Graphic 2 – Probability of being alive at age *x* and dying between ages *x* and *x+1*



Source: authors' calculations.

The continuous line on Graphic 2 shows the probabilities of the insured's death between ages x and $x+1$. The guarantee that the result is correct is evidenced by the integral of the dotted curve, which represents the accumulated mortality distribution, be equal to 1, since all the possibilities of his death until the end of the table are covered. Furthermore, the probability curve is a slightly asymmetric distribution to the left, favoring discretely the higher returns associated with younger ages.

From the point of view of the investor, to evaluate the viability of this market it is necessary to have a measure of expectation of return on the investment made in the *death bond*, as well as the uncertainty associated with this metric, which is represented by the standard deviation. Looking at the Equation 11, the rate of return expected by the investor $ERR_{x,t\ n}$ is the weighted average of the returns by age and respective probabilities of mortality. The standard deviation, that measures the degree of risk associated with the profitability, is obtained by the usual statistical mode: the difference between the second moment and the square of the first moment.

Performing the *ERR* calculation and its standard error, for this subsection's example, $ERR_{35,15,n}$ = 0.00001887%, with a standard deviation of 24.2%. In this way, the investor faces a market of return expectation apparently low, with a high risk.

5.2. Sensitivity analysis of the rate of return for the investor

Once executed the calculation of rate of return $RR_{x,t,n}$ for an individual with a given set of characteristics, analysis of the impact of changes in the parameters are made.[8] Similarly to the calculations carried out in the previous section, changes will be incorporated in the insured person's gender (male and female), at the age of hiring life insurance (x = 0, 20, 40, 60 and 80 years old), having paid a NLAP during t = 20 years,[9] constant, and changes in the age of secondary trading ($x + t$ = 20, 40, 60, 80 and 100 years old, respectively). The results are presented in Graphic 2. The curves in question show all the rates of return for each one of the possible ages of death of the insured, from the moment of negotiation. As an example, the curve of returns for a carrier of a bond from an insured woman who sold her insurance at 40 years old, whose contracting was made at 20 years old, is given by the dark green color. Similarly, the curve of returns for an insured man with the same characteristics is given by the color light green.

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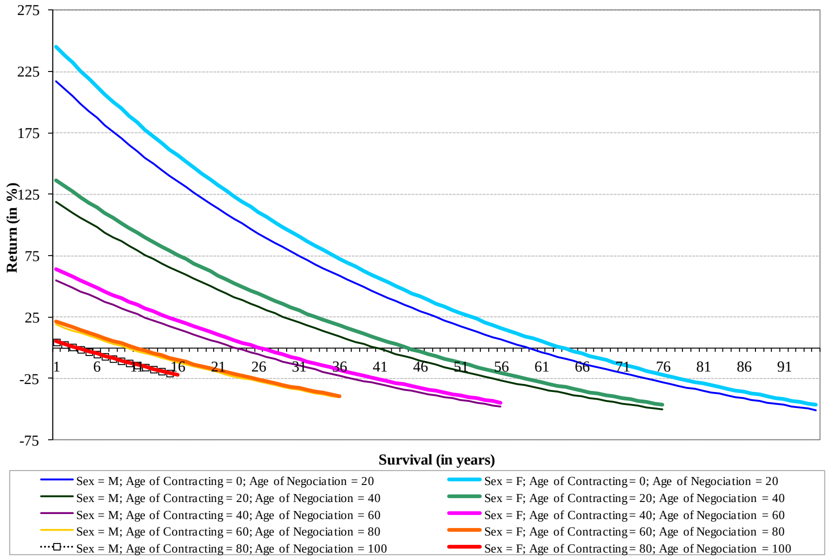
[8] It was opted for not simulating variations in the interest rate and mortality due to two corollaries derived from Lidstone Theorem, showing that increases in interest rate (or the probability of death) produce decrease in accumulated reserves (and vice versa) and, therefore, the amount paid by the purchaser of the bond. Those increases affect the magnitude of the returns, but not the general format of the curve. The proof of this theorem can be found in Jordan (1991, chap. 5)

[9] All the simulations were made by the revaluation of reserves at the date of secondary negotiation, in such a way that, regardless of how many portions of the *NLAP* were paid, the investor's future return remains constant, given the age of negotiation.

Graphic 3 – Rate of Return of the institution by age of death of the insured



Source: authors' calculations.

Observing Graphic 3, it is possible to reach an initial conclusion. There are differences in returns between the genres, so that the returns obtained by an insurance policy of an insured woman are always higher when compared to the returns of an insured man in equal conditions of contracting ages and negotiation. This phenomenon is explained by the difference between the mortality patterns, since women have higher survival expectation and this is reflected in lower probabilities of death, making the premiums and, therefore, the reserves and amounts paid by the buyer of *death bond*, smaller. Although the amount of the benefit, when revaluated, is also smaller, the relative reduction in the amount of the benefit is less intense than the reduction of the amount to be paid for the reserve, also due to the higher survival expectation of a woman.

The same explanation should be used to analyze the change of level of the other changes. As are acquired bonds from increasingly elderly people, the return by age decreases due to the fact that the amount to be paid by the investor will be very close to the value to be received. Another reason is that the primary pricing charges on more elderly insured because of the imminence of their death.

For the investor to be able to make a more accurate evaluation of the profitability of this bond it is necessary to have an estimate of the expected rate of return, pondering each return rate of each one of the curves presented in Graphic 3 by the respective probability of occurrence of death of the insured. This is done with the use of Equation 11, through which is calculated $ERR_{x,t\ n}$, using an analogous procedure to that used in the example of the previous sub-section. Also are reported the standard deviations. The results are presented in Table 2.

Table 2 – $ERR_{x,20,n}$ of the *death bond* for the simulated individuals of different ages of negotiation ($x+20$).

Age at negociation	**Gender**	
	M	**F**
20	0,000009% (32,99%)	0,000032% (30,46%)
40	0,000009% (27,11%)	0,000033% (25,63%)
60	0,000010% (20,21%)	0,000035% (20,01%)
80	0,000018% (11,76%)	0,000051% (12,04%)
100	0,000381% (4,47%)	0,000803% (4,71%)

Standard-deviation between parenthesis Source: authors' calculation.

The first conclusion that can be educed from Table 2 is that it is an investment that always shows positive rates, although extremely reduced, indicating little attractiveness in this product, since most fixed income securities may present quite higher returns than the found. One explanation for this result is the fact that the investor prices the insured's bond using the same mortality table used by the insurer. In this way, the investor's gain is given by the elapsed life time between the insurance contracting and the negotiation, which probability of death of the insured was low. This fact signals that the investor will have higher returns the more different is the expectation of death of the insured at the time of the transaction and what was predicted by the primary pricing.

Other relevant information is that, for both genders, the older is the insured more attractive is the rate of return for the investor. Finally, when you analyze the rates of return of a certain age of negotiation, for bonds from female lives, relative to male lives, it is possible to notice that they are always superior to 2, though they decrease as age increases. This fact is an evidence that the investor should have more interest in insured women than in insured men.

Analyzing the average dispersion associated with each one of the presented expected returns, it is possible to infer that the uncertainty of return decreases as the age of the life that originated the bond increases, regardless of genre. When comparing the genres, there's the perception that the bonds linked to women's lives are less volatile than those linked to men's lives up to a certain age (between 60 and 80 years old), when the situations are inversed. These results contradict what Gatzert (2009) and Doherty and Singer (2002) suggest to bonds of life of elderly: a *death bond*, priced according to an market standard table, is not an investment that has a low risk measure, since, although investors will certainly receive the benefit, it is unknown when this will occur. This implies that, in addition to investors may incur high opportunity costs and effective returns strongly negative, the expected return can be considered null, in such a way that its attractiveness is very low.

So, faced with a population whose mortality can be modeled with a known table, it is possible to infer that in case of an investor of a *death bond* wants to maximize the expected return, he would search for bonds linked to older women. However, if he searches for higher individual returns, he should acquire bonds from female and young individuals. Stands out, though, that expected rates can be null at almost any confidence level, indicating that it is not an interesting financial investment product, since the secondary pricing reflects nearly the same uncertainty as measured by the primary pricing, differing only by the probabilities of death until the period up to the negotiation of the bond.

Given this scenario, would there be more propitious conditions for the both sides to be interested in the transaction of the *death bond*? This will be done in Section 6.

6. Adverse selection: trying to identify the most profitable insured

In the previous section were identified the most interesting profiles for investors, when the probability of death is modeled by any standard mortality table. However, due to the asymmetry of information about the actual health condition (and hence, the probability of death) of the insured, the investor who acquires the *death bond* may estimate incorrectly the expected rate of return. Akerlof (1970) showed in a pioneer work that if all insurers have imperfect information about the individual risk, the insurance market could not exist and, in case of existence, could not be effective. This fact would justify, for example, insurances of people with more advanced age have higher prices as their probabilities of death increase and, therefore, end up being more interesting for the investors of *death bond*. Thus, investors have interest in knowing more characteristics of the individual than just their age.

According to Varian (1992), the *adverse selection* is a market failure in the principal (institution) has less information that the agent (insured), that can cause the non-existence of a equilibrium, with a unilateral change of behavior. As the investor is not able to estimate precisely the real probability of occurance of the event (in this case, the individual's death), the bond should be priced by the average price of all bonds. However, the insured with higher death probability would be those that investors would be more interested in purchasing their bonds, since they could receive higher financial return due to the early death of the insured.

Whereas people with some diseases, such as cancer and AIDS, for example, have higher probability of dying, the investor would like to identify them from healthy individuals, offering different prices in order to induce them to distinguish themselves from the others. This mechanism of selection from the part of investors could make individuals disclose their actual health conditions. A hypothetical way to select these policyholders would be finding them with their diseases already diagnosed.

In the literature were not found specific mortality tables for patients with serious diseases. Given this fact, the medical literature was used in order to get an estimative of the expected survival of a person with a serious illness. For this, it was used the information from the *EUROCARE* project, that has studied cancer survival in European countries. In this work from Coleman et al. (2003) is made a detailed analysis of a longitudinal study with 5 years duration, evaluating 1.8 million adults and 24,000 children diagnosed with cancer of various types, between 1990 and 1994 and monitored until 1999. Estimates of the probabilities of death that will be used to ponder the return rates were obtained from Coleman et al. (2003) with the data of the project *EUROCARE-3* and are presented in Table 3.

Table 3 – Categorized distribution of survival probabilities of types of cancer in men and women in Europe, diagnosed between 1990 and 1994 and monitored until 1999.

Category of probability	**Average point of category**	**Men**	**Women**	**Total**
≥80%	90%	2%	5%	4%
60–79%	70%	31%	45%	38%
40–59%	50%	25%	23%	24%
20–39%	30%	10%	12%	11%
<20%	10%	32%	14%	23%
Total		100%	100%	100%
Average probability of surviving 5 anos		**42,2%**	**52,5%**	**47,8%**

Source: adapted from Coleman *et al*. (2003).

The Table 3 presents the types of cancer in men and women by categories of survival probability within 5 years, follow-up study time. As an example, 32% of cancer types that occurred in men in the considered period had survival probability of analyzed individuals lower than 20%. For this range of survival probability, 14% of the cancer types occurred in women. This fact suggests that this disease is less fatal in women than in men.

Based on the information of Table 3, is reached an estimative of the probability of survival within 5 years by genre, pondering such probabilities by the observed frequencies in genres. It is possible to infer that, in general, 42.2% of men and 52.5% of women with cancer survive up to 5 years after the diagnosis of the disease. Based on this information, a linear interpolation was made to evaluate the annual probabilities of death until the 5th year of negotiation and, making a linear interpolation, up to the maximum period of survival after the 5th year of negotiation. Therefore, with this interpolation, it was found that the annual probability of death for men patients of cancer is 11.56%, constant. Similarly, the annual probability of death for women is of 9.50%. Therefore, with the linear extrapolations, a man with cancer would live up to 9 more years and a woman with cancer, 12 more years from the diagnosis.

Using the premise that the person negotiates their life insurance in the moment they discover the disease (assuming the purpose of negotiation of the bond is to get resources to the treatment), it is repeated the procedure from the subsection 5.2, using interpolated and extrapolated estimates of the probabilities of death. The results are presented in Table 4.

Table 4 – $ERR_{x,20,n}$ of the *death bond* to the simulated individuals of different ages of negotiation ($x+20$), using probabilities of death of cancer patients.

Age at negotiation	Gender	
	M	F
20	185,57% (12,64%)	203,25% (16,35%)
40	96,91% (8,71%)	107,39% (11,18%)
60	39,67% (6,18%)	44,29% (7,78%)
80	7,00% (4,73%)	6,66% (5,75%)
100	-5,65% (4,17%)	-7,77% (4,97%)

Standard-deviation between parenthesis Source: authors' calculations.

Analyzing the Table 4, it is possible to perceive the effect of adverse selection, so that the higher returns are associated with *young patients*, the most attractive profile, in this situation, for investors. This is a consistent result with the adopted assumptions, since the aggravated probabilities of death are those associated with the highest possible returns, given that the returns are monotonically decreasing. In addition, women provide higher returns than men. Coleman et al. (2003) justify the higher probability of survival expectancy of women due to the fact that the most common cancers in men (lung and stomach) have high rates of death and the most common in women (breast and uterus) have higher rates of survival and recovery.

The new expected returns rates presented in Table 4 are higher and their respective standard deviations are sensitively lower in relation to the reported results in Table 2. As negotiating with individuals who discover the disease later, both expected returns and the associated variances decrease, because it is expected that these individuals will die when they are older. This is the result that guarantees the attractiveness of this market on the part of investors.

A particularly interesting result is the reversal of the profile of interest between 60 and 80 years old (more precisely between 78 and 79 years old): although it is already possible to accept the hypothesis (with 95% of confidence and assuming normality) that the expected return can be considered null, the expected return to an elderly man is higher than for an elderly woman and also with less associated uncertainty. This situation is related to the fact that women with cancer have a higher survival expectation than men, which means that the aggravation is applied for a longer time and in lower returns, what justifies both the reduction of the expected rate of return and the increase of the uncertainty associated with this index.

7. Final Remarks

In this study it was made an analysis of the viability of secondary market for trading life insurance, through primary and secondary pricing simulations of life insurance, as well as to evaluate the possible market failures. By means of a standard mortality table, were evaluated expected return rates for investor and, considering that this is a certain receiving, but with uncertain time horizon, the standard deviations associated with the fees. The first comparisons between the main factors for pricing, gender and age, evidence that initially this is a low attractive investment once it presents positive rates, but quite low. Other relevant information is that, regardless of gender, the older is the insured, the higher is the rate, as also there is higher interest in women's lives than in men’s.

Evaluating a situation in which it is reasonable to assess an increased probability of premature death, was created a scenario based on medical literature. The results pointed to the high attractiveness of the product by the investors, once the *death bond* brings significantly positive and very high return rates if compared with the result obtained by the market standard mortality table used in the first scenario. Therefore, there is evidence of strong adverse selection in this market.

It is necessary to be cautious when making inferences about the validity of these results for different stages of the disease. Cancer is a disease that when diagnosed in late stages, can show lower probabilities of healing. For future studies, it is possible to think of the analysis for other types of diseases or in the more refined modeling of the curve of probability of death, as well as other possibilities of adverse selection.

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