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On improving pension product design

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Operations Research and Financial Engineering

- Large scale-optimization models and algorithms to assist the companies in making high-level decisions
  - Airport Operations Management, Maritime Optimization, Railway Optimization, Timetabling

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- Financial applications:
  - Risk Management and ALM, along with institutional constraints as well as uncertain cash flows, disbursements and taxes
  - Individual ALM – personal financial planning, savings management in DC pension plan

On improving pension product design

- Focus on DC pension plans (labor market pension and individual pension plans) as they are:
  - quickly expanding,
  - easier and cheaper to administer,
  - more transparent and flexible so they can capture individuals' needs.

- However,
  - if too much flexibility (e.g. U.S.), the participants do not know how to manage their savings,
  - if too little flexibility (e.g. Denmark), the product is generic and does not capture the individuals' needs.

What do we improve?

- Common questions regarding management of pension savings:
  - How to invest the savings?
  - How to spend the savings?
  - How much savings to leave to the heirs?

- Three main decisions:
  - Investment strategy
  - Payout profile
    - duration of the payments (lump sum, 10-25 years, or life long)
    - payout curve (constant, increasing, or decreasing)
    - level of payments
  - Level of death benefit

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Economical and personal characteristics

- Pension savings management is **individual** and should capture the individual’s characteristics:
  - Economical:
    - ✓ Current wealth
    - ✓ Pension contributions (mandatory and voluntary)
    - ✓ Expected state retirement pension
  - Personal:
    - ✓ Risk aversion
    - ✓ Lifetime expectancy
    - ✓ Preferable payout profile
    - ✓ Bequest motive
    - ✓ Preferences on portfolio composition

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Pension savings management should also be **optimal** for the given individual.

Multi-stage Stochastic Programming (MSP)

- Optimization software – **numerical solution**

✓ General purpose decision model with an objective function that can take a variety of forms

✓ Can address realistic considerations, such as transactions costs, surrender charges, taxes

✓ Can deal with details

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Multi-stage Stochastic Programming (MSP)

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✓ Can address realistic considerations, such as transactions costs, surrender charges, taxes

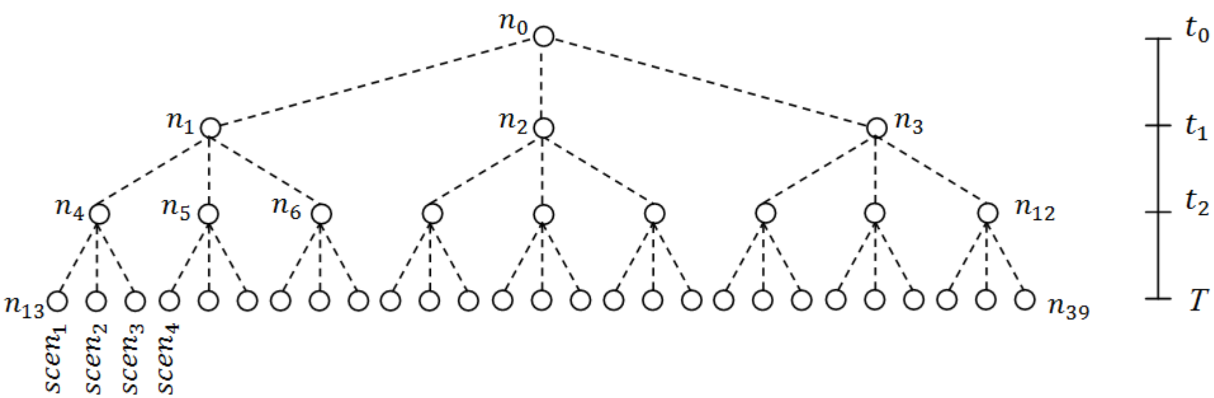
✓ Can deal with details

✗ Problem size grows quickly as a function of number of periods and scenarios

✗ Challenge to select a representative set of scenarios for the model

✗ May be difficult to understand the solution

MSP - Scenario tree



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MSP - Scenario tree

t_0

n0 prob = 1	Benefits	36.8
	Bequest	183.9

	Purchases	Sales	Pi
Cash			
Bonds	351.9		0.57
D.Stocks	214.6		0.35
I. Stocks	52.4		0.08

t_1

n1 prob = 0.069	Benefits	38.4
	Bequest	192.0

	Purchases	Sales	Pi	Returns
Cash				0.024
Bonds		75.4	0.61	0.179
D.Stocks		1.5	0.32	-0.164
I. Stocks		24.4	0.07	0.252

n2 prob = 0.073	Benefits	39.1
	Bequest	195.4

	Purchases	Sales	Pi	Returns
Cash				0.055
Bonds	8.8		0.61	-0.023
D.Stocks		88.2	0.32	0.238
I. Stocks		23.8	0.07	0.253

n3 prob = 0.071	Benefits	37.1
	Bequest	205.7

	Purchases	Sales	Pi	Returns
Cash				0.000
Bonds		25.5	0.61	0.130
D.Stocks		85.0	0.32	0.263
I. Stocks	1.2		0.07	-0.165

n4 prob = 0.05

	Purchases
Cash	
Bonds	
D.Stocks	
I. Stocks	

etc.

t_2

t_3

T

n311	n312	n313	n314	n315
Returns				
-0.0651	0.0404	-0.0496	0.1140	-0.0651
0.1691	0.0095	-0.1137	0.2038	0.2288
0.5928	-0.1697	0.5280	0.5349	-0.0593
-0.1853	-0.1769	0.4987	0.5005	0.5058

n316	n317	n318	n319	n320
Returns				
-0.0824	0.0384	0.1101	-0.0471	-0.0651
0.2369	0.0139	0.2048	-0.1155	0.1691
-0.0582	-0.1691	0.5137	0.5178	0.5928
0.5088	-0.1766	0.5004	0.4976	-0.1853

n321	n322	n323	n324	n325
Returns				
-0.0194	0.0116	0.2851	-0.0248	0.0322
0.0031	0.3143	0.2862	0.2502	-0.0104
0.5131	-0.1492	0.6583	0.4865	-0.1588
0.5017	0.4907	0.5134	-0.1585	-0.1827

n326	n327	n328
Returns		
-0.1295	0.0349	0.1421
-0.1241	0.0600	-0.0977
0.1899	-0.1516	0.5338
0.5546	-0.1344	0.5171

etc.

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MSP formulation

CRRA utility function:

$$u(t, B_{t,n}) = \frac{1}{\gamma} e^{-\rho t} B_{t,n}^{\gamma}$$

maximize:

$$\sum_{s=\max(t_0,T_R)}^{T-1} \sum_{n \in \mathcal{N}_s} {}_s\tilde{p}_y u\left(s, B_{s,n}^{tot}\right) \cdot prob_n$$

$$+ \sum_{s=t_0}^{T-1} \sum_{n \in \mathcal{N}_s} {}_s\tilde{p}_y \, \tilde{q}_{y+s} \, k \, u\left(s, Beq_{s,n}\right) \cdot prob_n$$

Parameters:

$1-\gamma$	risk aversion,
ρ	impatience factor,
T_R	retirement time,
T	end of decision horizon, and beginning of the period modelled by SOC,
$prob_n$	probability of being in node *n,*
k	weight on bequest motive,
${}_t\tilde{p}_y$, $\tilde{q}_y$	individual's expectations about survival and death probabilities

Variables:

$B_{t,n}^{tot}$	total benefits at time *t,* node *n*
$Beq_{t,n}$	bequest at time *t*, node *n*

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+ &\sum_{s=t_0}^{T-1} \sum_{n\in\mathcal{N}_s} {}_s\tilde{p}_y\, \tilde{q}_{y+s}\, k\, u\left(s, Beq_{s,n}\right) \cdot prob_n
\end{aligned}\right\} \text{MSP}$$

$$\left.+ \quad {}_T\tilde{p}_y \sum_{n\in\mathcal{N}_T} V\left(T, \sum_i X_{i,T,n}^{\rightarrow}\right) \cdot prob_n \right\} \text{SOC}$$

Parameters:

$1-\gamma$	risk aversion,
ρ	impatience factor,
T_R	retirement time,
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$X_{i,t,n}^{\rightarrow}$	amount allocated to asset i, period t, node n
$V(\cdot,\cdot)$	end effect; optimal value function calculated explicitly using SOC approach

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MSP formulation

CRRA utility function:

$$u(t, B_{t,n}) = \frac{1}{\gamma} e^{-\rho t} B_{t,n}^{\gamma}$$

maximize:

$$\left.\begin{aligned} & \sum_{s=\max(t_0, T_R)}^{T-1} \sum_{n \in \mathcal{N}_s} {}_s\tilde{p}_y u\left(s, B_{s,n}^{tot}\right) \cdot prob_n \\ + \quad & \sum_{s=t_0}^{T-1} \sum_{n \in \mathcal{N}_s} {}_s\tilde{p}_y \, \tilde{q}_{y+s} \, k \, u\left(s, Beq_{s,n}\right) \cdot prob_n \end{aligned}\right\} \text{MSP}$$

$$\left. + \quad {}_T\tilde{p}_y \sum_{n \in \mathcal{N}_T} V\left(T, \sum_i X_{i,T,n}^{\rightarrow}\right) \cdot prob_n \right\} \text{SOC}$$

subject to constraints:

(See p. 9-10 in the paper for the complete set of constraints)

Parameters:

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Optimal annuity payments and death sum

- Generalize Merton (1969, 1971) and Richard (1975) results:
  - Whole life annuity
  - The level of payments is proportional to the value of savings and to the present value of expected state retirement pension, and is defined by the optimal withdrawal rate that depends on the personal preferences and market parameters
  - The level of death sum is proportional to the level of payments

age	65	70	75	80	85	90
constant benefits, γ=-4, ρ=0.119	6,2%	6,8%	7,5%	8,5%	9,8%	11,4%
decreasing benefits, γ=-4, ρ=0.04	6,7%	7,2%	8,0%	8,9%	10,2%	11,7%
increasing benefits, γ=-4, ρ=0.15	5,1%	5,7%	6,5%	7,6%	8,9%	10,6%
constant benefits, γ=-2, ρ=0.132	8,1%	8,6%	9,3%	10,2%	11,3%	12,7%

Optimal withdrawal rates given optimal investment strategy

Optimal annuity payments and death sum

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  - Whole life annuity
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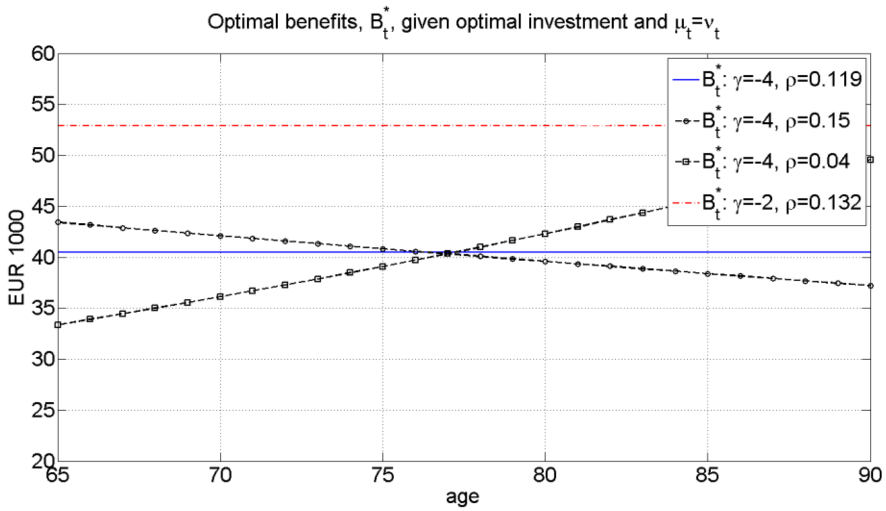
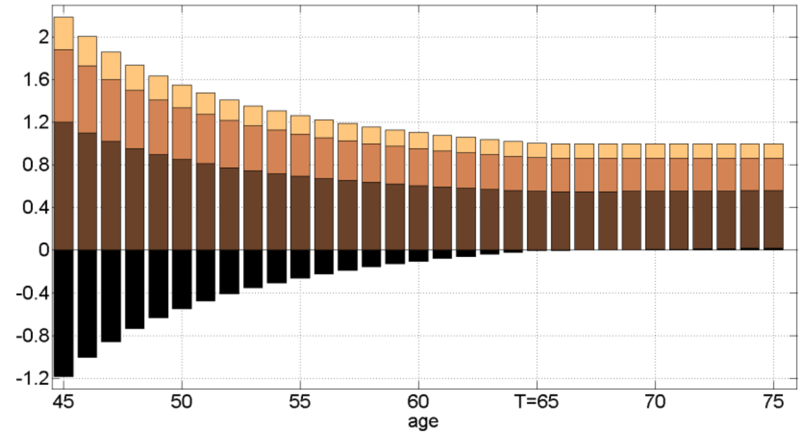
  - The level of death sum is proportional to the level of payments



Optimal benefits given optimal investment strategy

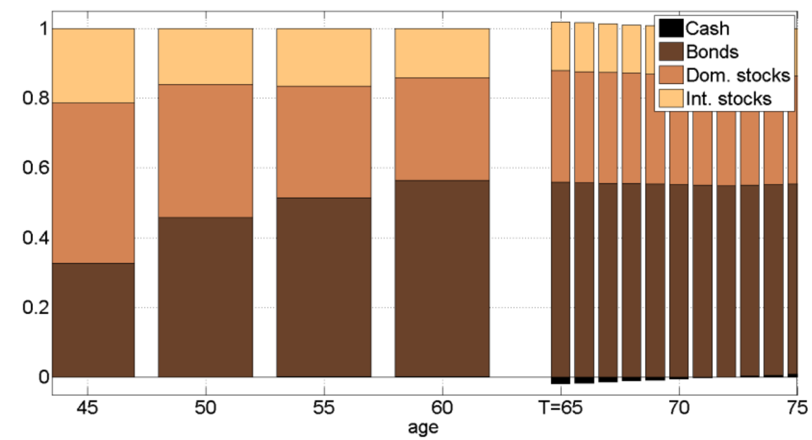
Optimal investment

- Generalize Merton (1969, 1971) and Richard (1975) results:
  - ➢ Equity-linked annuity
  - ➢ Optimal investment strategy depends on the value of savings, present value of expected state retirement pension, market parameters, and risk aversion



Optimal asset allocation - SOC approach

- A combination of MSP and SOC approaches ensures realistic solution



Optimal asset allocation
– a combined MSP and SOC approach

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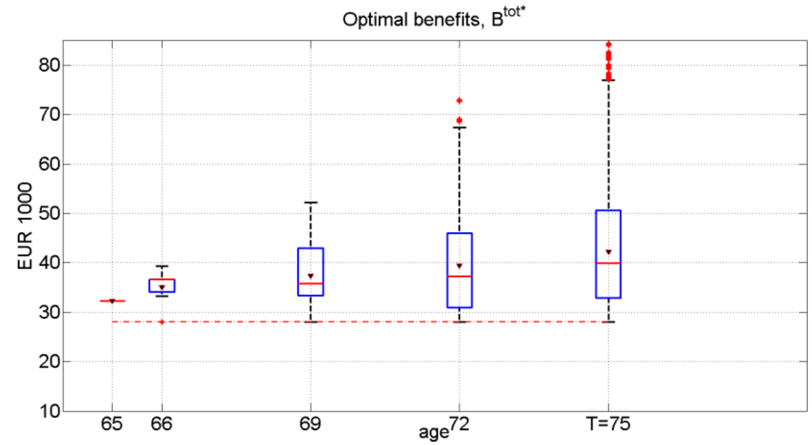
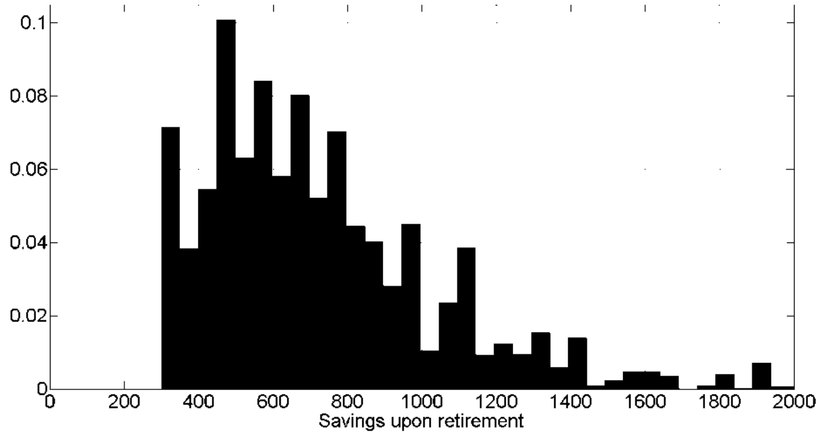
Other personal preferences

- Possible to set upper and lower bounds on variables (non-trivial to solve explicitly), e.g.:
  - Minimum level of the annuity payments, value of savings, death sum
  - Limits on portfolio composition



Optimal total benefits given minimum level of the benefits, EUR 28,000.



The value of savings upon retirement given additional premiums of 5% and a minimum level of savings upon retirement of EUR 300 000.

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One final thought…

Operations research methods have potential to stimulate new thinking and add to actuarial practice.

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Thank you

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