Australian Investment Performance 1959 to 2013 (and Investment Assumptions for Stochastic Models)

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Abstract

This paper analyses 54 years, or 216 quarters, of Australian investment performance from 30 June 1959 (and earlier for some sectors) to 30 June 2013.

The paper updates three previous papers presented in 2005, 2007 and 2009 to the Institute of Actuaries of Australia and to the ICA 2010. The aim is to assess whether the methodology for determining assumptions in the previous papers is still robust enough to produce reasonable financial assumptions now that a further four years of financial data, covering a tumultuous period in global markets, is being examined.

The analysis covers eleven investment classes and four key financial indicators:

Growth Securities	Interest Income	Financial Indicators
Australian shares	Australian fixed interest	CPI (price inflation)
Int'l shares (hedged)	Int'l fixed interest (hedged)	AWOTE (wage inflation)
Int'l shares (unhedged)	Government semis (0-3yrs)	90-day bill rates
Property trusts	Inflation linked bonds	10-year bond rates
Direct property	Loans/corporate credit	
	Cash	

For each of these 15 "sectors" the annualised average results are tabulated and summarised for:

- risk margins (over 10-year bond rates),
- coefficients of variation,
- skewness,
- kurtosis,
- cross-correlations, and
- auto-correlations.

From these results, assumptions are developed for the mean, standard deviation, skewness, kurtosis, cross-correlations and auto-correlations for each sector. The assumptions are intended for both medium-term (3 to 10-years) and long-term (10 to 40-years) modelling. These assumptions are primarily designed for use, until 2016, in stochastic investment and asset/liability modelling. After about two or three years they should be updated.

The paper also analyses:

- economic cycles using a sine curves technique,
- the impact of the Global Financial Crisis, comparing it with previous market downturns in 1974, 1987 and 2002/03, and
- auto-correlations, concluding that Australian share and bond 26-year auto-correlations ending 1986/87 were similar, after re-scaling the X-axis, to those ending 2012/13.

Keywords: *investment assumptions, stochastic, skewness, kurtosis, cross-correlations, auto-correlations, cycles*

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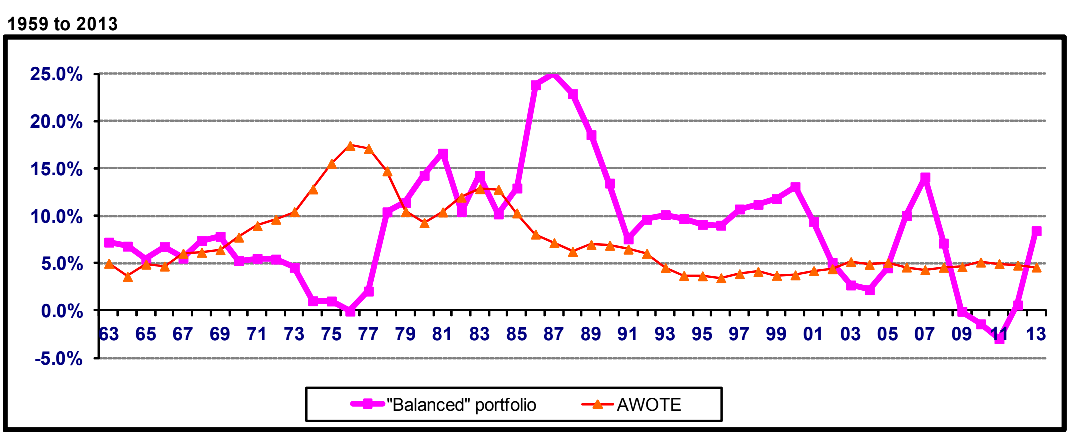

Historical 4-year (ending 30/6/63 to 30/6/13) compound average annual returns Source: *Austmod*, net of tax and fees

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1 Introduction

1.1 **Demand:** Most actuarial work requires assumptions about future investment returns. Often assumptions are also explicitly or implicitly made about future rates of inflation and/or salary increases.

1.2 **Supply:** However, often for commercial reasons, little information is publicly available for many sectors/asset classes about the historical data from which investment return assumptions, particularly long term assumptions, can be derived.

1.3 This paper attempts to bridge, at least partly, the gap between demand and supply.

1.4 Actuaries use investment assumptions in all practice areas for:

(a) premium or contribution rate calculations,
(b) valuations of liabilities,
(c) (sometimes) valuations of assets,
(d) capital assessments,
(e) benefit and other projections, and
(f) (often) investment strategy calculations.

1.5 In February 2009 the IAAust Life Insurance and Wealth Management Practice Committee announced the establishment of an Equity Risk Premium (ERP) Research Group whose objective is to "assist actuaries in forming their view of methods to determine the equity risk premium for use in practise". The explanatory note announcing the plans for the ERP Research Group commenced:

"***BACKGROUND***

1 The equity risk premium in which we are interested is the additional return (over government bonds of similar duration) that can be expected to be earned from investment in shares – to compensate for non-diversifiable risk. It is the only significant variable that cannot be observed or inferred from market prices.

2 Judgements as to the size of the equity risk premium, however, form an essential element of a number of actuarial tasks:

- *Valuation of risky cash flows that are not linked to listed assets or liabilities (such as with Embedded or Appraisal Values)*
- *All investment strategy problems where the liabilities are not guaranteed (such as the selection of benchmarks and asset liability management)*
- *Determination of capital required when guaranteed liabilities are backed by equity type assets*

3 Obtaining the best estimate of the equity risk premium is a public good. There is limited competitive advantage in being able to predict the risk premium accurately. An authoritative survey of expectations may however assist in the appropriate allocation of capital in equity markets and discourage excessive bubbles and panic."

1.6 Though important, the ERP is just one assumption for one sector. This paper aims to set many assumptions - for means, standard deviations, skewness, kurtosis, cross-correlations and auto-correlations - for each of the 15 sectors under consideration.

2 Methodology

2.1 The one methodology was used, with only slight variations, for calculating assumptions for risk margins, coefficients of variation, skewness, kurtosis and correlations.

2.2 **Step 1:** annual rates of return and forces of return for the 15 sectors were sorted into four separate arrays:

- **(i)** years ending 30 September,
- **(ii)** years ending 31 December,
- **(iii)** years ending 31 March, and
- **(iv)** years ending 30 June.

2.3 **Step 2:** determined the periods over which statistics are calculated. Periods of 44 years and 40 years were chosen for the reasons explained in Section 5 (in particular sections 5.18 to 5.20).

2.4 **Step 3:** the four 44-year periods were used to derive average annual risk margins, coefficients of variation, skewness, kurtosis and 105 ranked and unranked cross-correlations. These were calculated and tabulated for each of the forty-four year running periods for each quarter ending in the ten years between 30/9/03 and 30/6/13 inclusive. Average annual risk margins and coefficients of variation were also calculated and tabulated for 40-year periods.

2.5 **Step 4:** the four quarter-ending results were averaged to give sets of ten 44-year running period results. That is:

Table 2.1 Periods for 44-year calculations

Period	Average of four periods ending:
9	30/9/03, 31/12/03, 31/3/04 and 30/6/04
8	30/9/04, 31/12/04, 31/3/05 and 30/6/05
7	30/9/05, 31/12/05, 31/3/06 and 30/6/06
6	30/9/06, 31/12/06, 31/3/07 and 30/6/07
5	30/9/07, 31/12/07, 31/3/08 and 30/6/08
4	30/9/08, 31/12/08, 31/3/09 and 30/6/09
3	30/9/09, 31/12/09, 31/3/10 and 30/6/10
2	30/9/10, 31/12/10, 31/3/11 and 30/6/11
1	30/9/11, 31/12/11, 31/3/12 and 30/6/12
0	30/9/12, 31/12/12, 31/3/13 and 30/6/13

Thus for example, as at 30 June 2013:

0 represents periods ending between 0 and 0.75 years ago, and
9 represents periods ending between 9 and 9.75 years ago.

2.6 The same periods (9 to 0) were used for the 40-year calculations. For example, in these calculations period 9 represents the average of 40-year calculations ending 30/9/03, 31/12/03, 31/3/04 and 30/6/04.

2.7 **Step 5:** a weighted quadratic EXCEL trend function was fitted to each of the 10 results from section 2.5 (and to each of the 10 results from section 2.6) and projected forward six years. For the 44-year calculations the weights used were 50% for the earliest period, increasing linearly to 100% for period 0. A quadratic basis was chosen for the fitting to avoid spurious turning-points. For the cross and auto correlations the approach was slightly varied (see Sections 13 and 15). The number of fittings and projections was:

Table 2.2 Number of fittings and projections

Projection	Fittings & projections	Number	Bases
Risk margins	Two per sector except bonds	28	40 & 44 yrs
Coeff. of variation	Two per sector	30	40 & 44 yrs
Skewness	One per sector	15	44 years
Kurtosis	One per sector	15	44 years
Cross-correlations	One per combination	105	44 years
Rank cross-correlns.	One per combination	105	44 years
Auto-correlations	38 each for bonds and shares	76	40 years

For the 40-year calculations, that were only used for risk margin and coefficient of variation calculations, the trend weights used were 10% for the earliest period, increasing to 100% for period 0. Thus for these calculations data before 30/9/63 was not used and data before 30/9/68 was given a weight of 50% or less. This was done to achieve three objectives;

(a) Importantly, by starting with a low trend weight (i.e. 50% or 10%) the impact on running averages, period by period, from dropping off old data was minimised – thus the trends in the running averages are impacted far more by the new data being introduced each year and far less by the old data dropping off. This is important for the Step 6 two-year projection (or extrapolation).

(b) For the 40-year calculations it minimised the use of old data, much of which is "backdated" (see Section 4).

(c) By using 44-year averages with trend weights starting at 50% and 40-year averages with trend weights starting at 10%, the difference between both sets of results was maximised – so as to give a range of possible assumptions for the important risk margin and coefficient of variation calculations.

2.8 **Step 6:** recommendations of medium to long-term assumptions were made based primarily, but not totally, on the two-year projected results. **The rationale for this is that these can be considered as a 'best estimate' of the 44-year (or 40-year) four-quarter averages expected in two years time.** The two years might also be considered as the possible lifetime of these assumptions until they are subsequently updated. **This weighted trend plus two year approach has been designed to give greatest weight to the latest period with recognition of past trends assumed to continue for at least the next two years.** The projected results for year 2 were used as a guide (or "pointer") to set assumptions for the full term of the stochastic (or deterministic if applicable) modelling.

2.9 Pragmatically, the trend plus two year approach should also provide a reasonable response to the critic who argues that, because of such-and-such recent changes, use of (unadjusted) historical results is not appropriate for setting financial assumptions. The validity of the methodology increases if there are no discernible significant data changes over the past 54 years that are "outside the norm". Also, because Step 4 is based on running averages their trends (in Step 5) are impacted by the new data being introduced each year as well as by the old data dropping off. For the 44-year averages the old data dropping-off commences 30 September 1959 and ends 30 June 1968. For the 40-year averages the old data dropping-off commences 30 September 1963 and ends 30 June 1972. Fortunately, during the entire period 30 September 1959 to 30 June 1972 little happened that was "outside the norm", apart from CPI inflation which recorded its lowest 54-year annual rate of -0.70% in the year ending 30 June 1962. However any future applications of this methodology may have problems with the abnormal period from 30 September 1972 to 30 June 1975.

2.10 The projected results for years 3 to 6 of the projection were used:

- to check that the results are not trending to ridiculous levels such as negative coefficients of variation or correlations greater than 0.99, and
- to get 'smooth' correlation assumptions at 2 years by averaging the results for periods 0 to 4.

2.11 Consistent with accepted practice in Australia for input to stochastic projection models the following were calculated and tabulated:

(a) risk margins and means based on **arithmetic** averages of rates
(b) coefficients of variation and standard deviations based on **rates**
(c) skewness, kurtosis and correlations based on **forces**.

The inputs in (a) and (b) are based on arithmetic averages and rates so that they can be readily understood and judged by fund managers, economists, investment advisors and others who often seem to prefer and be familiar with such terms (though I do not support the practice – for example, the use of rates in (b) above can potentially hide risky sectors from proper consideration). These inputs are converted to forces for stochastic model calculations and then converted back to rates at the output stage. With the *Austmod* stochastic investment simulation model the output includes both compound and arithmetic averages. The possibility of inconsistencies arising from (a), (b) or (c) is discussed in Section 11.

2.12 Once assumptions relating to the results in section 2.11 (a), (b) and (c) above have been set, it is of course possible to calculate equivalent compound average rates (and forces) for each sector.

2.13 A deliberate consequence of section 2.5 above is that all individual results quoted in Sections 6 to 8 and 10 to 15 are the average of 4 results, **not** individual results for 44 or 40 years ending on one date.

3 The 15 Sectors

3.1 The 15 "sectors" included in the analysis are:

Table 3.1 Sector descriptions

B	Bill rate	90 day bank Bill rate (in middle of year)
C	Cash	UBS Australian Bank Bill Index, Cash sector NM/AXA No. 2 Fund prior 30/6/09
D	Bond rate	10 year bonD rate (in middle of year)
F	Fixed interest	UBS Australian Govt Bond 0+ yrs index, NM/AXA No. 2 Fund prior 30/6/09 (prev. G)
G	Govt semi	Government semis 0-3 years (SBC/UBS Warburg index SSG03)
H	In'l shares (h)	MSCI World ex Aust, net dividends reinvested (ndr), hedged
I	Int'l shares (uh)	MSCI World ex Aust (ndr), unhedged, NM/AXA No. 2 prior 30/6/09 (MSCIAI prior 30/6/88)
J	Int'l fixed	Citigroup World Govt Bond index, NM/AXA No. 2 Fund Int'l Bonds prior 30/6/09, hedged
L	Loans/credit	Corporate credit, UBS Credit 0+ yrs index, Loans NM/AXA No. 4 Fund prior 30/6/09
N	Infl'n linked	Inflation linked bonds (all maturities) UBS index
P	Property	Mercer Unlisted Property, one-third NM/AXA No. 2 and two-thirds AMP No. 2 prior 30/6/09
Q	Prop trust	Property trust accumulation index (from 31/1/01 S&P/ASX 300, from 30/6/02 GICS)
S	Shares	S&P/ASX 300 accum index, NM/AXA No. 2 prior 30/6/09 (All Ords accum prior 31/3/65)
W	AWOTE	AWOTE by quarter (= av 1.5 mths lag) , not seasonally adjusted, full-time adults (post 9/81), males original (pre 9/81), AWE males (pre 1/75)
X	CPI	CPI indeX by quarter end

3.2 The codes B to X above are used throughout the paper to denote each of the sectors.

3.3 The 15 sectors comprise 5 growth (or "equity") sectors (H, I, P, Q and S), 6 interest income sectors (C, F, G, J, L and N) and 4 financial indicators (B, D, W and X). The main database contains **annual forces at quarterly intervals**.

3.4 The financial indicator data (in red font above) has been collected at the same intervals and for the same periods as the investment data because they are useful in both deterministic and stochastic models for valuing, projecting and/or illustrating liabilities.

3.5 Bills and bonds are taken at mid-year because this gives a far better correlation against annual cash and loan rates. The lead between bond rates and annual investment returns for the F sector (Australian fixed interest) is considered in Section 6. The lag between bond rates and CPI and AWOTE is considered in Section 14.

3.6 The May 2013 AWOTE was published 15 August 2013 and was 1420.9. This value was used in all calculations for the year ending 30 June 2013. All "sector W" calculations involve an implicit average lag of 1.5 months (e.g. mid-May to end-June) as compared with AWOTE. Since the mid-May 2012 AWOTE, the index publishing cycle has been changed to half-yearly, so estimates have been used to obtain August 2012 and February 2013 values.

4 "Backdating"

4.1 The available data for the various sectors commence from different dates. The following is a summary of the historical start dates for **quarterly** data (for bonds and CPI, yearly data are available from earlier dates, and for some other sectors, one or two month's data is available prior the date shown).

Table 4.1 Historical start dates for data series

Series Start Date	Sector	Data series
30/9/41	W	AWE all males, total earnings
30/9/48	X	CPI (by quarter end)
1950	S	All ordinaries unweighted average dividend yield
31/3/58	D	Bond rate
31/3/58	S	All ordinaries price index
31/12/59	B	13 week treasury notes (see section 4.7)
31/3/65	S	Australian equities (EFG system) (see section 4.3)
31/3/65	F	Australian government securities (EFG system "G" sector)
30/9/69	B	90-day bank bills
30/6/71	P	AMP No. 2 Fund property sector
31/12/71	P	Property (EFG system)
31/12/71	S	Australian shares (EFG system)
1974	S	All ordinaries weighted average dividend yield
31/12/74	W	AWOTE males, ordinary time earnings
31/3/77	Q	Listed property trust accumulation index
30/9/79	C	Cash (EFG system)
30/9/81	W	AWOTE full-time adults, ordinary time, not seasonally adjusted
30/9/82	I	International shares (EFG system, see section 4.7)
30/9/85	F	Australian fixed interest (EFG system, previously "G" sector)
30/6/86	J	International bonds (ceased 30/6/87, recommenced 30/6/92)
30/6/86	L	Loans (market valued, NM/AXA No. 4 Fund)
31/12/89	G	Semi-government bonds SBC index (0 to 3 years)
31/3/91	N	Inflation-linked bonds WDR index (all maturities)
31/3/00*	H	Hedged international shares MSCI World ex Australia index
30/6/09**	S	Australian shares S&P/ASX 300 accumulation index
30/6/09**	I	Unhedged international shares MSCI World ex Australia index
30/6/09**	P	Mercer unlisted Australian property pre-tax asset weighted index
30/6/09**	L	Corporate credit, UBS credit 0+ yrs index
30/6/09**	F	UBS Australian government bond 0+ yrs index
30/6/09**	J	Citigroup world government bond index
30/6/09**	C	UBS Australian bank bill index

* This index commenced before 31/3/00 but the earlier index values are not widely published.

** These indices commenced before 30/6/09 but the earlier index values have not been used in the *Austmod* historical database; this gives consistency with the database used in Grenfell (2009).

4.2 It is evident from the above summary that data definitions for nine sectors, i.e. W, S, B, F, P, I, L, J and C have changed over time, though, apart from the 30 June 2009 changes, most of these changes relate to prior 1985.

4.3 It is also evident from the above summary that the National Mutual/AXA Australia, EFG ("E" for equities, "F" for fixed interest and "G" for government) investment unitisation system started on 31 March 1965. Sales brochures in the 1970's described it in these terms:

> *"Of major significance was the introduction in 1965 of a selective investment facility known as the EFG system. Evidence of the success and wide acceptance of this concept, which was pioneered by National Mutual in Australia, may now be seen in the fact that it has since been adopted by a number of other financial institutions as a medium for superannuation investment."*

4.4 There can be little doubt that the EFG system provoked the rapid development of the now enormous managed fund industry and the extensive use of unitisation throughout the Australian superannuation industry.

4.5 It is desirable with stochastic investment models to have consistency between assumptions both within sectors (e.g. risk margins and standard deviations) and across sectors (e.g. cross-correlations). An efficient way to gain such consistency is to have a 'complete' database for all sectors back to the one date, hence the term "backdating".

4.6 The chosen common start date for the complete database was 30 June 1959. The backdating was achieved primarily but not solely by the method of least squares, based on fitted parameters determined from data **after** the respective start dates and applying them to known data for other 'like' sectors **before** these start dates.

4.7 For each sector, the formulae used to backdate the annual forces prior to the start dates in Table 4.1, were:

$Q = 52.06\%F + 30.42\%S + 14.40\%C + 6.42\%L$

$P = 88.58\%C + 50.02\%X - 23.89\%F$

$L = 89.89\%C + 21.23\%F + 1.50\%P$

$G = 74.84\%C + 37.27\%F$

$J = 76.74\%F + 19.25\%C$

$C = 22.68\%B_{-2} + 27.44\%B_{-1} + 22.82\%B + 25.76\%B_{+1}$
where B_t = B in t quarter's time

$N = 71.38\%X + 62.99\%F - 195.05\%d$
where d = delta D force

B = in nominal terms, 13-week Treasury Note rate + 1.37%
where 1.37% = median excess of 90-day bank bill rate over 13-week Treasury Note rate 30/9/69 to 30/9/79.

13-week Treasury Note rates are available from 30/11/59. Rates for 30/6/59 and 30/9/59 were estimated by assuming that the ratio of 13-week Treasury Note rates to 2-year Government Security rates at these dates were the same as the ratio of 13-week rates to 2-year rates at 30/11/59. (2-year Government Security rates are available from 31/1/58.)

F = 87.09%D + 14.33%B - 673.02%d

S = approximate all ordinaries accumulation index prior 31/3/65 with June values as published by the RBA. September, December and March values based on all ordinaries price indices plus unweighted average dividend yields less 1.75% per annum (being the average difference between unweighted and weighted dividend yields between 1974 and 1984).

I = MSCI accumulation index from 30/6/70 to 30/6/88 (the EFG system I sector data from 30/9/82 to 30/6/88 was disregarded because for some of this period the sector was partially currency-hedged). Prior to 30/6/70 the S&P500 series plus an assumed 3% per annum average dividend yield was used. All index returns were then adjusted for $AU/$US exchange rate movements.

H = 108.54%I + 80.97%LN(E_0/E_{-4}) + 1.81%

where E_t = $AU/$US exchange rate t quarter's after year end.

4.8 The process of creating historical data by regression analysis means that several series must be correlated, at least for the period over which the data is created. The correlation coefficients between the series for any 44-year (or 40-year) period that extends back into the "backdated" period must therefore be illusionary in part. It is very difficult to unravel this effect. An examination was made to try and judge whether any significant bias was introduced. For the six most affected sectors (i.e. G, J, L, N, P and Q) 16 cross-correlations were calculated over the full period since 31/3/1965 to the calculation date and over the shorter periods from when each of these sectors commenced up to the calculation date. Each of these was compared with the relevant backdating parameter used to calculate the "backdated data". The analysis indicated that the parameters introduced very little, if any, bias overall.

4.9 Any bias introduced will also 'wear off' over time due to the trend-fitting, weighting and extrapolation inherent in the methodology described in Section 2. Nevertheless, users of the *Austmod* investment simulation model (refer Section 22 and Appendix C) and users of the cross-correlations assumptions tabulated in Sections 14 and 16, need to be aware that the "backdating" may have introduced some slight bias for those sectors where actual historical data started at later dates (see Table 4.1).

5 Economic Cycles

5.1 This section considers economic cycles. Though closely related, matters relating to auto-correlations are considered later, in Section 15. There are important reasons for identifying and considering economic cycles ‘up-front’. The length of **past** economic cycles is a key determinate for setting the periods over which statistics are calculated for the purposes of “Step 2” in section 2.3. Also, examining the persistency and characteristics of past economic cycles:

(1) helps the researcher understand the data, and

(2) when setting assumptions, guards against the danger of confusing a long term trend with the down-side or upside of a cycle, and

(3) may indicate the need to allow for **future** economic cycles in long term assumptions, and

(4) is essential for setting initial cycle positions if the long term assumptions allow for cycles or auto-correlations.

5.2 Spurred by the Global Financial Crisis, The International Actuarial Association (IAA) representing the global actuarial profession, has been active in addressing issues relating to the management and oversight of financial services industries. They have recommended that existing prudential capital requirements need to become **more dynamic and counter-cyclical**, rather than pro-active. A precursor for this initiative to be effective is to have robust and dynamic techniques for identification of economic cycles.

5.3 Dwonczyk (1993) identified 33 year cycles in Australian CPI inflation data. “A sine wave was fitted to the data which best estimated the actual [yearly rates] time series for the period 1956 to 1992”. He graphed the actuals, seven year moving averages and time series estimates over the 91-year period from 1 July 1901 to 30 June 1992.

5.4 For comparison, using the data described in Section 3 above with annual forces at quarterly intervals, the following curve-fitting results were obtained. Some curves are single others are combined. Each combined curve is **the sum of two sine curves**. The period of each sine curve is shown in Table 5.1 and only optimum results (with some minor rounding) are tabulated. The start points for Bonds (31 March 1958) and CPI (year-ending September 1949) correspond with when quarterly data first became available.

5.5 To give greater significance to more recent data, the data behind Table 5.1 was discounted on a compound basis for each quarter prior 30 June 2013. Thus, for example, with a data discount of 0.5% per quarter, data before December 1967 was given a weight of less than 40% and data before March 1992 was given a weight of less than 65%. By changing the data discount rate it is possible to get an indication of whether or not results are period dependent – in the following, results are shown for both a zero discount and for a discount of 0.5% per quarter.

5.6 Table 5.1 on the next page expands and updates the corresponding table in Grenfell (2009). For Bonds, results from the Grenfell (2009) are included for comparison.

5.7 The Institute of Actuaries of Australia Taskforce (2005), which was established to provide a tenth anniversary review of the Resilience Reserves used in the determination of statutory solvency and capital adequacy requirements of life insurance companies in Australia, stated in Section 4 of their report:

"Actuarial intuition, reflected in almost all actuarial models, is that dividend yields, real interest rates and inflation show a tendency to revert to some longer term mean. If true, this means that the probability of a future adverse shock depends, at least in part upon the current state of the market relative to its mean position."

In this context, it is relevant to note that sine curves (plus random noise) <u>are equivalent to mean reversion.</u>

Table 5.1 Economic cycles

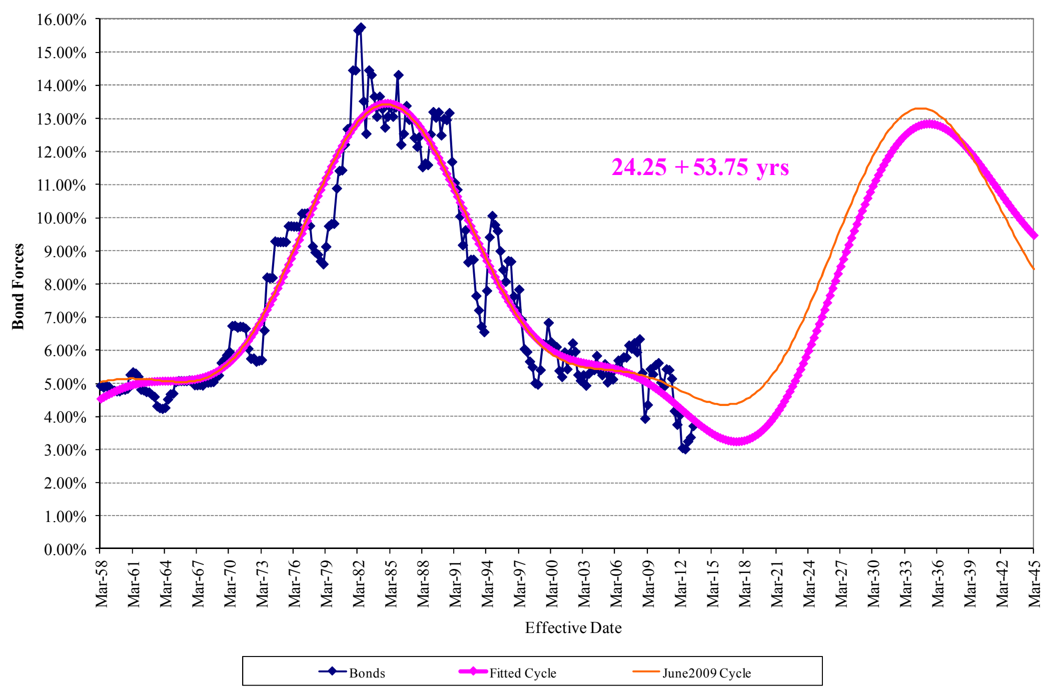
Periods of fitted sine curve cycles (years)	**Explained % of sum of squared residuals**				**Fitted curve mini-mum date**	**Fitted curve next mini-mum**	**Period bet-ween dates (years)**	**Refer Para.**
	Short cycle only	Long cycle only	**Both cycles**	Both cycles				
	Data discount per quarter							
	0.5%			0%				
BONDS	(Data from 31 March 1958 to **30 June 2009**)							
24.5 + 51	7.1	82.0	**91.7**	92.1	(flat at minimums)			5.8, 5.9
34.5 + 69	72.8	57.4	**90.5**	90.9	Mar 65	Dec 03	38.75	5.11
BONDS	(Data from 31 March 1958 to **30 June 2013**)							
24.25 + 53.75	0.3	81.4	**92.3**	92.4	(flat at minimums)			5.8, 5.9
26.5 + 53	3.8	81.7	**91.2**	91.6	(flat at minimums)			5.8, 0
49.0	(one curve)		**82.3**	83.2	Sep 60	Sep 09	49.0	5.11, 5.18
CPI	(Data from year-ending 30 September 1949 to **30 June 2013**)							
25.5 + 40	7.5	44.1	**57.6**	52.4	Mar 63	(flat at minimum)		5.12
40.25	(one curve)		**44.0**	39.1	Sep 61	Dec 01	40.25	5.14, 5.18
AWOTE	(Data from year-ending 30 September 1949 to **30 June 2013**)							
28.25 + 65	8.8	29.1	**49.8**	43.5	Mar 61	Sep 94	33.5	5.13
42.75	(one curve)		**34.1**	25.4	Jun 58	Mar 01	42.75	5.14, 5.18
SHARES	(Data from year-ending 30 June 1960 to **30 June 2013**)							
3.381 + 9.082	10.2	11.0	**21.6**	18.4	Dec 64	Dec 91	27	5.15
4.222 + 9.066	9.0	11.0	**20.3**	20.9	Sep 82	(four flat mins.)		5.16

For each sector the optimum result is illustrated in the charts on the following pages - the single cycles for Bonds, CPI and AWOTE are all illustrated together in Figure 5.4.

5.8 Comparing the 2013 Bond results with the 2009 Bond results shows that the short cycle for the first result is almost unchanged (a decrease from 24.5 to 24.25 years) but the long cycle has increased by 2.75 years (from 51 to 53.75 years). Comparing the first and third results shows that the explained percentage of the sum of squared residuals for both cycles has increased slightly (from 91.7 to 92.3). For the second and fourth results, the shapes of the combined curves have changed and are now flat at the minimums.

5.9 For Bonds, the 24.25 plus 53.75 year curve explained a very high 92.3% of the sum of squared residuals from the mean. However the curve is a strange shape with flat values near both minimums. The curve then cycles to a maximum force of 12.9% in 2035. This is consistent with a **high** yield/ high inflationary future period similar to the past (where the fitted curve reaches a maximum force of 13.5% in December 1984). Figure 5.1 below graphs this curve and the actual data. It also includes the corresponding June 2009 curve (in orange) from Grenfell (2009), which is the first curve in Table 5.1 above. Because it is based on an extra four years of data the purple curve always differs from the orange curve (except where the two curves cross), however after September 2009 the difference exceeds 0.2% and becomes evident on the graph.

Figure 5.1 Ten year bonds – economic cycles

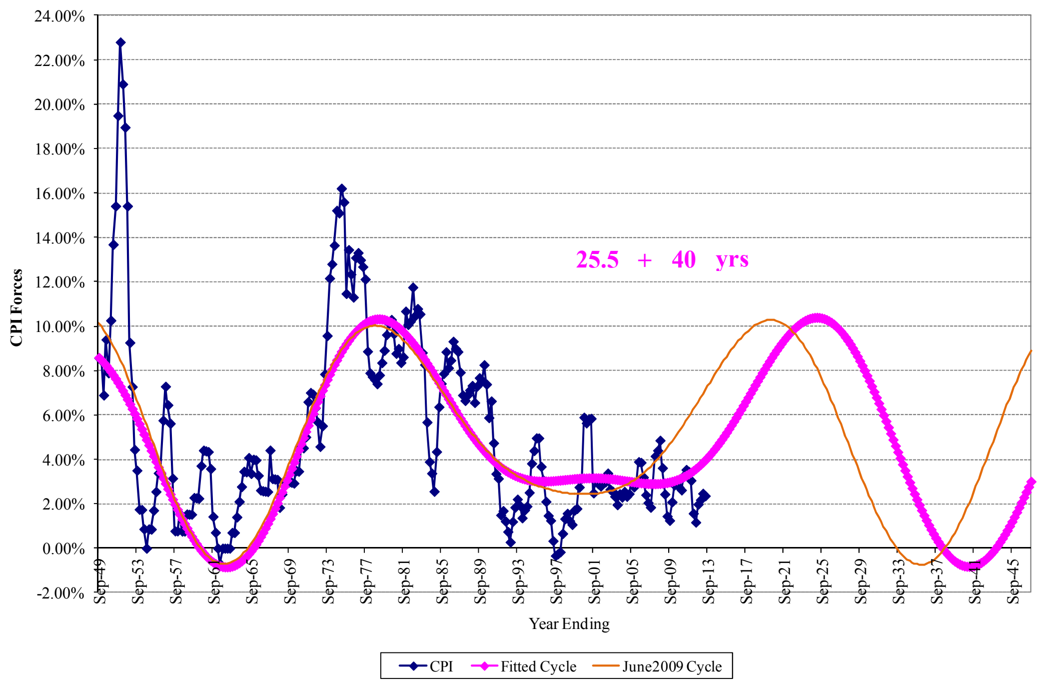


5.10 For Bonds, the 26.5 plus 53 year curve explained 91.2% of the sum of squared residuals. The curve, by design, has a long cycle twice the length of its short cycle. The curve is not illustrated, but has a shape and features very similar to the 24.25 plus 53.75 year curve.

5.11 Section 5.1 above explained that "The length of **past** economic cycles is a key determinate for setting the periods over which statistics are calculated for the purposes of "Step 2" in section 2.3". However it is now evident that the sum of two sine curves technique, by itself, is not always suitable for determining the length of past economic cycles for this purpose. This occurs because the combined cycles in sections 5.7 to 5.10 above (and section 5.12 below) are now flat at or near at least one minimum and are "strange" or distorted shapes. The sum of two sine curves technique remains suitable for the purposes (1) to (4) in Section 5.1 but something more is need to satisfy "Step 2" in section 2.3. Thus the single sine curves were added to section 5.7 and are commented on further in section 5.18 below. By design the single sine curves do not give flat results or have distorted shapes.

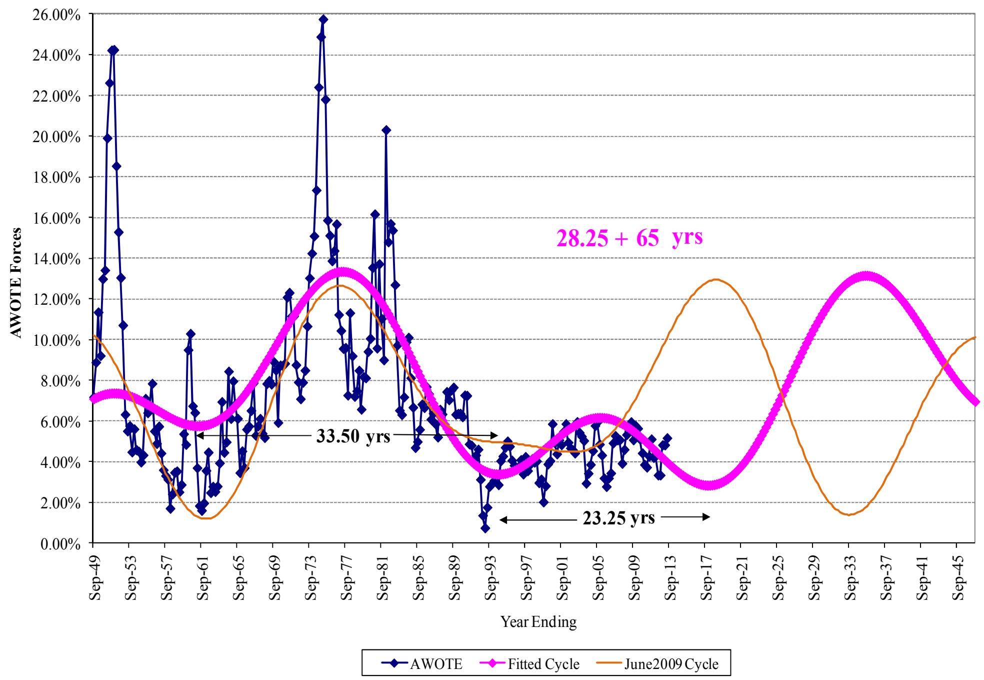
5.12 For CPI, the 25.5 plus 40 year curve explained 57.6% of the sum of squared residuals. This was the highest percentage explained of any one curve or two-curve combination. The CPI curve-fitting is, significantly, not as close to the data as for Bonds and is more period dependent. This curve is consistent with a **high** yield/ high inflationary future period similar to the past (the fitted curve reaches a maximum force of 10.3% in March 1979 and 10.4% in 2025). The fitted curve is almost flat (varying between 2.9% and 3.3%) from 1993 to 2011. Figure 5.2 below graphs this curve and the actual data.

Figure 5.2 CPI – economic cycles



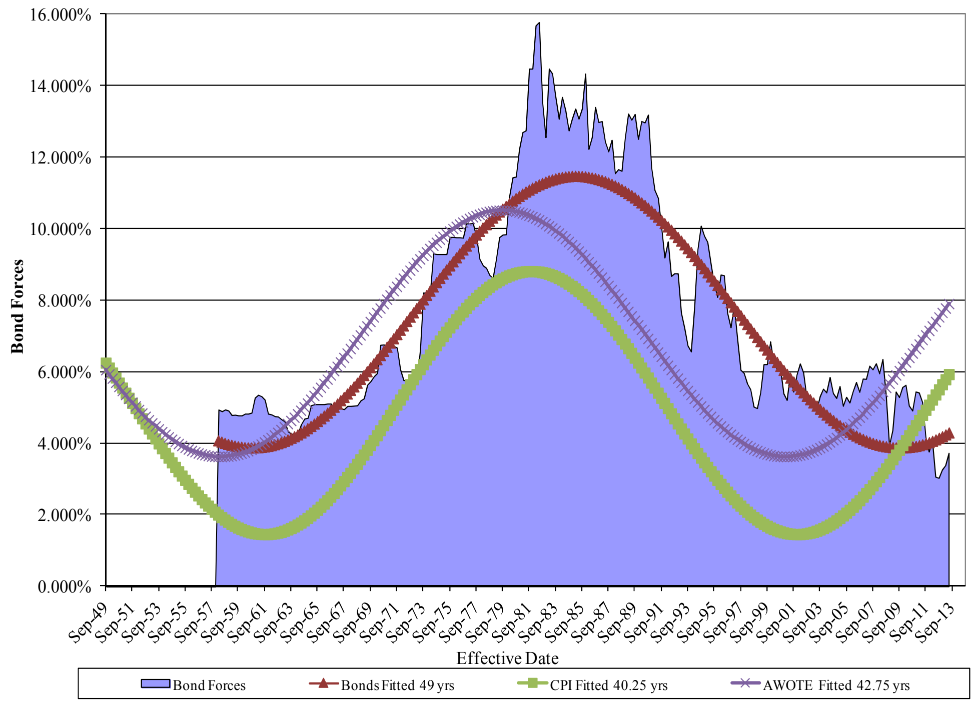
5.13 For AWOTE, the 28.25 plus 65 year curve explained 49.8% of the sum of squared residuals. The AWOTE curve-fitting is also, significantly, not as close to the data as for Bonds and is more period dependent. This curve is fairly smooth and is consistent with a **low** inflationary future period until about 2023, but then followed by a **high** yield/ high inflationary future period similar to the past. Figure 5.3 graphs this curve and the actual data. (Prior 1975 the W sector data was based on AWE total earnings rather than AWOTE. By increasing the data discount to 1% per quarter the weight given to this earlier period reduces significantly and it was noted that a 27.25 plus 57.5 year curve explained a much improved 58.5% of the sum of squared residuals.)

Figure 5.3 AWOTE – economic cycles



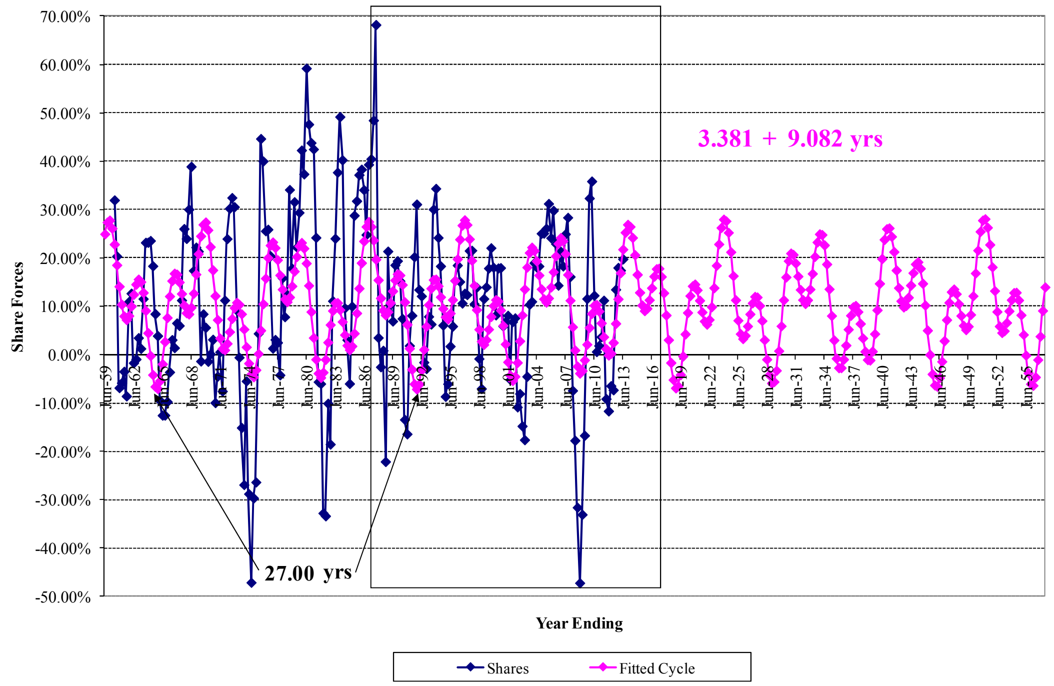
5.14 Figure 5.4 on the next page graphs the single cycle fitted curves for Bonds, CPI and AWOTE and the actual data for Bonds. For the 49 year fitted Bonds curve the minimum was reached in September 2009 but for the 40.25 year CPI curve and the 42.75 year AWOTE curve the minimums were reached in December 2001 and March 2001 respectively. Thus changes in CPI and AWOTE have lead changes in bond rates by about 7 to 8 years. This lead is analysed in more detail in Section 14.

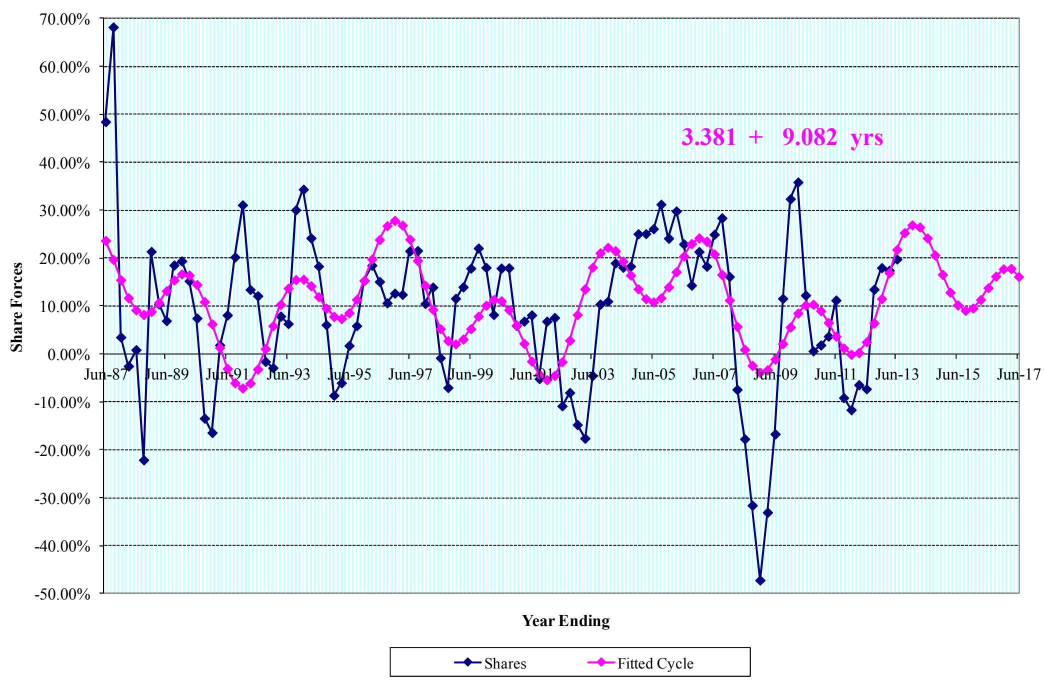
Figure 5.4 Bonds, CPI and AWOTE – economic cycles



5.15 For Shares, the 3.381 plus 9.082 year curve explained 21.6% of the sum of squared residuals from the mean. The Shares curve-fitting is, significantly, not as close to the data as for Bonds, CPI and AWOTE. This is primarily due to the more volatile share returns. The periods of 3.381 and 9.082 years are equal to 13 quarters plus 48 days and 36 quarters plus 30 days respectively. The period between the last two minimums of the fitted curve is 27 years. Figure 5.5 on the next page graphs the curve and the actual data. A "close-up" graph of the 30 years from June 1987 to June 2017 is also shown.

Figure 5.5 Shares – economic cycles





5.16 For Shares, the 4.222 plus 9.066 year curve explained 20.9% of the sum of squared residuals from the mean. The 4.222 year and 9.066 year periods are equal to 16 quarters plus 81 days and 36 quarters 24 days respectively. The results for this curve have been included in Table 5.1 primarily because the curve is similar to the 4.096 plus 9.343 year curve which was the best Shares curve in Grenfell (2007) and the 4.249 plus 9.036 year curve which was the second best Shares curve in Grenfell (2009). However the explained percentage of the sum of squared residuals for both cycles has decreased from 24.4% six years ago to 20.9% now. This fall is due to the fact that the 4.096 plus 9.343 year curve "predicted" that Share forces would start falling from December 2008 but the fall in Share prices actually commenced between September and December 2007 and was far deeper than indicated by the curve. If the explained percentage of the sum of squared residuals falls below say 20% then this curve would probably not be optimum.

5.17 In Grenfell (2007 and 2009) it was mentioned that the 4 or 4.096 year cycle, then evident in each of the Share curves, could perhaps be attributed to United States of America Presidential elections. This possibility is analysed in Nickles (2004). This shows that during the period 1942 to 2002 USA stock market cycles averaged 4.02 years. During this period, bull markets averaged about three years, while bear markets averaged less than a year.

5.18 Consideration of the second last column of Table 5.1 above shows that the periods between **past** minimums have ranged from 33.5 to 49.0 years (disregarding the fluctuating/changing Share curves) depending on the curve-fitting and whether one's focus is on Bonds, CPI or AWOTE. Much of the work in the following sections is based on 44-year past periods because:

- The lower extreme in the range mentioned above should be disregarded because it is for AWOTE where the explained proportion of the sum of squared residuals is low, and the 38.75 years for Bonds is a 30 June 2009 result – this leaves the following single cycle results:

	Years Between Minimums	Explained ppn. of Squared Residuals
Bonds	49.0	82.3%
CPI	40.25	44.0%
AWOTE	42.75	34.1%

- the analysis in sections 9.10 and 9.11 shows that the past period between maximums for the Balanced portfolio log "discounted unit price" trend line was 41.0 years and the explained proportion of squared residuals was 83.7%,

- **where-ever possible it is desirable for the setting of long-term assumptions to analyse results over at least one full economic cycle** – this is supported by Bernstein (1997) who "extracted the long-run rate of return on long-term bonds by searching for widely separated dates where yields are identical … and then calculated the annualised growth in the bond relative … between those dates",

- the curve-fitting for bonds is clearly superior (in terms of percentage of residuals explained and period dependency) to that for CPI, AWOTE and Shares,

- the risk margins in Section 6 are defined relative to bonds,
- **the weighted average of the four results from the first two dot points above** (where the weights are the explained proportion of the sum of squared residuals from the mean) is 43.8 years which, in view of the importance of Bonds, was rounded up to **44 years**.

5.19 However, for some sectors (e.g. those denoted G, H, J, L, and N) the 44 years chosen in section 5.15 will often extend well back into the "back-dated data" periods described in Section 4. Thus it was considered desirable, for comparison, to also consider a shorter period, at least for risk margins and coefficients of variation. A period of 40 years was chosen for this purpose because that is the minimum cycle period (after round-off) identified in the first two dot points of section 5.18. In Grenfell (2009) the two periods chosen were 39 years and 24 years, with a difference of 15 years. Experience has shown that the 15 year difference gave a very wide spread of results – the difference has therefore been reduced to 4 years for the current work.

5.20 It is considered desirable for the setting of long-term assumptions to always analyse past historical results over a full economic cycle (or in the more general sense, over an integral number of cycles). Thus 44 years was chosen for most calculations and 40 years was chosen for the second alternative set of risk margin and coefficient of variation calculations.

6 Risk Margins

6.1 For the purposes of this paper, risk margins are calculated relative to the ten-year bond rate. The risk margin is the excess of the sector annual investment return over the annualised effective point-in-time bond rate (but see paragraph 6.4).

6.2 For sector D (10-year bonds) the point-in-time bond rate is taken at mid-year, that is 6 months prior to the end of the period over which the annual investment returns are determined (but see paragraph 6.4).

6.3 Now consider the correlation between the F (Fixed Interest) sector forces and D sector forces for various lags:

Table 6.1 D and F sector correlations

24 yr Correlation of F against D lagged by Y years:									
Y	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2
End date									
30/06/1984	.333	.402	.462	.477	.482	.484	.461	.435	.402
30/06/1985	.370	.435	.492	.507	.511	.510	.490	.466	.432
30/06/1986	.416	.474	.528	.541	.5414	.5410	.525	.500	.468
30/06/1987	.468	.524	.580	.588	.589	.588	.570	.544	.515
30/06/1988	.487	.544	.602	.609	.608	.611	.594	.571	.545
30/06/1989	.477	.538	.597	.603	.599	.602	.581	.555	.528
30/06/1990	.493	.555	.616	.620	.615	.615	.595	.570	.542
30/06/1991	.519	.585	.645	.648	.642	.644	.622	.594	.562
30/06/1992	.513	.593	.667	.6775	.675	.6767	.655	.628	.596
30/06/1993	.452	.539	.627	.643	.643	.650	.628	.598	.566
30/06/1994	.393	.489	.588	.604	.604	.614	.591	.559	.527
30/06/1995	.441	.552	.655	.666	.660	.669	.650	.618	.584
30/06/1996	.514	.630	.735	.739	.718	.708	.673	.642	.616
30/06/1997	.362	.487	.629	.643	.625	.629	.605	.583	.556
30/06/1998	.274	.405	.539	.545	.521	.523	.498	.476	.455
30/06/1999	.295	.412	.534	.542	.521	.521	.496	.474	.453
30/06/2000	.356	.468	.578	.584	.563	.561	.539	.515	.491
30/06/2001	.366	.470	.577	.584	.562	.556	.527	.501	.481
30/06/2002	.414	.511	.610	.613	.593	.592	.565	.540	.520
30/06/2003	.430	.522	.6172	.6169	.597	.598	.576	.556	.542
30/06/2004	.500	.580	.664	.656	.631	.625	.599	.580	.568
30/06/2005	.586	.648	.723	.707	.671	.658	.627	.602	.586
30/06/2006	.593	.657	.728	.713	.680	.667	.640	.619	.605
30/06/2007	.597	.660	.732	.723	.691	.675	.647	.625	.612
30/06/2008	.609	.676	.749	.742	.714	.699	.666	.645	.640
30/06/2009	.618	.689	.766	.757	.730	.711	.676	.656	.649
30/06/2010	.603	.684	.760	.753	.718	.700	.664	.644	.636
30/06/2011	.561	.653	.735	.729	.694	.666	.629	.608	.601
30/06/2012	.539	.643	.736	.733	.693	.664	.629	.605	.590
30/06/2013	.596	.692	.780	.771	.736	.707	.667	.644	.630

6.4 Table 6.1 shows that when D sector forces are lagged by say a further 9 months (giving a total lag of 15 months) the correlation between D and F sector forces increases to a maximum, or close to the maximum. For the last 11 years the maximum occurs in the 6 month column but in the last 7 years the difference between the 6 month lag correlations and the 9 month ones are all less than 0.10. Except for five results prior to 1996, the maximum tabulated correlation between D and F sector forces has always occurred between:

- 6 months (giving a total lag of 12 months), and
- 12 months (giving a total lag of 18 months).

There is evidence that the lag shortened from 1993 to about 2003, probably primarily due to a contracting F sector average maturity "duration". What it might do in the future is uncertain, however for the specific purposes of Steps 3 to 5 of the methodology, past experience is relevant, not future predictions. Therefore, to give greater stability to F sector risk margins, and to a lesser extent other sector risk margins, **all the following risk margins are defined as**:

(annual investment return) less (bond rate lagged 15 months)

6.5 The choice in the previous paragraph of a further 9 months (giving a total of 15 months) has no significant effect on the general level of the following margins, other than introducing some stability to the results. This greater stability of the risk margins improves the accuracy of the quadratic trend fitting and the two-year projections in steps 5 and 6 of the methodology (see Section 2). The 9 month adjustment has no practical impact on the application of the resultant margins or on their use in stochastic or other models.

6.6 Once assumptions have been set for risk margins (for the 14 sectors other than bonds) and for the mean bond rate, assumptions for means for the 14 sectors can be determined from the formula:

Arithmetic mean rate = risk margin + mean bond rate

6.7 Figure 6.1 and Table 6.4 show the risk margin results. For the D sector (bonds) the means, after lagging the bond rate by 15 months, are shown.

EXPLANATION

6.8 In the following figures and tables the meaning of "Period" is:

Table 6.2 Statistics for each "period"

Period	Average statistic of four periods ending:
9	30/9/03, 31/12/03, 31/3/04 and 30/6/04
8	30/9/04, 31/12/04, 31/3/05 and 30/6/05
7	30/9/05, 31/12/05, 31/3/06 and 30/6/06
6	30/9/06, 31/12/06, 31/3/07 and 30/6/07
5	30/9/07, 31/12/07, 31/3/08 and 30/6/08
4	30/9/08, 31/12/08, 31/3/09 and 30/6/09
3	30/9/09, 31/12/09, 31/3/10 and 30/6/10
2	30/9/10, 31/12/10, 31/3/11 and 30/6/11
1	30/9/11, 31/12/11, 31/3/12 and 30/6/12
0	30/9/12, 31/12/12, 31/3/13 and 30/6/13
0	*30/9/12, 31/12/12, 31/3/13 and 30/6/13 (trend)*
-1	*30/9/13, 31/12/13, 31/3/14 and 30/6/14 (projections)*
-2	*30/9/14, 31/12/14, 31/3/15 and 30/6/15 (projections)*
-3	*30/9/15, 31/12/15, 31/3/16 and 30/6/16 (projections)*
-4	*30/9/16, 31/12/16, 31/3/17 and 30/6/17 (projections)*
-5 & -6	calculated as above, but not tabulated

6.9 The above trends and 'projections' (or extrapolations) are based on weighted quadratic EXCEL trend functions (refer section 2.7). The "period" is indicated on the X-axis of all figures.

6.10 The following codes are used:

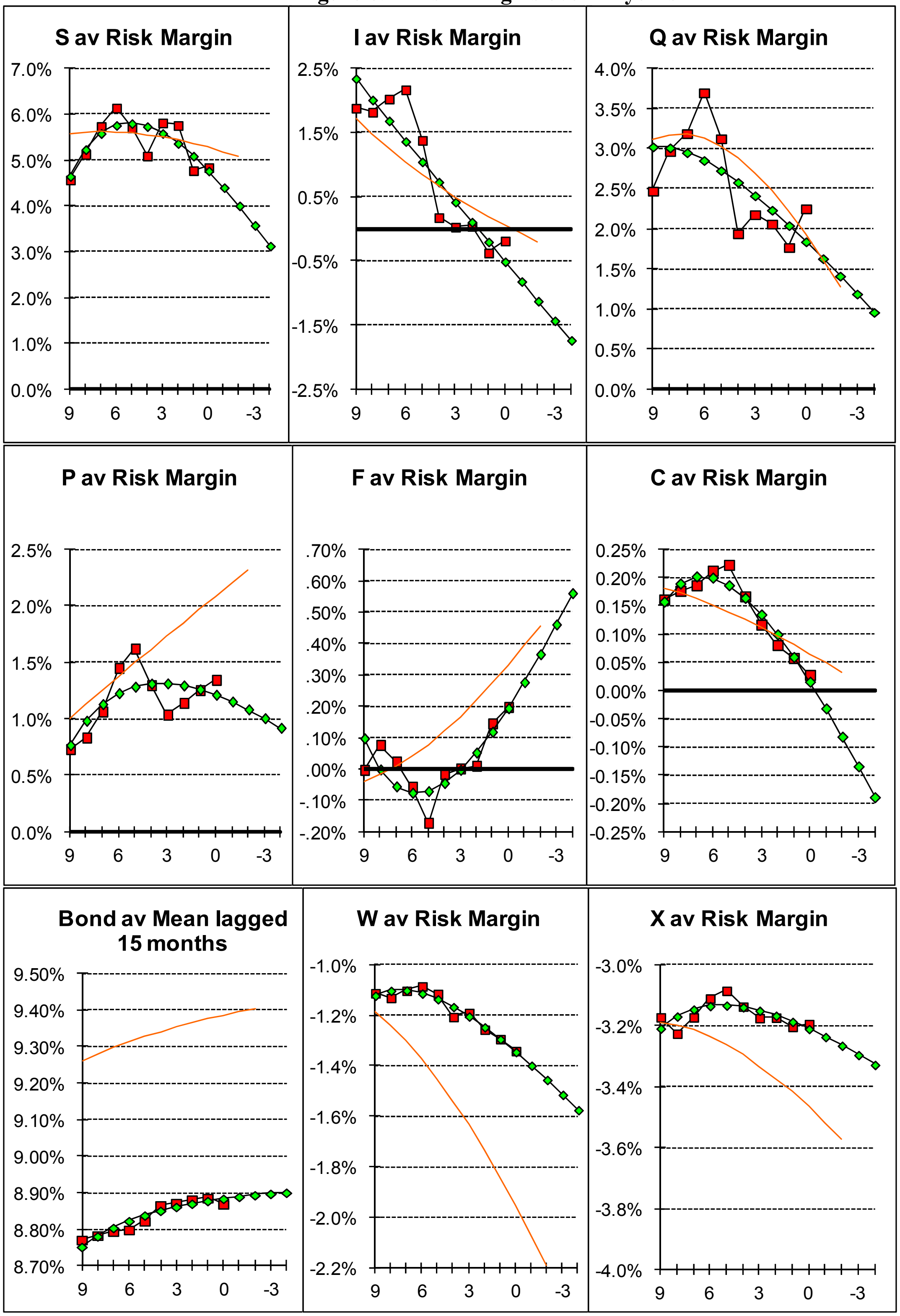
Sector	**Relates to**
D	ten-year bond rate in the middle of the year (i.e. lagged by 6 months)
D-9	ten-year bond rate lagged by 15 months (i.e. as for D but lagged by a further 9 months)
X	annual increase in the Consumer Price Index
X-8	annual increase in the Consumer Price Index lagged by 8 years
W	annual increase in AWOTE
W-8	annual increase in AWOTE lagged by 8 years
[square]	actual results based on 44-year averages
[diamond]	trend results based on 44-year averages
[orange line]	trend or projections from Grenfell (2009) based on 39-year averages
italics	*trend results or projections*

6.11 Taking into consideration the following three "pointers", the "NEW" assumptions are set (refer section 2.8) for risk margins and coefficients of variation:

Table 6.3 "Pointers"

Pointer	Refers to:
44 –2	Average 44-year projected statistic for "Period -2"
40 –2	Average 40-year projected statistic for "Period -2"
OLD	Previous assumptions based on data up to 30/6/2009

Figure 6.1 Risk margins over 44 years



X-axis: Period 9 = average 44 years ending 30/9/03, 31/12/03, 31/3/04 and 30/6/04
Period 0 = average 44 years ending 30/9/12, 31/12/12, 31/3/13 and 30/6/13 (see section 6.8)

Table 6.4 Risk margins over 44 years

Actual and *Quadratic Trend*

Period	S	I	Q	P	L	H	F	G
9	4.6%	1.9%	2.5%	0.7%	1.4%	3.6%	0.0%	1.0%
8	5.1%	1.8%	3.0%	0.8%	1.4%	3.6%	0.1%	1.0%
7	5.7%	2.0%	3.2%	1.1%	1.3%	3.7%	0.0%	0.9%
6	6.1%	2.2%	3.7%	1.4%	1.4%	4.0%	-0.1%	0.9%
5	5.7%	1.4%	3.1%	1.6%	1.4%	3.4%	-0.2%	0.9%
4	5.1%	0.2%	1.9%	1.3%	1.3%	2.2%	0.0%	1.0%
3	5.8%	0.0%	2.2%	1.0%	1.3%	2.4%	0.0%	0.9%
2	5.8%	0.0%	2.1%	1.1%	1.3%	2.7%	0.0%	0.9%
1	4.8%	-0.4%	1.8%	1.3%	1.4%	2.3%	0.1%	0.9%
0	4.9%	-0.2%	2.3%	1.3%	1.4%	2.5%	0.2%	0.9%
0	*4.8%*	*-0.5%*	*1.8%*	*1.2%*	*1.4%*	*2.3%*	*0.2%*	*0.9%*
-1	*4.4%*	*-0.8%*	*1.6%*	*1.2%*	*1.4%*	*2.2%*	*0.3%*	*0.9%*
-2	*4.0%*	*-1.1%*	*1.4%*	*1.1%*	*1.5%*	*2.1%*	*0.4%*	*0.9%*
-3	*3.6%*	*-1.4%*	*1.2%*	*1.0%*	*1.5%*	*1.9%*	*0.5%*	*0.9%*
-4	*3.1%*	*-1.7%*	*1.0%*	*0.9%*	*1.6%*	*1.8%*	*0.6%*	*0.9%*

J	C	N	B	D-9	W	X	Period
-0.3%	0.2%	0.9%	0.2%	8.77%	-1.1%	-3.2%	9
-0.1%	0.2%	1.1%	0.2%	8.78%	-1.1%	-3.2%	8
-0.2%	0.2%	1.1%	0.2%	8.80%	-1.1%	-3.2%	7
-0.2%	0.2%	1.1%	0.2%	8.80%	-1.1%	-3.1%	6
-0.2%	0.2%	1.1%	0.3%	8.82%	-1.1%	-3.1%	5
-0.3%	0.2%	1.1%	0.3%	8.87%	-1.2%	-3.1%	4
-0.2%	0.1%	1.1%	0.2%	8.87%	-1.2%	-3.2%	3
-0.1%	0.1%	1.1%	0.2%	8.88%	-1.3%	-3.2%	2
0.0%	0.1%	1.4%	0.1%	8.89%	-1.3%	-3.2%	1
0.1%	0.0%	1.5%	0.1%	8.87%	-1.3%	-3.2%	0
0.0%	*0.0%*	*1.4%*	*0.1%*	*8.89%*	*-1.3%*	*-3.2%*	*0*
0.1%	*0.0%*	*1.5%*	*0.1%*	*8.89%*	*-1.4%*	*-3.2%*	*-1*
0.2%	*-0.1%*	*1.6%*	*0.0%*	*8.90%*	*-1.5%*	*-3.3%*	*-2*
0.3%	*-0.1%*	*1.7%*	*0.0%*	*8.90%*	*-1.5%*	*-3.3%*	*-3*
0.4%	*-0.2%*	*1.8%*	*-0.1%*	*8.90%*	*-1.6%*	*-3.3%*	*-4*

Pointers (risk margins)

	S	I	Q	P	L	H	F	G
44 -2	4.0%	-1.1%	1.4%	1.1%	1.5%	2.1%	0.4%	0.9%
40 -2	4.6%	-0.3%	1.8%	1.0%	1.5%	2.5%	0.4%	0.9%
OLD	4.5%	3.0%	3.0%	2.0%	1.0%	3.2%	0.5%	0.1%
NEW	4.0%	2.6%	2.6%	1.5%	1.2%	2.7%	0.4%	0.4%

J	C	N	B	D-9	W	X	
0.2%	-0.1%	1.6%	0.0%	8.9%	-1.45%	-3.26%	44 -2
0.3%	-0.1%	1.7%	0.0%	9.0%	-2.04%	-3.48%	40 -2
0.2%	-0.3%	0.6%	-0.3%	6.0%	-2.25%	-3.50%	OLD
0.2%	-0.2%	1.0%	-0.2%	6.0%	-1.80%	-3.30%	NEW

6.12 In all cases the "NEW" risk margin assumptions are set in the range from:

a) the lowest of the first 3 pointers, to
b) the highest of the first 3 pointers.

However in many cases this is a wide range. Considerable weight was given to the "OLD" (see page 25) risk margin assumptions. The following further brief explanations should give some indication of the setting process:

S (shares)	Set equal to "44 –2" (4.02%) which was rounded to 4.0%.
I (int'l shares)	Combined with Q (because of the divergent results). Hence set equal to 2.6% (which equals the "OLD" pointers less 0.4%).
Q (prop trust)	Combined with I (because of the divergent results). The 'actual' 44-year average Q risk margins in Table 6.4 vary from a high of 3.7% (ending pre the GFC) to a low of 1.8% (ending post GFC) and average 2.57%, which was rounded to 2.6%. Also it was noted that this falls 44% of the way between the "NEW" P and S risk margins.
P (direct prop)	Set equal to the average of "OLD" and "44 –2" (1.08% + 2.00%)/2 = 1.54%, rounded down).
H (hedged i.s.)	Set equal to the average of "OLD" and "44 –2". This average (2.63%) was rounded down to 2.6%. This lowers by a half the 0.2% differential between hedged and unhedged international shares assumed in the "OLD" assumptions (which implies an assumed slight long-term upward trend in the Australian dollar with our economy running marginally stronger than the US and Europe).
F (fixed int)	Set equal to both "44 –2" and "40 –2".
G (govt semi)	Set equal to the average of "OLD" and "44 –2". This average (0.498%) was rounded down to 0.4% (i.e. equal to the risk margin for F).
J (int'l fxd int)	Set equal to F less 0.2% to cover the cost of hedging (including the direct transaction costs and indirect transaction costs arising from the re-investment of cash flows from expiry of forward foreign exchange contracts). Also noted that the difference between F and J was 0.14% for "44 –2" and 0.13% for "40 –2".
C (cash)	Set equal to the average of "OLD" and "44 –2" (and also equal to B). The average for C was -0.19% which was rounded down to -0.2%.
N (infln linked)	Set equal the average of "OLD" and "40 –2". This average (1.14%) was rounded down to 1.0%. Because of the short data term (22.25 years) the results for "44 -2" were disregarded. Noted that for the 40-year averages, N has consistently exceeded F by about 1.2%.
D (bonds)	The base was set equal to 6% per annum. This is higher than current effective 10-year government bond rates (e.g. 3.80% at 30 June 2013 and 4.06% at 19 August 2013) but it was noted that 40-year simulations with the "NEW" D sector means, CoV, skewness and kurtosis, result in Bond annual rates typically (i.e. based on medians) ranging from about 2.8% to 11%.
X (CPI)	Set equal to "44 –2" (-3.26%) rounded down to -3.3% which corresponds to a mean annual inflation rate of 2.7% per cent.
W (AWOTE)	The "productivity" difference between W and X for "44 –2", "40 –2" and "OLD" was 1.81%, 1.44% and 1.25% respectively. The average of these equals 1.50%. Hence, the W assumption was set 1.5% above X.

7 Coefficients of Variation

7.1 The coefficient of variation is equal to the standard deviation divided by the mean. Figure 7.1 and Table 7.1 show the results.

7.2 Once assumptions have been set for risk margins, the mean bond rate and for coefficients of variation, assumptions for standard deviations for the 15 sectors can be determined from the formula:

Standard deviation = (risk margin + mean bond rate) * (coefficient of variation)

7.3 Whether standard deviation is a reasonable surrogate for risk is an open question reaching far beyond the scope of this paper. However, bear in mind the quotation from Waite (2009) at the bottom of this page.

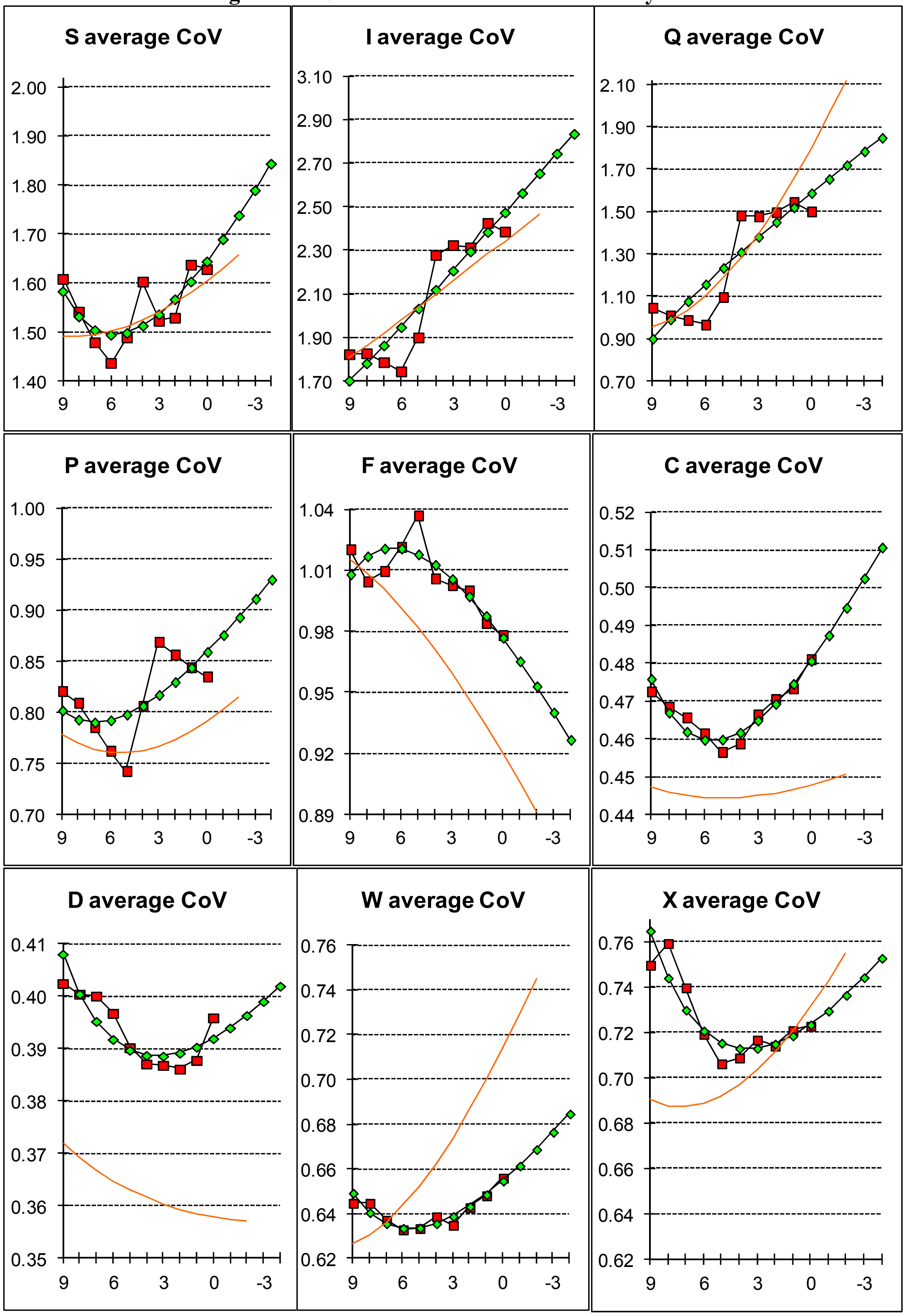
7.4 Brown (2013) refers to the "index of stability" or 'ios" which is the reciprocal of the coefficient of variation. In the engineering literature it is known as the "signal to noise ratio". In the application to investments we can assume that the mean return is positive. The standard deviation is always positive: hence it follows that the ios for investment returns will always be positive. Brown (2013) tabulates the following values of ios based on investment forces of return, before tax and fees, using the *Austmod* historical database for the period from September 1965 to September 2012:

I (int'l shares, unhedged)	0.33
H (hedged int'l shares)	0.48
S (shares, Australian)	0.52
Q (property trusts)	0.55
F (fixed interest)	0.98
P (direct property)	1.07
J (int'l fixed interest)	1.19
N (inflation linked)	1.19
G (semi-govt, 0-3yrs)	1.79
L (loans/credit)	2.20
C (cash)	2.20

REMEMBER

> *"There is an acceptable four letter word that is at the heart of the current financial crisis. It is "risk". Risk is at the centre of the actuarial profession and the area of ERM (Enterprise Risk Management) is gaining new ground in the actuarial world."*
>
> *Waite (2009)*

Figure 7.1 Coefficients of variation over 44 years



[red square] actual results [green diamond] trend results [orange line] 2009 trend 39-year avs. (see sec. 6.10)

X-axis: Period 9 = average 44 years ending 30/9/03, 31/12/03, 31/3/04 and 30/6/04
Period 0 = average 44 years ending 30/9/12, 31/12/12, 31/3/13 and 30/6/13 (see section 6.8)

Table 7.1 Coefficients of variation over 44 years

Actual and *Quadratic Trend*

Period	S	I	Q	P	L	H	F	G
9	1.61	1.82	1.05	0.82	0.49	1.52	1.02	0.58
8	1.54	1.83	1.01	0.81	0.49	1.51	1.00	0.58
7	1.48	1.79	0.99	0.79	0.49	1.48	1.01	0.59
6	1.44	1.74	0.97	0.76	0.49	1.43	1.02	0.59
5	1.49	1.90	1.10	0.74	0.49	1.50	1.04	0.59
4	1.60	2.28	1.48	0.81	0.49	1.77	1.01	0.58
3	1.52	2.32	1.48	0.87	0.49	1.76	1.00	0.58
2	1.53	2.31	1.50	0.86	0.49	1.70	1.00	0.58
1	1.64	2.43	1.55	0.84	0.48	1.76	0.98	0.58
0	1.63	2.39	1.50	0.83	0.48	1.74	0.98	0.59
0	*1.64*	*2.47*	*1.59*	*0.86*	*0.48*	*1.78*	*0.98*	*0.59*
-1	*1.69*	*2.56*	*1.65*	*0.88*	*0.47*	*1.81*	*0.97*	*0.59*
-2	*1.74*	*2.65*	*1.72*	*0.89*	*0.47*	*1.85*	*0.95*	*0.59*
-3	*1.79*	*2.74*	*1.78*	*0.91*	*0.47*	*1.88*	*0.94*	*0.59*
-4	*1.84*	*2.83*	*1.85*	*0.93*	*0.46*	*1.92*	*0.93*	*0.59*

J	C	N	B	D	W	X	Period
0.86	0.47	0.83	0.50	0.40	0.64	0.75	9
0.84	0.47	0.81	0.50	0.40	0.64	0.76	8
0.84	0.47	0.80	0.50	0.40	0.64	0.74	7
0.85	0.46	0.80	0.49	0.40	0.63	0.72	6
0.84	0.46	0.81	0.49	0.39	0.63	0.71	5
0.85	0.46	0.79	0.48	0.39	0.64	0.71	4
0.83	0.47	0.81	0.49	0.39	0.64	0.72	3
0.83	0.47	0.80	0.49	0.39	0.64	0.71	2
0.81	0.47	0.78	0.50	0.39	0.65	0.72	1
0.81	0.48	0.78	0.51	0.40	0.66	0.72	0
0.81	*0.48*	*0.78*	*0.50*	*0.39*	*0.65*	*0.72*	*0*
0.80	*0.49*	*0.78*	*0.51*	*0.39*	*0.66*	*0.73*	*-1*
0.79	*0.49*	*0.77*	*0.52*	*0.40*	*0.67*	*0.74*	*-2*
0.77	*0.50*	*0.77*	*0.52*	*0.40*	*0.68*	*0.74*	*-3*
0.76	*0.51*	*0.76*	*0.53*	*0.40*	*0.68*	*0.75*	*-4*

Pointers (CoV)

	S	I	Q	P	L	H	F	G
44 -2	1.74	2.65	1.72	0.89	0.47	1.85	0.95	0.59
40 -2	1.69	2.43	1.67	0.92	0.467	1.76	0.87	0.58
OLD	1.562	1.533	1.533	0.900	0.514	1.533	0.739	0.607
NEW	1.650	1.721	1.721	0.920	0.500	1.690	0.797	0.593

J	C	N	B	D	W	X	
0.79	0.49	0.77	0.52	0.40	0.67	0.74	44 -2
0.72	0.50	0.71	0.52	0.40	0.72	0.76	40 -2
0.694	0.508	0.788	0.543	0.417	0.640	0.720	OLD
0.709	0.500	0.714	0.534	0.417	0.667	0.740	NEW

7.5 In all cases the "NEW" coefficient of variation assumptions are set in the range from:

a) the lowest of the first 3 pointers, to
b) the highest of the first 3 pointers.

Considerable weight was given to the "OLD" (see page 25) coefficient of variation assumptions. The following brief explanations should give some indication of the setting process:

S (shares)	Set equal to the average of "44 –2" and "OLD" and rounded to 1.650, which gives a standard deviation (16.50%) equal to "OLD"+0.1%.
I (int'l shares)	Combined with Q (as for risk margins).
Q (prop trust)	Combined with I (as for risk margins) and equal to "44 –2" (for Q), rounded to 1.721 so that the standard deviation is an integral multiple of 0.1%.
P (direct prop)	Set equal to the maximum of "44 –2" and "40 –2" which gives a standard deviation equal to "OLD"-0.3%.
H (hedged i.s.)	Set equal to the average of "44 –2" and "OLD", rounded to 1.691 so that the standard deviation (14.70%) is an integral multiple of 0.1%.
F (fixed int)	Because of the shorter F sector average maturity "duration" noted in section 6.4, the 44-year average results were disregarded and the "NEW" value was set equal to the average of "40 –2" and "OLD".
J (int'l fxd int)	Also set equal to the average of "40 –2" and "OLD".
C (cash)	All pointers are close, so set equal to the average of "44 –2", "40 – 2" and "OLD". This left the standard deviation unchanged at 2.90%.
N (infln linked)	As for risk margins, the results for "44 –2" were disregarded. Set equal to "40 –2" rounded to .714 so that the standard deviation (5.00%) is an integral multiple of 0.1%.
B (bills)	Consistent with C, set equal to the average of "44 –2", "40 –2" and "OLD". This also left the standard deviation unchanged.
D (bonds)	Set equal to "OLD" (which is close to both "44 –2" and "40 –2"). .
W (AWOTE)	Set equal to "44 –2" rounded down to 0.667 so that the standard deviation is an integral multiple of 0.1%. Noted that 40-year simulations with the "NEW" W sector means, CoV, skewness and kurtosis, result in AWOTE annual rates typically (i.e. based on medians) ranging from about 1.3% to 16% reflecting in particular the very high positive skewness.
X (CPI)	All pointers are close, so set equal to the average of "44 –2", "40 –2" and "OLD".

8 Means and Standard Deviations

8.1 When formulating assumptions for stochastic models, risk margins and coefficients of variation from the previous two sections were used to determine corresponding means and standard deviations. The resulting assumptions are summarised in Section 16. In **this section** the 44-year and 40-year means and standard deviations are tabulated, **purely for historical interest.**

8.2 It should be noted that all the results in this section, and all results up to and including Section 16, are **gross of tax and gross of fees**.

8.3 Results are also included for "Balanced" and "Capital Stable" portfolios based on the following fixed asset allocations:

Sectors	**S**	**I**	**H**	**Q**	**P**	**F**	**J**	**C**	**N**
Balanced	36%	19%	6%	7%	2%	16%	7%	5%	2%
Capital Stable	15%	7%	2%	3%	2%	32%	12%	25%	2%

These asset allocations proportions are equal to the Mercer Pooled Fund asset-weighted average benchmarks at **30 September 2006** and at **30 June 2008** with some minor rounding. There were no significant changes between these two dates; the benchmarks have not been updated since 2008. No allowance has been made for changes in asset allocations over time. Except for increased allocations to International Shares and reduced allocations to Direct Property, typical "Balanced" asset allocations have not changed markedly over recent decades. For example, the Mercer Pooled Fund asset-weighted average benchmarks at **28 February 1997** used in Grenfell (1997) were:

Sectors	**S**	**I**	**Q**	**P**	**F**	**J**	**C**	**N**
Balanced	35%	20%	4%	7%	20%	6%	8%	0%

8.4 In the following three tabulations the sectors denoted G, H, J, L and N have been omitted because they started after 1985. The 44-year statistics for these sectors are dependent on the backdating described in Section 4 and might be misleading.

8.5 For sectors which started before 1985 (and for the section 8.3 "Balanced" and "Capital Stable" portfolios) the 44-year average arithmetic means have been:

Table 8.1 Arithmetic means over 44 years

Period	S	I	Q	P	F	C	B	D	W	X	Balncd	CapStb	Period
9	13.4%	10.7%	11.3%	9.5%	8.8%	8.9%	9.0%	8.8%	7.7%	5.6%	10.6%	9.4%	9
8	13.9%	10.6%	11.8%	9.6%	8.9%	9.0%	9.0%	8.8%	7.7%	5.6%	10.9%	9.5%	8
7	14.5%	10.8%	12.0%	9.9%	8.8%	9.0%	9.0%	8.8%	7.7%	5.6%	11.2%	9.7%	7
6	14.9%	11.0%	12.5%	10.2%	8.7%	9.0%	9.0%	8.8%	7.7%	5.7%	11.4%	9.7%	6
5	14.5%	10.2%	12.0%	10.4%	8.7%	9.0%	9.1%	8.9%	7.7%	5.7%	11.0%	9.6%	5
4	14.0%	9.1%	10.8%	10.2%	8.9%	9.0%	9.1%	8.9%	7.7%	5.7%	10.4%	9.4%	4
3	14.7%	8.9%	11.1%	9.9%	8.9%	9.0%	9.1%	8.9%	7.7%	5.7%	10.7%	9.5%	3
2	14.7%	8.9%	11.0%	10.0%	8.9%	9.0%	9.1%	8.9%	7.6%	5.7%	10.7%	9.5%	2
1	13.7%	8.5%	10.7%	10.1%	9.0%	8.9%	9.0%	8.9%	7.6%	5.7%	10.3%	9.3%	1
0	13.7%	8.7%	11.1%	10.2%	9.1%	8.9%	9.0%	8.8%	7.5%	5.7%	10.4%	9.4%	0

Note that for all Periods the average mean returns for Cash and Bills exceeded those for Bonds and for Periods 9 to 2 they both also exceeded those for Fixed Interest.

The 44-year average compound means have been:

Table 8.2 Compound means over 44 years

Period	S	I	Q	P	F	C	B	D	W	X	Balncd	CapStb	Period
9	11.4%	9.4%	10.6%	9.2%	8.5%	9.0%	9.0%	8.8%	7.7%	5.6%	10.2%	9.3%	9
8	12.0%	9.5%	11.1%	9.4%	8.5%	9.0%	9.0%	8.8%	7.6%	5.6%	10.4%	9.4%	8
7	12.6%	9.2%	11.3%	9.6%	8.5%	9.0%	9.0%	8.9%	7.7%	5.7%	10.4%	9.4%	7
6	13.0%	9.2%	11.7%	9.9%	8.5%	9.0%	9.0%	8.9%	7.7%	5.7%	10.6%	9.5%	6
5	12.5%	8.8%	11.5%	10.1%	8.4%	9.1%	9.1%	8.9%	7.7%	5.7%	10.4%	9.4%	5
4	11.8%	7.6%	10.1%	10.0%	8.6%	9.1%	9.1%	8.9%	7.7%	5.7%	9.9%	9.3%	4
3	12.5%	7.5%	10.3%	9.7%	8.6%	9.0%	9.1%	8.9%	7.6%	5.7%	10.2%	9.4%	3
2	12.5%	7.5%	10.3%	9.9%	8.6%	9.0%	9.1%	8.9%	7.6%	5.7%	10.1%	9.4%	2
1	11.5%	7.0%	9.7%	10.0%	8.7%	9.0%	9.1%	8.9%	7.6%	5.7%	9.7%	9.2%	1
0	11.6%	6.9%	9.6%	9.9%	8.8%	9.0%	9.0%	8.9%	7.5%	5.7%	9.6%	9.2%	0

Figure 8.1 Compound means over 44 years

2003/2004 (for example) indicates average of yrs. ending 30/9/03, 31/12/03, 31/3/04 & 30/6/04

8.6 The 44-year average standard deviations have been:

Table 8.3 Standard deviations over 44 years

Period	S	I	Q	P	F	C	B	D	W	X	Balncd	CapStb	Period
9	21.5%	19.4%	11.8%	7.8%	8.9%	4.2%	4.5%	3.5%	4.9%	4.2%	12.5%	7.6%	9
8	21.5%	19.4%	11.9%	7.8%	8.9%	4.2%	4.5%	3.5%	4.9%	4.2%	12.5%	7.5%	8
7	21.5%	19.3%	11.8%	7.7%	8.9%	4.2%	4.5%	3.5%	4.9%	4.2%	12.5%	7.5%	7
6	21.5%	19.1%	12.1%	7.8%	8.9%	4.2%	4.5%	3.5%	4.9%	4.1%	12.4%	7.4%	6
5	21.7%	19.4%	13.1%	7.8%	9.0%	4.1%	4.4%	3.5%	4.9%	4.1%	12.6%	7.6%	5
4	22.4%	20.6%	16.0%	8.2%	8.9%	4.1%	4.4%	3.4%	4.9%	4.1%	13.6%	7.9%	4
3	22.4%	20.7%	16.3%	8.6%	8.9%	4.2%	4.4%	3.4%	4.9%	4.1%	13.6%	7.8%	3
2	22.4%	20.7%	16.4%	8.6%	8.9%	4.2%	4.5%	3.4%	4.9%	4.1%	13.6%	7.8%	2
1	22.4%	20.7%	16.5%	8.6%	8.9%	4.2%	4.5%	3.4%	4.9%	4.1%	13.6%	7.8%	1
0	22.4%	20.7%	16.7%	8.5%	8.9%	4.3%	4.5%	3.5%	4.9%	4.1%	13.6%	7.8%	0

8.7 For comparison with Table 8.2, the 40-year average compound means **for all sectors** have been:

Table 8.4 Compound means over 40 years

Period	S	I	Q	P	L	H	F	G	J
9	11.8%	8.8%	11.4%	9.9%	10.7%	10.6%	8.9%	10.2%	8.8%
8	12.5%	8.6%	12.0%	10.1%	10.7%	10.6%	9.1%	10.2%	9.0%
7	13.4%	9.0%	12.2%	10.3%	10.6%	10.8%	9.0%	10.1%	8.9%
6	13.8%	9.2%	12.8%	10.6%	10.7%	11.2%	8.9%	10.1%	8.9%
5	13.1%	8.5%	12.1%	10.7%	10.7%	10.9%	9.1%	10.2%	9.1%
4	11.8%	7.1%	10.0%	10.4%	10.6%	9.6%	9.3%	10.3%	9.0%
3	12.3%	7.4%	10.2%	9.9%	10.6%	10.2%	9.4%	10.3%	9.3%
2	12.5%	7.2%	10.1%	9.9%	10.6%	10.4%	9.5%	10.2%	9.4%
1	11.9%	6.9%	9.7%	9.9%	10.6%	10.1%	9.4%	10.1%	9.3%
0	11.9%	7.3%	10.0%	9.8%	10.6%	10.2%	9.3%	10.0%	9.3%

C	N	B	D	W	X	Balncd	CapStb	Period
9.4%	10.2%	9.4%	9.3%	8.0%	6.1%	10.4%	9.6%	9
9.4%	10.4%	9.4%	9.3%	7.9%	6.0%	10.7%	9.8%	8
9.4%	10.4%	9.4%	9.3%	7.8%	6.0%	11.0%	9.9%	7
9.4%	10.3%	9.5%	9.3%	7.7%	6.0%	11.3%	10.0%	6
9.4%	10.4%	9.5%	9.3%	7.7%	5.9%	11.0%	10.0%	5
9.4%	10.4%	9.4%	9.3%	7.5%	5.8%	10.1%	9.7%	4
9.3%	10.4%	9.3%	9.2%	7.3%	5.7%	10.4%	9.8%	3
9.2%	10.5%	9.3%	9.2%	7.2%	5.6%	10.4%	9.8%	2
9.1%	10.5%	9.2%	9.1%	7.0%	5.5%	10.1%	9.7%	1
9.0%	10.4%	9.1%	9.0%	6.9%	5.4%	10.1%	9.6%	0

8.8 For each of the 12 sectors in Table 8.2 (except unhedged International Shares), the average 44-year returns are more stable than the average 40-year returns in Table 8.4. This is partly due to the longer averaging periods underlying Table 8.2 but, importantly, it is also due to Table 8.4 comprising relatively more of the down-slope of the economic cycles identified in Section 5. Because of the latter the 8.9% to 9% F sector returns in Table 8.4 are inflated by capital appreciation and therefore exceed those in Table 8.2 that range from 8.4% to 8.8%.

8.9 For the reasons explained in the previous paragraph, the 44-year statistics form much better indicators of long-term returns. Where-ever possible it is desirable for the setting of long-term assumptions to analyse results over at least one full economic cycle.

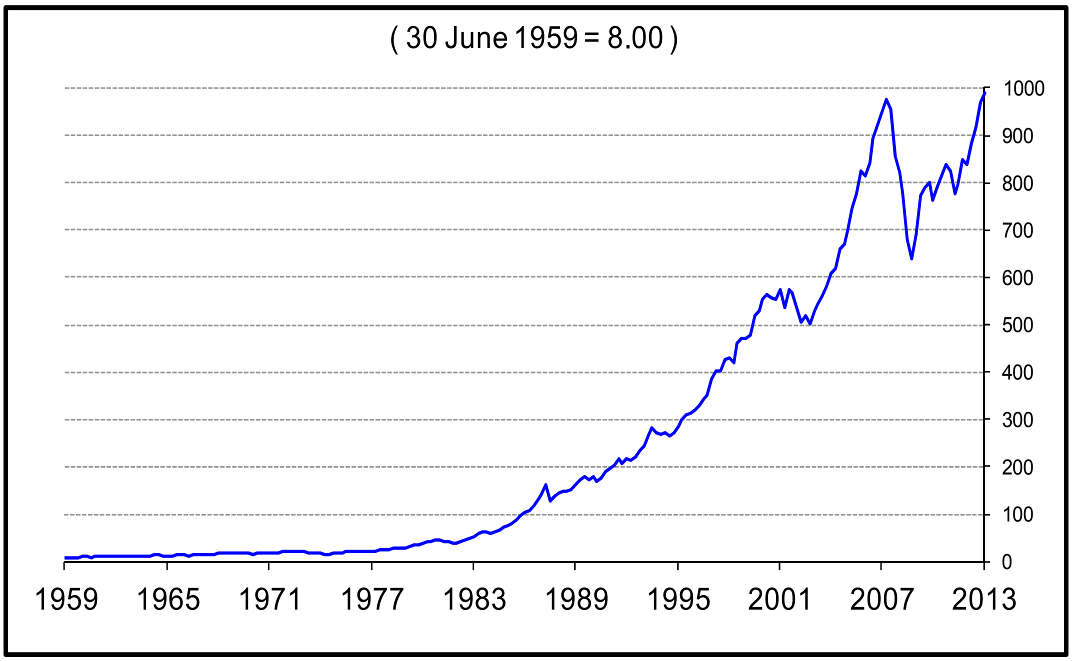
8.10 It should be noted that all individual results quoted in Sections 6 to 15, including those in this section, are the average of 4 annual results at quarterly intervals, **not individual results** for 44 or 40 years ending on one date.

9 The Global Financial Crisis

9.1 The "global financial crisis" is clearly the most relevant significant event during recent years. Was its impact right outside expectations, or was it predictable or consistent with past experience - never again or definitely again? How often can we expect the "perfect storm"?

9.2 Section 3.3 explained that the main database contains **annual forces at quarterly intervals**. Using this data and applying the fixed "Balanced" portfolio asset allocations explained in section 8.3, an array of quarterly unit prices was determined (starting from an initial value of 8.00 at 30 June 1959. Figure 9.1 below shows the results.

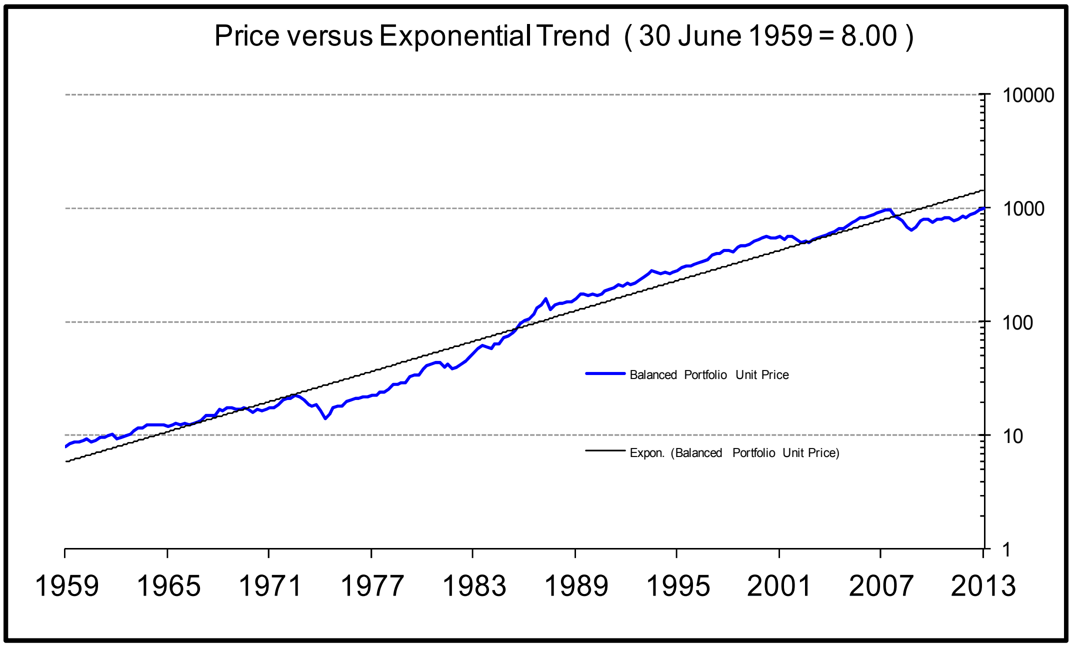
Figure 9.1 Balanced portfolio unit price



9.3 The Balanced portfolio unit price reached a quarterly maximum of 974.60 at 30 September 2007. In the following 18 months it fell 34.3% to a low of 639.87 at 31 March 2009. At 30 June 2013 the price had recovered 54.5% (from the low point of 4 years 3 months earlier) to 988.56.

9.4 Figure 9.2 on the next page shows the same results as in Figure 9.1 but this time using a logarithmic scale.

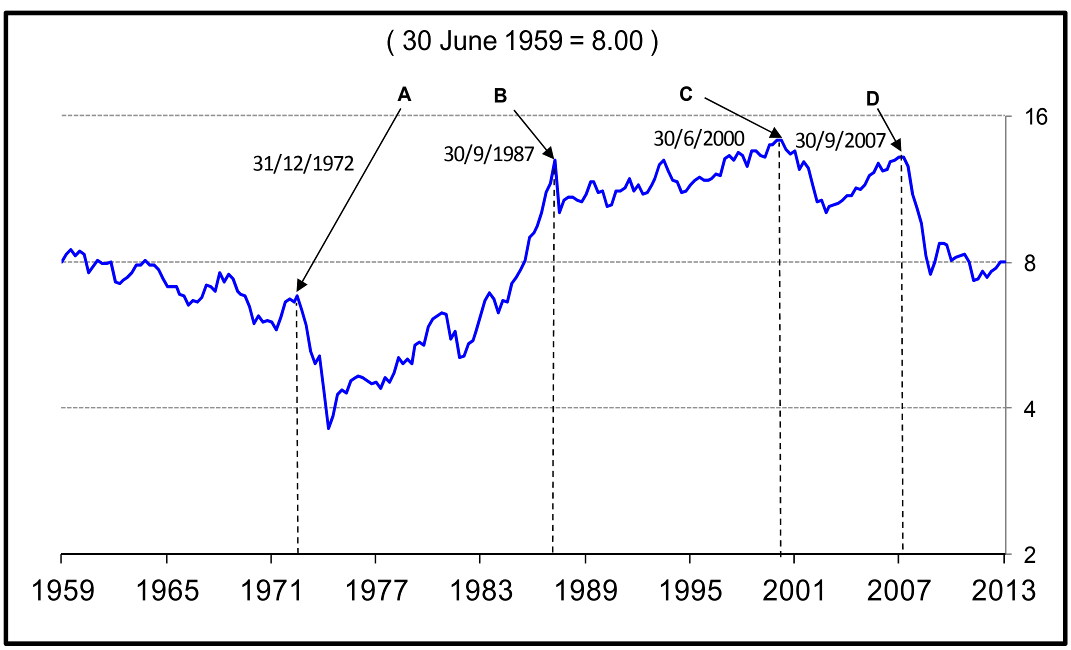
Figure 9.2 Balanced portfolio price on log scale



9.5 The exponential trend line in Figure 9.2, which presents as a straight line on this chart, indicates that the impact of the global financial crisis was not an isolated event.

9.6 In the **fifty four years** between 30/6/59 and 30/6/13 the Balanced portfolio return averaged **9.33% pa compound**. To help identify the peaks in the data, the Balanced unit prices were discounted by this long term rate. These "discounted unit prices" can be interpreted as a series of present values as at 30/6/59. Figure 9.3 below shows the results.

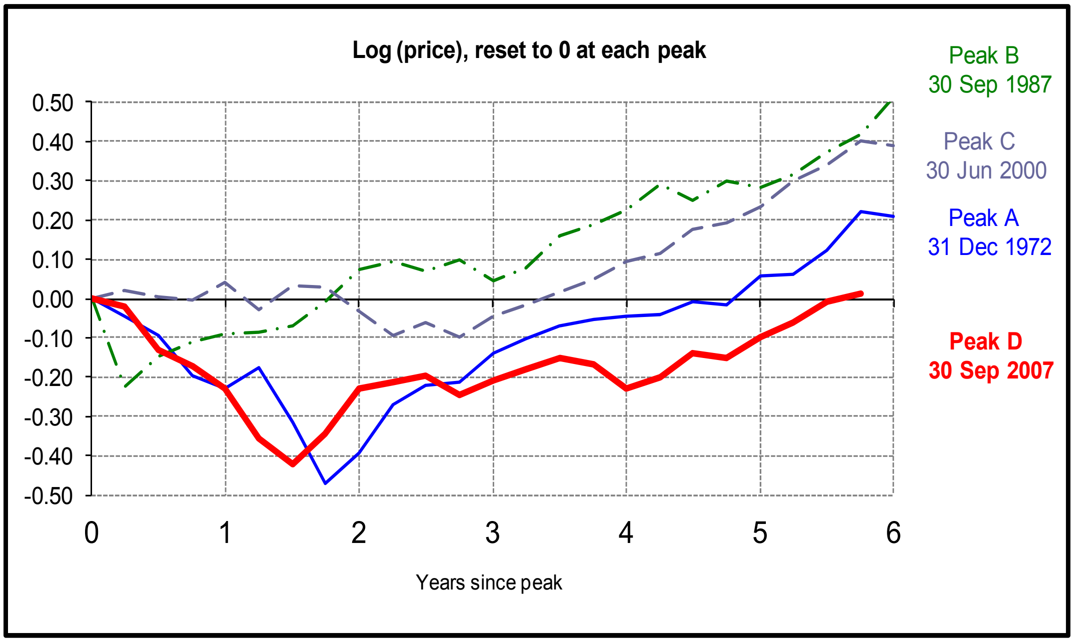
Figure 9.3 Balanced portfolio "discounted price"



9.7 The four peaks in the discounted price (identified as A, B, C and D in Figure 9.3) occurred at 31/12/72, 30/9/87, 30/6/00 and 30/9/07 respectively. There is another peak in Figure 9.3 which occurred at 31/3/81 but the 11.6% fall in price which occurred in the year following 31/3/81 is much less, in relative terms, than the 20.3% fall in price which occurred in the quarter following Peak B. The 31/3/81 peak has therefore not been specifically identified in Figure 9.3.

In Figure 9.4 below the log of the unit price (not the log of the discounted unit price) has been reset to 0 at each of the peaks A, B, C and D. A close examination of Figure 9.4 shows that the "peak C" curve based on the log of the unit prices actually increased in the first quarter after the peak, though the "discounted price" decreased during this quarter.

Figure 9.4 Balanced portfolio with price reset

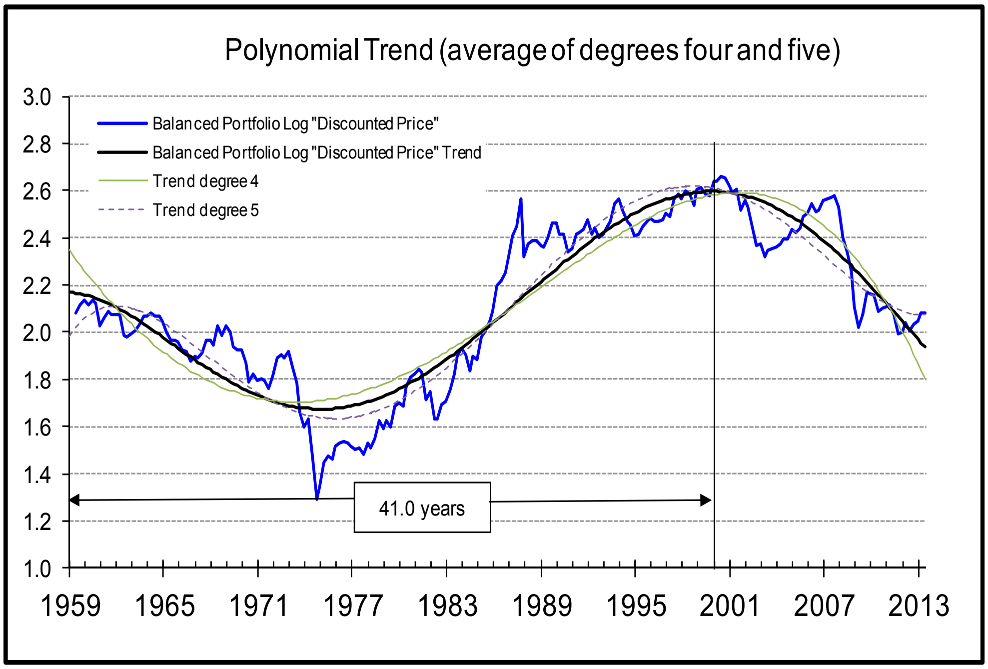


9.8 The chart above indicates that the impact of the global financial crisis on a Balanced portfolio was far more severe than the falls that occurred after the 30/9/87 and 30/6/00 peaks. However its impact and initial recovery were very similar to the 1974 market crash. The big economic difference between 1974 and 2009, which is not evident from the above charts, is that annual rates of salary increase (as measured by AWOTE changes) during the two years before and after 31/12/72 were 11.8%, 8.2%, 15.3% and 28.3% respectively. In the two years before and after 30/9/07 the annual rates of salary increase (as measured by AWOTE changes) were only 2.8%, 5.4%, 4.7% and 5.2%.

9.9 The other difference between 1974 and 2009, which is evident from the blue and red lines on the above chart, is that returns during the latter period of recovery (to 30 June 2013) have lagged significantly behind the post 1974 period (primarily due to low returns on Australian Fixed Interest investments and Cash).

9.10 As a consequence of its design, the "discounted unit price" described in section 9.6, starts and ends at 8.00. In the following Figure 9.5 a polynomial trend line of the log of the discounted price is shown. It is equivalent to the average of the trends for degrees four and five. This particular trend line was chosen because the degree four trend did not fit the data well at the start and end and the degree five trend did not fit the data well at the start.

Figure 9.5 Balanced portfolio log "discounted price" and trend



9.11 Figure 9.5 shows that the period between maximums in the trend line was 41.0 years (from 1 January 1959 to 1 January 2000). This adds support to the choice of 44 years in section 5.18 for the "average" past economic cycle. It should be noted that the methodology used to determine this cycle length is very different to that used in section 5.18. The latter is based on forces, sine curves and minimums, whereas this is based on logs of discounted prices, polynomial trends and maximums. Nevertheless the conclusion is very similar.

10 Outliers

10.1 Outliers can introduce bias into the sample means and standard deviations and in particular can have a very significant effect on skewness and kurtosis.

10.2 Grenfell (2007) considered the F sector (Australian fixed interest securities) and explained that the annual force for the year ending 30 September 1973 represented 3.39 standard deviations below the mean. If the F sector had a normal distribution this value had about one chance in 2,900 of occurring. It was compared with the most extreme outliers for Australian shares, in September 1974, of 2.69 standard deviations below the mean and, in September 1987, of 2.53 standard deviations above the mean.

10.3 However during the past five years a far more dramatic outlier has occurred.

10.4 Consider the following 44-year annual forces for years ending 30 September, 31 December, 31 March and 30 June for the Q sector (listed Australian Property Trusts). The period chosen corresponds to "Period 0" in section 2.5. The blue shading indicates backdated data.

Table 10.1 Property trust (Q sector) annual forces

Year Ending	30-Sep	31-Dec	31-Mar	30-Jun	Year Ending	30-Sep	31-Dec	31-Mar	30-Jun
1969/70	3.0%	1.2%	-3.0%	-1.0%	1991/92	4.6%	18.3%	10.0%	13.8%
1970/71	0.4%	-3.2%	0.9%	5.2%	1992/93	15.1%	6.8%	15.5%	15.8%
1971/72	2.9%	11.7%	19.6%	21.3%	1993/94	23.7%	26.3%	16.9%	9.4%
1972/73	21.7%	17.9%	7.1%	-2.5%	1994/95	-1.3%	-5.7%	2.5%	7.6%
1973/74	-15.6%	-18.6%	-11.6%	-17.0%	1995/96	9.4%	12.0%	3.7%	3.6%
1974/75	-12.7%	-7.2%	-5.9%	8.6%	1996/97	11.6%	13.5%	18.1%	25.1%
1975/76	18.1%	16.9%	12.0%	12.6%	1997/98	21.7%	18.5%	23.6%	9.5%
1976/77	13.3%	6.2%	6.7%	6.6%	1998/99	13.6%	16.5%	4.5%	4.2%
1977/78	4.6%	15.8%	25.2%	29.5%	1999/00	0.0%	-5.1%	1.0%	11.3%
1978/79	31.2%	26.4%	19.3%	14.4%	2000/01	8.7%	16.4%	12.0%	13.0%
1979/80	13.6%	10.0%	15.2%	10.6%	2001/02	15.1%	13.8%	16.0%	14.2%
1980/81	4.7%	6.2%	25.1%	22.0%	2002/03	11.3%	11.2%	13.0%	11.5%
1981/82	17.9%	27.8%	-3.0%	3.5%	2003/04	6.1%	8.4%	13.1%	15.9%
1982/83	13.8%	5.0%	18.1%	21.3%	2004/05	25.5%	27.9%	18.2%	16.9%
1983/84	29.9%	40.7%	34.6%	30.2%	2005/06	15.5%	12.0%	17.0%	16.6%
1984/85	22.0%	9.6%	12.5%	11.1%	2006/07	22.8%	29.3%	25.2%	23.4%
1985/86	5.4%	5.1%	14.3%	21.3%	2007/08	18.3%	-8.7%	-27.7%	-47.4%
1986/87	24.5%	30.3%	26.4%	34.6%	2008/09	-54.1%	-80.5%	-86.9%	-54.7%
1987/88	42.7%	5.6%	8.8%	-2.8%	2009/10	-26.2%	9.1%	35.1%	18.5%
1988/89	-20.6%	14.9%	1.2%	-1.1%	2010/11	-4.6%	-0.7%	4.6%	5.7%
1989/90	10.0%	2.3%	9.6%	14.2%	2011/12	-6.5%	-1.6%	1.7%	10.4%
1990/91	8.8%	8.3%	12.1%	7.4%	2012/13	25.4%	28.4%	26.6%	21.5%

30-Sep	31-Dec	31-Mar	30-Jun		
Statistics for all 44 years:					Average across
8.99%	9.07%	9.29%	9.44%	mu	9.20%
16.86%	18.33%	19.05%	16.51%	sigma	17.68%
-136%	-268%	-315%	-228%	skewness	-237%
373%	1274%	1487%	717%	kurtosis	963%

Modified statistics for all 44 years:					Average across
8.99%	9.07%	9.29%	9.44%	mu	9.20%
16.86%	18.33%	18.65%	16.51%	sigma	17.59%
-136%	-268%	-288%	-228%	skewness	-230%
373%	1274%	1249%	717%	kurtosis	903%

10.5 The annual force for the year ending 31 March 2009 was –86.9% (boxed). This represents 5.05 standard deviations below the mean. If the Q sector annual forces had a normal distribution (which they clearly haven't) this value has about one chance in 4,510,800 of occurring. The Q sector annual forces for the years ending 30 September 2008, 31 December 2008, 31 March 2009 and 30 June 2009 all represent more than 3.7 standard deviations below the mean.

10.6 The Q sector skewness and kurtosis for the 44 years ending 2012/2013 averaged **-237% and 963%** respectively. However the outlier of -86.9% (for the year ending 31 March 2009) significantly affects these results.

10.7 To indicate the sensitivity of the skewness and kurtosis to the March 2009 outlier, if this force had been 6% greater (i.e. –80.9%) and the force for the previous year had been 6% less (i.e. -33.7%), then the modified average skewness and kurtosis for the 44 years ending 2008/2009 would have been **-230% and 903%** respectively. This is equivalent to changing just one quarterly unit price (in this case, decreasing the 31 March 2008 unit price). The assumptions set out in Section 16 are not based on the modified statistics. Note that changes of 6% greater and 6% less are the largest integral percentage changes possible without changing the quarterly ranks of the tabulated forces.

10.8 By any reasonable yardstick, the experience of the Q sector five years ago was extreme. When setting assumptions for the future this must be carefully recognised. The specific adjustments now made when setting the Q sector risk margin and coefficient of variation have already been explained in Sections 6 and 7. The specific adjustments made for the Q sector skewness and kurtosis are explained in Sections 11 and 12. Section 13.4 explains why rank cross-correlations are not distorted by outliers and hence do not require specific adjustments.

10.9 Chan, Ng and Tong (2006) make a number of useful observations about outliers and explain why, whether or not it is appropriate to adjust the data for the outliers, depends on the purpose to which the model so derived will be used.

REMEMBER

> *"A **'black swan'** event is a seemingly inconceivable event that occurs infrequently and has massive impact. Quantitative risk modelling, with its bias in historic information would not be adequate to cover the risk of these future events that have not been conceived, especially when using nice smooth mathematical functions. The tails and extremes of events most likely don't follow nice smooth laws; across the range of possible outcomes it is extremely unlikely to get a 'one size fits all' distribution. Nice smooth laws tend to assume rational behaviour underlying them. The world can be discontinuous, which most mathematical models are not, and this is where unpredictability rules and chaos reigns. …*
>
> *Many actuarial methods involve backward looking, using past data which is out of date, sparse and perhaps no longer relevant. Future trends may not be predictable from the past (i.e. discontinuities / hysteresis)."*
>
> *Waite (2009)*

11 Skewness

11.1 Skewness characterises the degree of asymmetry of a distribution around its mean. Positive skewness indicates a distribution with an asymmetric tail extending towards more positive values. Negative skewness indicates a distribution with an asymmetric tail extending towards more negative values.

11.2 As explained in section 2.11, the skewness results shown in this section, and the kurtosis, cross-correlation and auto-correlation results shown in Sections 12 to 15 are based on annual forces.

11.3 When using stochastic models it is often desired to convert assumptions and/or model results from rates to forces and vice versa, and to combine the results from a number of sectors to form or approximate results for a portfolio or composite sector. Such changes are often based on the assumption, or an implied assumption, of a normal distribution for the forces and a lognormal distribution for the rates. However if non-zero values for skewness and kurtosis are used as input to a stochastic model, then formulae (based on a normal distribution for the forces and a lognormal distribution for the rates) will not hold.

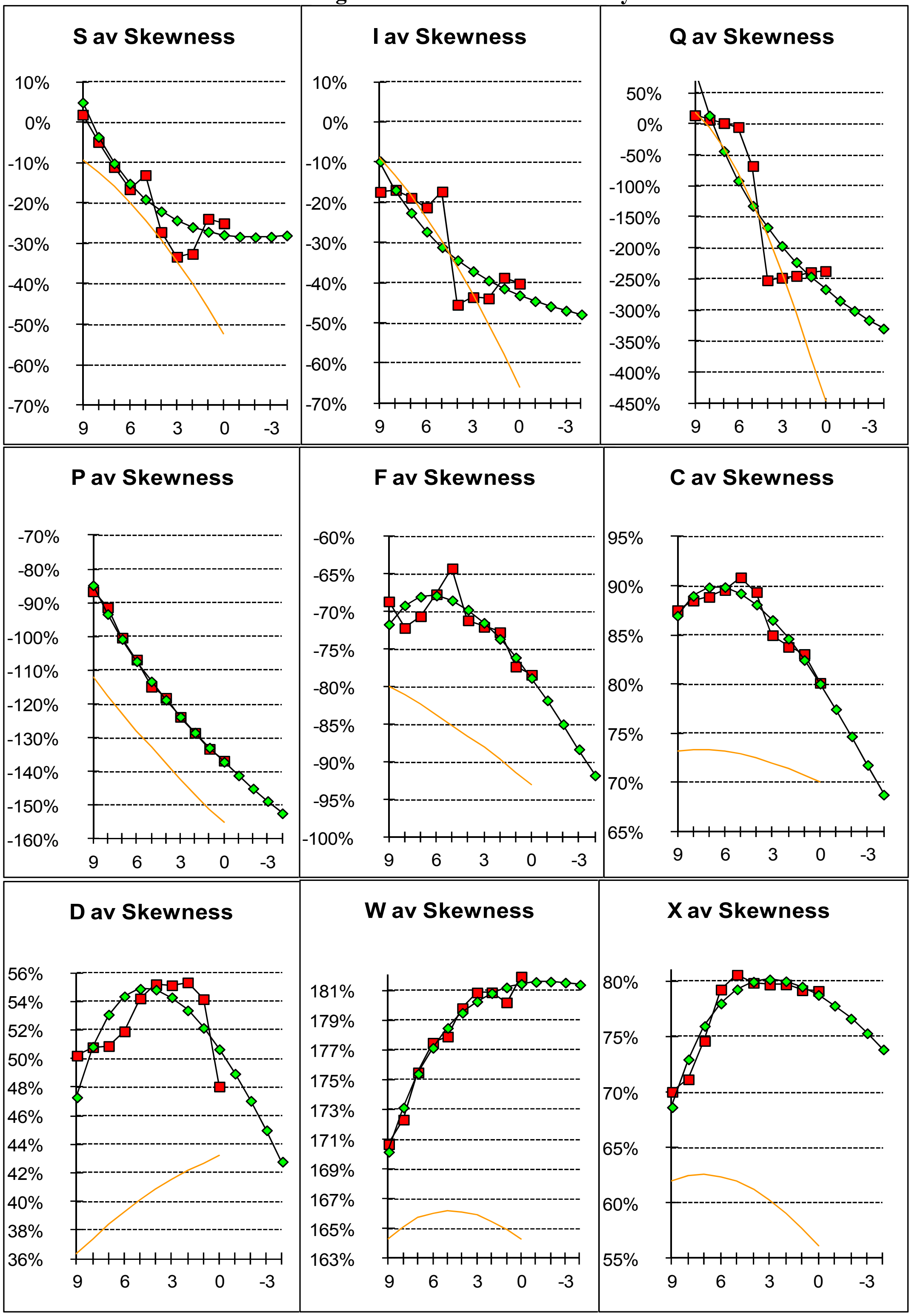
11.4 For example, when using dependent variables and non-zero values for skewness and kurtosis, it is not easy to determine:

- Long-term estimates of standard deviations of **portfolio or composite sector** returns from long-term estimates of standard deviations of returns for **each individual sector**, or
- Long-term estimates of means and/or standard deviations of **forces** for each individual sector from long-term estimates of means and/or standard deviations of **rates** for each individual sector.

11.5 However, under suitable assumptions, formulae can be determined to resolve these issues. For example, Appendix A of Grenfell (2005) contains formulae for statistical conversion between rates and forces of return for use when skewness and/or kurtosis are not zero.

11.6 The following Figure 11.1 and Table 11.1 illustrate the skewness results for each sector.

Figure 11.1 Skewness over 44 years



[red square] actual results [green diamond] trend results [orange line] 2009 trend (see section 6.10)

X-axis: Period 9 = average 44 years ending 30/9/03, 31/12/03, 31/3/04 and 30/6/04
Period 0 = average 44 years ending 30/9/12, 31/12/12, 31/3/13 and 30/6/13 (see section 6.8)

Table 11.1 Skewness over 44 years

Actual and *Quadratic Trend*

Period	S	I	Q	P	L	H	F	G
9	2%	-17%	14%	-87%	75%	-48%	-69%	34%
8	-5%	-17%	7%	-91%	76%	-50%	-72%	34%
7	-11%	-19%	1%	-100%	75%	-53%	-71%	34%
6	-17%	-21%	-5%	-107%	75%	-59%	-68%	35%
5	-13%	-17%	-68%	-115%	75%	-54%	-64%	35%
4	-27%	-45%	-252%	-118%	73%	-74%	-71%	30%
3	-33%	-44%	-248%	-124%	72%	-71%	-72%	30%
2	-33%	-44%	-245%	-128%	72%	-77%	-73%	30%
1	-24%	-39%	-239%	-133%	71%	-72%	-77%	30%
0	-25%	-40%	-237%	-137%	69%	-74%	-78%	31%
0	*-28%*	*-43%*	*-266%*	*-137%*	*69%*	*-75%*	*-79%*	*30%*
-1	*-28%*	*-45%*	*-285%*	*-141%*	*68%*	*-77%*	*-82%*	*29%*
-2	*-28%*	*-46%*	*-301%*	*-145%*	*67%*	*-78%*	*-85%*	*28%*
-3	*-28%*	*-47%*	*-316%*	*-149%*	*65%*	*-79%*	*-88%*	*28%*
-4	*-28%*	*-48%*	*-330%*	*-152%*	*64%*	*-81%*	*-92%*	*27%*

J	C	N	B	D	W	X	Period
-60%	88%	-19%	101%	50%	171%	70%	9
-65%	89%	-25%	101%	51%	172%	71%	8
-64%	89%	-27%	102%	51%	175%	75%	7
-62%	90%	-27%	102%	52%	177%	79%	6
-63%	91%	-26%	103%	54%	178%	81%	5
-63%	89%	-30%	103%	55%	180%	80%	4
-69%	85%	-30%	99%	55%	181%	80%	3
-71%	84%	-32%	98%	55%	181%	80%	2
-76%	83%	-39%	97%	54%	180%	79%	1
-78%	80%	-42%	93%	48%	182%	79%	0
-78%	*80%*	*-41%*	*94%*	*51%*	*181%*	*79%*	*0*
-82%	*77%*	*-44%*	*91%*	*49%*	*182%*	*78%*	*-1*
-86%	*75%*	*-48%*	*88%*	*47%*	*182%*	*77%*	*-2*
-91%	*72%*	*-51%*	*86%*	*45%*	*182%*	*75%*	*-3*
-95%	*69%*	*-55%*	*83%*	*43%*	*181%*	*74%*	*-4*

11.7 Except for the Q sector, the quadratic trend Period –2 results are the skewness assumptions that are tabulated in Section 16. For the Q sector (listed Australian Property Trusts) the actual Period 0 result (shaded red) was used because the actual results for the past 5 years have been very steady and it did not seem appropriate to extrapolate the already extreme impact of the GFC for a further 2 years.

REMEMBER

*"**'Spherical cow'** is metaphor for highly simplified models of reality. It is from a mathematical joke where the punch line is the mathematician's solution to improve milk production and starts with "Consider a spherical cow … ". The point of the joke is that model builders will often reduce a problem to its simplest form in order to make calculations feasible, even though such simplification may hinder the model's application to reality."*

Waite (2009)

12 Kurtosis

12.1 Kurtosis characterizes the relative peakedness or flatness of a distribution compared to the normal distribution. Positive kurtosis indicates a relatively peaked distribution. Negative kurtosis indicates a relatively flat distribution.

12.2 The tails of a distribution are significantly influenced by the kurtosis, and to a lesser extent the skewness.

12.3 Figure 12.1 and Table 12.1, on the next two pages, illustrate the kurtosis results for each sector.

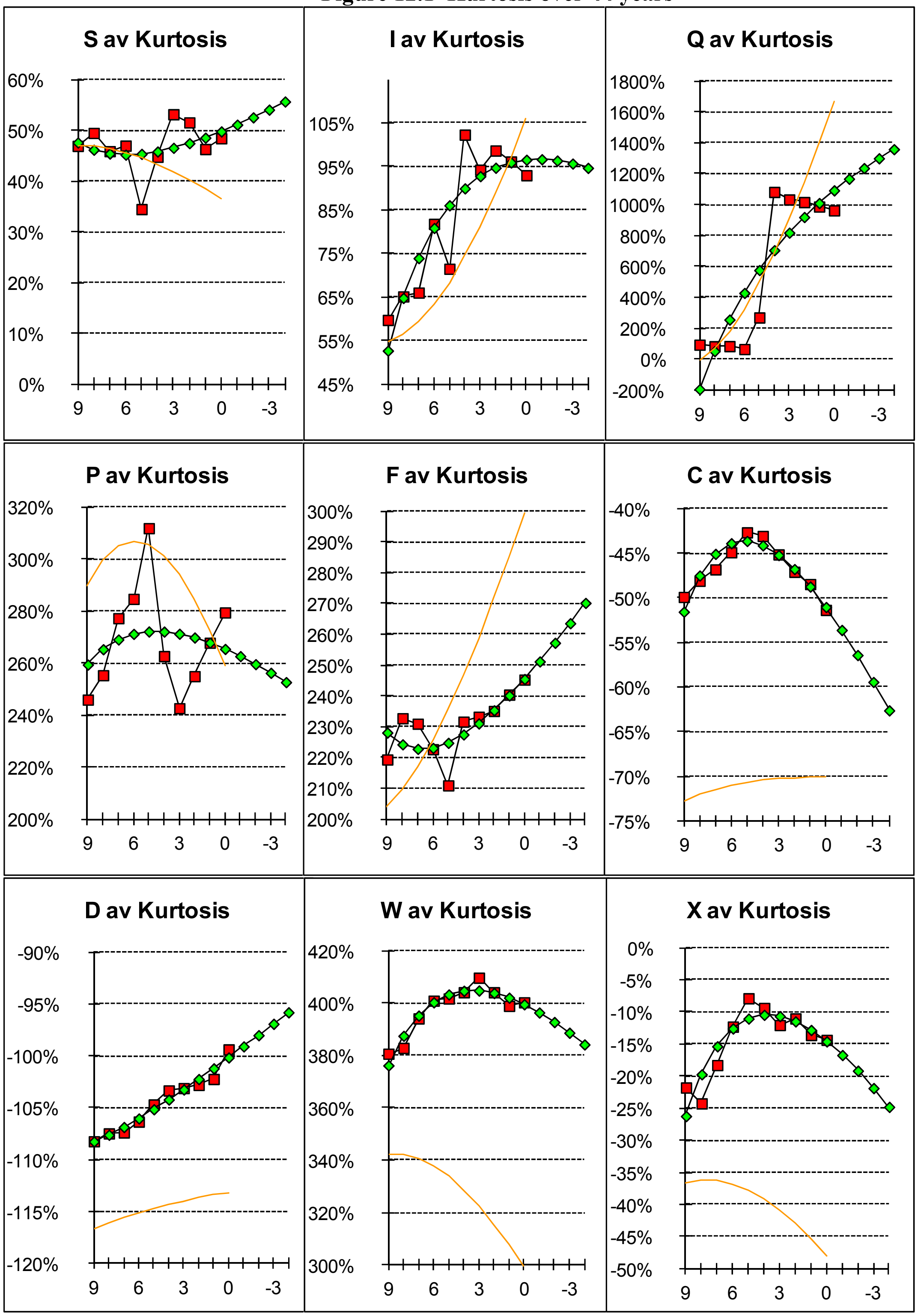
12.4 For illustrative purposes it is informative to classify and rank the skewness and kurtosis assumptions (tabulated in Section 16) as follows:

(a) **minimal** between -70% and +70% skewness and kurtosis.
(b) **moderate** between -71% and -200% skewness or kurtosis, or between +71% and +200% skewness or kurtosis.
(c) **extreme** over 200% skewness or kurtosis.

Sector		Skewness	Kurtosis	Classification
N	Inflation Linked Bonds	-48%	37%	Minimal
S	Australian Shares	-28%	53%	Minimal
G	Semi govt (0 to 3 yrs)	28%	-34%	Minimal
L	Loans/corp credit	67%	-3%	Minimal
H	Hedged Intl Shrs	-78%	113%	Moderate
Balncd	Balanced	-73%	111%	Moderate
CapStb	Capital Stable	-54%	99%	Moderate
I	Inter'nl Shares (uh)	-46%	96%	Moderate
D	10-Year Bonds	47%	-98%	Moderate
C	Cash	75%	-56%	Moderate
X	CPI	77%	-19%	Moderate
B	90-Day Bills	88%	-13%	Moderate
Q	Property Trusts	-237%	963%	Extreme
P	Direct Property	-145%	260%	Extreme
J	Inter'nl Fixed Interest	-86%	235%	Extreme
F	Austn. Fixed Interest	-85%	257%	Extreme
W	AWOTE	182%	393%	Extreme

Although this classification is only illustrative, the number of assumptions that fall outside the minimal classification sounds a warning for those who use normal or lognormal models in relation to investment performance.

Figure 12.1 Kurtosis over 44 years



[red square] actual results [green diamond] trend results [orange line] 2009 trend (see section 6.10)

X-axis: Period 9 = average 44 years ending 30/9/03, 31/12/03, 31/3/04 and 30/6/04
Period 0 = average 44 years ending 30/9/12, 31/12/12, 31/3/13 and 30/6/13 (see section 6.8)

Table 12.1 Kurtosis over 44 years

Actual and *Quadratic Trend*

Period	S	I	Q	P	L	H	F	G
9	47%	60%	94%	246%	-18%	56%	219%	-27%
8	49%	65%	84%	255%	-15%	64%	233%	-25%
7	46%	66%	85%	277%	-17%	73%	231%	-27%
6	47%	82%	66%	285%	-15%	95%	223%	-30%
5	34%	71%	270%	312%	-14%	88%	211%	-31%
4	45%	102%	1082%	263%	-14%	118%	232%	-25%
3	53%	94%	1033%	243%	-13%	105%	233%	-27%
2	52%	99%	1016%	255%	-12%	117%	235%	-31%
1	46%	96%	988%	268%	-9%	113%	240%	-30%
0	48%	93%	963%	279%	-6%	111%	245%	-33%
0	*50%*	*97%*	*1092%*	*265%*	*-7%*	*114%*	*245%*	*-32%*
-1	*51%*	*97%*	*1167%*	*263%*	*-5%*	*114%*	*251%*	*-33%*
-2	*53%*	*96%*	*1236%*	*260%*	*-3%*	*113%*	*257%*	*-34%*
-3	*54%*	*96%*	*1300%*	*256%*	*-1%*	*112%*	*264%*	*-35%*
-4	*56%*	*95%*	*1359%*	*253%*	*2%*	*110%*	*270%*	*-36%*

J	C	N	B	D	W	X	Period
183%	-50%	30%	-1%	-108%	381%	-22%	9
200%	-48%	39%	-1%	-107%	383%	-24%	8
197%	-47%	44%	0%	-107%	394%	-18%	7
193%	-45%	41%	2%	-106%	401%	-12%	6
195%	-43%	37%	6%	-105%	402%	-8%	5
189%	-43%	49%	7%	-103%	404%	-9%	4
201%	-45%	36%	3%	-103%	410%	-12%	3
206%	-47%	40%	1%	-103%	404%	-11%	2
213%	-48%	38%	-1%	-102%	399%	-14%	1
223%	-51%	42%	-6%	-99%	400%	-14%	0
220%	*-51%*	*39%*	*-5%*	*-100%*	*399%*	*-15%*	*0*
228%	*-54%*	*38%*	*-9%*	*-99%*	*396%*	*-17%*	*-1*
235%	*-56%*	*37%*	*-13%*	*-98%*	*393%*	*-19%*	*-2*
244%	*-59%*	*36%*	*-17%*	*-97%*	*389%*	*-22%*	*-3*
252%	*-63%*	*34%*	*-21%*	*-96%*	*384%*	*-25%*	*-4*

12.5 Except for the Q sector, the quadratic trend Period –2 results are the kurtosis assumptions that are tabulated in Section 16. For the Q sector (listed Australian Property Trusts) the actual Period 0 result (shaded red) was used because the actual results for the past 5 years have been very steady and it did not seem appropriate to extrapolate the already extreme impact of the GFC for a further 2 years.

REMEMBER

> *"Small changes in average behaviour do not give good evidence of what's happening in the tails, e.g. a smaller number of large banks may increase stability on average, but due to globalisation and interconnectedness, the tail risk becomes extreme (e.g. Fannie May and Freddie Mac). Even a fat-tailed distribution may understate the frequency and size of some risks."*
>
> *Waite (2009)*

13 Cross-correlations

13.1 The methodology used for risk margins, CoV's, skewness and kurtosis is detailed in Section 2. However for correlations it is essential that projected statistics are entirely consistent with each other over sectors (for cross-correlations) or over time (for auto-correlations). To obtain this greater consistency, the methodology was therefore slightly varied for correlations.

13.2 After step 2 in Section 2 the database was extended by adding on 6 years of "projected" annual forces for each quarter date. For this purpose the "projected" values were obtained from the projected trend risk margins (see Table 6.4) by adding back the fixed Bond base value (6% per annum) and converting to forces.

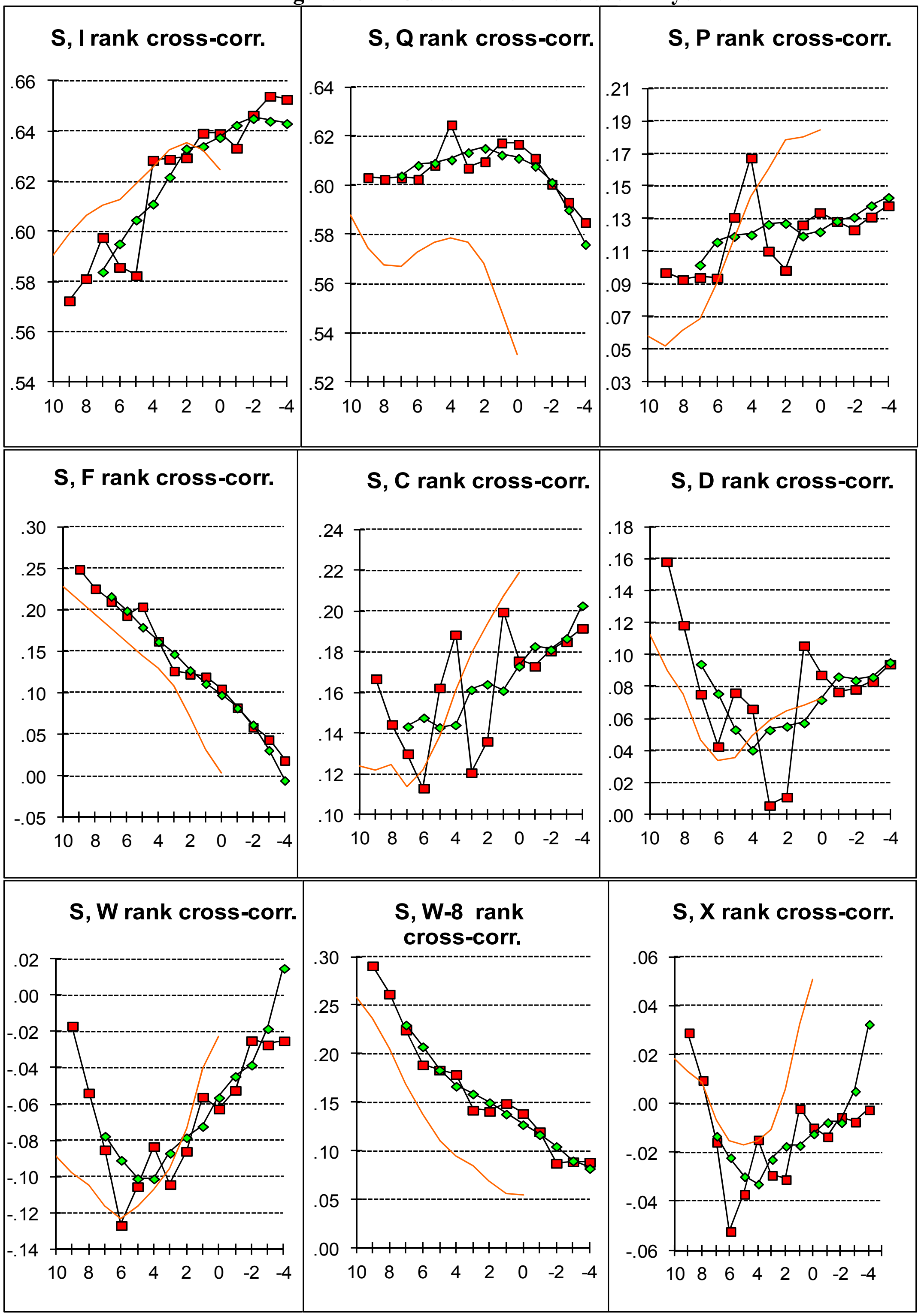
13.3 After the adjustments in 13.2 above, the extended database of each sector covered 59 years for each of the four quarter end-dates. (For the June quarter end-dates the extended database covered 60 years but the first year was disregarded). Steps 3 and 4 in Section 2 were then applied and step 5 was replaced by central 5-year running averages.

13.4 When values are changing rapidly from one period to the next the use of 5-year running averages may slightly distort results. An alternative might be to use a regression trend value at "Period -2". However having quadratic (or higher degree) trends at both steps 2 and 5 could result in hidden distortions in end results. Such an alternative was attempted in Grenfell (2009) for auto-correlations and found to produce results, up to a lag of 20 years, less than 3% (and usually less than 1%) different from using the more straightforward central 5-year average.

13.5 It is well documented that sample correlations can be distorted by outliers and non-zero skewness and kurtosis. To overcome this problem, for cross-correlations, 59-year ranks of the extended database forces were used as well as the actual forces. Thus two sets of cross-correlations were obtained, one based on ranks and one based on actual (i.e. actual plus backdated plus projected) forces. Correlations based on the former will be referred to as "rank" correlations, and correlations based on the latter will be referred to as "standard" correlations. In the *Austmod* investment simulation model, the rank cross-correlations are used because they are not distorted by outliers or by non-zero skewness or kurtosis (because the ranks of the forces are uniformly distributed) and the impacts of skewness and kurtosis are separately allowed for in the model.

13.6 Figure 13.1 and Table 13.1 illustrate the above process and show the results obtained for the S (Shares) sector rank cross-correlations.

Figure 13.1 Cross-correlations over 44 years



[red square] actual results [green diamond] 5-yr central avs [orange line] 2009 averages (see section 6.10)

X-axis: Period 9 = average 44 years ending 30/9/03, 31/12/03, 31/3/04 and 30/6/04
Period 0 = average 44 years ending 30/9/12, 31/12/12, 31/3/13 and 30/6/13 (see section 6.8)

Table 13.1 Rank cross-correlations over 44 years

Actual and Five-year Central Averages

Period	S I	S Q	S P	S L	S H	S F	S G	S J
9	.57	.60	.10	.28	.65	.25	.31	.23
8	.58	.60	.09	.24	.67	.23	.27	.22
7	.60	.60	.09	.19	.69	.21	.24	.19
6	.59	.60	.09	.17	.69	.19	.21	.17
5	.58	.61	.13	.21	.69	.20	.22	.18
4	.63	.62	.17	.24	.73	.16	.17	.19
3	.63	.61	.11	.20	.74	.13	.12	.18
2	.63	.61	.10	.20	.74	.12	.13	.17
1	.64	.62	.13	.21	.75	.12	.16	.18
0	.64	.62	.13	.21	.75	.11	.14	.17
0	*.64*	*.61*	*.12*	*.20*	*.75*	*.10*	*.13*	*.16*
-1	*.64*	*.61*	*.13*	*.20*	*.75*	*.08*	*.12*	*.15*
-2	*.65*	*.60*	*.13*	*.19*	*.76*	*.06*	*.10*	*.13*
-3	*.64*	*.59*	*.14*	*.16*	*.76*	*.03*	*.08*	*.10*
-4	*.64*	*.58*	*.14*	*.15*	*.76*	*.00*	*.06*	*.07*

S C	S N	S B	S D	S W	S X	S W-8	S X-8	Period
.17	.24	.16	.16	-.02	.03	.29	.21	9
.14	.23	.13	.12	-.05	.01	.26	.18	8
.13	.21	.10	.08	-.08	-.02	.23	.13	7
.11	.18	.08	.04	-.13	-.05	.19	.09	6
.16	.19	.11	.08	-.11	-.04	.18	.09	5
.19	.17	.09	.07	-.08	-.01	.18	.08	4
.12	.13	.03	.01	-.10	-.03	.14	.04	3
.14	.12	.05	.01	-.09	-.03	.14	.03	2
.20	.11	.11	.11	-.06	.00	.15	.07	1
.18	.10	.09	.09	-.06	-.01	.14	.06	0
.17	*.09*	*.09*	*.07*	*-.06*	*-.01*	*.13*	*.03*	*0*
.18	*.08*	*.10*	*.09*	*-.04*	*-.01*	*.12*	*.03*	*-1*
.18	*.06*	*.09*	*.08*	*-.04*	*-.01*	*.11*	*.02*	*-2*
.19	*.03*	*.10*	*.09*	*-.02*	*.01*	*.09*	*.01*	*-3*
.20	*.00*	*.11*	*.10*	*.02*	*.03*	*.08*	*.00*	*-4*

13.7 The five year central average Period –2 results are the S sector cross-correlation assumptions that are tabulated in the first of the two half-page matrices in Table 16.3.

13.8 Table 16.3 contains both the **rank** cross-correlation assumptions and the **standard** cross-correlation assumptions (defined in paragraph 13.4). Both of these were calculated using the methodology described in paragraphs 13.2 and 13.3.

13.9 Sweeting (2007) states,

"… Correlation gives one measure of the linkages, but assumes that the relationships between the marginal distributions are constant, whatever the levels of those distributions. It is only appropriate if the marginal distributions are jointly elliptical. Other measures such as Kendall's tau and Spearman's rho, which do not depend on the marginal distributions, might be more appropriate. …

If the degree of association is greater at extreme values of the marginal distributions (as it often is), then the above approaches understate the tail risk in the aggregate distribution. The solution is to use copulas. ... However, although there have been a number of papers on the subject of copulas, their use still appears to be limited. This is not least because the dependency structure is not straightforward. Since the dependency structure is largely defined by what happens at the tails, and as always, the tail of the distribution contains only a few observations, the form of the copula functions is not always clear. ...”

13.10 Alcock and Hatherley (2009) demonstrate a means of incorporating asymmetrical structures during the portfolio process using copula functions. Specifically, they investigate how asymmetrical return dependencies affect the efficient frontier and subsequent portfolio performance under a dynamic rebalancing framework assuming normally distributed marginal returns.

13.11 To give some insight into the extent to which the degree of association is greater at extreme values of the marginal distributions and when this might occur, Table 13.2 tabulates the **difference** between the standard cross-correlation assumptions and the rank cross-correlation assumptions.

Table 13.2 Standard less rank cross-correlations

	S	I	Q	P	L	H	F	G	J	C	N	B	D	W	X
S	0	.03	.04	.00	-.01	.01	.16	.06	.15	-.05	.11	-.01	.05	-.17	-.01
I	.03	0	.14	.04	.10	.05	.08	.07	.11	.07	.06	.06	.06	-.06	.01
Q	.04	.14	0	.14	.07	.15	-.01	.01	.04	.05	-.01	.08	.14	-.05	.08
P	.00	.04	.14	0	.04	.08	-.09	-.05	-.09	-.02	-.01	.00	.03	-.16	-.10
L	-.01	.10	.07	.04	0	.12	.12	.12	.11	.16	.10	.15	.12	-.10	-.03
H	.01	.05	.15	.08	.12	0	.13	.15	.15	.13	.12	.12	.11	-.08	.02
F	.16	.08	-.01	-.09	.12	.13	0	.08	.06	.08	.03	.03	.07	-.20	-.13
G	.06	.07	.01	-.05	.12	.15	.08	0	.14	.07	.08	.01	.06	-.22	-.12
J	.15	.11	.04	-.09	.11	.15	.06	.14	0	.10	.08	.08	.09	-.21	-.11
C	-.05	.07	.05	-.02	.16	.13	.08	.07	.10	0	.07	.02	.08	-.09	-.02
N	.11	.06	-.01	-.01	.10	.12	.03	.08	.08	.07	0	.04	.06	-.08	-.06
B	-.01	.06	.08	.00	.15	.12	.03	.01	.08	.02	.04	0	.04	-.11	-.05
D	.05	.06	.14	.03	.12	.11	.07	.06	.09	.08	.06	.04	0	-.09	-.03
W	-.17	-.06	-.05	-.16	-.10	-.08	-.20	-.22	-.21	-.09	-.08	-.11	-.09	0	.03
X	-.01	.01	.08	-.10	-.03	.02	-.13	-.12	-.11	-.02	-.06	-.05	-.03	.03	0
	Average	.027289													

13.12 The boxed cells in the upper right triangular matrix of Table 13.2 are all the differences greater than or equal to 0.15. Only 7 of the 105 cells are boxed - these indicate where the degree of association is likely to be greater at extreme values of the marginal distributions.

13.13 The boxed cells in the lower left triangular matrix of Table 13.2 are all the differences less than or equal to -0.15. Only 5 of the 105 cells are boxed – they all occur in the W sector (AWOTE) row. In fact the differences for the entire AWOTE row (except for CPI) are all negative. The rank cross-correlation assumptions for W (see Table 16.3) against the S, I, Q, H and F sectors are themselves negative. For these combinations any negative differences indicate that the degree of negative association is more extreme at extreme values of the marginal distributions. The rank cross-correlation assumptions for W (see Table 16.3) against all other sectors are positive. For these combinations any negative differences indicate that the degree of association is less at extreme values of the marginal distributions.

13.14 93 of the 105 cells in each triangular matrix are not boxed (in either triangle) - these indicate where the degree of association at the tails is likely to be similar to that for the central values.

13.15 It should be noted that all the correlations in this paper are annual. For example, the annual rank cross-correlation assumption for S against W is -0.04 and the annual standard cross-correlation assumption for S against W is -0.21. At first sight these negatives seem to contradict the common actuarial assumption of positive long term correlation between share returns and salary increases. However for correlation periods longer than a year this might still hold.

14 Leads and Lags

14.1 Consider the correlation between D sector (Bond) forces and CPI forces for various lags, with blue shading to indicate the maximum cross-correlation in each row:

Table 14.1 D sector and CPI correlation

24 yr Correlation of D against CPI lagged by Y years:

Y / End date	0	6.25	6.75	7	7.25	7.5	7.75	8	8.75	9	10
30/6/84	.687	.804	.816	.813	.806	.801	.790	.771			
30/6/85	.591	.790	.805	.812	.813	.815	.812	.799	.766	.744	
30/6/86	.550	.820	.808	.804	.801	.801	.800	.794	.789	.776	.704
30/6/87	.513	.828	.848	.852	.843	.832	.815	.797	.783	.771	.729
30/6/88	.459	.822	.843	.853	.857	.861	.862	.852	.801	.778	.726
30/6/89	.414	.818	.838	.848	.852	.8571	.857	.851	.842	.827	.732
30/6/90	.347	.817	.841	.850	.853	.858	.856	.848	.837	.828	.790
30/6/91	.238	.763	.828	.852	.863	.868	.863	.850	.834	.824	.791
30/6/92	.124	.726	.797	.832	.857	.877	.885	.874	.825	.802	.764
30/6/93	.060	.667	.751	.792	.823	.853	.869	.871	.846	.814	.708
30/6/94	.081	.607	.666	.704	.739	.772	.794	.799	.810	.797	.685
30/6/95	.090	.579	.641	.675	.704	.732	.751	.756	.770	.758	.665
30/6/96	.080	.523	.609	.652	.685	.713	.734	.732	.711	.690	.587
30/6/97	.227	.478	.551	.593	.635	.678	.702	.705	.678	.647	.479
30/6/98	.441	.521	.563	.590	.610	.634	.652	.643	.624	.598	.429
30/6/99	.579	.603	.639	.656	.667	.677	.667	.645	.576	.537	.389
30/6/00	.675	.653	.694	.713	.723	.732	.726	.702	.604	.558	.367
30/6/01	.674	.686	.736	.756	.770	.782	.779	.761	.679	.634	.408
30/6/02	.703	.693	.748	.773	.791	.805	.804	.788	.721	.682	.471
30/6/03	.742	.728	.764	.784	.798	.809	.810	.800	.752	.719	.536
30/6/04	.765	.783	.794	.806	.816	.828	.832	.822	.771	.747	.610
30/6/05	.758	.803	.816	.820	.824	.832	.835	.830	.823	.806	.666
30/6/06	.713	.791	.807	.814	.818	.819	.824	.819	.828	.829	.773
30/6/07	.688	.758	.786	.799	.808	.812	.815	.815	.817	.814	.782
30/6/08	.726	.759	.769	.771	.780	.787	.795	.801	.802	.801	.756
30/6/09	.689	.749	.774	.781	.7860	.7855	.781	.766	.768	.778	.737
30/6/10	.638	.730	.758	.771	.785	.794	.795	.784	.744	.741	.711
30/6/11	.562	.709	.743	.753	.767	.775	.779	.774	.757	.758	.667
30/6/12	.491	.660	.711	.730	.748	.762	.765	.756	.738	.744	.677
30/6/13	.291	.559	.603	.637	.675	.699	.722	.728	.731	.729	.659

14.2 Also consider the correlation between D sector forces and AWOTE forces for various lags:

Table 14.2 D sector and AWOTE correlation

24 yr Correlation of D against AWOTE lagged by Y years:

Y / End date	0	6.25	6.75	7.25	7.5	7.75	8	8.5	8.75	9	10
30/6/84	.527	.743	.786	.826	.835	.822	.789				
30/6/85	.445	.706	.751	.798	.8059	.8061	.789	.756	.747	.728	
30/6/86	.330	.673	.716	.766	.777	.773	.762	.732	.737	.730	.690
30/6/87	.242	.668	.707	.750	.761	.759	.750	.719	.721	.716	.698
30/6/88	.169	.649	.695	.742	.758	.753	.741	.713	.718	.714	.688
30/6/89	.096	.638	.674	.724	.741	.738	.730	.711	.715	.711	.692
30/6/90	.003	.622	.682	.723	.737	.722	.712	.690	.703	.702	.683
30/6/91	-.085	.565	.647	.722	.746	.742	.722	.679	.681	.679	.670
30/6/92	-.147	.509	.605	.692	.726	.730	.734	.710	.706	.672	.611
30/6/93	-.137	.470	.561	.667	.705	.707	.704	.685	.695	.690	.571
30/6/94	-.057	.468	.550	.644	.673	.677	.683	.668	.673	.652	.571
30/6/95	-.008	.453	.554	.635	.669	.661	.663	.639	.643	.625	.522
30/6/96	.006	.420	.520	.638	.686	.688	.684	.649	.635	.607	.475
30/6/97	.130	.414	.520	.621	.664	.669	.680	.664	.662	.623	.446
30/6/98	.359	.446	.526	.614	.656	.663	.667	.642	.637	.608	.458
30/6/99	.516	.517	.566	.637	.665	.660	.658	.633	.632	.600	.456
30/6/00	.605	.563	.615	.671	.692	.687	.683	.653	.644	.610	.471
30/6/01	.614	.598	.651	.701	.724	.721	.718	.685	.676	.644	.496
30/6/02	.617	.613	.671	.736	.757	.754	.747	.716	.709	.679	.530
30/6/03	.620	.643	.686	.745	.768	.768	.768	.742	.735	.706	.577
30/6/04	.644	.715	.707	.755	.778	.779	.781	.757	.749	.723	.611
30/6/05	.656	.777	.784	.776	.779	.776	.782	.775	.779	.761	.636
30/6/06	.625	.758	.767	.779	.783	.782	.784	.767	.775	.770	.710
30/6/07	.616	.761	.764	.766	.771	.768	.788	.794	.802	.778	.711
30/6/08	.557	.764	.765	.768	.764	.759	.775	.779	.785	.772	.709
30/6/09	.518	.783	.789	.771	.771	.760	.774	.770	.772	.754	.686
30/6/10	.424	.753	.793	.796	.799	.793	.792	.793	.779	.755	.663
30/6/11	.388	.729	.763	.774	.791	.801	.819	.826	.815	.777	.663
30/6/12	.309	.743	.755	.753	.765	.778	.802	.814	.815	.794	.672
30/6/13	.099	.739	.683	.641	.661	.703	.754	.781	.764	.749	.707

14.3 Tables 14.1 and 14.2 have some hidden columns but the hidden columns do not contain any maximums. Both tables are calculated from the main database, which contains annual forces at quarterly intervals (refer section 3.3).

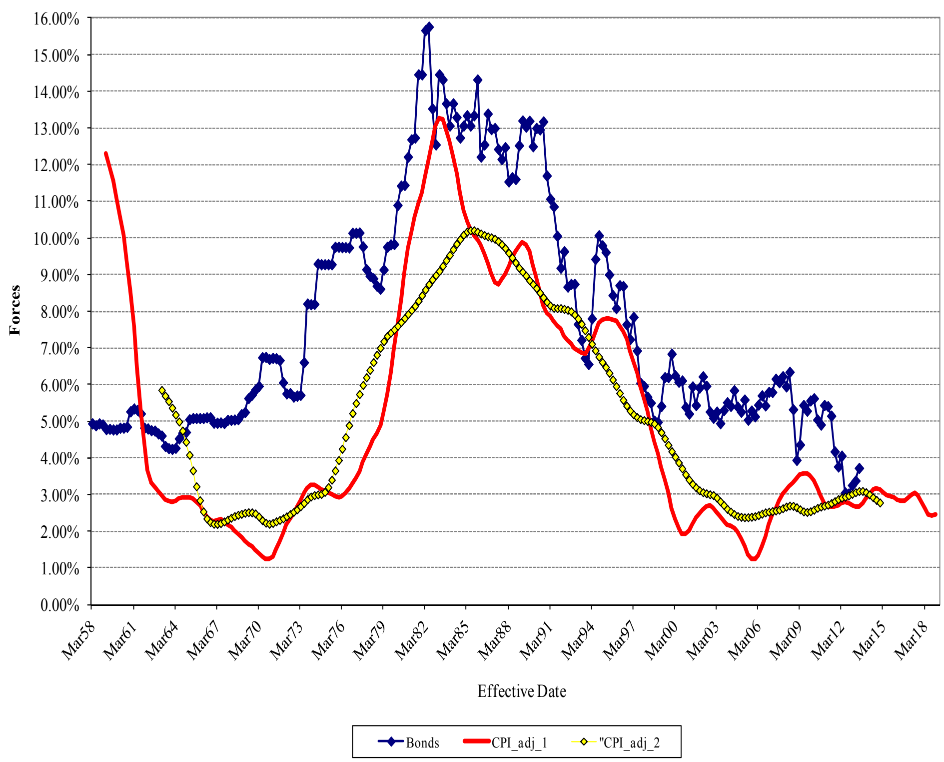
14.4 Table 14.1 and Table 14.2 show that when CPI and AWOTE forces are lagged by about 7.5 to 8.5 years their correlation with D forces increases to a maximum. Further, these relationships appear to hold reasonably consistently over time. It should be noted that the 8-year lagged cross-correlations are also far more stable than the non-lagged results.

14.5 Sections 3.1 and 6.10 explain that the "D sector" is by definition equal to the 10-year bond rate lagged by 6 months. Hence the 8-year lagged cross-correlations in Table 14.1 correspond to CPI lagged 7.5 years relative to the actual bond rate.

14.6 It is difficult to find an economic justification as to why changes in CPI and AWOTE appear to lead changes in bond rates by about 7.5 years (with a correlation of between 63% and 88%). Figure 14.1 below explores this relationship for CPI. It includes the actual bond forces from Figure 5.1 together with adjusted CPI forces. If the actual CPI forces are averaged over 4 years they will be lagged on average by 2 years. Thus the first adjusted CPI curve (labelled "CPI_adj_1") is CPI forces averaged over 4 years and then advanced by 5.5 years, giving a total lead averaging 7.5 years. If the actual CPI forces are averaged over 12 years they will be lagged on average by 6 years. Thus the second adjusted CPI curve (labelled "CPI_adj_2") is CPI forces averaged over 12 years and then advanced by 1.5 years, again giving a total lead averaging 7.5 years.

14.7 Both the adjusted CPI force curves in Figure 14.1 visually show a reasonably close relationship with the bond forces. If this relationship continues to hold it suggests that bond rates might fall by a further 0.2% to 0.3% during the next few years.

Figure 14.1 Bond and CPI forces



14.8 The increase in correlation with the D sector (and the increased stability of the correlations) as the lag changes from zero to say 8 years is significant for both CPI and AWOTE. The W-8 and X-8 columns of Table 14.3 show that a similar significant increase occurs when lagged CPI (sector X) and lagged AWOTE (sector W) are correlated against the forces of most other sectors.

14.9 The following table compares a wider range of correlations both with and without the 8 year lag.

Table 14.3 Alternative cross-correlation assumptions

RANK CROSS-CORRELATIONS
@ 2 YEARS (5-point average, rounded)

	W	X	W-8	X-8
S	-.04	-.01	.11	.02
I	-.04	.04	.25	.17
Q	-.04	-.09	.08	.01
P	.42	.54	.26	.12
L	.34	.43	.70	.56
H	-.12	-.06	.13	.11
F	-.04	-.01	.40	.40
G	.38	.44	.69	.60
J	.02	.03	.40	.37
C	.55	.66	.57	.44
N	.06	.18	.36	.25
B	.56	.68	.60	.50
D	.47	.59	.72	.66
W	1	.81	.39	.17
X	.81	1	.39	.19
W-8	.39	.39	1	.78
X-8	.17	.19	.78	1

STANDARD CROSS-CORRELATIONS
@ 2 YEARS (5-point average, rounded)

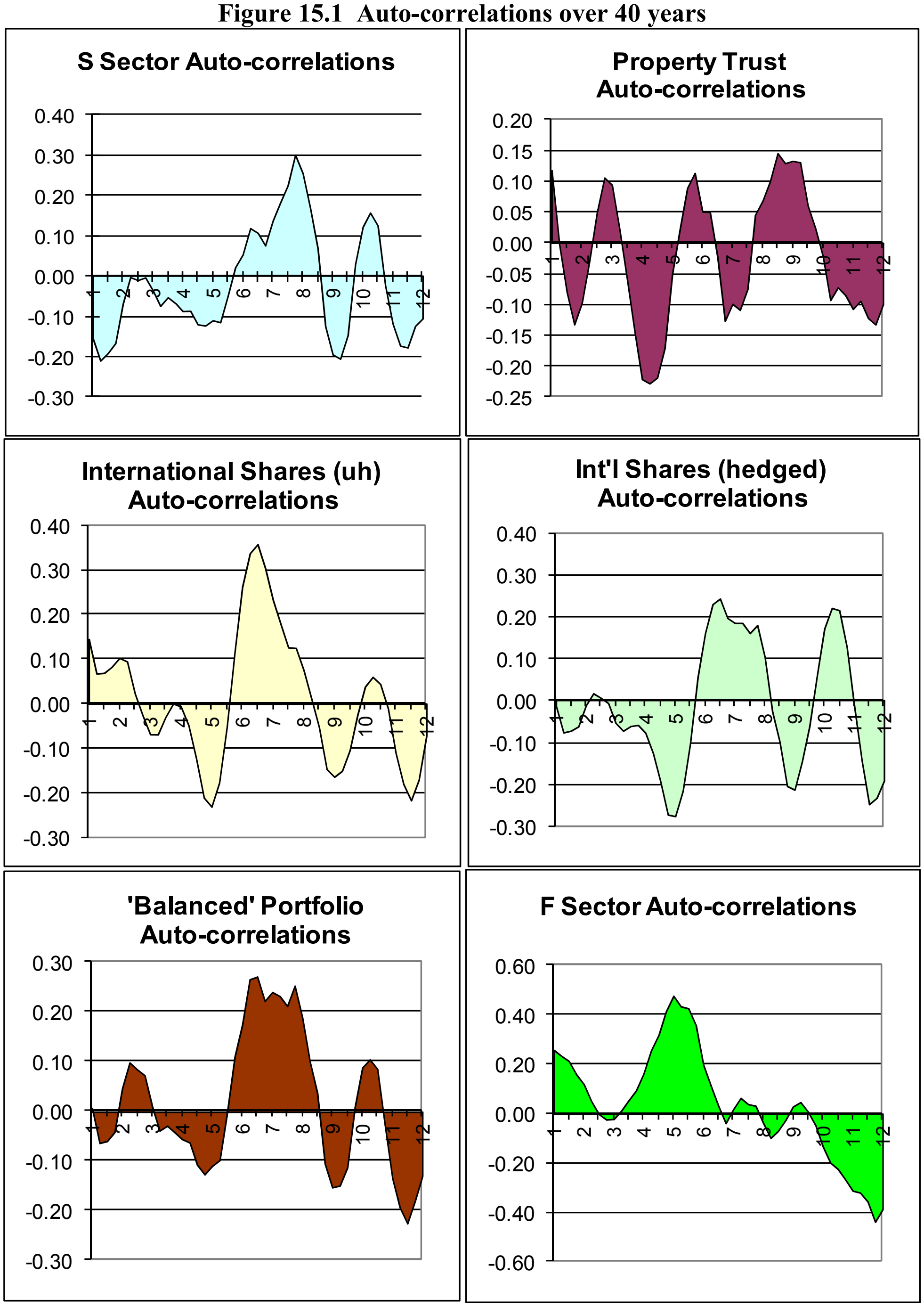
	W	X	W-8	X-8
S	-.21	-.02	.00	.01
I	-.10	.05	.25	.25
Q	-.09	-.01	.15	.09
P	.26	.44	.29	.18
L	.24	.40	.73	.69
H	-.20	-.04	.15	.16
F	-.24	-.14	.38	.41
G	.16	.32	.69	.67
J	-.19	-.08	.41	.40
C	.46	.64	.75	.69
N	-.02	.12	.39	.32
B	.45	.63	.71	.66
D	.38	.56	.78	.80
W	1	.84	.30	.11
X	.84	1	.40	.20
W-8	.30	.40	1	.81
X-8	.11	.20	.81	1

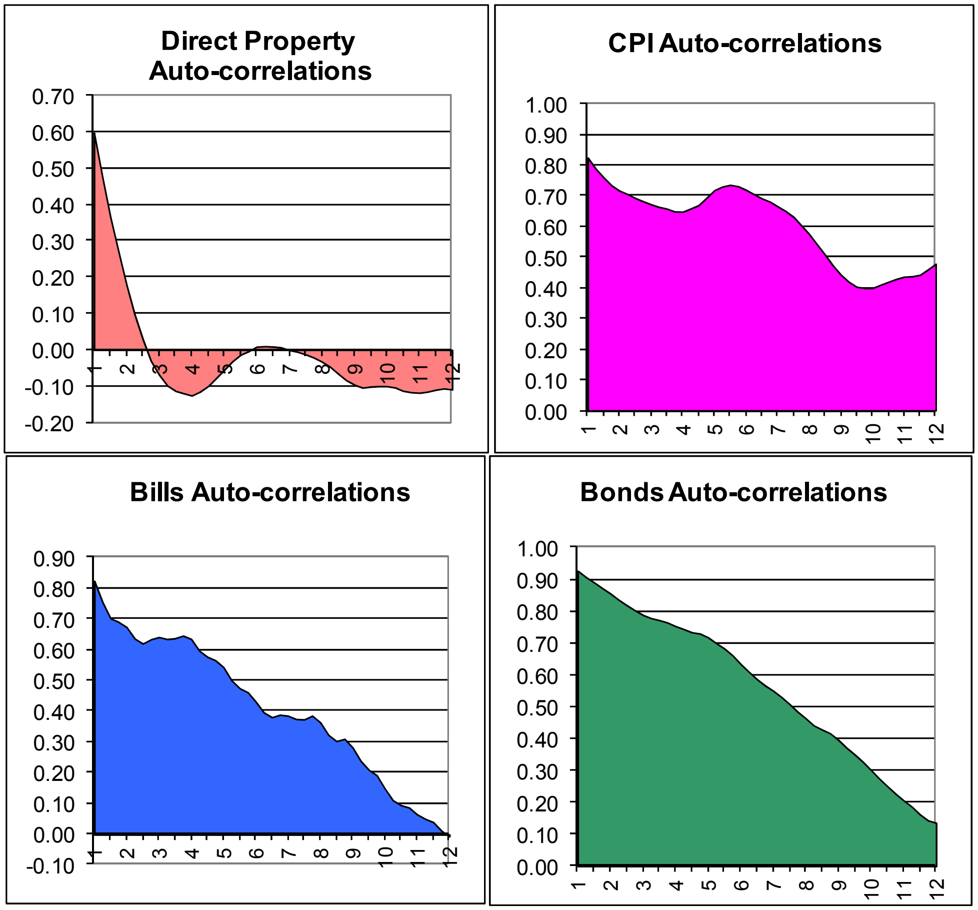
14.10 The results in this section suggest that stochastic investment models might be more realistic if allowance for an eight year lag is built in for CPI and AWOTE. Some further comments on this are included in Appendix B.

15 Auto-correlations

15.1 Auto-correlations vary significantly between the 15 sectors examined in this paper. To produce the following auto-correlation charts the annual forces for each sector were tabulated at quarterly intervals for the forty-year period from 30/6/1973 to 30/6/2013. From this, forty-year or 160 quarter database, auto-correlations were calculated for lags of 4 to 48 quarters for nine of the sectors plus the "Balanced" portfolio composite sector. Figure 15.1 shows the results. The X-axis of these charts is the lag in years.

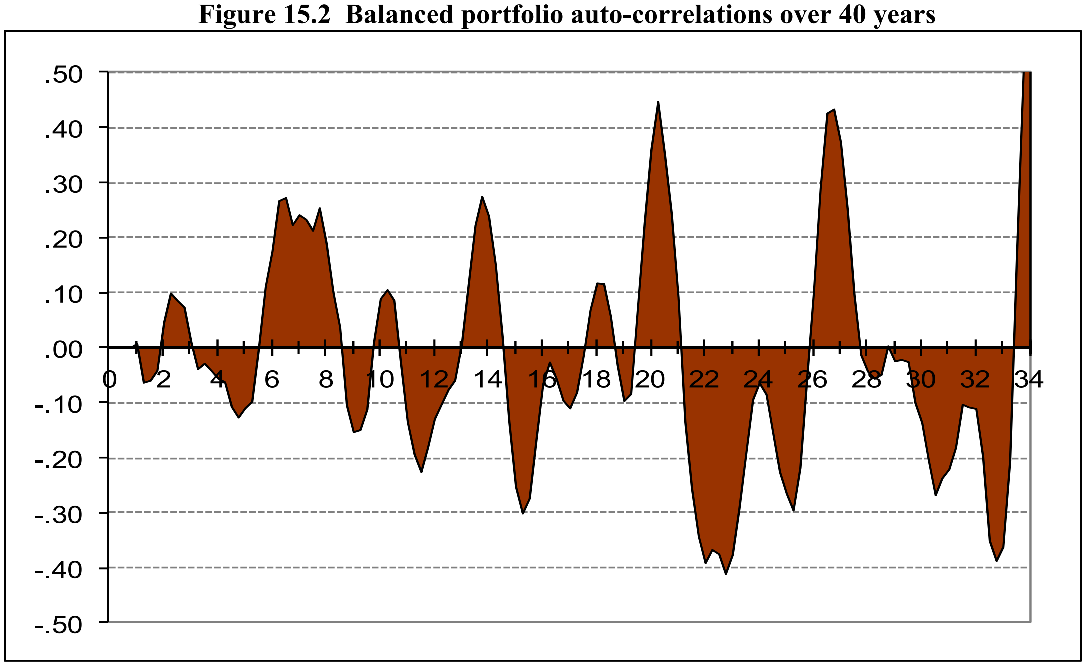
Figure 15.1 Auto-correlations over 40 years





15.2 In Figure 15.2 below the above 40-year "Balanced" portfolio auto-correlations are shown, this time for lags from one year to 34 years (i.e. from 4 to 136 quarters).

Figure 15.2 Balanced portfolio auto-correlations over 40 years



15.3 For the shorter lags, there is a slight negative bias in the results in Figure 15.2. Negative auto-correlations indicate that there is a tendency for market prices to "bounce back" after falls and/or to "drop again" after price increases. This tendency is somewhat similar, but different, to mean reversion. Of the 77 observations up to the central lag of 20 years, 41 (or 53%) are negative. Of the 19 observations up to a lag of 5.5 years, 13 (or 68%) are negative. For lags beyond 5.5 years there is generally a slight positive bias in the results.

15.4 A key consideration is whether auto-correlations are stable or fluctuating over time. To judge this requires a lot of data. Asher (2006), when commenting on the analysis of auto-correlations in Grenfell (2005), states:

"While there appears to be no theoretical justification – nor adequate data – for some of the longer term correlations reported in that paper, the shorter term cycles for share markets are more suggestive of some persistent relationships. In particular, the four year cycle for equities does coincide with US presidential elections. ... This would also be consistent with other research, reported for instance in Cutler et al (1991) which shows that many markets overshoot in the short run and correct in the second (or perhaps fourth) year.

...The consensus opinion of experts is also of interest. There is no definitive answer here. ... Over a third [of financial and economics professors surveyed in 2000] thought that markets showed negative auto-correlations over a three to five-year horizon ...."

15.5 More data is now at hand. What does it indicate? Since the greatest contribution to Balanced (or Growth) portfolio auto-correlations comes from Australian shares, the following analysis measures auto-correlations for the S sector, and also for the D sector (10-year bonds), over the two <u>non-overlapping</u> 26-year periods described below:

Period	Average of four periods ending:
0	30/9/12, 31/12/12, 31/3/13 and 30/6/13
26	30/9/86, 31/12/86, 31/3/87 and 30/6/87

These periods were chosen because:

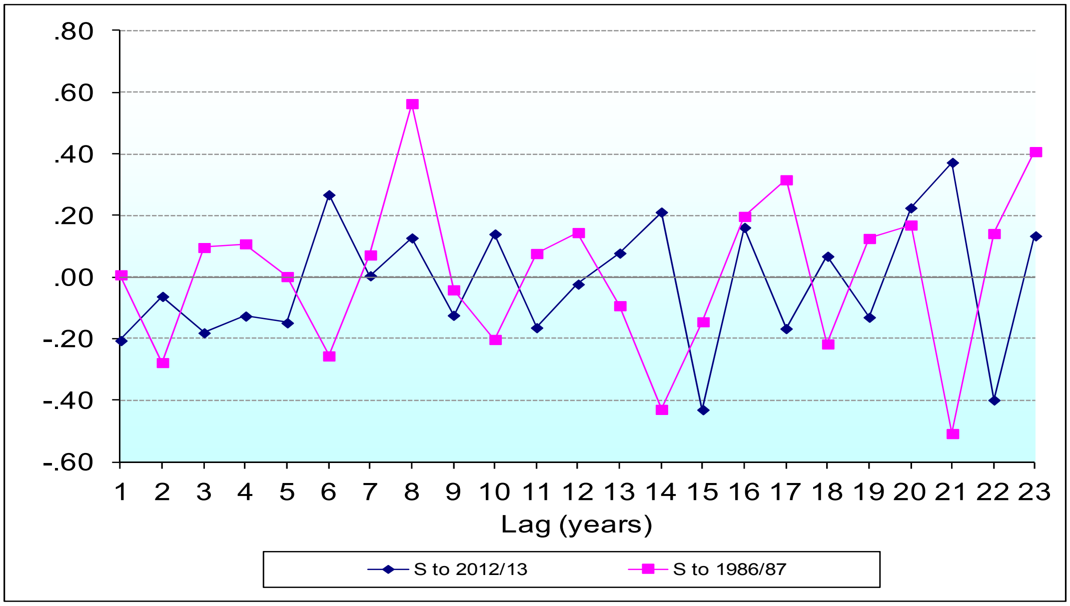
(a) all the available data is included except for the 3 months from 30 June 1959 to 30 September 1959,

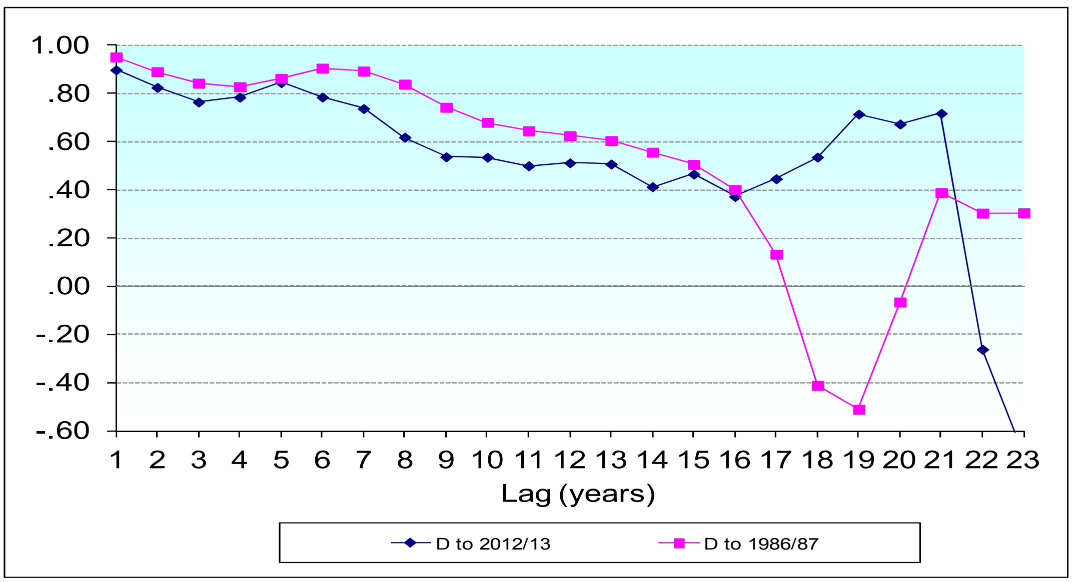
(b) they provide 104 data observations for each "period", and

(c) 26 years is consistent with the evidence of 24.5, 24.25, 26.5, 25.5 and 28.25 year cycle influences for Bonds, CPI and AWOTE in the first column of Table 5.1.

15.6 Figure 15.3 shows the average results for both 26-year periods.

Figure 15.3 Auto-correlations over 26 years



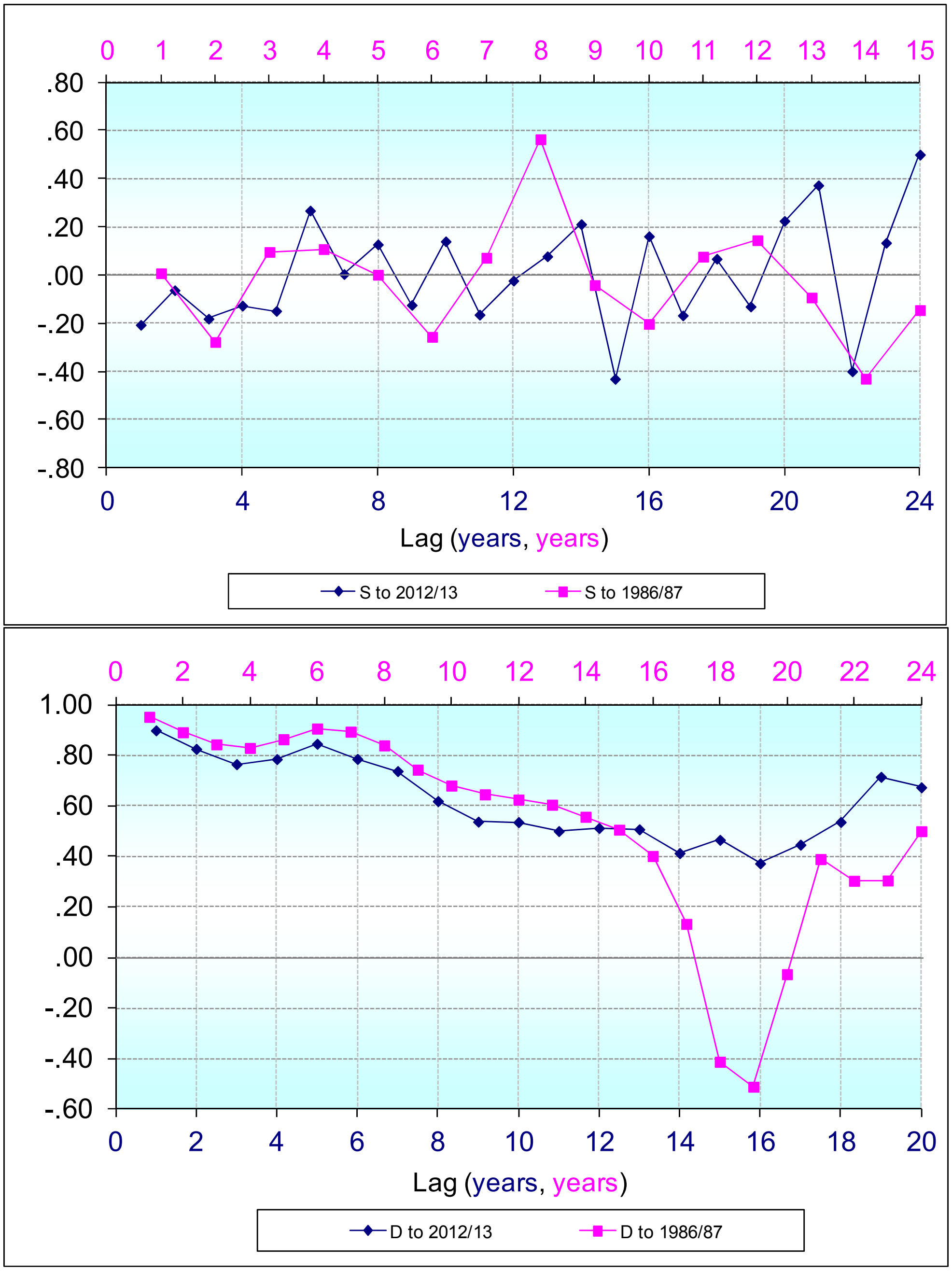


15.7 Examination of Figure 15.3 shows that if the X-axis scale:

- of the S sector correlations to 1986/1987 (shown in purple) is expanded by 60% (and hence progressively moved to the right) - indicating that the S sector auto-correlation features for the last 26 years occurred more slowly than those for the previous 26 years, and
- of the D sector correlations to 1986/1987 (shown in purple) is contracted by one-sixth (and hence progressively moved to the left) - indicating that the D sector auto-correlation features for the last 26 years occurred more quickly than those for the previous 24 years,

then both modified results ending 1986/1987 are similar to the (unmodified) results ending 2012/2013. This conclusion is illustrated in the following Figure 15.4.

Figure 15.4 Re-scaled auto-correlations over 26 years



15.8 Figures 15.3 and 15.4 indicate that the 26-year average auto-correlations ending 1986/87 are similar, after re-scaling, to those ending 2012/13. This similarity seems to be a reasonable justification for maintaining the Section 2 methodology trend-extrapolation approach for S and D sector auto-correlations.

15.9 Auto-correlations are dependent on the period over which they are measured. For some distributions this applies even if the underlying distribution has been consistent over time. For example, if annual forces follow say an exact 35-year sine-curve over an infinite number of years, then auto-correlations measured over any say 30, 35 or 40-year periods will produce different auto-correlations for any given lag.

15.10 When the *Austmod* investment simulation model was first developed in the early 1990's, it was based on 32 year projections partly because at that time less than 30 years' investment performance data was available for the main sectors. In view of the longer data period now available the model now produces 40-year projections.

15.11 For these reasons it was decided to measure auto-correlations over the following 40-year periods:

Table 15.1 Periods for auto-correlations

Period	Average of four periods ending:
13	30/9/99, 31/12/99, 31/3/00 and 30/6/00
12	30/9/00, 31/12/00, 31/3/01 and 30/6/01
11	30/9/01, 31/12/01, 31/3/02 and 30/6/02
10	30/9/02, 31/12/02, 31/3/03 and 30/6/03
9	30/9/03, 31/12/03, 31/3/04 and 30/6/04
8	30/9/04, 31/12/04, 31/3/05 and 30/6/05
7	30/9/05, 31/12/05, 31/3/06 and 30/6/06
6	30/9/06, 31/12/06, 31/3/07 and 30/6/07
5	30/9/07, 31/12/07, 31/3/08 and 30/6/08
4	30/9/08, 31/12/08, 31/3/09 and 30/6/09
3	30/9/09, 31/12/09, 31/3/10 and 39/6/10
2	30/9/10, 31/12/10, 31/3/11 and 39/6/11
1	30/9/11, 31/12/11, 31/3/12 and 39/6/12
0	30/9/12, 31/12/12, 31/3/13 and 39/6/13

15.12 The results were then projected forward using the same methodology to that used for cross-correlations in Section 13.

15.13 Appendix B explains that the auto-correlations for Australian shares and 10-year bond rates "form two extremes". The following Figures 15.5 and 15.6 and Tables 15.2 and 15.3 therefore illustrate the auto-correlations results for only the S and D sectors.

Figure 15.5 S sector auto-correlations over 40 years

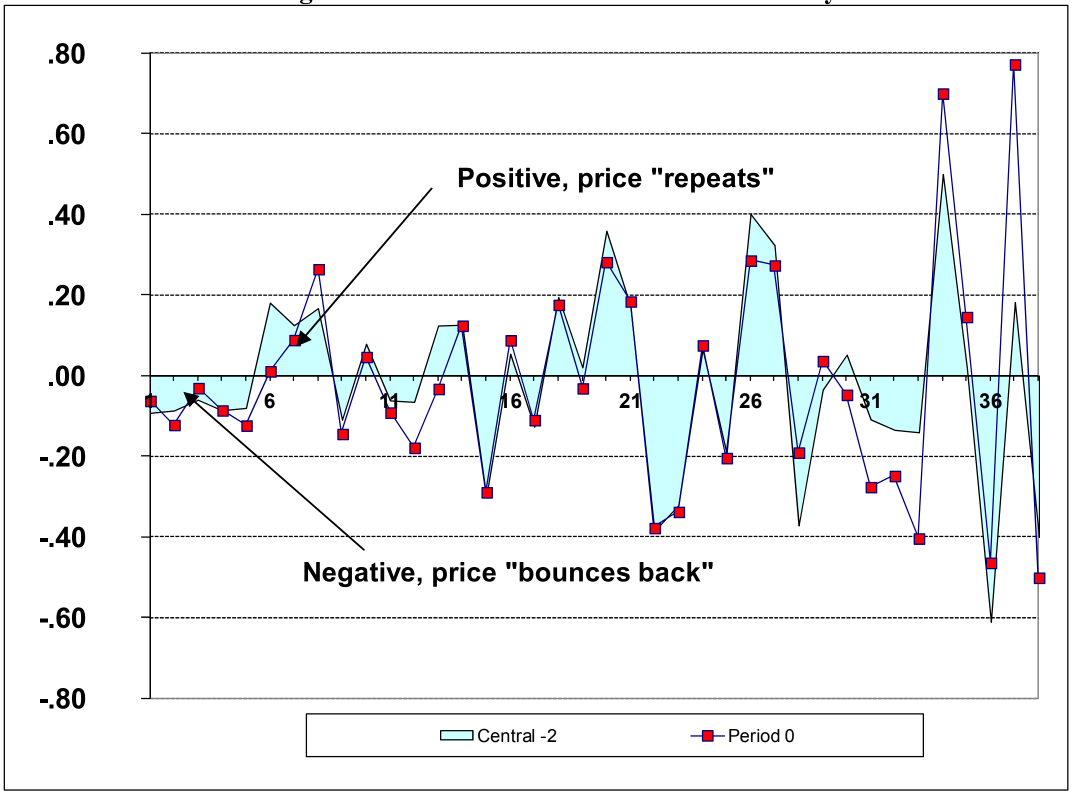


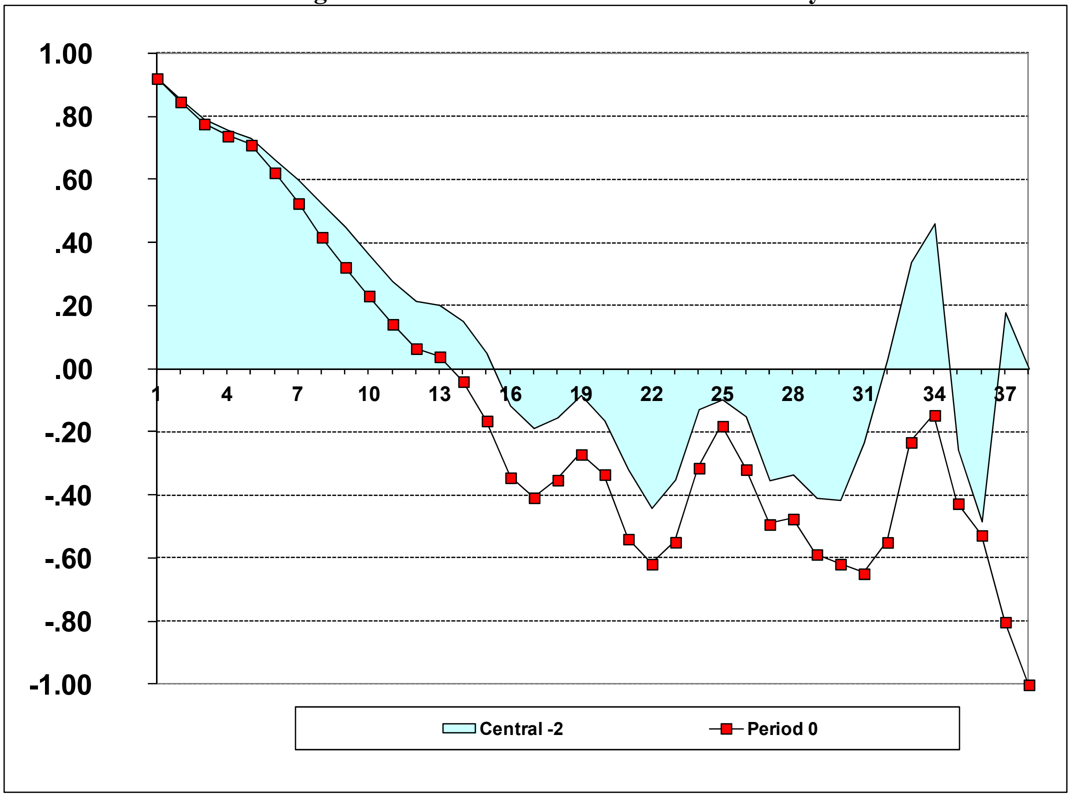
Table 15.2 S sector auto-correlations over 40 years

Period:	13	12	11	10	9	8	7	6	5	4	3	2	1	0	-1	-2	-3	-4	-2
Lag																			central
1	-.09	-.09	-.09	-.07	-.06	-.07	-.07	-.06	-.07	-.04	-.08	-.08	-.06	-.06	-.14	-.09	-.09	-.09	-.09
2	-.17	-.18	-.18	-.18	-.17	-.19	-.17	-.16	-.16	-.17	-.17	-.13	-.14	-.12	-.08	-.09	-.08	-.07	-.09
3	.00	.01	.01	-.01	-.01	.01	-.01	-.01	-.01	-.05	-.04	-.08	-.02	-.03	-.05	-.06	-.09	-.07	-.06
4	.11	.09	.09	.08	.07	.07	.04	.02	.01	-.04	-.05	-.09	-.07	-.09	-.10	-.05	-.10	-.09	-.09
5	-.06	-.08	-.09	-.08	-.08	-.09	-.12	-.13	-.10	-.11	-.13	-.12	-.12	-.12	-.08	-.02	-.08	-.10	-.08
6	-.12	-.12	-.11	-.11	-.10	-.12	-.12	-.12	-.06	.04	.06	.04	.01	.01	.11	.21	.28	.29	.18
7	.01	.03	.04	.03	.04	.05	.06	.05	.10	.13	.10	.10	.07	.09	.17	.09	.13	.14	.12
8	.32	.33	.31	.32	.32	.32	.30	.30	.28	.26	.25	.28	.26	.27	.18	.15	.12	.12	.17
9	-.06	-.08	-.08	-.10	-.11	-.16	-.23	-.23	-.23	-.21	-.21	-.17	-.13	-.14	-.17	-.09	-.08	-.06	-.11
10	-.11	-.13	-.13	-.11	-.09	-.15	-.11	-.12	-.09	-.08	-.06	-.02	.04	.05	.11	.10	.05	.09	.08
11	-.03	-.02	-.01	-.04	-.01	.03	.00	.00	-.03	-.02	.00	-.05	-.06	-.09	-.08	.01	-.06	-.08	-.06
12	.03	.02	-.01	.00	-.02	-.05	-.04	-.04	-.11	-.12	-.17	-.18	-.19	-.18	-.10	.00	-.03	-.02	-.07
13	-.03	-.05	-.07	-.06	-.05	-.05	-.04	-.02	-.05	-.05	-.06	-.02	-.03	-.03	.10	.17	.19	.19	.12
14	-.10	-.15	-.14	-.13	-.12	-.12	-.07	-.06	.00	.08	.11	.11	.11	.12	.15	.13	.11	.11	.13
15	-.14	-.15	-.19	-.18	-.19	-.15	-.12	-.15	-.14	-.21	-.23	-.20	-.24	-.29	-.33	-.32	-.29	-.28	-.30
16	.10	.08	.03	-.03	-.06	-.03	-.11	-.13	-.15	-.08	-.03	.04	.07	.09	.12	.04	.01	.01	.05
17	.11	.09	.09	.04	.03	-.07	-.12	-.11	-.10	-.13	-.11	-.12	-.10	-.11	-.17	-.13	-.12	-.10	-.13
18	-.06	-.05	-.06	-.02	.03	.07	.14	.13	.09	.18	.19	.20	.17	.18	.23	.22	.18	.16	.20
19	-.06	.02	.03	-.06	-.04	.02	.05	.07	-.01	-.02	-.05	-.04	-.03	-.03	-.06	.04	.07	.08	.02
20	.00	.02	.06	.07	.03	.05	.16	.26	.31	.30	.31	.30	.29	.28	.40	.37	.37	.38	.36

Period 0: average 40 years ending 30/9/12, 31/12/12, 31/3/13 and 30/6/13 (see section 6.8)

The green-shading marks the first tabulated occurrence of auto-correlations in the range -1% to +1%. For Shares, where the curve crosses the X-axis just before (but not shown above) lag 1, the green shading is an indication of one-half of a full cycle. For Bonds, where the curve for lags 0 to at least lag 8 is positive, the green shading is an indication of one-quarter of a full cycle.

Figure 15.6 Bond auto-correlations over 40 years



Up to the half-way point, the "central -2" curve lies to the right of the "period 0" curve, indicating that Bond auto-correlation features are now occurring more slowly. This is confirmed by the slope of the green shading below, between periods 5 and -4.

Table 15.3 Bond auto-correlations over 40 years

Period:	13	12	11	10	9	8	7	6	5	4	3	2	1	0	-1	-2	-3	-4	-2
Lag																			central
1	.92	.92	.92	.92	.92	.92	.92	.91	.91	.91	.91	.91	.92	.92	.92	.92	.92	.93	.92
2	.83	.83	.83	.82	.82	.82	.82	.82	.81	.82	.82	.82	.83	.85	.86	.85	.86	.86	.86
3	.75	.74	.73	.73	.72	.72	.71	.71	.72	.72	.72	.73	.75	.78	.79	.80	.80	.80	.79
4	.69	.67	.67	.66	.65	.64	.64	.65	.65	.65	.66	.68	.70	.74	.76	.76	.76	.76	.76
5	.64	.62	.60	.58	.58	.57	.57	.58	.57	.59	.62	.64	.67	.71	.72	.73	.74	.75	.73
6	.54	.50	.47	.44	.42	.43	.43	.43	.44	.47	.52	.55	.59	.62	.64	.66	.68	.71	.66
7	.39	.37	.32	.28	.26	.25	.25	.26	.30	.34	.39	.44	.47	.53	.57	.60	.64	.67	.60
8	.23	.18	.16	.11	.07	.03	.04	.08	.13	.18	.24	.28	.33	.42	.49	.54	.57	.60	.52
9	.06	-.01	-.05	-.06	-.13	-.17	-.16	-.11	-.05	.00	.05	.10	.18	.32	.42	.47	.50	.54	.45
10	-.09	-.18	-.24	-.31	-.32	-.33	-.32	-.29	-.25	-.19	-.13	-.05	.07	.23	.33	.38	.42	.45	.36
11	-.21	-.31	-.43	-.49	-.50	-.46	-.45	-.44	-.44	-.39	-.27	-.16	-.03	.14	.24	.30	.34	.37	.28
12	-.33	-.45	-.54	-.60	-.62	-.61	-.56	-.57	-.57	-.52	-.40	-.25	-.11	.07	.18	.24	.28	.31	.21
13	-.45	-.54	-.59	-.64	-.68	-.70	-.70	-.66	-.64	-.57	-.46	-.34	-.17	.04	.16	.22	.27	.32	.20
14	-.55	-.62	-.66	-.69	-.74	-.80	-.81	-.78	-.70	-.64	-.55	-.45	-.29	-.04	.09	.16	.23	.31	.15
15	-.63	-.69	-.73	-.77	-.82	-.87	-.88	-.86	-.81	-.70	-.62	-.53	-.40	-.16	-.02	.07	.16	.20	.05
16	-.73	-.78	-.82	-.87	-.92	-.93	-.93	-.93	-.90	-.85	-.72	-.65	-.54	-.34	-.18	-.07	-.01	.02	-.12
17	-.80	-.85	-.90	-.94	-.95	-.95	-.95	-.95	-.94	-.91	-.84	-.72	-.63	-.41	-.24	-.15	-.09	-.06	-.19
18	-.83	-.88	-.91	-.93	-.93	-.93	-.93	-.92	-.91	-.89	-.85	-.78	-.60	-.35	-.20	-.12	-.08	-.03	-.16
19	-.83	-.87	-.87	-.87	-.88	-.88	-.87	-.87	-.85	-.83	-.81	-.76	-.64	-.27	-.13	-.09	.00	.07	-.08
20	-.82	-.84	-.84	-.83	-.83	-.83	-.83	-.82	-.80	-.77	-.74	-.73	-.61	-.33	-.26	-.15	-.07	-.01	-.16

Period 0: average 40 years ending 30/9/12, 31/12/12, 31/3/13 and 30/6/13 (see section 6.8)

15.14 Tables 15.2 and 15.3 include only the first 20 year lags because beyond the half-way point the auto-correlations often fluctuate.

16 Assumptions

16.1 The following assumptions, developed as explained in previous sections, are gross of (i.e. before) tax and fees and are before additions for imputation credits. They have not yet been "road-tested", but are not significantly different from the previously tested ("OLD") assumptions.

Table 16.1 Investment assumptions

Sector		Risk margin (arithmetic average)	Mean rate (arithmetic average)	Compound average	Coefficient of variation	Standard deviation of rates	Skewness	Kurtosis
S	Shares	4.0%	10.0%	8.8%	1.650	16.5%	-28%	53%
I	Int'l Shrs	2.6%	8.6%	7.6%	1.721	14.8%	-46%	96%
Q	Prop Trust	2.6%	8.6%	7.4%	1.721	14.8%	-237%	963%
P	Direct Prop	1.5%	7.5%	7.3%	0.920	6.9%	-145%	260%
H	Hedged IS	2.7%	8.7%	7.7%	1.690	14.7%	-78%	113%
L	Loans/credit	1.2%	7.2%	7.1%	0.500	3.6%	67%	-3%
F	Fixed Int	0.4%	6.4%	6.3%	0.797	5.1%	-85%	257%
G	Semi-govt	0.4%	6.4%	6.3%	0.593	3.8%	28%	-34%
J	Int'l Fxd Int	0.2%	6.2%	6.1%	0.709	4.4%	-86%	235%
C	Cash	-0.2%	5.8%	5.8%	0.500	2.9%	75%	-56%
N	Infln Linked	1.0%	7.0%	6.9%	0.714	5.0%	-48%	37%
Balncd	Balanced	2.03%	8.03%	7.60%	1.207	9.69%	-73%	111%
CapStb	Cap Stable	0.81%	6.81%	6.69%	0.725	4.94%	-54%	99%
B	Bills	-0.20%	5.80%	5.76%	0.534	3.10%	88%	-13%
D	Bonds		6.00%	5.97%	0.417	2.50%	47%	-98%
W	AWOTE	-1.80%	4.20%	4.16%	0.667	2.80%	182%	393%
X	CPI	-3.30%	2.70%	2.68%	0.740	2.00%	77%	-19%

Table 16.2 40-year auto-correlation assumptions

Lag (years)	S sector	D sector	Lag (years)	S sector	D sector
1	-.09	.92	21	.17	-.32
2	-.09	.86	22	-.39	-.44
3	-.06	.79	23	-.32	-.35
4	-.09	.76	24	.07	-.13
5	-.08	.73	25	-.19	-.10
6	.18	.66	26	.40	-.15
7	.12	.60	27	.32	-.35
8	.17	.52	28	-.37	-.34
9	-.11	.45	29	-.04	-.41
10	.08	.36	30	.05	-.42
11	-.06	.28	31	-.11	-.23
12	-.07	.21	32	-.13	.03
13	.12	.20	33	-.14	.34
14	.13	.15	34	.50	.46
15	-.30	.05	35	.02	-.26
16	.05	-.12	36	-.61	-.48
17	-.13	-.19	37	.18	.18
18	.20	-.16	38	-.40	.00
19	.02	-.08			
20	.36	-.16			

Table 16.3 Cross-correlation assumptions

RANK CROSS-CORRELATIONS @ 2 YEARS (5-point average, rounded)

	S	I	Q	P	L	H	F	G	J	C	N	B	D	W	X
S	1	.65	.60	.13	.19	.76	.06	.10	.13	.18	.06	.09	.08	-.04	-.01
I	.65	1	.47	.17	.28	.86	.27	.25	.28	.25	.31	.19	.26	-.04	.04
Q	.60	.47	1	.14	.29	.44	.34	.21	.42	.17	.35	.03	.06	-.04	-.09
P	.13	.17	.14	1	.26	.14	-.03	.17	.07	.44	.04	.38	.30	.42	.54
L	.19	.28	.29	.26	1	.17	.52	.78	.58	.71	.55	.66	.73	.34	.43
H	.76	.86	.44	.14	.17	1	.13	.06	.19	.08	.13	.03	.09	-.12	-.06
F	.06	.27	.34	-.03	.52	.13	1	.73	.89	.31	.86	.31	.34	-.04	-.01
G	.10	.25	.21	.17	.78	.06	.73	1	.66	.75	.69	.78	.75	.38	.44
J	.13	.28	.42	.07	.58	.19	.89	.66	1	.34	.80	.30	.36	.02	.03
C	.18	.25	.17	.44	.71	.08	.31	.75	.34	1	.37	.94	.85	.55	.66
N	.06	.31	.35	.04	.55	.13	.86	.69	.80	.37	1	.35	.37	.06	.18
B	.09	.19	.03	.38	.66	.03	.31	.78	.30	.94	.35	1	.87	.56	.68
D	.08	.26	.06	.30	.73	.09	.34	.75	.36	.85	.37	.87	1	.47	.59
W	-.04	-.04	-.04	.42	.34	-.12	-.04	.38	.02	.55	.06	.56	.47	1	.81
X	-.01	.04	-.09	.54	.43	-.06	-.01	.44	.03	.66	.18	.68	.59	.81	1

Average .383289

STANDARD CROSS-CORRELATIONS @ 2 YEARS (5-point average, rounded)

	S	I	Q	P	L	H	F	G	J	C	N	B	D	W	X
S	1	.68	.64	.13	.18	.77	.22	.16	.28	.13	.17	.08	.13	-.21	-.02
I	.68	1	.61	.21	.38	.91	.35	.32	.39	.32	.37	.25	.32	-.10	.05
Q	.64	.61	1	.28	.36	.59	.33	.22	.46	.22	.34	.11	.20	-.09	-.01
P	.13	.21	.28	1	.30	.22	-.12	.12	-.02	.42	.03	.38	.33	.26	.44
L	.18	.38	.36	.30	1	.29	.64	.90	.69	.87	.65	.81	.85	.24	.40
H	.77	.91	.59	.22	.29	1	.26	.21	.34	.21	.25	.15	.20	-.20	-.04
F	.22	.35	.33	-.12	.64	.26	1	.81	.95	.39	.89	.34	.41	-.24	-.14
G	.16	.32	.22	.12	.90	.21	.81	1	.80	.82	.77	.79	.81	.16	.32
J	.28	.39	.46	-.02	.69	.34	.95	.80	1	.44	.88	.38	.45	-.19	-.08
C	.13	.32	.22	.42	.87	.21	.39	.82	.44	1	.44	.96	.93	.46	.64
N	.17	.37	.34	.03	.65	.25	.89	.77	.88	.44	1	.39	.43	-.02	.12
B	.08	.25	.11	.38	.81	.15	.34	.79	.38	.96	.39	1	.91	.45	.63
D	.13	.32	.20	.33	.85	.20	.41	.81	.45	.93	.43	.91	1	.38	.56
W	-.21	-.10	-.09	.26	.24	-.20	-.24	.16	-.19	.46	-.02	.45	.38	1	.84
X	-.02	.05	-.01	.44	.40	-.04	-.14	.32	-.08	.64	.12	.63	.56	.84	1

Average .410578

17 Assumptions Gross/Net of Tax

17.1 All results in previous sections are gross of (i.e. before) tax and imputation credits. The following results illustrate the impact of tax and imputation credits on the Section 16 assumptions for mean rates for superannuation in the accumulation stage.

Table 17.1 Gross/net of tax

Sector		Mean rate (arithmetic average) Before tax	After tax and imputation credits	Average tax rate	Compound Average rate After tax and imputation credits
S	Shares	10.00%	9.65%	3.5%	8.66%
I	Int'l Shrs	8.60%	7.83%	8.9%	7.02%
Q	Prop Trust	8.60%	7.69%	10.6%	6.67%
P	Direct Prop	7.50%	6.40%	14.7%	6.21%
H	Hedged IS	8.70%	7.92%	8.9%	7.09%
L	Loans/credit	7.20%	6.12%	15.0%	6.08%
F	Fixed Int	6.40%	5.44%	15.0%	5.34%
G	Semi-govt	6.40%	5.44%	15.0%	5.39%
J	Int'l Fxd Int	6.20%	5.27%	15.0%	5.19%
C	Cash	5.80%	4.93%	15.0%	4.90%
N	Infln Linked	7.00%	6.02%	14.0%	5.93%
Balncd	Balanced	8.03%	7.41%	7.8%	7.06%
CapStb	Cap Stable	6.81%	6.02%	11.5%	5.93%
B	Bills	5.80%	4.93%	15.0%	4.90%
D	Bonds	6.00%	5.10%	15.0%	5.08%

17.2 The above table allows for income tax at the 15% superannuation rate and, on an approximate basis, for imputation credits and the lower rates of tax on realised capital gains after 12 months.

18 Assumptions Gross/Net of Fees

18.1 All results in previous sections are gross of (i.e. before) fees. The following results illustrate the impact of wholesale passive investment fees on the Section 16 and 17 assumptions.

Table 18.1 Gross/net of fees

Sector		Mean rate (arithmetic average)			Compound Average rate
		Before tax Before fees	Before tax After fees	After tax & IC's After fees	After tax & IC's After fees
S	Shares	10.00%	9.74%	9.43%	8.44%
I	Int'l Shrs	8.60%	8.31%	7.59%	6.77%
Q	Prop Trust	8.60%	8.31%	7.44%	6.42%
P	Direct Prop	7.50%	6.80%	5.81%	5.62%
H	Hedged IS	8.70%	8.41%	7.68%	6.85%
L	Loans	7.20%	6.91%	5.87%	5.83%
F	Fixed Int	6.40%	6.22%	5.29%	5.18%
G	Semi-govt	6.40%	6.22%	5.29%	5.24%
J	Int'l Fxd Int	6.20%	6.02%	5.12%	5.04%
C	Cash	5.80%	5.65%	4.80%	4.77%
N	Infln Linked	7.00%	6.81%	5.86%	5.76%
Balncd	Balanced	8.03%	7.78%	7.19%	6.84%
CapStb	Cap Stable	6.81%	6.60%	5.85%	5.76%
B	Bills	5.80%	5.80%	4.93%	4.90%
D	Bonds	6.00%	6.00%	5.10%	5.08%

18.2 The "Balanced" portfolio compound average rate in the right hand column of the above table and the compound average rates for AWOTE and CPI in Table 16.1 are comparable with the actuarial projections for long-term investment returns, wage growth and growth in the Consumer Price Index (CPI) reported by superannuation funds and published in APRA (2009). For the three years to June 2006, June 2007 and June 2008 (which are the latest to be published), Figures E, G and J of the APRA Bulletin indicate the following averages:

206 defined benefit and hybrid plans	2006	2007	2008
Actuarial projection for investment return	7.0%	7.1%	6.7%
Actuarial projection for wage growth	4.5%	4.5%	4.6%
Actuarial projection for CPI growth	3.0%	3.1%	3.3%

174 non-public sector plans	2006	2007	2008
Actuarial projection for investment return	7.0%	7.1%	6.6%
Actuarial projection for wage growth	4.5%	4.4%	4.6%
Actuarial projection for CPI growth	3.1%	3.1%	3.3%

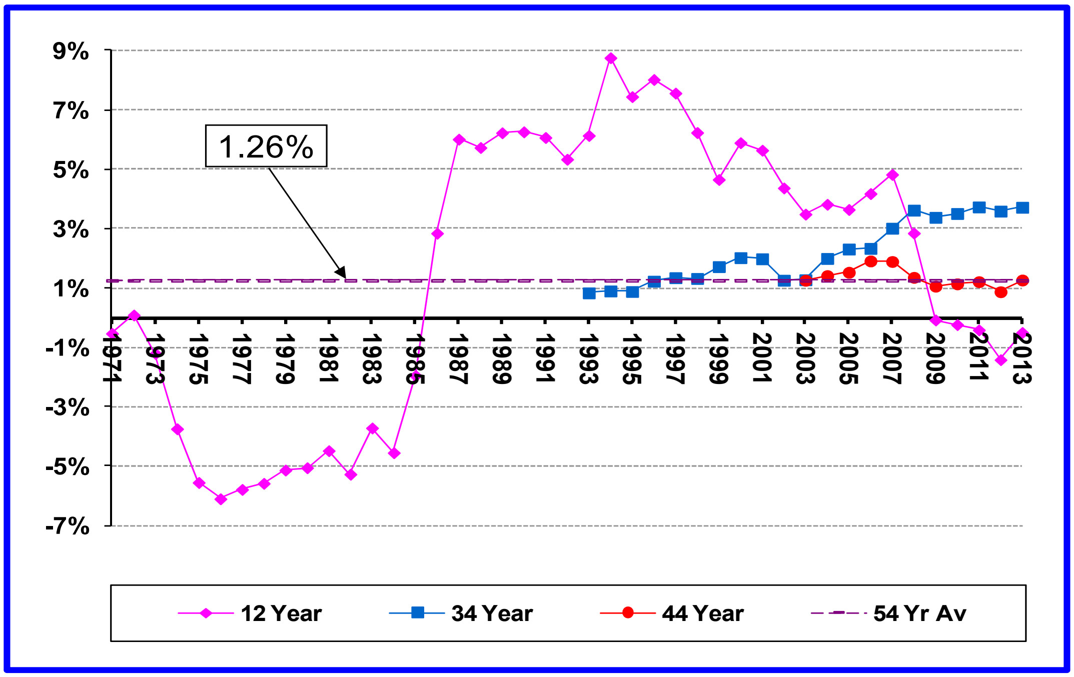
32 public sector plans	2006	2007	2008
Actuarial projection for investment return	7.2%	7.1%	7.1%
Actuarial projection for wage growth	4.6%	4.7%	4.9%
Actuarial projection for CPI growth	2.7%	2.9%	3.1%

19 Historical Returns Net of Tax and Fees

19.1 In this section all returns are net of tax and net of investment fees. The analyses allow for income tax at the current 15% superannuation accumulation stage rate and, on an approximate basis, for imputation credits and the lower rates of tax on realised capital gains after 12 months. The calculations assume that the current tax basis has applied at every year in the past, even though, in reality, tax on superannuation investment income did not commence until 1 July 1988.

19.2 Figure 19.1 shows compound average net annual real returns (over increases in AWOTE) over 12, 34, 44 and 54 years ending 30 June 2013 for the Balanced portfolio described in section 8.3. The average real annual returns over 12 and 34 years fluctuate between -6.1% and 8.8%. The average real annual returns over 44 years fluctuate within a narrow range between 0.9%% and 1.9%. The average real annual return over 54 years was 1.26%.

Figure 19.1 Balanced portfolio average net real returns



19.3 The 54 years of historical net returns can be used to illustrate, see Figure 19.2, the adequacy (and the inadequacy) of Australia's proposed 12% superannuation guarantee contribution rate – in a far more effective way than the hundreds of deterministic calculators and projections that appear on many Australian internet websites. Figure 19.2 is based on the section 8.3 Balanced portfolio, and the following assumptions:

(a) 12% SG contribution rate (with no further member or employer contributions),

(b) administration fees and costs and group life premiums aggregating 1.5% salary,

(c) 15% contribution tax (hence the net amount accumulating towards retirement is (12% less 1.5%) times 0.85 = 8.925% of salary),

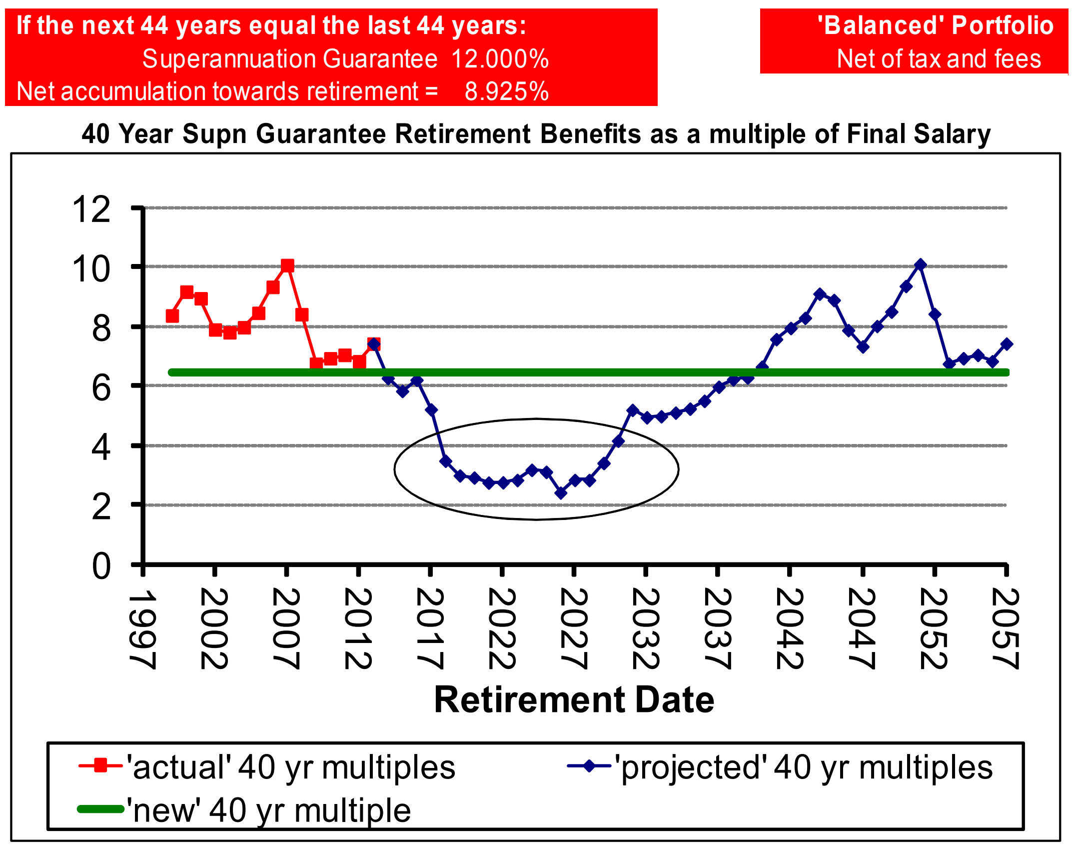
(d) salary increases equal to increases in AWOTE, and

(e) investment returns and AWOTE increases for each of the 44 years commencing 30 June 2013 at the same rates as for the 44 years ending 30 June 2013.

19.4 Each of the benefit multiples is the result of the accumulation of 40 years of net contributions and the accumulated benefit is then expressed as a multiple of final salary at retirement. The results illustrate how lump sum retirement benefits can fluctuate depending on the chosen retirement date. This is a consequence of a **fixed input** (based on 12% of salary) producing a **variable output**. Less variation would result if income benefits were illustrated and less still if those income benefits allowed for the means-tested Age Pension.

19.5 Figure 19.2 also shows, as a solid green line, the 40-year multiple that would result from using the 'new' Balanced portfolio and AWOTE assumptions, instead of those in sub-paragraph 19.3(e) above. The resultant benefit multiple is 6.45. The investment return and salary increases assumed are 6.84% and 4.16% per annum (compound) respectively (which imply a 40-year real annual return of 2.57%).

Figure 19.2 Superannuation 40-year benefit multiples



19.6 The thirteen very low multiples (encircled above) for retirement dates from 2018 to 2030 inclusive are primarily due to the very high salary increases assumed to occur between these two dates. This period corresponds (on the assumptions used) to the period from 30 June 1973 to 30 June 1986 when AWOTE increased by an average of 12.1% per annum compound. Disregarding these thirteen years, the 40-year multiples fluctuate between about 5 and 10.0.

20 Concluding Remarks

20.1 An objective of this paper was to help bridge the gap between the demand from actuaries (and consumers) for robust assumptions in respect of future investment returns across a broad range of investment sectors, and the limited supply of data readily available for a range of investment sectors.

20.2 The advent of unit price data in Australia 48 years ago, through National Mutual's "EFG" system, supplemented with other published indices and rates, provided a substantial database of investment performance statistics, across a broad range of investment sectors.

20.3 The paper developed and summarized assumption sets for medium to long-term use in stochastic (and deterministic) investment models, across the range of investment sectors and economic indicators that were examined. In determining those assumptions, the paper has illustrated techniques for 'backdating' incomplete data series for use, particularly, with cross-correlation and auto-correlation analyses.

20.4 In addition, the paper adopted the technique used by Dwonczyk in fitting sine waves to CPI data, and applied an extended version of this technique to 10-year bonds and to CPI, AWOTE and Australian shares. This analysis appears to provide support for the existence of 'long-term' economic cycles extending over 40.25 to 49 years (but shorter and fluctuating for Australian shares). A similar cycle length (41.0 years) is found in section 9.10 – this time using a different analysis of Balanced portfolio log "discounted" unit prices and measuring the period between trend line maximums. Economic cycles and auto-correlations appear to be areas that warrant further research.

20.5 The paper emphasises that where-ever possible it is desirable for the setting of long-term assumptions to analyse results over at least one full economic cycle - and explains why 44-year periods are considered suitable for this purpose. Other observations include:

(a) care is needed when using running averages because their trends are impacted by both new data being introduced each year and by the old data dropping off (sections 2.7 and 2.9),

(b) the impact of the global financial crisis was not an isolated event (section 9.5),

(c) the number of skewness and kurtosis assumptions that fall outside a minimal classification sounds a warning for those who use normal or lognormal models in relation to investment performance (section 12.4),

(d) there is a slight negative bias in Balanced portfolio auto-correlations up to a lag of 5.5 years, which illustrates the tendency for market prices to "bounce back" after falls and/or to "drop again" after price increases - somewhat similar, but different, to mean reversion (section 15.3),

(e) for Australian shares and 10-year bond rates, 26-year average auto-correlations ending 1986/87 were similar, after re-scaling the X-axis, to those ending 2012/13 (section 15.8),

(f) historical net returns can be used to illustrate the adequacy (and the inadequacy) of Australia's proposed 12% superannuation guarantee contribution rate – in a far more effective way than the hundreds of deterministic calculators and projections that appear on many Australian internet websites (section 19.3), and

(g) as a consequence of fluctuating investment performance and salary increases, a **fixed input** (such as 12% of salary) will produce a **variable output**, with significant variation in lump sum retirements benefits depending on your chosen retirement date (section 19.4).

20.6 Significant differences in the assumptions now recommended, as compared to those recommended 4 years ago, include:

		Now Recommended	**Previously Recommended**
Risk Margin	Sector S	4.0%	4.5%
	I	2.6%	3.0%
	Q	2.6%	3.0%
	Balanced	2.03%	2.36%
Coefficient of Variation	Sector Q	1.721	1.533
Standard Deviation	Sector Q	14.8%	13.8%
	I	14.8%	13.8%
Skewness	Sector Q	-237%	-179%
	Cap Stable	-54%	-72%
	Inflation	77%	59%
Kurtosis	Sector Q	963%	683%
	W	393%	316%
Rank Cross-corr.	Sectors W & P	42%	47%
	S & P	13%	18%
	Q & P	14%	19%
Auto-correlation	S @ lag 6 yrs	18%	4%
	D @ lag 14 yrs	15%	-43%

Often there were no or insignificant changes. For example, the number of assumptions for which the absolute change was zero or between -0.5% and 0.5% inclusive (for means and standard deviations) or between -5% and 5% inclusive (for skewness, kurtosis and cross-correlations) was:

Now: arithmetic mean 14 (of 14), standard deviation 12 (of 14),
skewness 8 (of 14), kurtosis 1 (of 14), cross-correlations 82 (of 91)

Previously: arithmetic mean 13 (of 15), standard deviation 13 (of 15),
skewness 9 (of 15), kurtosis 2 (of 15), cross-correlations 71 (of 105).

21 Acknowledgments

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- The designers, in 1965, of the National Mutual EFG investment system.

- The Institute of Actuaries of Australia for their financial support via the Australian Actuarial Research Grant Program.

21.2 I am also indebted to Jeremy Waite for his article "ERM – Black Swans, Fat Tails and Spherical Cows" which was published in the April 2009 edition of *Actuary Australia*. Extracts from the article appear in various sections of this paper (in *italics*, within a border and under the heading "REMEMBER").

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Appendix A: Modelling Skewness and Kurtosis

A1 This appendix describes a method for adding skewness (denoted g) and kurtosis (denoted k) to a 40-year stochastic projection.

A2 Let x represent a uniform random variable (or "step") with the values 0.5, 1.5, 2.5, 3.5 … 39.5.

A3 Let z represent a standardised normal random variable. For example, using an EXCEL function, z = NORMSINV(x/40).

A4 Applying the Normal Power Approximation the variable y given by:

y = z + g*(z^2-1)/6 + k*(z^3-3*z)/24 – (g^2)*(2*z^3-5*z)/36 [1]

will be found over the 40 steps from 0.5 to 39.5 inclusive to have a mean of approximately 0, standard deviation of approximately 1, skewness of approximately g and kurtosis of approximately k. By trial-and-error (or by using EXCEL Solver) it is possible to change the value of g to *g* and the value of k to *k* so that the variable *y* (note *italics*) given by:

y = z + g*(z^2-1)/6 + k*(z^3-3*z)/24 – (g^2)*(2*z^3-5*z)/36 [2]

has, over the 40 steps from 0.5 to 39.5, skewness of exactly g and kurtosis of exactly k. When the variable *y* is standardised it will still have skewness of g and kurtosis of k.

A5 The Normal Power Approximation, applied as described above works well for those sectors with negative skewness and positive kurtosis and for those with low skewness and kurtosis. This group includes all sectors **where modest to significant negative returns are observed.** The forces for this group have positive kurtosis and include:

S, I, Q, P, H, F, J and N [section 3.1 explains the sector codes]

A6 However the above method is unsatisfactory (because the resultant variable does not increase for all steps from 0 to 40) for those sectors **where negative returns are not observed or where only occasional small negative returns are observed.** The forces for this group have negative kurtosis and include:

L, C, B, D and X [section 3.1 explains the sector codes]

A7 For this latter group it was found that instead of [1] above the following gamma exponential variable, y, expressed in EXCEL functions, could be used:

y = EXP(1-GAMMAINV(1-x/40,alpha,beta)) [3]

A8 Again, by trial-and-error (or by using EXCEL Solver) it is possible to change the value of alpha to *alpha* and the value of beta to *beta* so that the variable *y* given by:

y = EXP(1-GAMMAINV(1-x/40,*alpha*,*beta*)) [4] has, over the 40 steps from 0.5 to 39.5, skewness of exactly g and kurtosis of exactly k. When the variable y is standardised it will still have skewness of g and kurtosis of k. The resultant variable also increases for all steps from 0 to 40.

A9 Sector W (AWOTE) does not fit naturally into either of the two groups described on the previous page. It has high positive skewness and high positive kurtosis and (depending on the coefficient of variation assumed) can be expected to have occasional modest negative values. Interestingly, it was found that relationships [2] and [4] both produce satisfactory and almost identical values of y for this sector (except at the extreme tails). Relationship [2] was favoured because the relationship [4] left hand tail was very flat.

A10 Sector G (Government semis 0-3 years) also does not fit naturally into either of the two groups described on the previous page. For this Sector it was found that relationship [2] produced satisfactory values of y for the April 2005 calculations but not for the July 2007, the July 2009 (i.e. "OLD") nor for the July 2013 (i.e. "NEW") calculations. For the latter three calculations, relationship [4] produced satisfactory values of y.

A11 The process described above produces standardised variables for all the 15 sectors with the desired skewness and kurtosis. However this alone does not mean that the entire **shape of the distribution** of returns is realistic. To test this, the following analysis was performed:

(a) Forty years of forces for each sector ending 30/9/12, 31/12/12, 31/3/13 and 30/6/13 were each **sorted and standardised** – giving four datasets.

(b) The skewness and kurtosis of each sector and dataset was calculated.

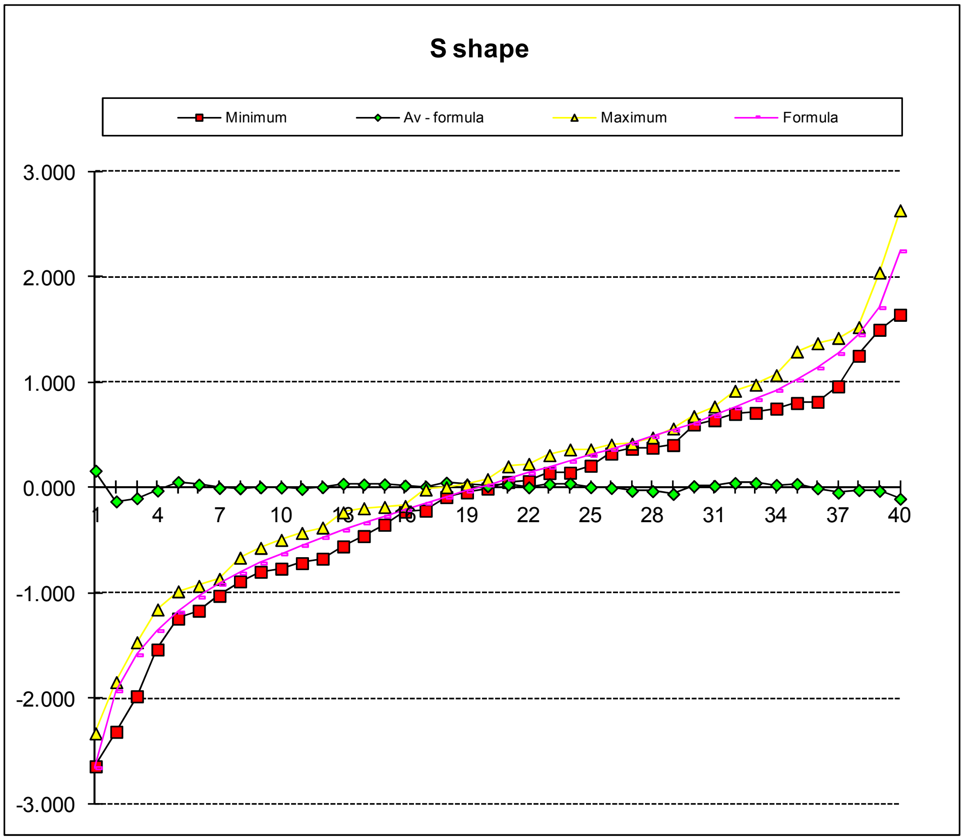
(c) The average skewness and kurtosis of each sector was calculated.

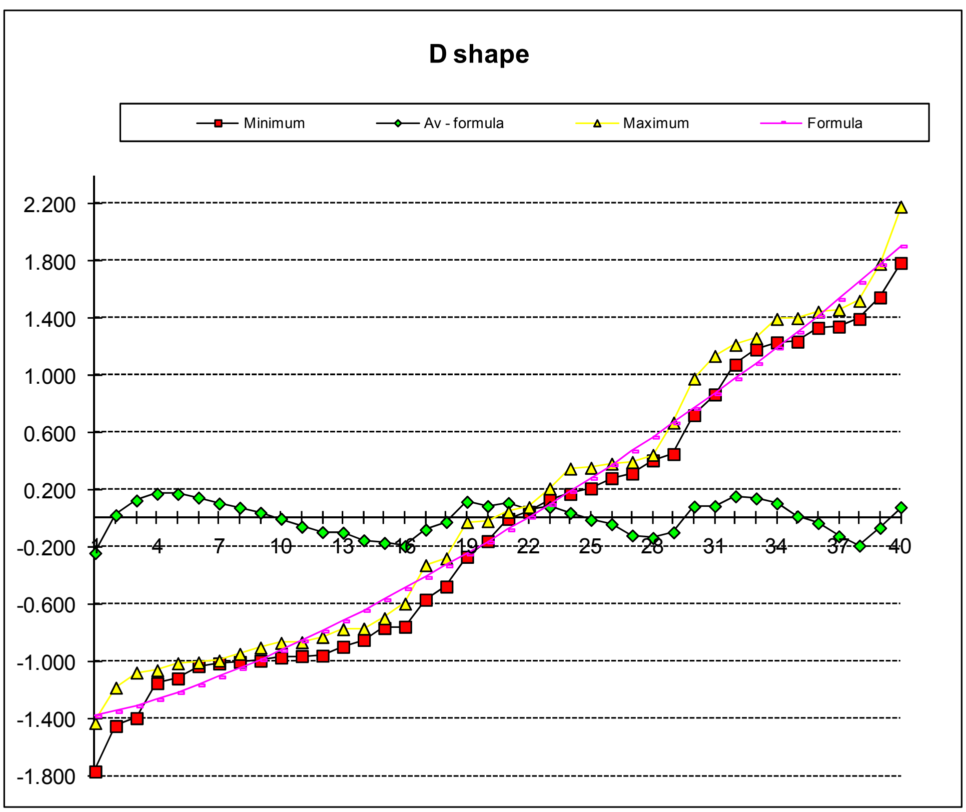
(d) Relationships [2] and [4] were applied based on the average skewness and kurtosis of each sector from (c) above, **not** based on long-term estimates of g and k.

(e) The resultant variables from (d) above (after standardisation) were compared with the four datasets from (a) above. The comparison focused, in particular, on whether the tail values for the resultant variables were:

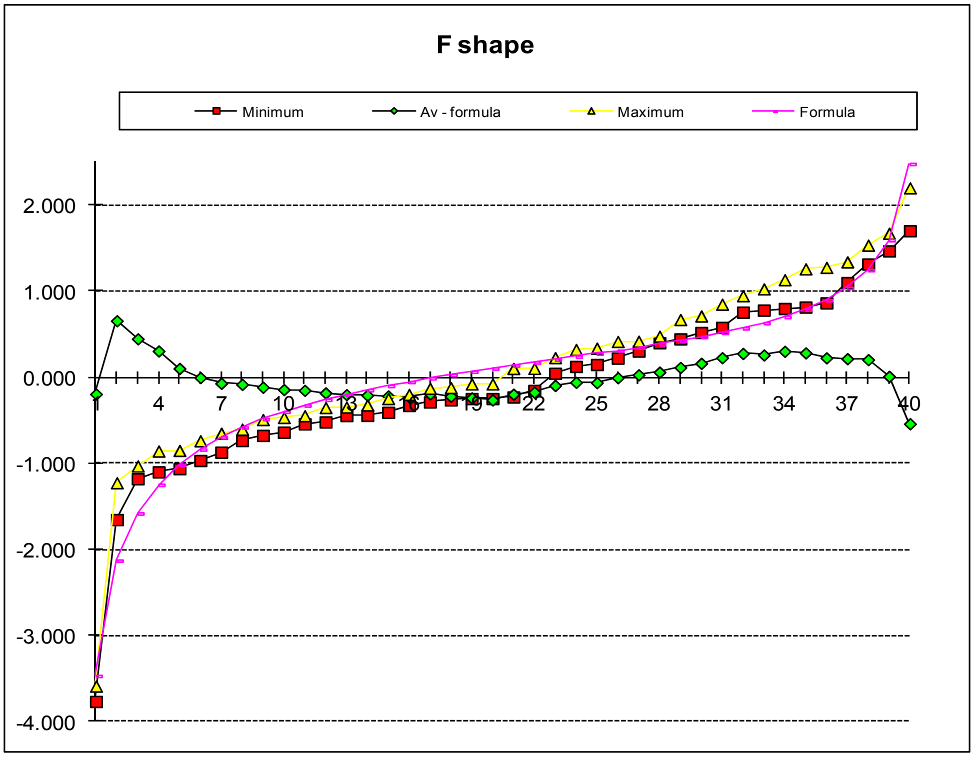
- greater than the minimum of the tail values from the four datasets in (a) above, and
- less than the maximum of the tail values from the four datasets in (a) above.

(f) Two examples of where (e) was satisfied are shown in the following charts, one for the S (Australian shares) sector based on relationship [2], and one for the D (10-year bonds) sector based on relationship [4]. Each chart contains the maximum and minimum sorted standardised forces from the four datasets and also includes a graph (denoted "Av – formula") of the difference between the average of the four datasets and the formula result. A very close study of the chart for the D sector will show that the relationship [4] formula values marginally exceeded the maximum extreme bottom left hand tail value, but they were judged as satisfactorily capturing the bottom left hand tail values when considered as a group.


S shape
Minimum
Av - formula
Maximum
Formula
3.000
2.000
1.000
0.000
-1.000
-2.000
-3.000
1
4
7
10
13
16
19
22
25
28
31
34
37
40


D shape
Minimum
Av - formula
Maximum
Formula
2.200
1.800
1.400
1.000
0.600
0.200
-0.200
-0.600
-1.000
-1.400
-1.800
1
4
7
10
13
16
19
22
25
28
31
34
37
40

(g) An example of where (e) was not satisfied is shown in the following F (Australian fixed interest) sector chart.



A12 The chart in (g) above shows that relationship [2] has satisfactorily captured the bottom left hand tail value for the F sector. This same outlier is referred to in section 10.2. It relates to the year ending 30 September 1973. However relationship [2] has not satisfactorily captured the top right hand tail value for this sector since the formula result at point 40 exceeds the maximum. Because of the "backdating" formulae in Section 4, a similar problem occurred for the L, G, J and N sectors.

A13 To rectify the problem identified in the previous paragraph, it was found that an effective pragmatic solution was to reduce **both** the skewness and kurtosis for the Q, L, F, G, J, C, N and B sectors by 0.11, 0.26, 0.27, 0.20, 0.26, 0.10, 0.14 and 0.15 respectively. With these reductions, relationships [2] and [4] satisfactorily captured both the bottom left hand and top right hand tail values for the eight sectors and the shape for the non-tail values was insignificantly changed.

A14 The skewness and kurtosis assumptions tabulated in Table 16.1 are **gross** of (i.e. before) the reductions in the previous paragraph.

A15 Relationship [4] has been shown to provide a good fit to the observed data for some sectors. However care needs to be taken when applying these observations to asset risk management when the focus is on left hand tail distributions. In the last 54 years there has not been in Australia an occasion where a significant level of default on bonds, loans or mortgages has occurred. It may be fallacious to assume that such an event will not occur in the future. Yet this is a (well-hidden) assumption within relationship [4].

Appendix B: Modelling Auto-correlations

B1 An examination of Figure 15.1 shows that auto-correlations are of three broad types:

(a) Share sectors, Property Trusts and the "Balanced" portfolio

(b) Fixed interest sector, Direct Property and CPI (and AWOTE)

(c) Bills and Bonds.

B2 Also of interest is the similarity between the auto-correlations of the S (shares) sector and that of the "Balanced" portfolio. This is primarily due to the greater volatility of the S sector, which dominates the shape of the "Balanced" portfolio auto-correlations. Though not included in Figure 15.1, the auto-correlations of a "Capital Stable" portfolio are quite different from those for the "Balanced" portfolio. The "Capital Stable" portfolio auto-correlations are similar to those for the F sector.

B3 A closer examination of Figure 15.1 reveals that there is a gradual progression from the **S sector**, to Property Trusts, to the "Balanced" portfolio, to the F sector, to Direct Property, to CPI, to Bills and then to **Bonds**.

B4 **The auto-correlations of the S sector and Bonds form two extremes**. This hypothesis is supported by the following analysis of the auto-correlations underlying Figure 15.1:

i) for lags of 1 to 1.75 and 2.5 years, the S sector has the lowest auto-correlations of all the 15 sectors. In stochastic modelling the shorter term lags are most important because there are far more observations at the short end than at the long end. Further, the longer results are to some extent repeats of the shorter results.

ii) for lags of 1 to 5 years, the D sector (i.e. Bonds) has the highest auto-correlations of all the 15 sectors.

iii) for lags of 9.25 to 9.5 years, 14.75 and 35.25 years the S sector again has the lowest auto-correlations of all the 15 sectors.

iv) for lags of 20.5 to 20.75 years, 21.75 to 22 years, 23.25 years, 28.5 to 32.25 years and 35.5 to 37.5 years the D sector has the lowest auto-correlations of all the 15 sectors.

B5 Intuitively, the S sector is significant because of its higher volatility and expected investment returns. In addition, the D sector is significant because of its influence on current market investment yields.

B6 The above analysis forms the background as to why the *Austmod* stochastic investment simulation model focuses on the S and D sector autocorrelations.

B7 Briefly, the *Austmod* model sets D sector auto-correlations by sorting results into 40-year "bumpy" cycles (but note paragraph B12 on the next page) as follows:

- subdividing (126 times) D sector zero auto-correlated stochastic results for 20 years out of 120 into six groups of 20 years,
- sorting each group of 20 year D sector results,
- 63 times, positioning each D sector result, first of first 20 to first of 40, second of second 20 to second of 40, third of second 20 to third of 40, fourth of first 20 to fourth of 40, etc., and

- 63 times, positioning each D sector result, first of first 20 to first of 40, second of second 20 to second of 40, third of first 20 to third of 40, fourth of second 20 to fourth of 40, etc.,
- retaining each result for the 14 other sectors with their original D sector value (thus leaving cross-correlations undisturbed).

B8 Briefly, the *Austmod* model then sets S sector autocorrelations by discarding all but one of the above D sector combinations, and retaining the D sector combination which is associated with the S sector auto-correlation closest to that desired. The method of least squares is used to determine the closest S sector auto-correlation, with the auto-correlations weighted towards the shorter lags (which have the greatest number of observations). The weight used is the square of (39 less the lag in years).

B9 Having obtained the S and D sector auto-correlations as above the (retained) cross-correlations carry through these auto-correlation distributions to the other sectors. The Cholesky decomposition formula is used to create the cross-correlation distributions.

B10 Without lagging CPI (X) and AWOTE (W) values, the process described in the previous paragraph works reasonably well for all sectors except X and W, where only about one-third of the desired auto-correlation distributions was obtained for X and significantly less than this for W. In view of the D and X (0.59) and D and W (0.47) rank cross-correlations in Figure 16.3, this result is not surprising.

B11 The higher D and X-8 (0.66) and D and W-8 (0.72) cross-correlations in Figure 14.3 have been found to produce much better results if lagged X and W forces are used throughout the modelling. By increasing these two lagged cross-correlations by a further 0.12 (beyond which the Cholesky decomposition fails) it was found that about 75% of the desired auto-correlation distributions were obtained for both X and W.

B12 The results in Table 16.2 indicate that the D sector auto-correlation is zero at a lag of about 15.29 years whereas model results currently indicate a D sector auto-correlation of zero at a lag of about 10.21 years. This analysis indicates that the 40-year cycle referred to on the previous page should be increased by about 4 times 5.08 or say 20 years (to 60 years). However a study of the blue-shaded area in Figure 15.6 indicates that the D sector autocorrelations between say lags 15.5 years and 31.5 years are oscillating between about -0.10 and -0.40. Thus to produce the D sector auto-correlation assumptions in Table 16.2 will require cycles more complex than a simple 60 year "bumpy" cycle.

B13 For further information about the appendices, the historical database or *Austmod*, the author can be contacted at colin.grenfell@supereasy.com.au or colnbarb@hotmail.com.

Appendix C: The *Austmod* Investment Simulation Model

C1 *Austmod* is a stochastic and historical investment simulation model. The model is an EXCEL workbook that displays up to 54 years of historical and 40 years of simulated investment performance for the 15 "sectors" described in Section 3.

C2. This appendix describes the input, algorithms and output for the latest versions of the model (version V based on the "OLD" assumptions and version W based on the "NEW" assumptions).

C3 The 26 inputs to the model are:

Which	What	Input
TAX	**0** = Nil **1** = Ordinary **2** = Superannuation **3** = Exempt **4** = other	[1]
MODEL	Type : 0 = stochastic 1 = historical random start date	[2]
	Start seed : Enter a number from 1 to 8000 (or enter R for random)	[3]
PPNS	**Balanced >>** 36% Shares	
	19% Int' Shares (uh)	
	Usually — **Composite** Press — 7% Property Trust	
	enter 0% : — portfolio F9 if — 2% Property Direct	
	(enter 0, 1 or 2) : 'manual' — 0% Loans/credit	
	Bill Rate — [5] — 6% Int'l Shares (h)	[6]
	Bond Rate — [4] — **0** = Input primes — 16% Fixed Interest	
	AWOTE — **1** = Capital stable — 0% Semi Government	
	CPI — **2** = Balanced — 7% International Bonds	
	3 = Growth — 5% Cash	
	2% Inflation Linked	
PLAN	Assets at start $	[7]
	Liabilities (or accumulated retirement benefits) at start $	[8]
	Annual net Enter the net cash flow per annum at start (as a number)	[9]
	cash flow : ~ varies W = ~ AWOTE X = ~ CPI $ = dollar P = % assets	[10]
	Driver: A=3yr geo av B=bills C=cash D=bonds E=earnings W=AWOTE	[11]
	Crediting rate formula: 1 = standard 2 = driver 3 = other	[12]
INVEST	**Means:** 1 = based on standard 2 = other	[13]
	Additional return: (all investment sectors, input additional rate before tax)	[14]
	Net of fees: 0 = no 1 = standard 2 = other	[15]
	Standard deviations: 0 = nil 1 = standard 2 = other	[16]
	Skew & kurtosis: 0 = nil 1 = std (skew only) 2 = std (both)	[17]
	Cross correlations: 0 = nil 1 = standard 2 = std+WX 8 yr lags 3 = other	[18]
	Auto correlations: 0 = nil 1 = C cycles 2 = S+D trends	[19]
	Seasons start: Enter number of years since bottom (0=bot, 20=top, R=random)	[20]
	Historical results: Enter number of years (between 1 and 54)	[21]
	Historical results ending: Enter 31/3, 30/6, 30/9 or 31/12 dates only	[22]
INITIAL	Fund earning rate in the year prior [22] (= assumed start date)	[23]
	Fund earning rate in 2nd year prior [22] (needed for driver)	[24]
	Nominal bill rate at start (if Driver = B and end or start year based)	[25]
	Nominal bond rate at start (if Driver = D and end or start year based)	[26]

C4 Inputs [1] to [26] are briefly explained on the next three pages.

TAX (Taxation)

[1] Enter 0 for no taxation and no imputation credits.
Enter 1 for life company "Ordinary" taxation.
Enter 2 for superannuation tax (in the accumulation stage).
Enter 3 for exempt from taxation but with imputation credits (for example, pensions and annuities).
Enter 4 for another taxation basis (the "watch" column, not illustrated above, will then indicate where the new taxation basis is entered).

MODEL (Modelling basis)

[2] Enter 0 for stochastic modelling.
Enter 1 for "historical random start" modelling. With this basis, historical data for the period specified in [21] and [22] on pages 80 and 83 will be stacked end-on-end to form a recurrent sequence and then 1, 8, 40, 200 or 1000 40-year scenarios will be reported from this sequence, each based on a random start date. This is a form of bootstrapping – the following extract from Sweeting (2007) explains:

"In stochastic modelling the first distinction is between bootstrapping and forward-looking approaches. For bootstrapping, all that is needed is a set of historical data for the asset classes being modelled. For example, monthly market returns for the last 20 years could be used. However rather than simply using this as a single 'run' of data, modelling is carried out by selecting a slice of data randomly, in this case the results from a particular month. This forms the first observation. This observation is then 'replaced' and another month is chosen randomly. This means that a relatively small data set can be used to generate a large number of random observations.

The main advantage of bootstrapping is that the underlying characteristics of the data and linkages between data series are captured without having to resort to parameterisation."

Unlike bootstrapping, "historical random start" modelling captures most of the auto-correlation of the historical data as well as most of the cross-correlation. For the chosen period (specified in [21] and [22]) and for all sectors, all of the historical auto-correlation and cross-correlation is captured except for when one sequence stops and another starts.

[3] Enter a "seed" number between 1 and 8000 or enter "R" for a random series.

PPNS (Proportions)

[4] For bills, bonds, AWOTE and CPI the default 0% proportion will normally apply since it is not possible to "invest" in these sectors. However for model-testing it may be desired to enter 100% in one of these cells.

[5] Enter 0 if proportions for "prime" investment sectors are being entered in [6] below.
Enter 1 if the default "Capital Stable" portfolio proportions are to be used.
Enter 2 if the default "Balanced" proportions (illustrated in C3 above) are to be used.
Enter 3 if the default "Growth" portfolio proportions are to be used.

[6] If the default composite portfolio proportions are not being used, enter the desired proportions for each sector. These proportions must total 100%.

PLAN (Plan details]

[7] Enter the market value of assets at the projection start date (i.e. at the date [22]).
[8] Enter the liabilities or the accumulated retirement benefits at the projection start date.

[9] Enter the annual net cash flow at the projection start date.

[10] Enter “W” if the annual net cash flow varies proportional to AWOTE changes.
Enter “X” if the annual net cash flow varies proportional to CPI changes.
Enter “$” if the annual net cash flow is a constant dollar amount.
Enter “P” if the annual net cash flow is a proportion of plan assets (the “watch” column, not illustrated, will then indicate where the required proportion/s is/are entered).

[11] The “driver” is a component of the crediting rate basis or of the growth in liabilities.
Enter “A” if the driver is the geometric 3 year average of past plan investment returns.
Enter “B” if the driver is based on the Bill rate.
Enter “C” if the driver is based on the Cash investment return.
Enter “D” if the driver is based on the Bond rate.
Enter “E” if the driver is the plan investment return (earnings).
Enter “W” if the driver is based on changes in AWOTE.

[12] Enter 1 if the crediting rate formula is “standard” (this is unlikely to apply but a standard formula is built into the model which can be used, varied or ignored).
Enter 2 if the crediting rate equals the driver.
Enter 3 if neither 1 nor 2 applies (the “watch” column, not illustrated, will then indicate where the “other” basis is entered – similarly, when “other” is chosen for [13], [15], 16] or [18] below, the “watch” column will indicate where the basis is entered).

INVEST (Investment assumptions)

[13] Enter 1 if the mean is “standard” (see Table 16.1).
Enter 2 if the assumption for the mean is “other” (note the “watch” column).

[14] Enter any “additional return” (before tax) to be added to [13] above (for all investment sectors).

[15] Enter 0 if investment returns are to be before investment fees.
Enter 1 if investment returns are to be net of “standard” investment fees.
Enter 2 if investment returns are to be net of “other” investment fees.

[16] Enter 0 if the standard deviation of all returns is zero (in which event all projections will be “deterministic”).
Enter 1 if the standard deviation is “standard” (see Table 16.1).
Enter 2 if the standard deviation is “other”.

[17] Enter 0 if the skewness and kurtosis of all returns is zero (in which event all projections will be based on log-normal distributions).

Enter 1 if L, C, B, D, W and X sector returns (only) are to be skewed on a "standard" basis so as to avoid negative returns (or significantly negative returns for W and X sectors) and for the 8 other sectors log-normal distributions are required.
Enter 2 if the skewness and kurtosis of all returns is "standard" (see Table 16.1 and Appendix A).

[18] Enter 0 if the cross-correlations of returns are zero (i.e. independent).
Enter 1 if the cross-correlations of returns are "standard" (see Table 16.3 and section 13.4).
Enter 2 if the cross-correlations of returns are "standard" with an 8-year lag for W and X sectors (see Table 14.3 and Section 14).
Enter 3 if the cross-correlations of returns are "other".

[19] Enter 0 if the auto-correlations of returns are zero (i.e. serially independent).
Enter 1 if the auto-correlations of C sector returns are based on a fixed 4-year cycle.
Enter 2 if the auto-correlations of S and D sector returns are based on the method explained in Appendix B.

[20] When 1 or 2 is entered in [19] above, the returns for the C or D sector will be based on cycles (also referred to as "seasons"). The code entered in [20] then determines when these cycles start. If 1 is entered in [19] then:

Enter D if the cycle starts with a downward movement (half-way from top to bottom).
Enter B if the cycle starts at the bottom.
Enter U if the cycle starts with an upward movement (half-way from bottom to top).
Enter T if the cycle starts at the top.
Enter R if the cycle starts from a random position.

Alternatively, if 2 is entered in [19] then:
Enter 0 if the cycle starts at the bottom, or otherwise
Enter the number of years from the bottom to the desired start, or

Enter R if the cycle starts from a random position.

[21] Enter the number of years (between 1 and 54) over which historical returns are required.

[22] Enter the end-date (31 March, 30 June, 30 September or 31 December dates only) for historical returns. This date is also the start date for projections.

INITIAL (Initial information)

If further information is unnecessary, the description for [23] to [26] will be blank and any input will be irrelevant (as explained, but not illustrated, in the "watch" column). In most situations the input in [11] will **not** require further "INITIAL" information.

[23] Enter the fund earning rate in the year prior [22].
[24] Enter the fund earning rate in the year prior to the year prior [22].
[25] Enter the nominal Bill rate at the date in [22].
[26] Enter the nominal Bond rate at the date in [22].

C5 The model has an annual time scale. The mathematical structure of the underlying *AustmodW* algorithms is summarized below:

Normal. First, the model generates independent Normal random variables for each sector.

Cross-correlation. These random variables are then converted to dependent Normal random variables using the *Cholesky* decomposition formula (refer Wilkie A D, 1988, JIA 115. Part 1, page 51).

Skewness and kurtosis. These random variables may then be converted to dependent non-Normal random variables using formula [2] or [4] as described in Appendix A.

Shape. For sectors Q, L, F, G, J, C, N and B the shape of the distribution may then be improved by slightly reducing both the skewness and kurtosis - refer Appendix A, paragraph A13.

Taxation. The input for each sector includes two tax rates and information for imputation credits. The *income tax rate* is applied to the long term expected income yield and imputation credits. The *deferred tax rate* is applied to the total before tax yearly return less the long term expected income yield. This is equivalent to assuming, for tax purposes, that all fluctuations in investment returns are due to fluctuations in the capital appreciation component. The *after tax standard deviation* for each sector equals the *before tax standard deviation * (1 - deferred tax rate)*.

Additional return. The before tax standard investment return for each investment sector may be increased (or decreased if negative) by adding an *additional return*. For tax purposes the *additional return* is treated as a capital appreciation component and taxed at the *deferred tax rate*.

Investment fees. The *investment fee* for each sector is deducted from the before tax long term expected income yield; the result is then taxed at the *income tax rate*.

Forces. After allowing for taxation and any additional return and investment fees, the standardized random variables (denoted *srv*), are then converted to annual forces using the formula *force = mu + sigma*srv*.

Mixture. The annual force for the mixture or portfolio is then determined by weighting the sector forces by the proportions for each sector. The proportions may be specified individually, or default proportions for “Growth”, “Balanced” or “Capital Stable” portfolios may be used.

Rates. The annual forces are then converted to rates using the formula
rate = EXP(force) - 1.

Repeats. The above 10 steps are repeated 120 times to give a 120-year single scenario with no auto-correlation.

Auto-correlation. A 40-year scenario with auto-correlation may then be generated using the methodology described in Appendix B, paragraphs B7 and B8.

Lags. CPI and AWOTE may then be lagged (refer Section 14 and Appendix B paragraphs B10 and B11).

Refinement. The CPI and AWOTE auto-correlations may be further improved by increasing their cross-correlation with the D sector by 0.12 (refer paragraph B11).

C6 The output for *Austmod* includes the following:

Investment Output	Historical and simulated annual returns (and their ranks) for the 15 sectors and the mixture or portfolio
Investment Return Charts	Historical and simulated annual returns and auto-correlation charts
Plan Output	Output for a superannuation or other plan, product or member showing: (1) accumulated retirement benefit at end of each year, (2) "real" accumulated retmt. ben. at end of each year, (3) salary-deflated accumulated retirement benefits, (4) accumulated retmt. ben. as a multiple of salary, (5) equivalent retirement benefit scale, (6) decumulation account balance at end of each year, (7) salary-deflated decumulation account balances, or (8) reserve ratios (F-A)/A at end of each year
Plan Charts	Various charts of the plan output and distributions
Crediting Rate Charts	Charts of simulated crediting rates and investment returns
Research Output	Output for a superannuation or other plan, product or member showing: (1) mixture annual returns each year, (2) L, C, B, D, W, and X minimums and maximums, (3) S sector force auto-correlations for lags 1 to 38 years, (4) D sector force auto-correlations for lags 1 to 38 years, (5) force cross-correlations (samples – 1971/2009 trends), (6) decumulation annual drawdowns, (7) salary-deflated decumulation annual drawdowns, or (8) LN fund ratios LN (F/A) at end of each year
Research Chart	Chart of research output
Skewness and Kurtosis	Worksheet of Appendix A, paragraph A1 to A10 calculations
Assumptions and Analysis	Comparisons of mean, standard deviation, skewness, kurtosis, rank cross-correlation and auto-correlation assumptions with the average simulated results for each sector and mixture across 8, 40, 200 or 1000 simulations.
Sector Long Term Estimates	"Ordinary" (for Life Insurance) and "Superannuation" long term estimates and taxation calculations
Actual Forces (Gross of Tax)	Historical annual forces at quarterly intervals from 30/6/1959 to 30/6/2013 and auto-correlations
Macros	Eight macros for simulations, correlations, "season" calculations, menu, plan and research output options, random number and random seed calculations (seeded minimum/maximum event probabilities are one in 3000)

C7 For further information about the *Austmod* investment simulation model, see www.supereasy.com.au/welcome.asp?pg=investmentsimulation or www.supereasy.biz/investment_simulator/investment_simulator.htm

REMEMBER

> *"**There is a great deal of belief in mathematical models – though some models are useful, all models are wrong.***
>
> *This is not to say models aren't helpful, but that they need to be combined with experience, business acumen and judgement, and ... used properly. It is important to know what is not included. They also have a strong part to play in 'what-if' analysis. ...*
>
> *Models have a useful part to play and clear communication of the parts of the risk that are not modelled is crucial information in order to be able to use the models as a useful input to making strategic decisions. ...*
>
> *The communication skills of the actuarial profession have never been more needed."*
>
> *Waite (2009)*