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Institut für Finanz- und  
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# Participating Life Insurance Contracts under Risk Based Solvency Frameworks

How to increase Capital Efficiency by Product Design

Andreas Reuß

Institute for Finance and Actuarial Sciences (ifa)

Jochen Ruß

Institute for Finance and Actuarial Sciences (ifa) and Ulm University

Jochen Wieland

Institute for Finance and Actuarial Sciences (ifa) and Ulm University

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# Introduction

## Motivation

- **Participating life insurance products** play a major role in old-age provision.
- **Key problem:** significant financial risk due to cliquet-style guarantees
  - impact of low interest rates and volatile asset returns
- Currently, risk analysis of interest rate guarantees particularly important!
  - market consistent valuation (e.g. **MCEV**)
  - capital requirements under risk based solvency frameworks (e.g. **Solvency II**)
- Aims from insurer's view:
  - stabilize profits and reduce capital requirements
  - but preserve main product features perceived and requested by policyholders

**Not by „model arbitrage“,  
but by real reduction of  
economic risks!**



This paper presents alternative product designs, and analyses  
“**Capital Efficiency**”, i.e. relation of profits and capital requirements.

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# Considered products

## Traditional contract design

### ■ Guaranteed benefit $G$

- constant interest rate  $i = 1.75\%$  applied to annual premium payments (after deduction of charges)

$$\sum_{t=0}^{T-1} (P - c_t)^t \cdot (1 + i)^{T-t} = G$$

- annual charges  $c_t = \beta \cdot P + \alpha \cdot \frac{T \cdot P}{5} \mathbb{1}_{t \in \{0, \dots, 4\}}$  with  $\beta = 3\%$ ,  $\alpha = 4\%$

### ■ prospective actuarial reserve (based on the same interest rate $i$ )

$$AR_t = G \cdot \left(\frac{1}{1+i}\right)^{T-t} - \sum_{k=t}^{T-1} (P - c_k) \cdot \left(\frac{1}{1+i}\right)^{k-t}$$

- **yearly surplus**  $s_t$  (e.g. 90% of book value returns) is credited to a bonus reserve, and the interest rate  $i$  is also applied to the bonus reserve:

$$BR_t = BR_{t-1} \cdot (1 + i) + s_t$$

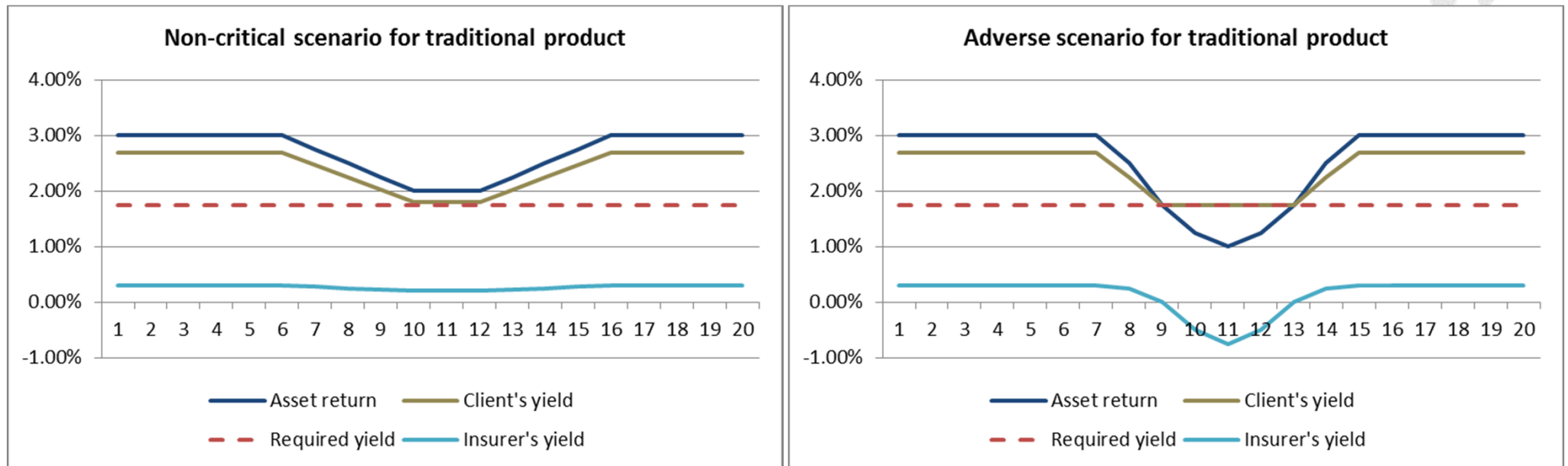
- client's **account value**  $AV_t$ : sum of actuarial and bonus reserve



$i$  is a **year-to-year minimum guaranteed interest rate**, i.e. (in book value terms) at least this rate has to be earned each year on the assets backing the account value (cliquet-style guarantee).

# Considered products

## Traditional contract design



in adverse scenarios: **significant shortfall** for the insurer  
major driver for **high capital requirements** (Solvency II, Swiss Solvency Test (SST)).

# Considered products

## Alternative contract design

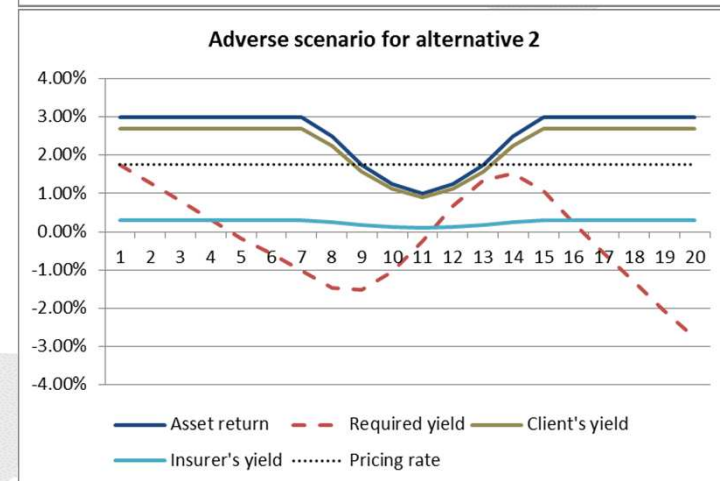
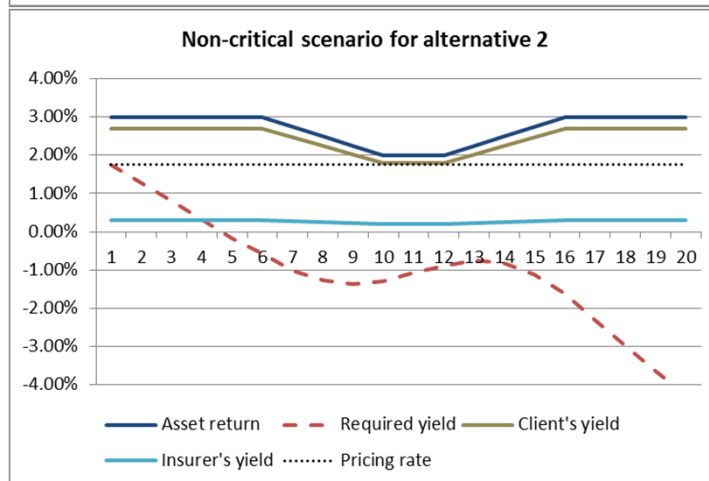
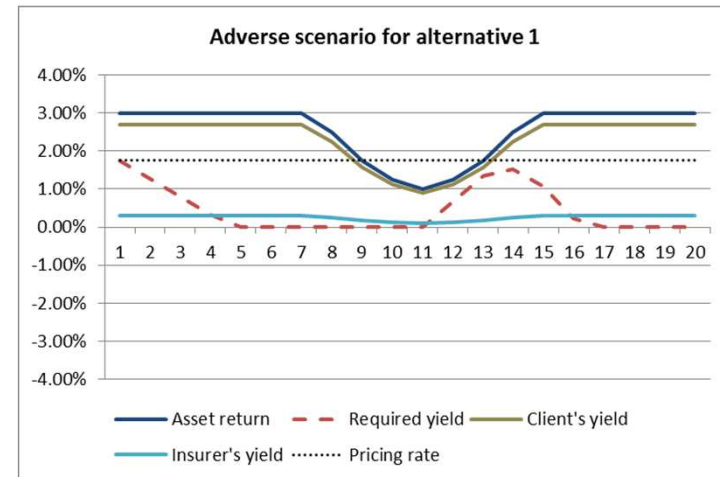
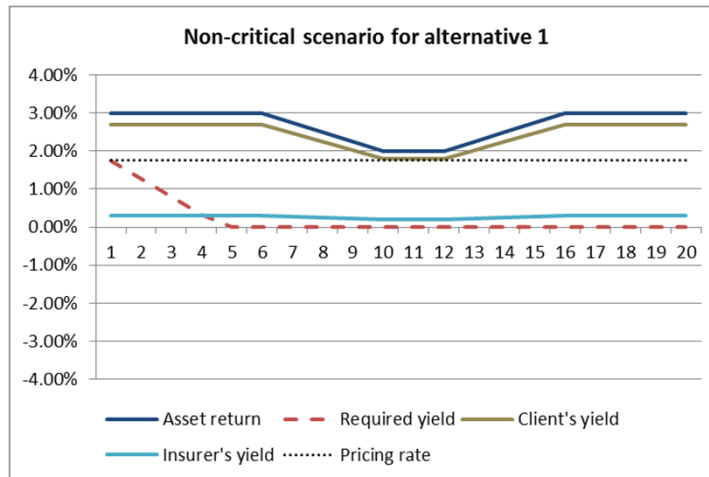
- The technical rate  $i$  plays 3 different roles
  - the **pricing** interest rate (i.e. for the calculation of  $P$ )
  - the **reserving** interest rate (i.e. for the calculation of  $AR_t$ )
  - the **year-to-year minimum guaranteed interest** rate on the account value
- alternative contract designs: split in three variables  $i_p$ ,  $i_r$  **and**  $i_g$  which can take different values
  - The minimum rate to be earned on the account value (= **required yield**) is then

$$z_t = \max \left\{ \frac{\max\{AR_t, 0\}}{(AV_{t-1} + P - c_{t-1})} - 1, i_g \right\}$$

- $P$  based on  $i_p$ ,  $AR_t$  based on  $i_r$
- In the paper, two alternative products are considered:
  - **Alternative 1:**  $i_g = 0\%$  (i.e. guarantee that account value cannot decrease)
  - **Alternative 2:**  $i_g = -100\%$  (i.e. no additional guarantee on the account value)
  - ( $i_p = i_r = 1.75\%$ )

# Considered products

## Alternative contract design



Alternative contract designs reduce the required yield after „good“ years.  
**Lower financial risk** for insurer in subsequent **adverse years**; shortfalls are prevented!



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# Stochastic modeling and analyzed key figures

## The financial market model

- Insurer's assets are invested in a portfolio consisting of **stocks** and **coupon bonds**.
- Short rate process follows a classical Vasicek model, stock market index follows a geometric Brownian motion:

$$dr_t = \kappa(\theta - r_t)dt + \sigma_r dW_t^{(1)}$$

$$\frac{dS_t}{S_t} = r_t dt + \rho\sigma_S dW_t^{(1)} + \sqrt{1 - \rho^2}\sigma_S dW_t^{(2)}$$

- probability space  $(\Omega, \mathcal{F}, \mathbb{Q})$  with a filtration  $\mathbb{F} = (\mathcal{F}_t)$  and a risk-neutral measure  $\mathbb{Q}$
- Bank account given by  $B_t = \exp\left(\int_0^t r_u du\right)$ , and used for investment of cash flows during the year.
- valuation using **Monte Carlo methods**
- parameter values:

	$r_0$	$\theta$	$\kappa$	$\sigma_r$	$\sigma_S$	$\rho$
basis	2.5%	3.0%	30.0%	2.0%	20.0%	15.0%
stress	1.5%	2.0%				

- (Source:  $r_0, \theta$  corresponding to current observations in the German market; other parameters from **Graf et al. (2011)**)

# Stochastic modeling and analyzed key figures

## The asset-liability model

- simplified balance sheet:

Assets	Liabilities
$BV_t^S$	$X_t$
$BV_t^B$	$AV_t$

- **book-value accounting rules** following German GAAP are applied.
  - $BV_t^S / BV_t^B$ : book value of stocks / coupon bonds
  - $X_t$  : shareholders' profit or loss
  - $AV_t$ : sum of actuarial and bonus reserves
- **rebalancing** strategy with a **constant stock/bonds ratio**
  - stock ratio  $q=5\%$  in the base case
- **portion of total asset return credited to the policyholders** :  $p=90\%$ 
  - but at least the required yield
  - surplus distribution such that total yield is the same for all policyholders (may not be possible in all cases)
- further management rules regarding asset allocation (reinvestment, rebalancing) and handling of **unrealized gains or losses** etc.
- projection of sample book of business over **20 years**

# Stochastic modeling and analyzed key figures

## Key figures for capital efficiency

- proposed **measure for “Capital Efficiency”**: distribution of  $\frac{\sum_{t=1}^{\tau} \frac{X_t}{B_t}}{\sum_{t=1}^{\tau} \frac{RC_{t-1} \cdot CoC_t}{B_t}}$ 
  - $RC_t$ : required capital under some risk based solvency frameworks
  - $CoC_t$ : cost of capital rate
  - Distribution of this ratio contains a lot of information, but requires complex calculations.

- Therefore, we focus on the following **key figures**:

- Present Value of Future Profits:  $PVFP = \frac{1}{N} \sum_{n=1}^N \sum_{t=1}^{\tau} \frac{X_t^{(n)}}{B_t^{(n)}} = \frac{1}{N} \sum_{n=1}^N PVFP^{(n)}$

- $X_t^{(n)}, B_t^{(n)}, PVFP^{(n)}$  the realizations of  $X_t, B_t, PVFP$  in scenario  $n$

- Time Value of Options and Guarantees:  $TVOG = PVFP_{CE} - PVFP$

- $PVFP_{CE}$  from a so-called “certainty equivalent” scenario

- $\Delta PVFP = PVFP(basis) - PVFP(stress)$

- approximation for the solvency capital requirement (SCR) for interest rate risk

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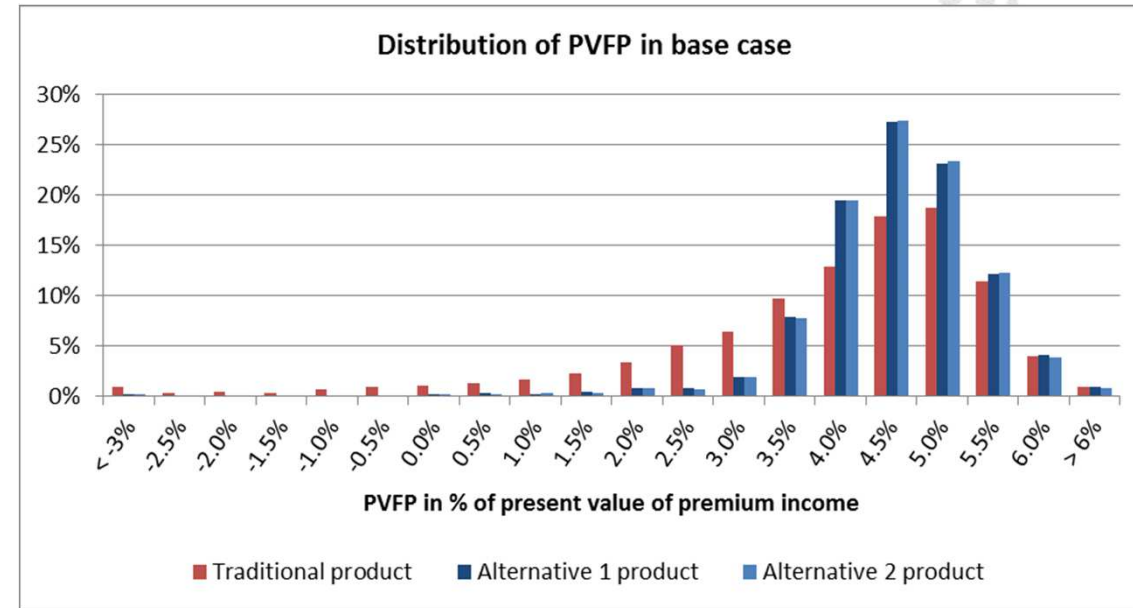
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# Results

## Comparison of product designs

	Traditional product	Alternative 1	Alternative 2
<i>PVFP</i>	3.63%	4.24%	4.25%
<i>TVOG</i>	0.63%	0.02%	0.01%
<i>PVFP(stress)</i>	0.90%	2.58%	2.60%
$\Delta PVFP$	2.73%	1.66%	1.65%



- Alternative products: 17% increase of profitability; **> 90% TVOG reduction**
- Distribution of  $PVFP^{(n)}$  changes from highly asymmetric to symmetric, i.e. **more stable profit perspective**
- Reduction of PVFP under stress significantly lower, i.e. **SCR decreases**

# Results

## Interesting questions / Sensitivities

- Type of guarantee vs. level of guarantee
  - reduce the level of guarantee in the traditional product setting such that the PVFP is the same as for the alternative products:  **$i=0.9\%$**  instead of 1.75%
  - significant reduction of level of guarantee can be avoided by using a different type of guarantee
- Market stress equivalent to considered change of type of guarantee
  - If interest rates decrease by **50 bps**, the alternative products have the same PVFP as the traditional product in the basic setting.
- **Sensitivities:**
  - **interest rate** sensitivity ( $\theta, r_0$ : -100 bps)
  - **stock ratio** sensitivity ( $q=10\%$  instead of 5%, i.e. more risky asset allocation)
  - **initial buffer** sensitivity (initial bonus reserve doubled for all contracts)

# Results

## Sensitivities

Base case	Traditional product	Alternative 1	Alternative 2
<i>PVFP</i>	3.63%	4.24%	4.25%
<i>TVOG</i>	0.63%	0.02%	0.01%
<i>PVFP(stress)</i>	0.90%	2.58%	2.60%
$\Delta PVFP$	2.73%	1.66%	1.65%
<b>Interest rate sensitivity</b>			
<i>PVFP</i>	0.90%	2.58%	2.60%
<i>TVOG</i>	2.13%	0.78%	0.76%
<i>PVFP(stress)</i>	-4.66%	-1.81%	-1.76%
$\Delta PVFP$	5.56%	4.39%	4.36%
<b>Stock ratio sensitivity</b>			
<i>PVFP</i>	1.80%	3.83%	3.99%
<i>TVOG</i>	2.45%	0.43%	0.26%
<i>PVFP(stress)</i>	-1.43%	1.65%	1.92%
$\Delta PVFP$	3.23%	2.18%	2.07%
<b>Initial buffer sensitivity</b>			
<i>PVFP</i>	3.74%	4.39%	4.39%
<i>TVOG</i>	0.64%	<0.01%	<0.01%
<i>PVFP(stress)</i>	1.02%	2.87%	2.91%
$\Delta PVFP$	2.72%	1.52%	1.48%

### Interest rate sensitivity:

- Also alternative products exhibit significant TVOG
- However, PVFP/TVOG changes much less pronounced, i.e. alternative products still much more profitable and less volatile .
- SCR reduction compared to traditional product: > 1 percentage point

### Stock ratio sensitivity:

- PVFP decreases /TVOG increases, but stronger for traditional product
- More pronounced differences between Alternative 1 and 2 → Guarantee on account value more risky with higher volatility of asset returns

### Initial buffer sensitivity:

- TVOG/SCR remains approx. the same for traditional product, but significantly reduced for alternative products → larger surpluses from previous years create a "buffer" reducing risk in future years



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## Conclusion and outlook

- **Results** confirm that products with a **typical year-to-year** guarantee are rather **risky**.  
→ **high capital requirement**
- Proposed **product modifications** significantly **enhance “Capital Efficiency”**, reduce the insurer’s risk, and increase profitability.
  - Policyholder receives less **only in extreme scenarios**, but these scenarios drive the capital requirements (Solvency II, SST).
- Areas for **additional research**:
  - additional participation of policyholders in reduced capital requirements
  - optimal strategic asset allocation for modified products
  - analysis of a change in new business strategy (traditional product in the past, modified products in new business)
  - product modifications for the annuity payout phase



Importance of “**risk management by product design**” will increase.

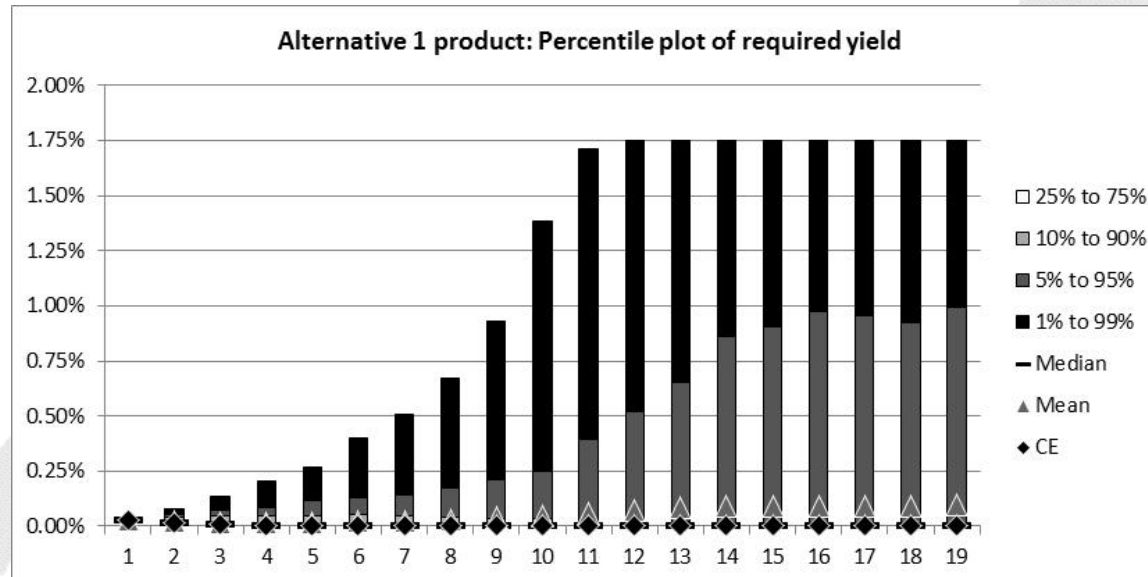
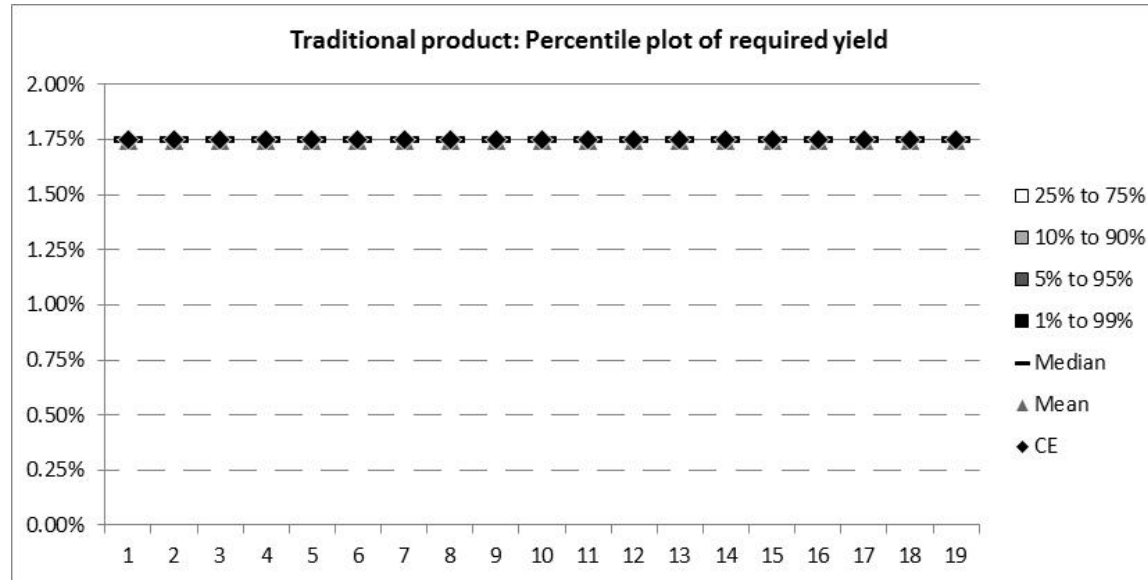
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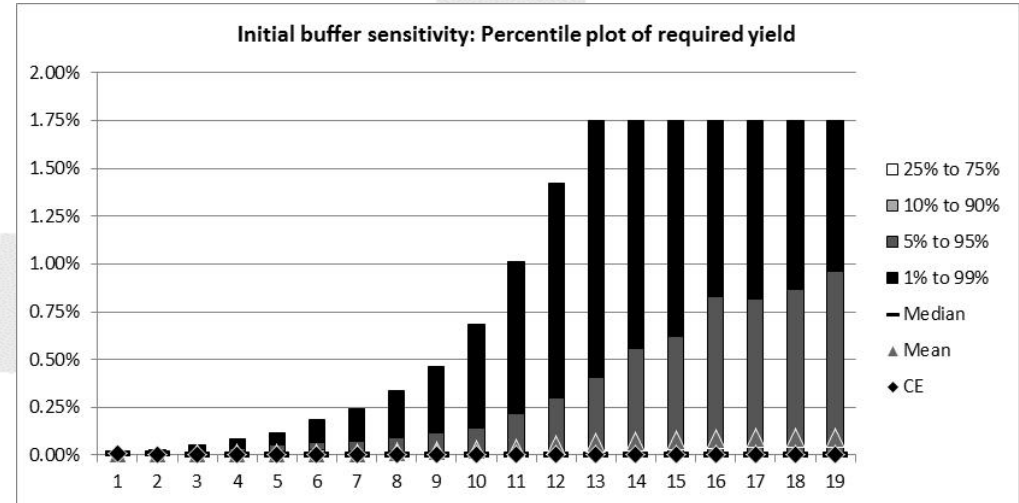
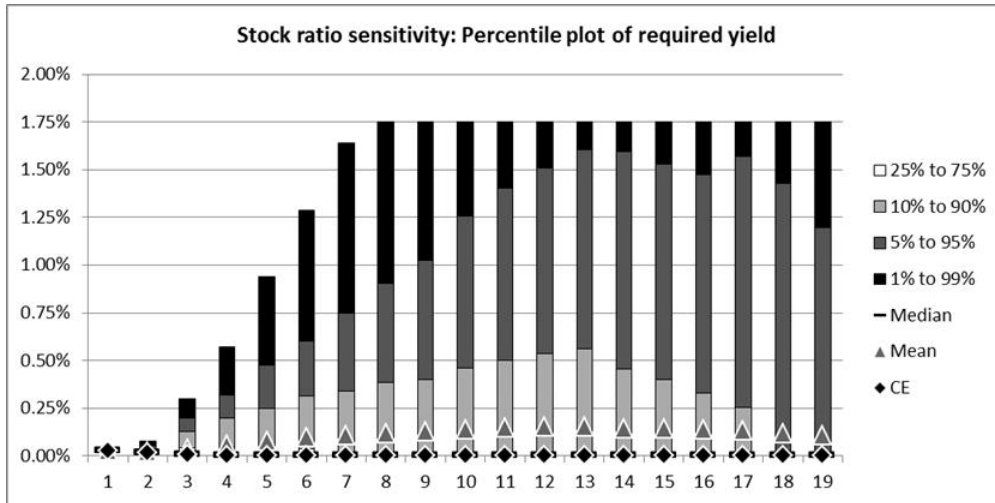
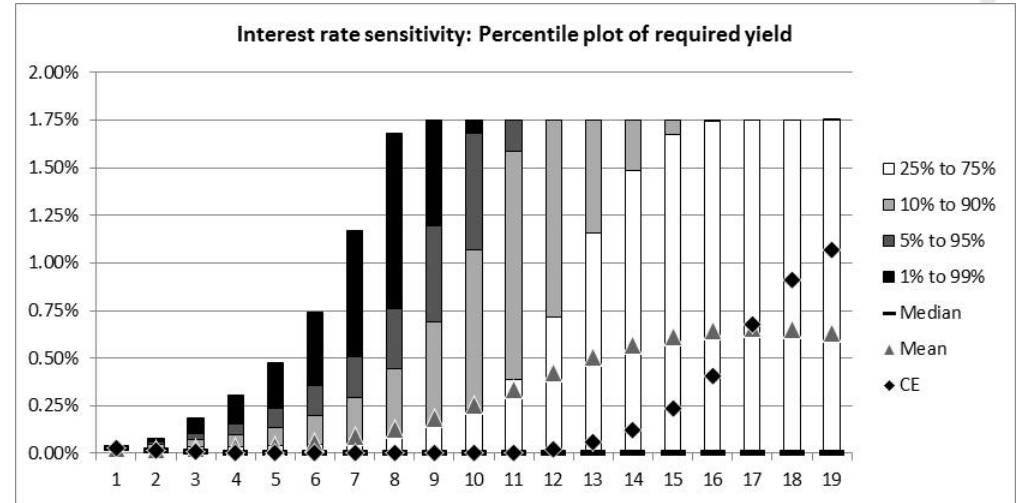
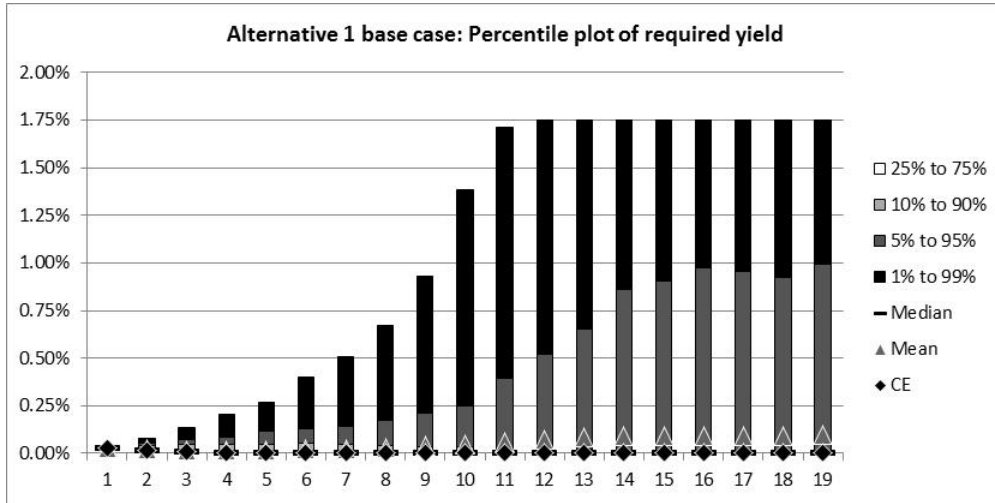
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## Percentile plots: Base case



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## Percentile plots: Alternative 1 sensitivities



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## Percentile plots: Alternative 2 sensitivities

