UNIVERSITY of ST. THOMAS

Optimal Policyholder Behavior for Withdrawal Guarantees in Variable Annuities

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Policyholder Behavior

- Many modern life insurance products depend on policyholder behavior
  - Surrender options
  - Withdrawal guarantees (GMWBs) in Variable Annuities (VAs)
  - Implicit option to (re-)allocate money in different subaccounts
- But: Policyholder behavior is not well understood
  - Relatively new products $\Longrightarrow$ Lack of data
  - Many insurers suspended their VA business in recent years
    - Or made substantial adjustments to its guarantees
  - Moody's (June 2013):

    "Unpredictable policyholder behavior challenges US life insurers' variable annuity business"
- Today: Policyholder behavior for withdrawal guarantees in VAs

VA + GMWB: A Simple Example

- Policyholder invests $100,000 in VA for 15 years
  - Money invested in mutual fund
  - Adds on a **Guaranteed Minimum Withdrawal Benefit (GMWB)**
    - PH has the right (but no obligation) to withdraw $7K each year
    - If VA account depletes, withdrawal amount comes out of insurer's pocket
    - Until $100K have been withdrawn on aggregate
    - PH can withdraw more than $7K, if account value permits
    - But guarantee covers only $7K per year
  - Pays X basis points (as % of account value) annually for this guarantee
  - At death: beneficiaries receive account value
  - If alive at maturity: PH receives account value
- Can we find the fair guarantee fee X?
  - X depends on withdrawal behavior !!
    - If PH withdraws less $\Longrightarrow$ Insurer less likely to make payment
    - If PH withdraws less $\Longrightarrow$ Insurer collects more fees

Policyholder Behavior for GMWBs – What can we do about it?

- Find withdrawal strategy that is **financially optimal**
  - Similar to pricing/early exercise of American options
    - Continuation value vs. exercise value
    - Choose withdrawal amount that maximizes w/d amount + VA continuation value
    - Recursively, year by year
  - Problem: fair GMWB fee way above what is charged in practice
- **Behavioral Economics**
  - Young science
  - Lots of (different) opinions
  - Theory not well developed yet
  - Not much help (yet!) for a product this complicated
- Our approach: Find a middle ground ...
  - How should policyholder withdraw optimally, under various conditions?

How do you price a financial option?

- **Textbook:**
  - Use Arbitrage Pricing
    - ⋆ Option price = initial value of replicating portfolio ($\rightarrow$ e.g. Black-Scholes)
  - (If given a choice:) When should you exercise your option?
    - ⋆ When exercise value > continuation value !!
  - Requires a complete, frictionless market
    - ⋆ All assets can be traded at competitive market prices
    - ⋆ No transaction costs, no taxes
- Are VA policyholders value-maximizers?

Introduction

Are VA policyholders value-maximizers?

- Why wouldn't they be?
  - VAs cannot be sold in the market, cannot be "split up"
  - VAs grow tax-deferred
    - Replicating portfolio does not
    - This is why people buy VAs, so it might impact their withdrawal decisions
- How can we model this?
  - Market frictions only
    - → **Subjective Risk-Neutral Valuation**
    - PH withdraws in order to maximize expected after-tax payout
  - Market frictions and market incompleteness
    - → **Life-Cycle Model**
    - PH withdraws in order to maximize expected lifetime utility of consumption

How do Market Frictions Affect Optimal W/d Behavior?

How do Market Frictions Affect Optimal W/d Behavior?

Risk-Neutral Valuation from Policyholder's Perspective

- Under standard RNV, withdrawing always optimal; **but :**
  - ▶ VAs popular because of preferential tax treatment
  - $\Rightarrow$ Taxes might impact withdrawal decisions
- Tradeoff with taxes: Withdrawing means ...
  - $\oplus$ Making use of guarantee
  - $\oplus$ Reducing fee payments
  - $\ominus$ Foregoing tax benefits

$\Rightarrow$ Develop **"subjective" risk-neutral valuation (SRNV)** approach
  - ▶ Takes into account differences in taxation
- When cash-flow is taxed differently than replicating portfolio:
  - ▶ Ross (JPE, 1986): No universal pricing measure exists
  - ▶ Valuation of cash-flows *locally* (i.e. agent-specific / subjective)

How do Market Frictions Affect Optimal W/d Behavior?

Subjective Risk-Neutral Valuation (SRNV) approach

- Determine time-t value (X_t) of post-tax cash flow X_{t+1}
  - Define X_t as amount needed in **replicating portfolio** (at time t)
    - ...to attain X_{t+1} at time $t+1$ after taxes
    - Gains in replicating PF taxed at rate κ
    - Assume complete pre-tax market
  - For given (assumed) value of X_t:
    - Find pre-tax cashflow Y_{t+1} that yields X_{t+1} after taxes
    - "Discount" Y_{t+1} to time t with (unique) pre-tax measure $\mathbb{Q}$
  - Iterate over X_t

Proposition 1.

Any post-tax cash flow X_{t+1} can be valued uniquely at time t as X_t, where

$$X_t \cdot e^r = \mathbb{E}_t^{\mathbb{Q}}[X_{t+1}] + \frac{\kappa}{1-\kappa} \cdot \mathbb{E}_t^{\mathbb{Q}}[\max\{X_{t+1} - X_t, 0\}].$$

How do Market Frictions Affect Optimal W/d Behavior?

The Policyholder's Optimization Problem

- Implement using **recursive dynamic programming**
  - For all times and states, (recursively) determine optimal w/d amount w_t
  - To maximize expected after-tax payout from the VA:

$$V_t(y_t) \;=\; \max_{w_t} \; [w_t - (\text{fees+taxes}) + V_t^+ \;] \,, \tag{1}$$

    - ⋆ y_t: time-t state vector
  - where the continuation value V_t^+ is given implicitly by

$$V_t^+ \cdot e^r \;=\; \mathbb{E}_t^Q[Y] + \frac{\kappa}{1-\kappa} \cdot \mathbb{E}_t^Q[\max\{Y - V_t^+, 0\}] \,, \tag{2}$$

    - ⋆ r: risk-free interest rate κ: capital gains tax rate
  - and where

$$Y \;=\; q_{x+t} \cdot b_{t+1} \;+\; p_{x+t} \cdot V_{t+1}\,(y_{t+1}) \;. \tag{3}$$

    - ⋆ b_{t+1}: time $t+1$ death benefit payment

How do Market Frictions Affect Optimal W/d Behavior?

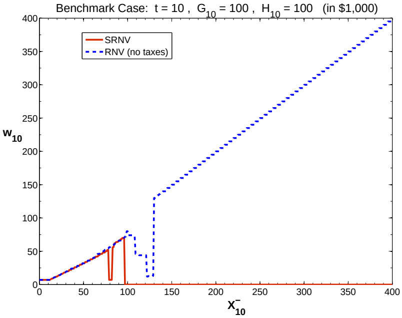
Parameter Assumptions

- To implement the "simple example" from above (benchmark case):

Description	Parameter	Value
Policyholder & contract specification		
Age at inception	x	55
VA principal	P_0	100,000
Years to maturity	T	15
Annual guaranteed amount	g^W	7,000
Excess withdrawal fee	s_t	$8\%, 7\%, \ldots, 1\%, 0\%, 0\%, \ldots$
Financial market parameters		
Interest rate	r	0.05
Volatility	σ	0.19
Tax rates		
Income tax rate	τ	30%
Capital gains tax rate	κ	23%
Early withdrawal penalty	s^g	10%

How do Market Frictions Affect Optimal W/d Behavior?

Optimal Withdrawal Behavior



	SRNV	RNV
$\mathbb{E}^{\mathbb{Q}}$**[Fees]**	2,555	1,394
$\mathbb{E}^{\mathbb{Q}}$**[GMWB]**	2,498	4,059
Insurer's Profit	57	-2,665
Agg. Withdrawals	5,260	265,870

- Either way: withdraw when account goes down
- With taxes: no surrender, even when guarantee is worthless

$\Rightarrow$ With tax considerations: insurer collects more fees

- Can charge less: $X = 20$ bps (without taxes: 64 bps)

$\leadsto$ Taxation clearly matters !

How do Market Frictions Affect Optimal W/d Behavior?

Sensitivities

- Fair GMWB fee (in bps) for different market parameters
  - Fair fee without taxes in parentheses

r \ σ	16%	19%	22%
3%	35 (105)	55 (146)	79 (198)
5%	11 (43)	20 (64)	31 (89)
7%	3 (18)	7 (30)	13 (45)

- Fair GMWB fee (in bps) for different tax rates
  - Fair fee without taxes: 64 bps

κ \ τ	25%	30%	35%
20%	20	22	24
23%	17	20	22
25%	15	17	20

How do Market Frictions Affect Optimal W/d Behavior?

Analysis of an Empirical Product

- Implement VA offered in U.S. market
  - *ASL II* by Prudential Annuities Life Assurance Corporation
- Key differences to simple GMWB example
  - Charges of 165 bps (of account value) p.a. (for M&E risk and Admin.)
  - Basic death benefit included
  - GMWB eligible for additional 35 bps p.a.
    - ⋆ Includes step-up option
    - ⋆ At maturity or death of PH: option to receive remaining benefits base, annuitized with zero interest
    - ⋆ Guarantee fee waived after 7 years, if no withdrawals are made
  - Investment in riskiest eligible fund: *Pro Fund VP Bull*
    - ⋆ Returns similar to *S&P500*
- Implement optimization with SRNV approach

How do Market Frictions Affect Optimal W/d Behavior?

Results for Prudential's ASL II

- Valuation Results for *ASL II*:

	With GMWB	W/o GMWB	Δ
GMWB fees collected	3,473		3,473
Other fees collected	25,053	22,242	2,811
Costs of guarantees	7,541	2,866	4,675
Insurer's profit (NPV)	20,985	19,376	**1,609**
Years under contract	20.37	16.01	
Surrender rate	$<$ 0.01%	41.1%	
V_0	101,574	100,859	
V_0 without taxes	99,053	98,420	

- Marginal value of GMWB to insurer: $1,609
  - Guarantee not under-priced
- Without tax considerations, VA not worth buying

Does Market Incompleteness Play a Role As Well?

Does Market Incompleteness Play a Role As Well?

A Life-Cycle Model

- Frictions matter, but what about incompleteness ?
  - Need to build a bigger (economic) model . . .
- Develop **life-cycle model**:
  - Risk-averse policyholder maximizes life-time utility
  - Can invest in outside account
  - Drawbacks:
    - ⋆ Complex model, requires simplifying assumptions
    - ⋆ Not preference independent
- Bellman Equation:

$$V_t(y_t) = \max_{C_t, w_t, \nu_t} u_C(C_t) + \beta \cdot \mathbb{E}_t^P \left[q_{x+t} \cdot u_B \left(b_{t+1} \,\middle|\, \frac{S_{t+1}}{S_t} \right) + p_{x+t} \cdot V_{t+1} \left(y_{t+1} \,\middle|\, \frac{S_{t+1}}{S_t} \right) \right] \tag{4}$$

  - . . . subject to a whole bunch of constraints . . .
  - Solve (again) by *recursive dynamic programming*

Does Market Incompleteness Play a Role As Well?

Optimal Withdrawal Behavior – Preliminary Results

- PH behaves very similar to SRNV model
  - Preferences have little impact
    - PH can attain desired risk exposure by adjusting outside account
    - Outside investment opportunity “completes market”

  ⇒ Optimal behavior driven by (subjective) value maximization

- One source of market incompleteness remains: **Biometric risk**
  - Not very significant for GMWBs (age of PH: 55–70)
  - Even less relevant if PH has access to life-contingent products
  - Markets more incomplete for older ages / unlimited durations
    - E.g., lifetime withdrawal guarantees
    - Pension annuities offer protection against biometric risk
    - But don’t protect simultaneously against long-tailed biometric & investment risk

Subjective Mortality Risk

Subjective Mortality Risk

- GMWB valuable only while policyholder is alive
- Withdrawal behavior depends on PH's perception of his/her mortality risk
  - Even in SRNV model:

$$V_t(y_t) = \max_{w_t} \left[w_t - (\text{fees+taxes}) + V_t^+ \right],$$

★ where the continuation value V_t^+ is given implicitly by

$$V_t^+ \cdot e^r = \mathbb{E}_t^Q[Y] + \frac{\kappa}{1-\kappa} \cdot \mathbb{E}_t^Q[\max\{Y - V_t^+, 0\}], \quad (5)$$

★ and where

$$Y = q_{x+t}^{subj} \cdot b_{t+1} + p_{x+t}^{subj} \cdot V_{t+1}(y_{t+1}). \quad (6)$$

- Since payouts in *"death"* and *"alive"* state differ, optimal w_t depends on probability weight that PH assigns to each state

Subjective Mortality Risk

- Undergraduate research project (Summer 2013)
  - Impact of subjective mortality risk on policyholder behavior
  - *University of St. Thomas* undergrads: Clem Foltz, Nathan Kent, Yabing Yang
  - Sponsored by National Science Foundation (CSUMS grant)
- How do people's subjective mortality perceptions differ from objective mortality risk?
  - Data sources:
    - *Health and Retirement Study* (HRS)
    - *Survey of Health, Aging, and Retirement in Europe* (SHARE)
    - Individual surveys (e.g. Harrison & Rutström, 2006; Jarnebrant & Myrseth, 2013)
  - Academic studies in demography & economics literature

Subjective Mortality Risk

Literature Summary

- Difficulties understanding and quantifying one's own mortality risk
  - Poor understanding of the concept of *Probability*
  - One's mortality is harder to visualize than other probabilistic events
  - Lack of experience (Harrison and Rutström, 2006)
  - Large amount of focal responses ("0", ".5", and "1")
- Substantial Heterogeneity in Subjective Mortality Beliefs
  - Gender gap
    - ⋆ Males slightly overestimate their survival probabilities to age 75, while females significantly underestimate theirs.
  - Other factors:
    - ⋆ Cognitive abilities, socio-economic status, health, education, ethnicity, marital status, etc. *beyond objective differences* (Hurd and McGarry, 1995, 2002; Peracchi and Perotti, 2012)

Subjective Mortality Risk

Literature Summary

- Variation in systematic ways
  - Flatness bias / constant hazard rate
    - ⋆ Individuals tend to underestimate their survival probability to age 75, but overestimate their survival to age 85 (Hamermesh, 1985; Elder, 2013)
  - Optimism & pessimism
    - ⋆ "Much of the heterogeneity in subjective survival risks is related to a general optimism/pessimism factor." (Hill et. al, 2004)
  - Longevity risk
    - ⋆ It's difficult to predict medical advances and quantify mortality improvements
  - Population averages
    - ⋆ People tend to absorb information from the entire population without accounting for individual characteristics (Hurd and McGarry, 2002; Andersson, 2011)
  - Equal survival rates across time
    - ⋆ People in particular age groups across time might have a similar framework for thinking about mortality (Elder, 2007)

Subjective Mortality Risk

Implementation

- Objective mortality based on: Annuity 2000 Basic Table (ABT)
- Hill, Perry and Willis (2004)
  - Based on HRS 2002 data
  - Life-table survival rate: 59% (for a person aged 50 to 64, with target age 75)
  - Average subjective survival response: 66% . ($\rightsquigarrow$ $q_x^{subj} = 0.83 * q_x^{ABT}$)
  - Mode of the "Optimist's Beliefs": 80%. ($\rightsquigarrow$ $q_x^{opt} = 0.48 * q_x^{subj}$)
  - Mode of the "Pessimist's Beliefs": 46%. ($\rightsquigarrow$ $q_x^{pess} = 1.29 * q_x^{subj}$)
- Elder (2013)
  - Based on HRS 2006 data
  - Average subjective survival response: 59.1% (life table: 67.56%)
  - Constant subjective hazard: ($\rightsquigarrow$ $q_x^{subj} = 1.691\%$)
- More extreme beliefs:
  - Focal response of 100% survival rate: ($\rightsquigarrow$ $q_x^{subj} = 0$)
  - Highly pessimistic (hypochondriac) person ($\rightsquigarrow$ $q_x^{hypoch} = 7.12 * q_x^{subj}$)
    - ⋆ Not likely to purchase GMWB.

Subjective Mortality Risk

Impact on Optimal Withdrawal Behavior

Table: Valuation results based on Hill, Perry and Willis (2004) ($\phi = 19$ bps).

	Objective	Subj. BM	$q = 0$	Opt.	Pess.	Hypoch.
Fees collected	2,444.6	2,508.5	2,500.3	2,504.7	2,507.8	2,534.4
Costs of GMWB	2,441.8	2,589.9	2,648.9	2,611.3	2,591.8	2,276.8
Insurer's profit	2.8	-81.4	-148.6	-106.6	-84.0	257.6

- Subjective mortality beliefs have minor impact
- But: Tend to *reduce* insurer's profit
  - More optimistic policyholder has more incentives to withdraw
  - Investors pessimistic about their mortality unlikely to purchase GMWB
  - Over- and under-estimations do not cancel out
- Increasing guarantee fee by 1-2 bps seems sufficient
  - Perhaps more in utility-based framework ($\rightsquigarrow$ bequest motive)
  - Add a death benefit guarantee

Can VA Guarantees Have Negative Prices?

Can VA Guarantees Have Negative Prices?

- *Arbitrage Pricing Theory*: An option cannot have a negative price!
  - Holder can always choose to ignore option
  - Issuer has nothing to gain, should charge positive price
- Result breaks down for products with preferential tax treatment
  - Valuation of PH and insurer no longer opposites
  - Third party involved: tax collector
- Example: VA + GMWB
  - Consider adding death benefit guarantee (GMDB)
    - At no extra charge !!
    - Reduces incentives to withdraw / surrender policy
    - Good for insurer: more fees, less guarantee!
    - Also: Delaying / foregoing withdrawals reduces tax payments
  - Both policyholder and insurer may be strictly better off
    - At "expense" of government

Can VA Guarantees Have Negative Prices?

Implications

- Not just a blackboard curiosity
  - 2-period model
  - Prudential's *ASL II*
- Insurer willing to give away GMDB for free
  - In competitive insurance market: price of GMDB could be negative!
  - Might explain why GMDBs are now standard features of most VAs
- "New" role for life insurers
  - Design long-term savings products that best take advantage of investors' tax benefits
    - Insurer and PH can "share" the tax savings
  - Financially savvy policyholders more profitable to insurers??
  - Lots of $$ to be made ☺

Conclusion and Outlook

Conclusion and Outlook

- **To sum up:** Looking at withdrawal guarantees in VAs, we understand (a little better) what factors drive optimal policyholder behavior:
  - **Subjective** value maximization
    - We develop valuation framework
    - Tractable & preference independent
  - Can cause some guarantees to have negative prices in equilibrium
    - Might (partially) explain why GMDBs are now standard in most VAs
  - Unobservable PH characteristics don't matter too much
    - Risk aversion; marginal tax rates; etc.
- **Future research:** When is market incompleteness important?
  - For lifetime withdrawal guarantees??
  - Can we find a "measure" for the incompleteness of savings products?

THANK YOU!

Questions?

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