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Cash Balance Plans: Valuation and Risk Management

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Outline

1. Background
2. Framework, assumptions, notation
3. The valuation formulas
4. Some results
5. Funding and Valuation
6. Final thoughts



Cash Balance Pensions

- Look like DC
 - contribution (% of salary) paid into participant's account
 - account accumulates to retirement
 - lump sum retirement benefit
 - withdrawal benefit = account value (after vesting)
- Regulated like DB
- Participant accounts are nominal

Crediting rates

- Participant's account accumulates at specified *crediting rate*.
- For example
 - Yield on 30-year government bonds
 - Yield on 10-year government bonds
 - Yield on 5-year government bonds + 25bp
 - Yield on 1-year government bonds + 100bp
 - Fixed rate, eg 5% p.y.
 - CPI rate

Some statistics...

- In 2010, 12 million CB participants in US
- Early popularity with sponsors, late 1990s
 - Simple transition from traditional DB to CB
 - Compared with DB to DC transition
 - Tax benefits
 - More transparent (apparently)
 - Less contribution volatility (apparently)
- With participants..
 - More portable, more transparent
 - But transition problems for older members

Framework, assumptions, notation

- Participant with n years service at valuation date.
- At valuation $t=0$.
- Retires at T with $n+T$ years
- Ignore exits, annuitization.
- Value future benefit arising from past contributions
- Use market valuation methods
 - Generates the cost of transferring the pension liability to capital markets

Framework, assumptions, notation

- F_t denotes the participant's fund at t
- $i^c(t), r^c(t)$ denote the crediting rates at t
- $r_k(t)$ denotes the k -year spot rate at t
- $r(t)$ denotes the short rate at t
- $p(t, t + k)$ denotes the price at t of a \$1, k -year zero coupon bond.

Framework, assumptions, notation

- Recall that

$$p(t, t+k) = e^{-kr_k(t)}$$

- Using financial valuation principles, we also have

$$p(t, t+k) = \mathbf{E}_t^Q \left[\exp \left\{ - \int_t^{t+k} r(s) ds \right\} \right]$$

Framework, assumptions, notation

- Assume continuous crediting, given F_t

$$F_T = F_t \exp \left\{ \int_t^T r^c(s) ds \right\}$$

- This is a random variable unless the crediting rate is constant.

The Valuation Formula

- The market value at $t=0$ of the benefit F_T is

$$\begin{aligned} {}_0V &= E_0^Q \left[F_T e^{-\int_0^T r(s) ds} \right] = F_0 E_0^Q \left[\left(e^{\int_0^T r^c(s) ds} \right) \left(e^{-\int_0^T r(s) ds} \right) \right] \\ &= F_0 E_0^Q \left[e^{\int_0^T (r^c(s) - r(s)) ds} \right] \end{aligned}$$

The Valuation Formula

- We let

$$V(t, T) = E_t^Q \left[\exp \left\{ \int_t^T r^c(s) - r(s) ds \right\} \right]$$

That is

- $V(t, T) =$ market value at t of CB benefit at T
 - per \$1 of nominal fund at t
 - No exits
 - No future contributions
 - With continuous compounding

Fixed crediting rate

➤ Suppose $r^c(t)$ is constant, $=r^c$, say

➤ Then

$$\begin{aligned}V(0, T) &= E_0^Q \left[\exp \left\{ \int_0^T r^c(s) - r(s) ds \right\} \right] \\ &= \exp(Tr^c) E_0^Q \left[\exp \left\{ -\int_0^T r(s) ds \right\} \right] \\ &= \exp(Tr^c) p(0, T)\end{aligned}$$

➤ The T-year zcb price $p(0, T)$, is known at $t=0$

Fixed crediting rate

➤ For example, $r^c = \log(1.05)$

➤ Using US yield curve at 1/April/2013

$$V(0,5) = (1.05)^5 (0.96256) = 1.2285$$

$$V(0,10) = (1.05)^{10} (0.82250) = 1.3398$$

$$V(0,20) = (1.05)^{20} (0.58889) = 1.5626$$

➤ That is, with a 10-year horizon to retirement, every \$1 of fund or contribution costs \$1.4375

➤ Model-free valuation result.

Crediting with the short rate

- Suppose the crediting rate is the short rate plus a fixed margin m
- That is $r^c(t) = r(t) + m$, then

$$\begin{aligned} V(0, T) &= E_0^Q \left[\exp \left\{ \int_0^T r^c(s) - r(s) ds \right\} \right] \\ &= E_0^Q \left[\exp \left\{ \int_0^T r(s) + m - r(s) ds \right\} \right] \\ &= e^{mT} \end{aligned}$$

Crediting with the short rate

- For example, $r^c(t) = r(t) + m$, with $m = 0.0175$
 - Then
$$V(0,5) = e^{5m} = 1.09144$$
$$V(0,10) = e^{10m} = 1.19125$$
$$V(0,20) = e^{20m} = 1.41908$$
- This will be \approx to the valuation for 3-month T-bill +175bp crediting rates.
- Model-free

Crediting with k -year spot rates

- Crediting with $r^c(t) = r_k(t) + m$
- We need a market model for $r_k(t)$
- We use one-factor Hull-White / ext Vasicek model

$$dr(t) = a(\theta(t) - r(t))dt + \sigma dW_t$$

$$p(t, t+k) = \exp\{A(t, t+k) - B(t, t+k)r(t)\}$$

- Where $B(t, t+k)$ is a function of a, k
- $A(t, t+k)$ is a function of yield curve at t and H-W parameters

Crediting with k -year spot rates

- After some manipulation....

$$V(0, T) = e^{mT} \exp\left(-\int_0^T \frac{A(t, t+k)}{k} dt\right) E_0^Q \left[\exp\left(-\int_0^T \gamma r(t) dt\right) \right]$$

where $\gamma = 1 - \left(\frac{1 - e^{-ak}}{ak}\right)$

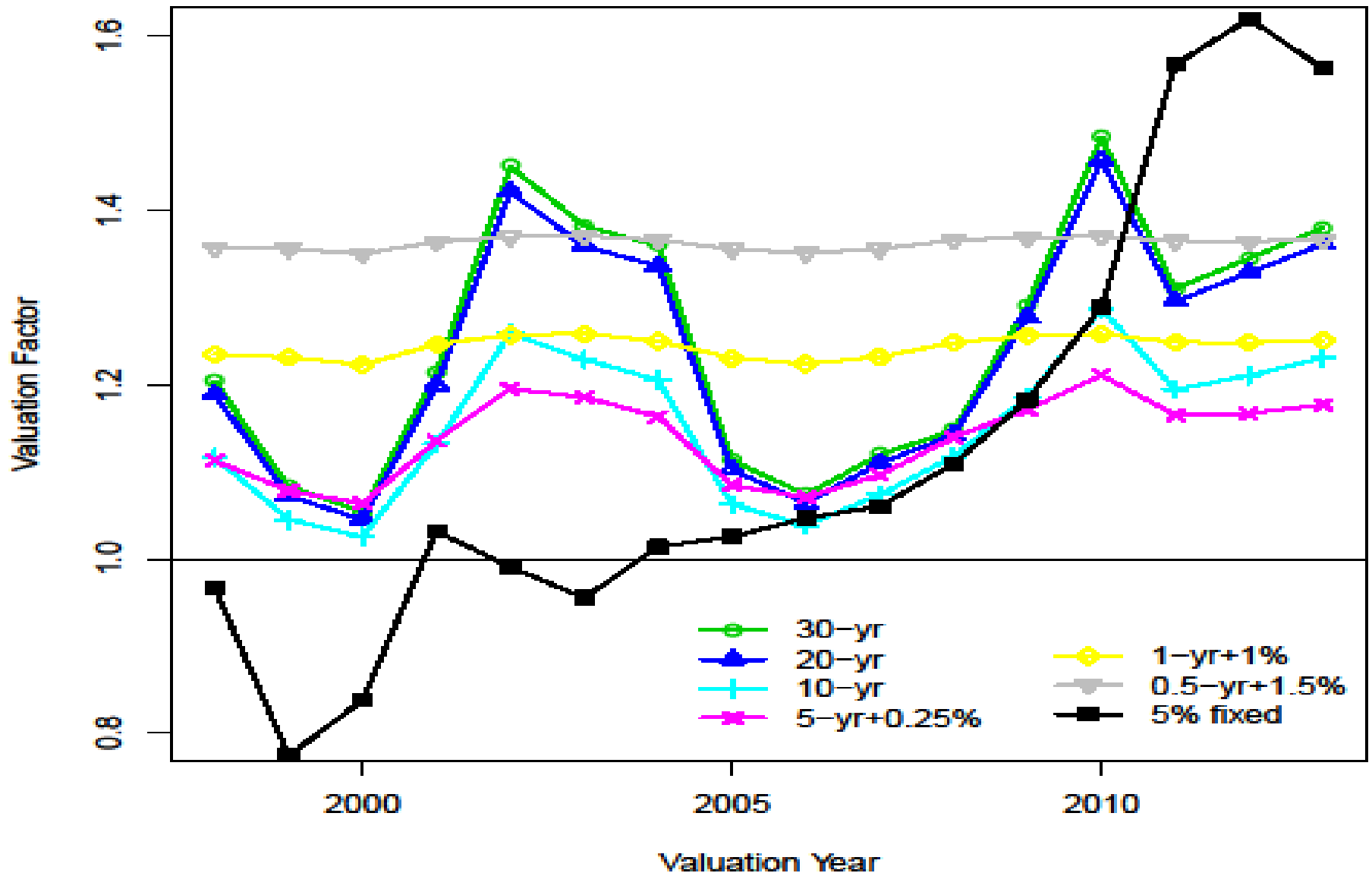
- The second term is evaluated using numerical integration (partly).
- The third term can be solved analytically – similar to the case $\gamma=1$

Crediting with k-year spot rates

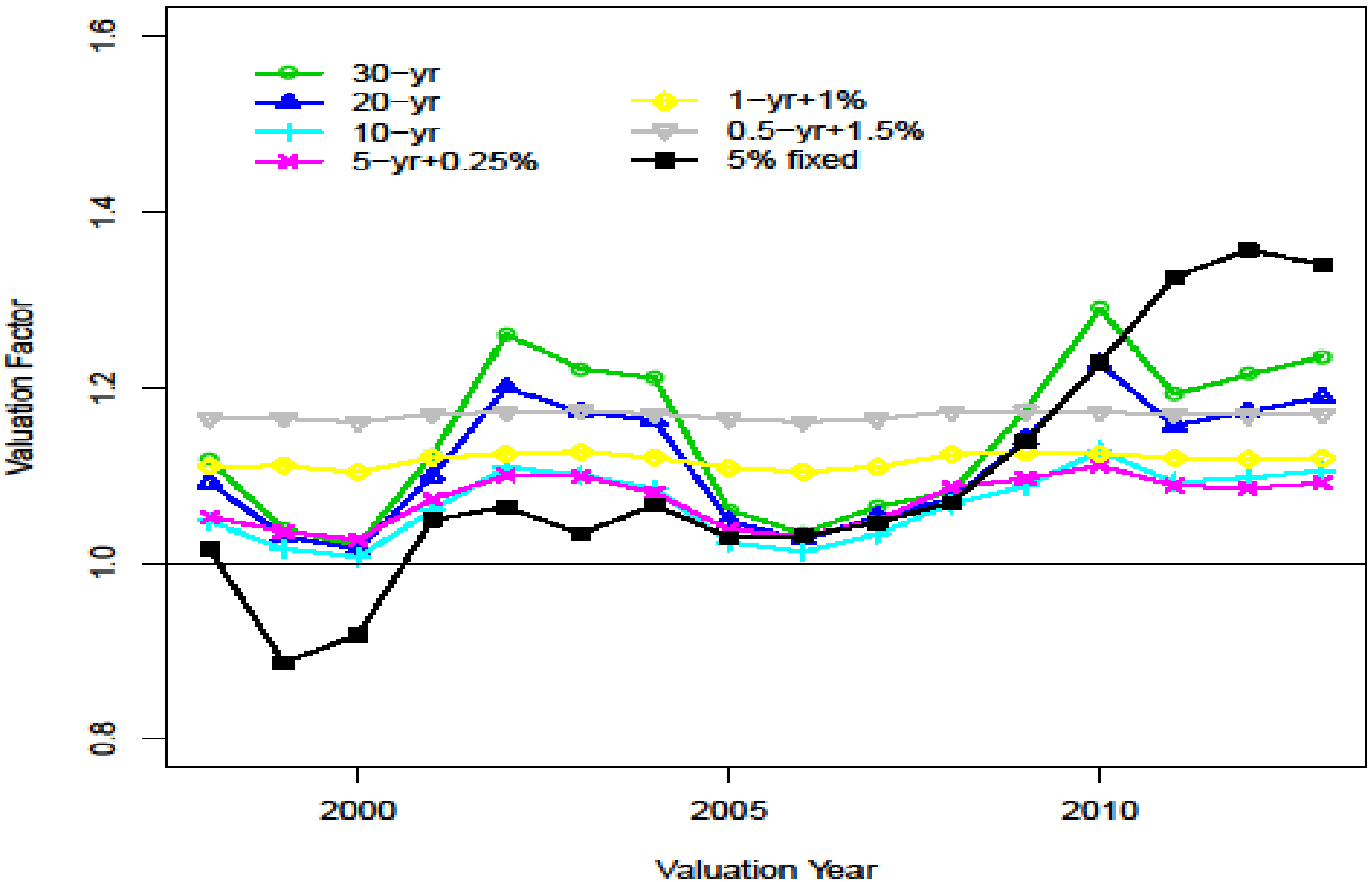
- For illustration we use
 - $a = 0.02, \sigma = 0.006$
 - $T=5, 10, 20$ years
 - $r^c(t) =$

30-yr spot rate	20-yr spot rate
10-yr spot rate	5-yr + 25bp
1-yr + 100bp	0.5-yr+150bp
 - Yield curve from US treasuries 1998, ..., 2013

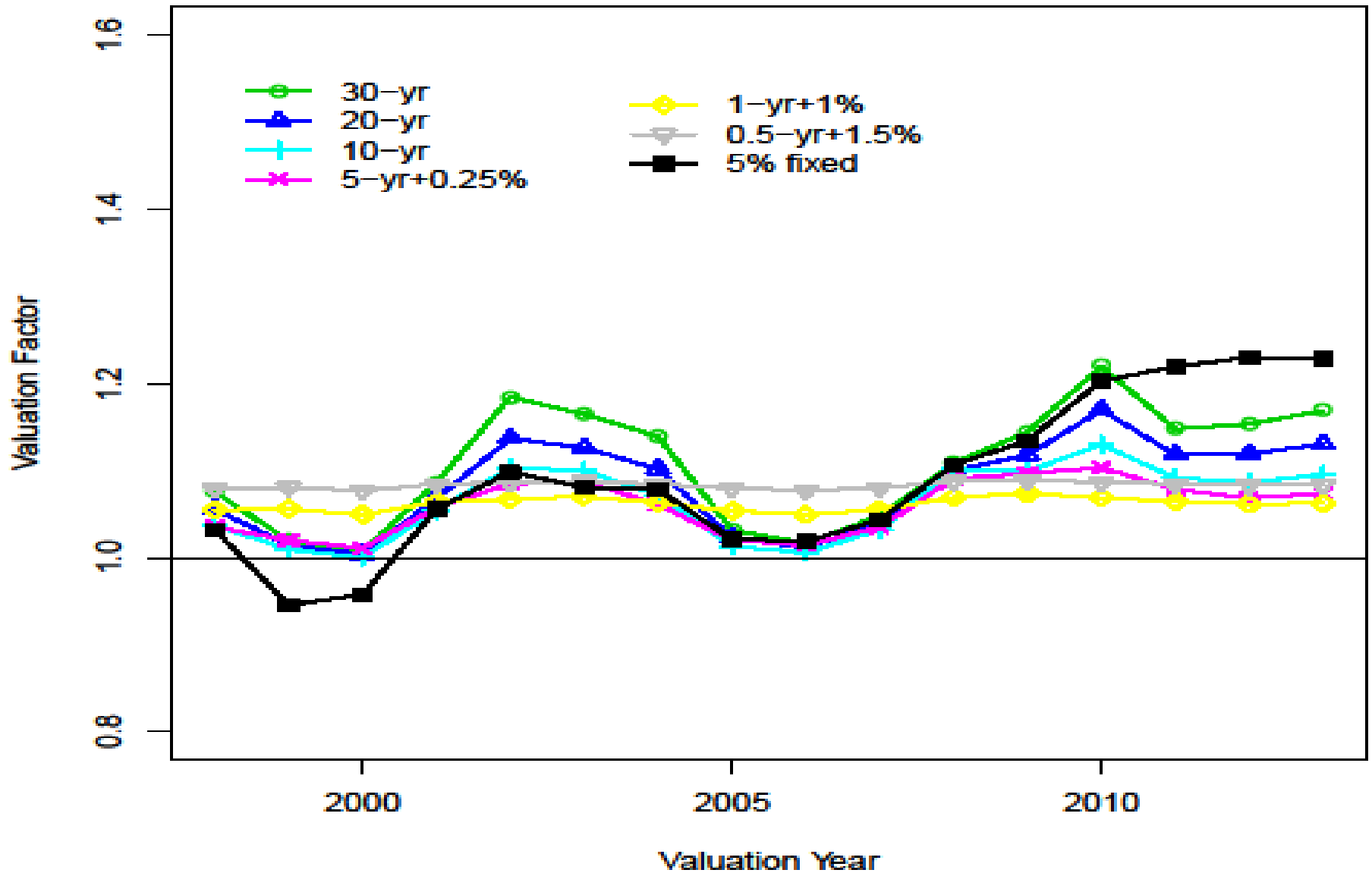
T=20-years



T=10-years



T=5-years



Comments

- Long rates and constant rates produce more volatility than short rates.
- For fixed rates -- costs have risen through the crisis
- For market based rates – it's more complicated
 - Interest rates were high in 1999, $r_{30} \approx 6.3\%$
 - But the cost is low
 - The risk is from the spread, $r_k(t) - r(t)$ not from the absolute values

Comments

Has the cost risen since the early transitions in 1998?

- For fixed rates – yes
- For market based rates – it's more complicated
 - Interest rates were high in 1999, $r_{30} \approx 6.3\%$
 - But the cost is low because short rates were also high.
 - The risk is from the spread, $r_k(t) - r(t)$ not from the absolute values

Actuarial valuations

- Review traditional approaches
- Consider three CB methods
- Principles and notation:
 - AL_t = actuarial liability = target asset requirement
 - NC_t = Normal Contribution = contribution needed to fund the expected increase in AL , t to $t+1$
- Under valuation assumptions, ignoring exits

$$(AL_t + NC_t)(1 + i_t) = AL_{t+1}$$

Actuarial valuation for final-salary DB

- Accruals based \Rightarrow past service earned benefits are included in the valuation
- Accruals methods are PUC and CUC(=TUC)
 - Projected accrued \Rightarrow benefits from past service indexed to retirement by salary scale.
 - Current accrued \Rightarrow benefits from past service valued assuming no further increases.

CB Valuation 1:

Past service, projected credited interest

- Past service \Rightarrow no allowance for future contributions to participant's fund
- This is the method used above, with market rates and models

$$AL_t = F_t V(t, T)$$

$$NC_t = cS_t V(t, T)$$

CB Valuation 2:

Past service, current credited interest

- Past service \Rightarrow no allowance for future contributions to participant's fund
- Current credited interest \Rightarrow no allowance for future credited interest
- $v_i(s)$ denotes the valuation discount factor for s-yrs ahead

$$AL_t = F_t$$

$$NC_t = cS_t + (F_t + cS_t) \left((1 + i^c(t)) v_i(1) - 1 \right)$$

CB Valuation 3:

Full service, projected credited interest, pro-rata accrual

- Let $\widetilde{B}_t(T)$ denote the projected final benefit, and let n denote service at the valuation date
- Deterministic salary growth and crediting rate assumptions

$$AL_t = \left(\widetilde{B}_t(T) v_i(T-t) \right) \frac{n}{n+T-t}$$

$$NC_t = \frac{AL_t}{n}$$

Example

- Employee A

- 1 year service
- 19 years to retirement
- $S = 50\,000$; $F = 4\,000$
- $c = 6\%$

- Employee B

- 10 years service
- 10 years to retirement
- $S = 60\,000$; $F = 55\,000$
- $c = 6\%$

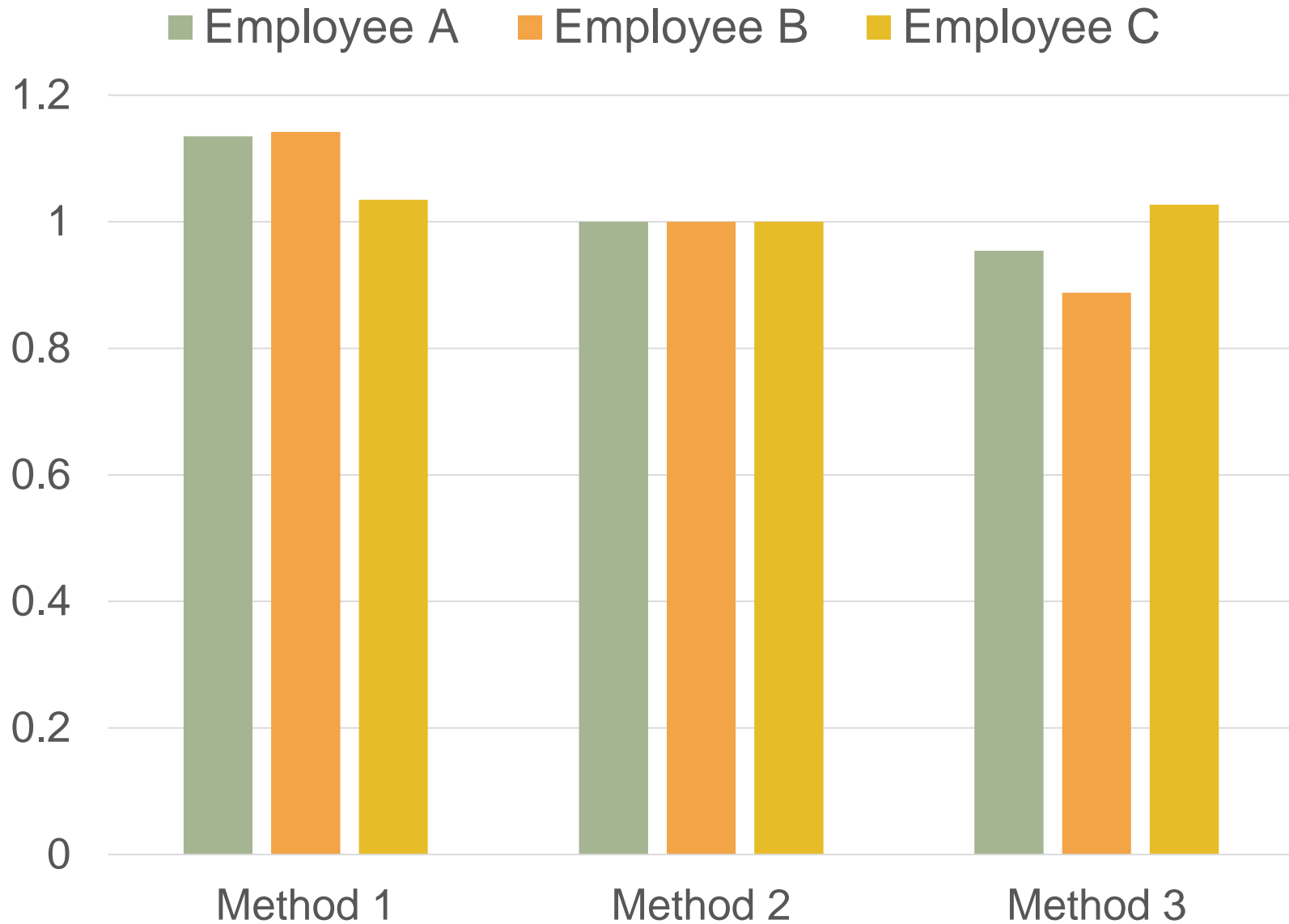
- Employee C

- 19 years service
- 1 year to retirement
- $S = 75\,000$; $F = 100\,000$
- $c = 6\%$

Example

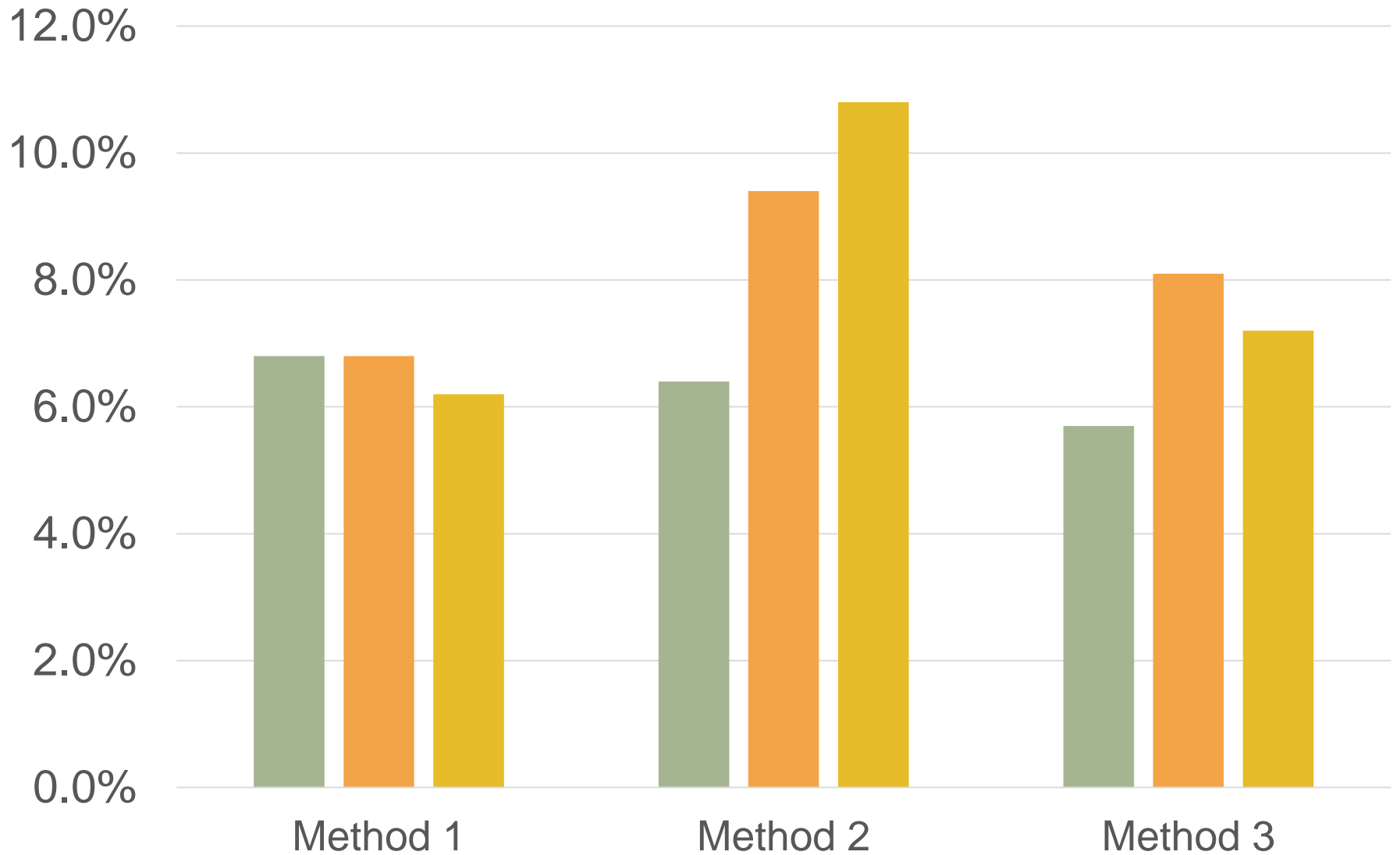
- Assume Corporate Bond valuation interest rates
- Crediting rate = 0.036 (30-year rate)
- Future crediting rate assumption (for method 3)
 $i^c(s) = 0.036$
- Future salary growth assumption 2% p.y. (method 3)

Valuation Factors AL/F



Contribution Rate NC/S

Employee A Employee B Employee C



Comments 1

- Method 1 is a PUC method
 - Projecting benefit increases through future service period
- Method 2 is a TUC method
 - Valuation does not project future benefit increases
- Method 3 is not an accruals method
 - But is sometimes called PUC as it uses future salaries.

Comments 2

- Valuation Factors:
 - Method 1: $AL_t \geq F_t$
 - Method 2: $AL_t = F_t$
 - Method 3: $AL_t \leq F_t$
- Contribution Rates:
 - Method 1: $NC \geq c$
 - Method 2: $NC \geq c$ ($NC \gg c$ for B and C)
 - Method 3: $NC \leq c$

Method 3 – pro-rata projected benefits

- Method 3 is adapted from traditional DB valuation
 - Not accruals based
 - Gives perverse results
 - Inconsistent with financial theory
 - Cannot be “100% Funded” at less than aggregate notional funds
 - Implies benefit is less for stayers than leavers
 - Very sensitive to assumed salary and crediting rate assumptions
 - Not suited to CB design

Concluding thoughts

- The CB benefit isn't as simple as we thought
- This benefit isn't as cheap as we thought/think
- DB valuation methods do not adapt to CB
 - Needs a new approach
- Design is important
 - Short rates are more stable for crediting
 - Short rates are easier to hedge

Concluding thoughts

- Do participants understand the difference between CB and DC?
 - Significant difference in benefit security when assets < notional accounts
 - Every exiting participant diminishes the security of the remainder
 - Even for a fund which is “100% funded” under Method 3
- **There is no justification for valuation factors less than 100% under any acceptable valuation methodology.**

Final question

- Does the Cash Balance Pension really meet the objectives of sponsors or participants?
 - Costs are volatile.
 - Hedging is complex.
 - Commonly used funding methods obfuscate costs.
 - Benefit security may be significantly compromised, even for “100% Funded” plan.
 - Disadvantages of lump sum benefit design from employee perspective.

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