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| Investigation of Hedging Strategies between Assurances And Annuities for the Purpose of Mitigating Longevity Risk |
| by  Frans F. Koning  Fellow of the Actuarial Society of South Africa  Fellow of The Institute and Faculty of Actuaries UK |
| THE FACULTY OF NATURAL AND AGRICULTURAL SCIENCE  DEPARTMENT OF MATHEMATICAL STATISTICS AND ACTUARIAL SCIENCE  UNIVERSITY OF THE FREE STATE  BLOEMFONTEIN, SOUTH AFRICA  MAY 2013 |
| (UFS), BLOEMFONTEIN, 9300, Republic of South Africa.  [koningf@ufs.ac.za](mailto:koningf@ufs.ac.za) |

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# Abstract

Longevity risk is an increasing risk factor in the world of increasing pensions and annuity business. If declining the business is not an option, a method is required to minimise the risk of longevity. This research considers the natural hedge between annuities and assurances. The stability of the hedge is investigated, how often rebalancing is required, and the effect that a change in interest rates will have on it. The hedge is illustrated for various ages, both for annuities and assurances. Finally, this is expanded and a simple method is suggested to apply the hedge to a portfolio of annuities or assurances.

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# Introduction

The natural hedge is not a new concept, yet it is rarely put to use. Various practicalities serve as a barrier for active trade in such risk, such as finding a suitable longevity index, as described by Blake et al (2006). Improvement in mortality is a slow process, with medical advances taking years to filter down to the man on the street. If we compare it to the much more volatile processes such as interest rates and currency movements, the question arises: why could we hedge these, but not mortality? Is the longevity index really a requirement?

Most companies have been allowing for mortality improvements for a very long time. This may be done one dimensional using a single percentage over all ages and calendar cohorts, or could allow for ages, calendar year, male/female and other factors. These individual company and product complexities may also seem to be barriers to a simple solution.

In this paper the natural hedge is illustrated by applying a simple method, in which company specific complexities could be allowed for, and mortality indices could possibly be done away with.

We assume that companies already allow for improvements in some way, complex or not. Our method provides a hedge for improvements over and above what was expected, i.e. 0.5% more than expected or 0.5% less than expected. The underlying assumption for the hedge is that if mortality improves it will improve in the same direction, i.e. that our best estimates used for pricing was either too low or too high for both annuities and assurances. Surely a flu pandemic will increase mortality across the board, and a new medical cure for cancer will improve mortality of all. In the unlikely event that improvements move in different directions, the hedge will be ineffective.

Typically a company will be cautious in pricing and assume light mortality for annuities and heavy mortality for life cover. Margins may be a lot smaller for life cover than for annuities. However, whatever the bases on these products, if mortality moves in a direction over the full age range, then the present values of annuities and assurances will move in opposite directions. The profits/losses on life cover will be offset by losses/profits on the annuity side. The aim is to illustrate a way to quantify this effect under various scenarios, for the use of internal hedging, swaps and even securitisation.

Lapses on the life side tend to occur frequently, with practically none on the annuity side. This does not present a problem and could be allowed for. Although mortality changes and portfolio changes are slow, like any hedge, the natural hedge does not last forever. Frequent rebalancing is required to optimise the hedge, as was also pointed out by Cox and Lin (2004).

This paper starts with a bit of background on longevity and the natural hedge. It then provides the theoretical background of the hedging process. Finally it introduces the Longevity Hedge Ratio (LHR) with various results under various conditions.

1. Background

The insurance market in SA has managed to effectively shift most longevity risk for both annuities and assurances to the insured, making us administrators of customer risks, rather than insurers of it.

For annuities a product was created called the living annuity. The accumulation of life savings is invested at retirement in this living annuity. It is similar to a bank account from which monthly or annual amounts are withdrawn by the annuitant, under certain rules. The percentage withdrawn is chosen by the annuitant at the start of each year, and can vary between 2.5% and 17.5%. Annuitants enjoy the flexibility but carry the longevity risk, which are likely to byte on high annual fund withdrawals and/or longer than expected lifetimes.

For life cover products the longevity risk is also shifted in a clever way. The choice still resides with the insured to opt for constant premium, increasing or age-related premiums. However, age-related (or age-aggressive as it is called) are the ones pushed by financial advisers due to the very low premium early on. The premium for the next year is simply the risk premium plus margins and profit. The mortality experience is then updated annually over the course of the contract to allow for changes, including improvements. Since there are no cross subsidy to older ages, premiums are very low early on in life, and very marketable and competitive, but become unaffordable later on at higher ages. This age-aggressive life cover is similar to a one year renewable life cover for life, or limited term. This reduces longevity risk considerably, as shown by Wong, Sherris and Stevens (2013). However, this also reduces the hedging ability of the life cover portfolio, leaving the internal natural hedge of a company weighted towards the annuity side.

If we consider what is happening in the world, see table 1, there is definitely an improvement in mortality every year. At the moment we measure this as a fixed annual increase over a certain period, however, it may become exponential for all we know if there are any truth in the theories of Aubrey de Grey from the Department of Genetics, University of Cambridge.

Table 1: Worldwide change in mortality

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Country | Year | Age | nqx | Annual compound  change in mortality |
| Australia | 1990 | 25 | 0.00460 |  |
|  | 2008 | 25 | 0.00293 | -2.54% |
|  | 1990 | 55 | 0.03524 |  |
|  | 2008 | 55 | 0.02063 | -3.02% |
| Brazil | 1990 | 25 | 0.01096 |  |
|  | 2008 | 25 | 0.00847 | -1.44% |
|  | 1990 | 55 | 0.06932 |  |
|  | 2008 | 55 | 0.04954 | -1.88% |
| China | 1990 | 25 | 0.00781 |  |
|  | 2008 | 25 | 0.00436 | -3.29% |
|  | 1990 | 55 | 0.06092 |  |
|  | 2008 | 55 | 0.04151 | -2.15% |
| Germany | 1990 | 25 | 0.00396 |  |
|  | 2008 | 25 | 0.00215 | -3.45% |
|  | 1990 | 55 | 0.04448 |  |
|  | 2008 | 55 | 0.02968 | -2.27% |
| South Africa | 1990 | 25 | 0.01746 |  |
|  | 2008 | 25 | 0.05516 | 6.19% |
|  | 1990 | 55 | 0.08081 |  |
|  | 2008 | 55 | 0.12480 | 2.39% |
| United Kingdom | 1990 | 25 | 0.00339 |  |
|  | 2008 | 25 | 0.00273 | -1.21% |
|  | 1990 | 55 | 0.04197 |  |
|  | 2008 | 55 | 0.02790 | -2.29% |
| United States of America | 1990 | 25 | 0.00623 |  |
|  | 2008 | 25 | 0.00491 | -1.33% |
|  | 1990 | 55 | 0.04577 |  |
|  | 2008 | 55 | 0.03473 | -1.55% |

Source: World Health Statistics 2010

Figure 1 below shows the change in life expectancy for different regions from 1950 to 2050. It can be seen that life expectancy is expected to increase rapidly over the next 40 years.

Figure 1: Life expectancy at birth in different regions



(United Nations, 2004 cited in Crawford *et al.*, 2008)

Antolin (2007) argues that as long as these increases in life expectancy are taken into account when planning retirement it would have a very small effect on retirement finances. However, he also states that the problem with increases in life expectancy is that it is uncertain. Therefore, he defines longevity risk as the risk that future life expectancy outcomes turn out different than expected.

More specifically, Crawford *et al.* (2008) defines longevity risk from the perspective of an insurance company as the risk that the company will have to face unexpected decreases in mortality.

According to Milevsky and Promislow (2003) this view is supported by Moody’s investor services:

“Moody’s believe that the two main risks to insurance companies from payout annuities are embedded equity guarantees and inaccurate longevity assumptions ... Aggressive longevity assumptions relate to mortality risk assumed, or the risk that annuitants will on average live longer than originally assumed by the insurer, thus extending the period for which the insurer is obligated to make these monthly payments.” (Moody’s investor services cited in Milevsky and Promislow, 2003)

This view is supported by Willets *et al.* (2004). They state that a cohort effect can be observed in mortality data. They define the cohort effect as the influence of year of birth on mortality improvement rates. This will mean that before deciding what will be the correct improvement shock to apply to the mortality table, the year of birth of the annuitant or assured life have to be taken into account. They have also shown that the greatest increase in life expectancy over the past century was experienced by people who were born between 1925 and 1945.

Crawford *et al.* (2008) agrees with this by stating that the most important implication of the cohort effect is that mortality rates for a population does not improve at a constant rate. Furthermore, they state that this could have significant implications for companies when the future pricing of products is considered.

Cox and Lin (2004) found that the natural hedge might improve the profitability of an insurance company, or improve pricing and competitiveness. This view is supported by Mungan (2004):

However, Milevsky and Promislow (2003) state that even though on a purely theoretical level, life insurance and annuity liabilities are sensitive in opposing directions to changes in the entire mortality table the traditional economic response of using this difference in reactions to obtain a hedge is dismissed by the life insurance industry. This might be changing rapidly with developments in applying the natural hedge.

Cox and Lin (2004) state that even though most insurance companies issue whole of life annuities as well as life assurance contracts, the combination of these two types of products in the portfolio of a single company may not be optimal in order to obtain a natural hedge. They state that it may be too expensive and difficult for an insurer to develop new product lines and obtain more of the business that is needed to obtain a natural hedge.

Furthermore, Cox and Lin (2004) also foresee that the natural hedge will not be static. If it is possible to balance internally by means of selling the extra products required to obtain a natural hedge remains an open question.

It was shown by Stevens *et al.* (2009) that the natural hedge effect can significantly reduce the amount of reserves required. However, the product mix determines the size of such a reduction.

Regarding possible ways to determine the optimal mix of annuities and assurances to use in order to obtain a natural hedge several papers investigate different possibilities. For example, Wang *et al.* (2009) extend the immunization theory proposed by Redington in 1952 to deal with longevity risk. They argue that this is possible because the effect that changes in mortality rates has on the liability of life insurers is similar to that of an interest rate change.

Cox and Lin (2004) on the other hand propose a mortality swap between two companies, a life insurer and an annuity insurer, without considering the possibility of a natural hedge within one company on its own.

In another paper, Cox and Lin (2007) gives more empirical evidence to prove that a natural hedge is the ideal solution for a company facing longevity risk and develops a pricing strategy for a mortality swap between two companies. They argue that mortality swaps will make the option of a natural hedge more widely available.

The little correlation between capital markets and mortality makes it an excellent diversification tool in investment portfolios, as noted by Crawford *et al.* (2008). Many role players in the capital market may also be interested in the hedging effect of longevity securities, for example medical companies.

Although we are not at the point of having standardised actively traded longevity securities yet, it may not be too far off. Certain uncertainties have to be sorted out, and fears addressed, to generate confidence in the capital market. The rest of the paper hopes to contribute in this direction. The principles underlying the method are explained next, followed by an application and results.

# **Methodology**

## 3.1 The Theory

Different methods exist to apply mortality improvements. In practice the improvement is usually applied to the mortality rate . For example, let the annual improvement rate = 2%, then someone aged who entered 3 years ago would have received 3 improvements by age . So the mortality rate at this age including the improvements would be . The rate could allow for male or female, year of birth and other factors if need be. The q-rate is read from a normal mortality table.

Cox and Lin (2004) applies the improvement to the survival probability as , where *r ’* is the improvement rate equivalent to applied on the -rate above.

In this paper the method applied is a conversion of the applied on the q-rate. The reminds us of an interest rate, so we use the known result to convert it. Let the force of improvement be , then

or (3.1)

But since it is an improvement, will typically be negative if applied to a survival probability as in the Cox and Lin application.

We know that

(3.2)

And if we consider a single year and assume constant force of mortality, then

(3.3)

and

(3.4)

Thus if we have to apply the improvement it would be

(3.5)

Similarly, to allow for other decrements, like lapses or surrenders, it could also be written in the form of forces of transition, incorporated into .

Now let us consider a portfolio of independent lives, and let . Let = time under observation or simply exposed to risk. Let = 1 if life died and zero otherwise, an indicator variable. Then without proof, for the exponential distribution, for a sample we know that the probability of observing the particular data set, for life only

(3.6)

And the likelihood function for all N lives is proportional to:

(3.7)

where and

And we can use this result to find a maximum likelihood estimator (MLE) of as

(3.8)

This result does not consider different ages, sex or smoker status, and is a universal result for a cohort/portfolio of lives. We can therefore easily calculate the MLE of the force of mortality for any give portfolio of lives, or even parts of portfolios with certain groupings. The portfolio of lives can be seen as a new entity with a single force of mortality over the next year. In the same way the force of mortality could be estimated for future years. This is under the assumption that it would be constant over each individual life-year, or between ages for each life. The mortality improvements can then be applied on the survival probability of the portfolio as a whole, using (3.5) above.

The portfolio force of mortality could already include allowance for lapses. Annual rebalancing could be done. The application of the hedge will be illustrated shortly.

Another complication is that the sum assured is not the same for each life, and that annuity payments also differ between pensioners. To cross this barrier we use a portfolio of assurances with a single constant sum assured, and similar for annuity payments. Everything above this constant could be reinsured or retained, but will not form part of the hedge on a group basis. If the hedge is calculated on an individual basis[[1]](#footnote-1) the differences in amount does not pose a problem. It would however introduce more volatility and risk due to the correlation. Two different lives with $100,000 sum assured are independent, but a single life with $200,000 sum assured has perfect correlation.

Figure 2 - Split of assurances to create a constant sum assured



Another method to allow for differences in sum assured, is to incorporate weights into the forces of transition, allowing for larger sums assured to have heavier mortality. Let the sums assured , then to calculate the additional weight we solve

(3.9)

to get

(3.10)

The argument to view the portfolio as a single entity can now be expanded. Consider two lives and then we know that . The theory holds even if lives and are different in year of birth, sex, age and smoker status. We can generalise the idea as follows: let then . This could incorporate the weights in (3.10) to allow for differences in sums assured.

## 3.2 The Longevity Hedge Ratio (LHR)

The theory of section 3.1 allows us to develop approximations for and for a total portfolio with some grouping characteristics. The Longevity Hedge Ratio (LHR) can now be introduced, building onto the above.

(3.11)

The LHR could in this way be applied to individual lives, or portfolios of lives. It considers the change in value of the assurance portfolio compared to a change in value of the annuity portfolio after a change in mortality of some kind. Each portfolio PV could be calculated on its own basis, with the only common factor the improvement in mortality over all ages. In (3.11) the LHR would not be defined if there were no change in the assurance portfolio (division by zero).

## 3.3 Assumptions used in LHR calculation

The following assumptions were made to illustrate the method:

1. The term structure of interest rates is flat at 6%, a variable rate incorporating the full term structure if interest rates could also be used.
2. Death benefits are paid at the end of the year of death.
3. A male life aged 25 for the assurance and male life aged 65 for the annuity.
4. Annuity payments are annual and made at the beginning of every year.
5. Mortality rates used when pricing annuities follow those specified in the PMA80 and PFA80 tables, for pensioner’s mortality, for males and females respectively. Other mortality tables can also be used.
6. Mortality rates used when pricing life assurances follow those specified in the AM80 and AF80 tables, for males and females respectively. Other mortality tables can also be used.
7. Mortality changes by a constant annual compound percentage over all lives and all calendar cohorts. This could also be relaxed to allow for exact changes.

In the context of this paper one unit of a whole of life annuity to a life currently aged x, denoted as , is defined as a product that pays an amount of 1 at the beginning of each year provided the life is then alive.

Furthermore, one unit of a life assurance to a life currently aged x, denoted as , is defined as a product that pays an amount of 1 at the end of the year of death.

## 3.4 Example application of LHR

Using the AM80 tables and an interest rate of 6% pa, the present value of 1 unit of an assurance to a male aged 25 can be calculated as R0.072481.

Using the PMA80 tables and an interest rate of 6% pa, the present value of 1 unit of an annuity to a male aged 55 can be calculated as R12.312183.

The improvement in mortality can now be applied. Any reference to a change in mortality should be considered as a compound annual change, i.e. from a specified age, the mortality rate changes by a fixed percentage every year. This change in the mortality rate can be regarded as extra divergence from the mortality rate used to price a product when it is issued.

To illustrate this concept, consider the mortality rate of a life currently aged 25. In order to price a product issued to this life standard mortality rates will be used, that is *etc.* If mortality rates decrease by 1% each year then the adjusted mortality rates to use can be calculated as follows: *etc.*

The natural hedge is then obtained by taking the quotient of the change in present values, where the numerator is the change in present value of the product to be hedged and the denominator is the change in present value of the hedging product.

As an example, consider one unit of an annuity that has a change in present value of R0.35. If this product is to be hedged by an assurance that has a change in present value of R0.02, then:

The natural hedge amount can be interpreted as follows: For each R1 of an annuity, payable at the beginning of each year, to a life currently aged x, provided the life is then alive, an assurance contract to the value of R17.50 is necessary to hedge the insurance company against the pre-specified change in mortality.

Using this method in order to obtain a natural hedge and then calculating the total present value, first under the assumptions used in pricing, with or without improvements, and then under the assumption of a possible compound annual change in mortality over and above that assumed in pricing, it can be seen that the total present value does not change.

The alternative view of this is that if a certain number of assurances are available, the hedging ability could be calculated if the annuity portfolio improves more than expected in pricing, i.e. it could withstand an additional improvement of say 0.2% per annum above pricing.

We are also interested in how long the hedge will last. In order to test this, the hedge can be kept constant while increasing the ages of the two lives. However, the stability of the hedge is very dependent on the initial ages used to obtain the hedge. Furthermore, how and when it will be necessary to restore the hedge will depend on the exposure that the company under consideration is willing to face.

The change in the present value of one unit of an assurance issued to a life aged 25-years old can be calculated.

Table 2 - Change in present value for an assurance issued to a 25-year old male

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Whole life assurance: Male Aged 25, Payable at end of year | | | | |
| Change | **PV** | **% Change** | **Variance** | **% Change** |
| 0.00% | 0.072481 | 0.00% | 0.006454 | 0.00% |
| -0.50% | 0.066186 | -8.69% | 0.006313 | -2.18% |
| -1.00% | 0.059952 | -17.29% | 0.006163 | -4.51% |
| -1.50% | 0.053796 | -25.78% | 0.006004 | -6.97% |
| -2.00% | 0.047720 | -34.16% | 0.005839 | -9.53% |
| 0.50% | 0.078821 | 8.75% | 0.006585 | 2.04% |
| 1.00% | 0.085194 | 17.54% | 0.006707 | 3.93% |
| 1.50% | 0.091588 | 26.36% | 0.006819 | 5.67% |
| 2.00% | 0.097990 | 35.19% | 0.006921 | 7.25% |

Using the PMA80 and PFA80 tables, for males and females respectively, a given range of possible changes in mortality rates and an interest rate of 6%, the change in the present value of one unit of an annuity issued to a life aged 65-years old can be calculated.

Table 3 - Change in present value for an annuity issued to a 65-year old male

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Whole life annuity: Aged 65, Payable in advance | | | | |
| Change | **PV** | **% Change** | **Var** | **% Change** |
| 0.00% | 9.639131 | 0.00% | 14.037590 | 0.00% |
| -0.50% | 9.787840 | 1.54% | 14.591661 | 3.95% |
| -1.00% | 9.943421 | 3.16% | 15.185084 | 8.17% |
| -1.50% | 10.106437 | 4.85% | 15.822295 | 12.71% |
| -2.00% | 10.277495 | 6.62% | 16.509589 | 17.61% |
| 0.50% | 9.496789 | -1.48% | 13.519192 | -3.69% |
| 1.00% | 9.360357 | -2.89% | 13.033228 | -7.15% |
| 1.50% | 9.229426 | -4.25% | 12.576820 | -10.41% |
| 2.00% | 9.103627 | -5.56% | 12.147408 | -13.47% |

In the above two tables, a negative percentage change denotes an annual compound decrease in mortality rates. The first row in each table, where a change of 0.00% is indicated, is calculated using the mortality rates assumed when pricing the product.

Assume an annual compound decrease in mortality of 1%, and that an insurance company wants to use a life assurance contract issued to a male life aged 25 to hedge a whole of life annuity issued to a male life aged 65. The hedge amount can then be calculated as follows:

Therefore, for every unit of a whole life annuity issued to a male life aged 65, 24.287 units of life assurance to a male life aged 25 should be issued in order to protect the issuing company against an annual compound decrease of 1% in mortality rates.

In order to test the sensitivity of the hedge once it is in effect, the following table can be constructed and calculated: (explanation follows beneath the table)

Table 4 - Sensitivity of a natural hedge

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Row | Type | Change | Mortality Age | PV Age | PV | | Change |
| 1 | Assurance | Original | 25 | 25 | 0.072481 | |  |
| 2 | Annuity | Original | 65 | 65 | 9.639131 | |  |
| 3 | Total | Original | 25 & 65 | 25 & 65 | 11.399426 | |  |
| 4 | Assurance | 1% Improvement | 25 | 25 | 0.059952 | | -17.29% |
| 5 | Annuity | 1% Improvement | 65 | 65 | 9.943421 | | 3.16% |
| 6 | Total | 1% Improvement | 25 & 65 | 25 & 65 | 11.399426 | 0.00% | |
| 7 | Total | Original | 25 & 65 | 26 & 66 | 11.201403 |  | |
| 8 | Total | 1% Improvement | 25 & 65 | 26 & 66 | 11.206053 | 0.04% | |
| 9 | Total | Original | 25 & 65 | 27 & 67 | 11.007780 |  | |
| 10 | Total | 1% Improvement | 25 & 65 | 27 & 67 | 11.016502 | 0.08% | |
| 11 | Total | Original | 25 & 65 | 28 & 68 | 10.819575 |  | |
| 12 | Total | 1% Improvement | 25 & 65 | 28 & 68 | 10.831616 | 0.11% | |
| 13 | Total | Original | 25 & 65 | 29 & 69 | 10.637858 |  | |
| 14 | Total | 1% Improvement | 25 & 65 | 29 & 69 | 10.652288 | 0.14% | |
| 15 | Total | Original | 25 & 65 | 30 & 70 | 10.463678 |  | |
| 16 | Total | 1% Improvement | 25 & 65 | 30 & 70 | 10.479380 | 0.15% | |

Note that for the sake of simplicity some of the rows in the above table have been removed. Table 4, shown above, can be interpreted as follows, considering a 1% improvement hedge:

The first two rows show the present values of the two products under the assumption of no change in the mortality rates. This will be the price charged by the insurance company. The third row shows the total present value using the ratio calculated earlier. Recall that the ratio can be expressed as.

The fourth and fifth rows show the present value of the two products calculated under the assumption of an annual compound decrease of 1% in mortality rates. In the last column it can be seen that without any hedging the present value of the annuity increases by 3.16% while the present value of the assurance decreases by 17.29%.

The sixth row then shows the total present value under the hedge and the assumption of an annual 1% compound improvement in mortality rates. It can be seen that the total present value is now the same under the assumption of no change in row three the improvement of 1% after the hedge in row six.

The third column “Mortality Age” can be interpreted as the age at which the change in mortality rates start. In the example the ages shown in the third column stays the same because that is the age at which the products were issued.

The fourth column “PV Age” shows the ages at which the total present value is calculated. For the first five years after the products were issued the total present value is calculated under the assumption of the hedge calculated earlier. The total present value is first calculated under the assumption of no change in the mortality rates and then under the assumption of an annual compound decrease of 1% in the mortality rates.

From the table it can be seen that after 1 year, when the assured life is now 26 years old and the annuitant is now 66 years old, the total present value, or hedged position, has changed by 0.04% from the total present value if mortality rates do not change. This change of 0.04% amounts to extra liability of only 0.00465 currency units[[2]](#footnote-2).

Even more importantly, after five years the total present value under the hedge and the assumption of an annual compound decrease of 1% in mortality rates have only deviated by 0.15% from the total present value if mortality rates were not to change at all. Therefore, the natural hedge significantly reduces the company’s total exposure to longevity risk. The absence of the hedge would have meant the following: the expected change in annuities PV after 5 years is from 9.639131 to 8.178181, while with an additional unexpected 1% improvement it would reduce by a lot less to only 8.600211, which is 5.16% more than expected, which could be a significant amount on an annuity portfolio. Table 5 shows the difference between what was expected (no improvement above priced for) vs. each potential improvement.

Table 5 - Annuity PV After 5 years (age 70)

|  |  |  |
| --- | --- | --- |
| Change | PV | % Change |
| Original | 8.178181 | 0.00% |
| 0.50% | 8.384289 | 2.52% |
| 1.00% | 8.600211 | 5.16% |
| 1.50% | 8.826754 | 7.93% |
| 2.00% | 9.064788 | 10.84% |
| 2.50% | 9.315151 | 13.90% |
| 3.00% | 9.578427 | 17.12% |
| 3.50% | 9.854629 | 20.50% |
| 4.00% | 10.142902 | 24.02% |

If the hedge was not in place and even a 0.5% improvement occurred above expected, the PV will be 2.52% higher than expected, compared to the 0.15% of a hedged position.

Whether or not the company will choose to rebalance the hedge after five years depends on the company’s policy regarding the amount of longevity risk that it is willing to face as well as the actual amounts insured and the availability of extra products that can be used to rebalance the hedge.

# The LHR Applied

## 4.1 Hedging principles

The initial hedge as considered above allowed for changes in present values of annuities and assurances over their full future lifetimes, thus the total Present Value (PV). It is clear from the graphs below that with improvements in longevity, the PVs move in opposite directions:

Figure 3 - PV of a Whole Life assurance for different longevity improvements for a life aged 25

Figure 4 - PV for an annuity to a life aged 65 for different improvements in longevity

However, a difficulty arises in that annuities have an average age much older than assurances, since they are normally paid in the form of a pension from the age of 60 or 65 with future expected payment period of approximately 20+ years, for example. Assurances in the form of life cover or term assurance are normally taken out much earlier in life, to cover bonds for example, and thus may have a future lifetime of 60+ years. The total hedge works if both portfolios are kept until the last policyholder dies, but this may simply not be practical. Annuities may typically be kept constant with no rebalancing, but life cover products can lapse and re-enter, or with the age aggressive pricing, lose all hedging ability past a one-year period, since pricing is updated annually. When the changes in Present values are considered to calculate the LHR, nothing stops us from doing a one year hedge only, by calculating the PVs over the next year only.

Some theoretical effects of longevity improvements are illustrated below.

## 4.2 Effects of increased longevity

Longevity will create losses on annuities since more payments are made to annuitants over time as they live longer.

Figure 5 - Cash-flows for an annuity aged 65 with and without improvements

The red line lies to the right for the full term, showing losses which accumulate for the full term of the portfolio. The effect on assurances is similar:

Figure 6 - Cash-flows for an assurance for a life aged 25 with and without improvements

Deaths are delayed and paid out later in the term, causing more premium income. The combined effect can be seen below:

Figure 7 - Differences in the cash-flows for an annuity and assurance

The differences in the cash-flows of the annuities and the assurances are initially negative, then turn positive, then later turn negative again. After approximately 50 years, the red and green lines fall on the same line. Due to the LHR the sum of the red line is zero. However, for this to be zero we need to keep the hedge for 95 years. Again, this is not practical, creates interest rate and other risks. A shorter hedge is required.

If we consider only the first few years and try to smooth our cash-flows and hedge against a possible longevity improvement, we need to pull the red line upwards by means of more assurances, thus increasing the LHR. The theoretical approach to calculate the LHR would simply be to make use of the changes in PV over a shorter period of time.

## 4.3 Hedging for different time horizons

In an attempt to show the practical implications of the LHR, we assumed a portfolio of 100000 annuities of R60000 per annum issued to males aged 65 and 100000 life assurances issued to males aged 25. We made use of PMA80 mortality for the annuitants and AM80 for the policyholders with life assurance. The sum assured on the life products was calculated using the LHR. Interest was ignored in the calculations of the present values. These assumptions were used in all of the following illustrations.

If assurance rates are changed, the LHR would change and the hedge would thus need to be reset. Key assumptions made in the following illustrations are that assurance rates cannot change and policyholders cannot lapse and re-enter within the time period considered. We considered a closed portfolio with death as the only decrement. These assumptions could be allowed for in the hedge if required.

It should be noted that the aim of hedging is not to create profits, but merely to stabilise cash-flows. A hedge attempts to smooth the cash-flows by offsetting profits on the one product with losses on the other product. In this section, the terms profits and losses will refer to positive and negative cash-flows respectively. These terms differ from usual business net profits, since these take expenses, the cost of rebalancing, management charges and other components of net profit into account.

We investigated the hedge over several terms or time horizons. For a term of years, we calculated the present values of the assurance and annuity using different increases in longevity. We then calculated the difference between the present values of each product for two different longevity improvements and divided these differences to find the amount of assurances required as defined by the LHR. For example, we assumed a 1% improvement in longevity in pricing and hedged for another 1% (total 2% improvement). As such, we subtracted the present value of the assurance calculated assuming a 2% improvement from the present value of the assurance calculated assuming a 1% improvement. The change in the annuity value was calculated similarly. We then calculated the LHR and used this value to determine the number of units of life assurance required.

The following graphs show the effect of the different terms on the cash-flows resulting from the hedge.

This graph shows the cash-flows for the hedge where the hedge was done for a term of 2, 3, 4 and 5 years. The next graph show the cash-flows for the hedge where the hedge was done for a term of 5, 10, 15, 20, 25 and 55 years. Assuming a terminal age of 120, all annuitants will be dead in 55 years time.

As can be seen from the graphs, hedging in this manner will allow the firm to break even or make profits if the actual longevity experience lies between the longevity assumed in pricing and the longevity hedged for. We assumed longevity improvements of 1% in pricing and hedged for another 1%. The above graphs thus show zero profits at 1% and 2% and profits greater than zero if the longevity improvements are between 1% and 2%. If the improvements are less than 1% or greater than 2%, losses are incurred.

Another observation that can be made is that the profit and losses incurred depend on the term of the hedge. Larger losses are incurred for longer terms, but larger profits are made for longer terms as well. The risk appetite of the firm and the certainty of their estimates of future longevity increases may thus have a large impact on the decision regarding the term of the hedge. The following table shows the large difference in the size of the cash-flows for hedges of different terms.

Table 6 - Cash-flows resulting from the different longevity improvements

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Term in years | Cash-flows resulting from the different longevity improvements, on 100 000 annuities of R60 000pa | | | | | | |
|  | 0.0% | 0.5% | 1.0% | 1.5% | 2.0% | 2.5% | 3.0% |
| 2 | -7,560 | -2,835 | 0 | 945 | 0 | -2,835 | -7,560 |
| 3 | -36,844 | -13,788 | 0 | 4,577 | 0 | -13,674 | -36,388 |
| 4 | -109,017 | -40,709 | 0 | 13,456 | 0 | -40,026 | -106,283 |
| 5 | -254,035 | -94,649 | 0 | 31,143 | 0 | -92,225 | -244,341 |
| 10 | -4,194,472 | -1,543,214 | 0 | 495,212 | 0 | -1,430,436 | -3,743,191 |
| 15 | -26,307,038 | -9,555,539 | 0 | 2,989,354 | 0 | -8,420,619 | -21,762,904 |
| 20 | -97459,022 | -35,046,791 | 0 | 10,743,748 | 0 | -29,650,557 | -75,849,548 |
| 25 | -242,351,698 | -86,736,115 | 0 | 26,304,409 | 0 | -71,718,644 | -182,279,136 |
| 55 | -664,508,381 | -256,879,401 | 0 | 88,104,514 | 0 | -263,794,107 | -696,816,137 |

Table 6 above shows that losses are incurred if actual longevity improvements are below those priced for or above those hedged for, as was seen on the graph. It is clear that the cash-flows for shorter terms are a great deal less than those for longer terms. The possible losses incurred when hedging for longer terms, such as terms longer than 20 years, may be too large for these terms to be a feasible option for smaller insurers. Not hedging at all has the same risk.

An important consideration may be the cost implications relating to the term of the hedge. The cost of acquiring the amount of assurances needed for the hedge for a specific term may outweigh the benefits of that hedge. The table below shows the assurances needed for the hedges for various terms and ages of policyholders with assurances. The annuitants are aged 65 in all columns.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Assurances required per 1 annuitant of age 65 | | | |
| Term in years | Age 25 | Age 35 | Age 45 | Age 55 |
| 2 | 12.7073 | 2.2244 | 0.7763 | 0.3694 |
| 3 | 26.6163 | 5.5311 | 1.9572 | 0.9474 |
| 4 | 41.5279 | 9.6527 | 3.4675 | 1.7077 |
| 5 | 57.1470 | 14.3463 | 5.2386 | 2.6257 |
| 10 | 132.9090 | 39.8655 | 16.0030 | 8.7777 |
| 15 | 175.2038 | 58.2888 | 25.9977 | 15.6785 |
| 20 | 172.9677 | 64.4059 | 31.9715 | 21.3641 |
| 25 | 143.6045 | 60.2385 | 33.2897 | 24.9484 |
| 55 | 27.7738 | 24.2982 | 26.8835 | 33.9814 |
| Whole life | 27.3120 | 27.7920 | 29.5216 | 34.9381 |

As can be seen, in the second column, the assurances required increase steadily as term increases up until 15 years. After 17 years, the assurances required decrease as term increases, but the rate of this decrease is not as large as the rate of the initial increases. The cost of attaining the required amount of assurances for longer terms may be high and as a result the insurer would need to find the perfect cost/hedge benefit within its own limits.

The graph below gives us a graphical representation of the table above. It is clear from the graph that the hedge will be much more stable when older ages are used in the LHR. This is a useful aspect as the average age of assurance portfolios is usually much larger than age 25.

Table 7 - Assurances required for the hedges of various terms at various ages

Since no portfolio of assurances or annuities is static, more frequent hedging will be required. Generally annuity portfolios only change by means of new entrants and deaths. Assurance portfolios are much more volatile with lapses, lapse and re-entry, new entrants and deaths. Assurance premiums may even be adjusted regularly for existing policyholders to counter lapses. This change in internal mortality used on existing policyholders will of course change the LHR and require a rebalance of the hedge. A change in interest rate will also have a small effect, but for the small time period considered it is almost negligible.

The above hedge for a specific term of say 5 years, will of course mean that before or after the five years the hedge will not be exact. With this strategy the movement of the LHR is anticipated over time and the required hedge set up to be exact not at present but at some point in future.

## 4.4 Hedging for different spreads of improvement

Consider annuities and assurances priced for a 1% improvement. If we then hedge for further improvements in steps of 0.5% each, using a term of 5 years, the cash-flows will be as follows:

Figure 8 - Cash-flows resulting from the hedges for different additional longevity improvements above pricing

This graph shows that the choice of the size of the margin may be linked to the risk appetite of the insurance company, and may thus have a significant financial effect. As can be seen above, a small margin, such as 0.5%, does not enable the company to make significant profits on anticipated improvements, but has lower downside risk. A larger margin increases the range over which the company can make a profit. It also increases the amount of loss possible, should the improvement be lower than priced for (expected). A margin of 2% allows profits to be made if the longevity improvements are between 1% and 3%, as well as allowing the highest possible profits to be made at any rate between 1% and 3%.

The downside of a larger margin is the increased losses that are possible. This happens when improvements turn out to be less than that hedged for. The graph above clearly shows that the losses incurred by using a 2% margin are the largest and those incurred by the 0.5% margin are the smallest.

The assurances required in these hedges may need to be considered. The table below shows how many assurances are required if increases in longevity of 1% are assumed and the various additional longevity increases are hedged for.

Table 8 - Hedging requirements for additional longevity improvement above pricing

|  |  |
| --- | --- |
| Additional longevity improvement above pricing | Assurances required |
| 0.5% | 56.9886 |
| 1% | 57.1470 |
| 1.5% | 57.3053 |
| 2% | 57.4637 |

The assurances required increase as the additional longevity improvements increase. The increases shown in the table are rather small, but it should be noted that these are the assurances required to hedge R1 of annuities. The actual amounts of assurances required may vary greatly when calculated for an entire portfolio and may then result in a significant financial decision for the insurer. Hedging for more additional longevity improvements may initially seem more profitable, but the cost of attaining the required amount of assurances may make hedging for less additional improvements more profitable. The cost of attaining the required amount of assurances may be so large as to make hedging for larger additional longevity improvements impossible for smaller insurance companies.

A similar effect is observed when holding the longevity improvement to be hedged constant and varying the longevity improvement assumed for pricing. In the following graph, we made use of a term of 5 years and hedged a longevity increase of 2%.

Figure 9 - Cash-flows resulting from the hedges for different longevity improvements used for pricing

The graph shows the effect of pricing for different longevity improvements, but hedging for a fixed 2% longevity improvement. A similar shape to the previous investigation occurs. If the actual longevity improvements fall in between the improvements priced and hedged for, a profit is made. If longevity improvements are more than the improvements hedged for or less than those priced for, a loss is incurred.

Once again a margin of 0.5% allows the company to break even or make losses. This strategy does not really allow the company to make profits. Larger margins allow larger ranges over which profits can be made as well as larger profits. However, larger losses could also be incurred by making use of larger margins.

The assurances required in these hedges may need to be of significance. The table below shows how many assurances are required if increases in longevity of 2% are hedged for and the various longevity increases are priced for.

|  |  |
| --- | --- |
| Longevity improvement used for pricing | Assurances required |
| 0% | 56.8260 |
| 0.5% | 56.9865 |
| 1% | 57.1470 |
| 1.5% | 57.3074 |

The values in the table above represent the amounts of assurances needed to hedge R1 of annuities. These values increase as the longevity improvement priced for increases and could thus result in a significant financial decision for the insurer. The actual amounts of assurances required may vary greatly when an entire portfolio is considered.

This table and the graph above suggest that, although pricing for 0% longevity improvements and hedging for 2% allows the possibility of the largest losses if actual longevity improvements are above 2%, it may be the most cost effective option. This option allows the largest range over which profits can be made as well as the largest profits to be made. It also requires the least amount of assurances by the LHR. It thus minimises the cost of attaining the required amount of assurances, making this a more viable option for many insurers.

## 4.5 Changes in interest rates

At the low rates of interest, such as below 4%, there is very small effect on the LHR. At larger rates, more assurances are required for the hedge since a change in the interest rate has a larger effect on the longer term of the assurances. The graph below shows the number of assurances required for the hedge assuming the full term is hedged. The life assured is aged 25 and the annuitant is aged 65.

Figure 10 - Assurances needed per R1 of annuity for different rates of interest

## 4.6 A practical example using simulation

Apart from the fact that the portfolios are not static, different portfolios will also have different structures. We may not expect a uniform or normal distribution for age or for the sums assured/annual annuity payments. However, this could be achieved by the means of reinsurance. An example of how this can be achieved may be to reinsure all sums above a certain amount, as mentioned earlier. If portfolios are large, it may create a more stable structure for age and sums assured in the portfolios. Policy options may also help determine the structure. An example of such an option may be to allow sums assured in multiples of R100000 only. However, if we simply allow for an average age and an average sum assured the effects on the LHR is as follows:

Figure 11 - Assurances of a life aged 25 required for hedging R1 of annuity at each age

This graph assumes a constant age of 25 for the assured life and different ages for the annuitant. If we keep the annuitant age constant and consider different ages for assured lives the LHR quantities are as follows:

Figure 12 - Assurances required at each age to hedge a single 65 year old annuity

Putting them together gives the following graph:

Figure 13 - # Assurances to hedge for different annuity and assurance ages

Thus, if we would like to hedge an annuity portfolio with an assurance portfolio, we can simply calculate the average ages and average amounts, calculate the LHR based on those and hedge on a portfolio basis. We would however need sufficiently large numbers of policies to make it stable, or even make the smoothing of cash-flows due to the hedge visible. A practical example will show this clearly. If we make use of a portfolio of 10000 assurances uniformly distributed between R500k and R3.5m with average age 45 then a 1% improvement may do the following to cash-flows:

Figure 14 - Cash-flow simulation for a portfolio of assurances for different longevity improvements

The red and blue line cross since the lives will die eventually with payment of the sum assured merely postponed. We did the same with annuities with the following effect:

Figure 15 - Cash-flow simulation for a portfolio of annuities for different longevity improvements

We considered a portfolio of 10000 annuities uniformly distributed between 60k and 240k per annum with an average age of 65 in the above graph. The lines look much smoother than for the assurances but the scale on the y-axis is different. If we put these together we get the following combined effect on cash-flows, as shown by the green line below:

Figure 16 - Difference in cash-flows for a simulated portfolio of assurances and annuities

If we consider the amounts then cash-flows may not be sufficiently stable, depending on the viewpoint of the insurer. This clearly shows that large portfolios are required to create a smoother cash-flow. It is therefore suggested that the smaller portfolios may use this method in conjunction with reinsurance to create the desired smoothness.

# Conclusion

The changes in present value can easily be calculated for two portfolios with opposite reactions to a change in mortality rates. These changes in the present values can then be used to calculate one method of obtaining a natural hedge to mitigate longevity risk.

The stability of the above method for obtaining a natural hedge can easily be investigated for any two particular portfolios. Once the hedge described above has been implemented it seems to be stable for a few years, but this will depend on the makeup of the portfolios involved.

Using this method an underlying mortality index is not required. Under this method a portfolio becomes a new entity under which its own complete future mortality table could be estimated. The LHR could then be calculated using the mortality of the opposite portfolio in the hedge. Since the portfolio becomes an entity on its own, it could be sold with its own inherent mortality including a certain allowance for improvements, in the form of a security. Excess assurances or annuities within a company could be reinsured or swapped.

The natural hedge by means of the LHR, as suggested in this paper, may be used to effectively smooth cash-flows and hedge against mortality improvements. The hedge proves to work well for large portfolios. Since many insurers have both assurances and annuities, it makes sense to consider the combined effect of mortality improvements on the total portfolio. If this is considered in conjunction with reinsurance, it may actually reduce the reinsurance cost, reduce risk of unexpected longevity changes, or even change the total reinsurance strategy of a company.

More research could be done on securitisation of such portfolios and parts thereof. Swapping the mortality table with a mathematical function may produce eloquent solutions to the LHR, and improve pricing of such securities. Current improvements are applied to tables with a fixed maximum age, where in reality the maximum age will be extended due to the improvements. This creates an underestimation of the extra number of payments on largely the annuities side. Research is required to quantify this effect, and may be used to improve the above hedge results.

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1. Pairing individual annuities with individual assurances, then what is left after the first pairing forms part of the second pairing, and so on. [↑](#footnote-ref-1)
2. The currency unit for the rest of the paper will be South African Rands where say five Rands = R5. [↑](#footnote-ref-2)