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Investigation of Hedging Strategies between Assurances And Annuities for the Purpose of Mitigating Longevity Risk

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- Actuarial theory
- The Longevity Hedge Ratio (LHR)
    - LHR
    - Assumptions
    - Example
- Varying assumptions and conditions
- LHR applied on a simulation
- Conclusion

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Introduction

- We all know that nowadays people are living longer, this could be celebrated
- Mortality improvements are very difficult to model, in fact, is it not impossible? They vary over time for different populations and in different age ranges, can occur in jumps with medical advances:
  - antibiotics, heart transplant/bypass, cure for cancer, HIV/AIDS
- Many drivers of longevity are complex –
  - Obesity, genetic engineering, pollution, global warming: will longevity slow down, stabilise, decrease?

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Introduction

CountryES	Year	Age	$_nq_x$	Change p.a.	Country	Year	Age	$_nq_x$	Change p.a.
Australia	1990	25	0.00460		SA	1990	25	0.01746	
	2008	25	0.00293	-2.54%		2008	25	0.05516	6.19%
	1990	55	0.03524			1990	55	0.08081	
	2008	55	0.02063	-3.02%		2008	55	0.12480	2.39%
Brazil	1990	25	0.01096		China	1990	25	0.00781	
	2008	25	0.00847	-1.44%		2008	25	0.00436	-3.29%
	1990	55	0.06932			1990	55	0.06092	
	2008	55	0.04954	-1.88%		2008	55	0.04151	-2.15%
USA	1990	25	0.00623		Germany	1990	25	0.00396	
	2008	25	0.00491	-1.33%		2008	25	0.00215	-3.45%
	1990	55	0.04577			1990	55	0.04448	
	2008	55	0.03473	-1.55%		2008	55	0.02968	-2.27%
UK	1990	25	0.00339						
	2008	25	0.00273	-1.21%					
	1990	55	0.04197						
	2008	55	0.02790	-2.29%					

Introduction

- Vaupel et al (2009) predicted that ½ of babies born in 2007 in Germany will reach age 102
- According to Institut National d'Etudes Démographiques (INED) mortality for ages 40-70 reduced by 50% in Western Europe from 1952 to 2006
- The # of centenarians doubled every 10 years from 1950 to 1990, i.e. in France there were more than 20,000 centenarians in 2008, compared to only 200 in 1950

Introduction

- To consider the natural hedge between Assurances and Annuities
- We know that these two move in opposite directions due to mortality improvements and that this hedge exists, but questions are:
  - can it be used, how
  - What time frames should be considered
  - What sizes of portfolios are needed
  - Stability of the hedge over time
  - Format of cash-flows

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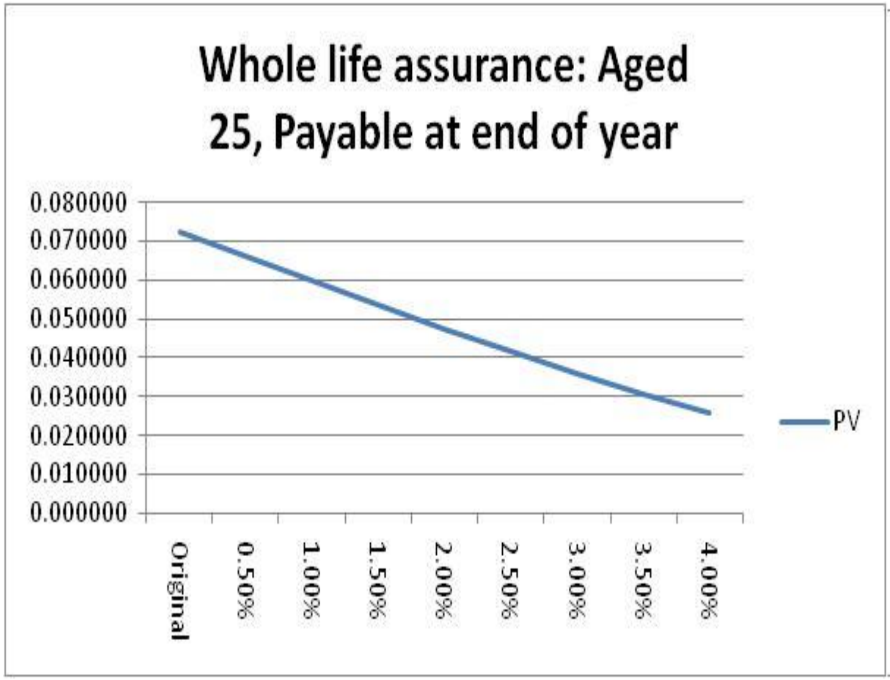
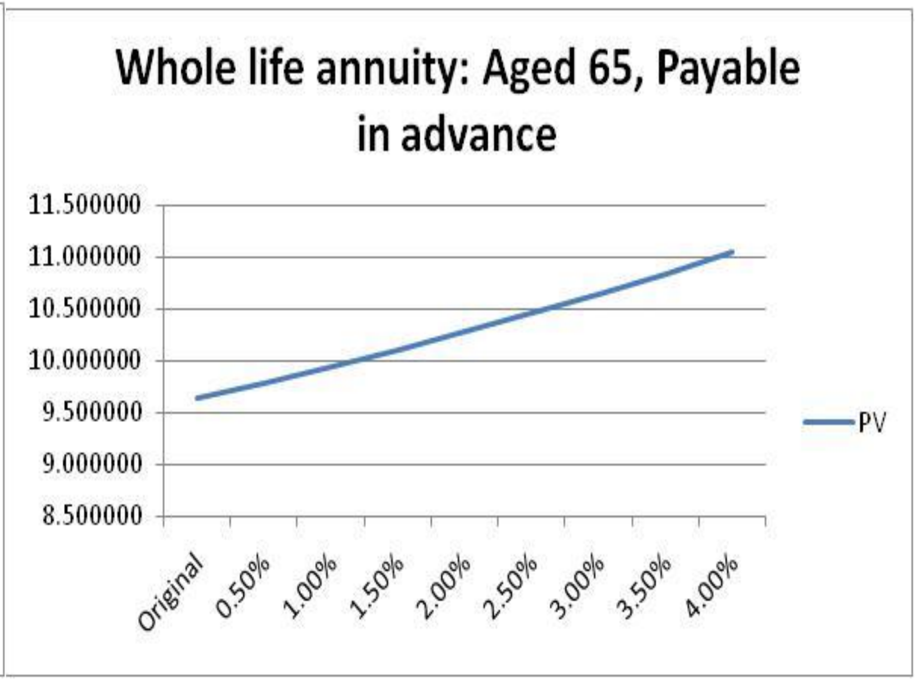
Introduction





Decrease in mortality rate

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Introduction

- Avoid the longevity risk, many companies in South Africa are now selling annuities as a "Living Annuity", funds that can be depleted, investment and longevity risks shifted to policyholder
- Securitising annuity portfolios and selling them on the capital markets have been met with limited success. Longevity risk, can provide great diversification to asset portfolios, since this risk is not linked to the fuel crisis, war in the Middle East…
    - Reinsurance or co-insurance / swap agreements could work…

Introduction

- Normally the natural hedge considers an annuity and life cover to the same life, taking one of two common forms:
  - “cash-back annuities”, with decreasing life cover to the annuitant
  - Deferred annuities, with life cover initially, followed by the annuity
- We consider this on portfolio basis below, where the annuity holder and the person with life cover may be very different
- Mortality is comonotonic: a single factor affecting mortality would normally affect all lives, and in the same direction

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Actuarial theory

Actuarial theory

- In practice the improvement is usually applied to the mortality rate q_x.
- Let the annual improvement rate r = 2%,
- May be $q_{x+3} \times (1-r)^3 = q_{x+3} \times (0.98)^3$
- $(1-r)^n = e^{-n \cdot \bar{r}}$ or $\bar{r} = -ln(1-r)$
- Where the force of improvement is $\bar{r}$
- Remember that $_1p_x = e^{-\mu_x}$ (CFM assumed)
- And $_1q_x = \mu e^{-\mu_x}$
- Then $_tp_x^* = e^{-\mu_x(1-\bar{r})t} = e^{-\tilde{\mu}_x t}$ with $\bar{r} < 0$, $0 < t < 1$

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Actuarial theory

- Portfolio of N independent lives, $i = 1, \dots, N$
- Let v_i= time under observation or exposed to risk
- Let d_i= 1 if life i died and zero otherwise
- probability of observing the particular data set, for life i only is $f_i(d_i, v_i) = e^{-\mu v_i}\mu^{d_i}$
- likelihood function $\prod_{i=1}^{N} e^{-\mu v_i}\mu^{d_i} = e^{-\mu v}\mu^{d}$
- where $v = \sum_{i=1}^{N} v_i$ and $d = \sum_{i=1}^{N} d_i$
- (MLE) of μ as $\hat{\mu} = \frac{d}{v}$
- This result does not consider different ages, sex or smoker status, and is a universal result for a cohort/portfolio of lives

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Actuarial theory

- An alternative is to remember that $\mu_{xy} = \mu_x + \mu_y$
- x and y are different in year of birth, sex, age and smoker status
- let $X = \{x_1, \ldots, x_N\}$ then $\mu_X = \mu_{x_1} + \cdots + \mu_{x_N}$
- Complication: different sums assured
  - Let the sums assured $S_1 < S_2$
  - $S_1 e^{-\mu^1 t} = S_2 e^{-\mu^2 t}$
  - Solve to get $\mu^2 = \mu^1 + \frac{1}{t}\ln(\frac{S_2}{S_1})$
- We see the portfolio as a single entity with single μ

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The Longevity Hedge Ratio (LHR)

- LHR
- Assumptions
- Example

The Longevity Hedge Ratio (LHR)

- develop approximations for A_X and $\ddot{a}_X$ for a total portfolio
- $LHR = \dfrac{|Change\ in\ annuity\ portfolio\ PV|}{|Change\ in\ assurance\ portfolio\ PV|}$
- $LHR = \#Assurances\ required\ per\ annuity$

- The PV could be for single lives or portfolios

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The Longevity Hedge Ratio (LHR)

Assumptions

- The term structure of interest rates is flat at 6%
- Death benefits are paid at the end of the year of death.
- A male life aged 25 for the assurance and male life aged 65 for the annuity.
- Annuity payments are annual and made at the beginning of every year.
- Mortality rates PMA80 and PFA80 tables, for pensioner's mortality, for males and females respectively.

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The Longevity Hedge Ratio (LHR)

<u>Assumptions</u>

- Mortality rates used when pricing life assurances follow those specified in the AM80 and AF80 tables, for males and females respectively.
- Mortality changes by a constant annual compound percentage over all lives and all calendar cohorts.
- All assumptions could be relaxed to allow for exact changes.

The Longevity Hedge Ratio (LHR)

Example

PV of 1 unit of an assurance to a male aged 25 = R0.072481
PV of 1 unit of an annuity to a male aged 55 = R12.312183

Now assume some annual improvement and recalculate

PV of assurance reduced by R0.02
PV of annuity increased by R0.35

$$LHR = Assurances\ required = \frac{0.35}{0.02} = 17.5$$

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The Longevity Hedge Ratio (LHR)

Row	Type	Change	Mortality Age	PV Age	PV	Change
1	Assurance	Original	25	25	0.072481	
2	Annuity	Original	65	65	9.639131	
3	Total	Original	25 & 65	25 & 65	11.399426	
4	Assurance	1% Improvement	25	25	0.059952	-17.29%
5	Annuity	1% Improvement	65	65	9.943421	3.16%
6	Total	1% Improvement	25 & 65	25 & 65	11.399426	0.00%
7	Total	Original	25 & 65	26 & 66	11.201403	
8	Total	1% Improvement	25 & 65	26 & 66	11.206053	0.04%
9	Total	Original	25 & 65	27 & 67	11.007780	
10	Total	1% Improvement	25 & 65	27 & 67	11.016502	0.08%
11	Total	Original	25 & 65	28 & 68	10.819575	
12	Total	1% Improvement	25 & 65	28 & 68	10.831616	0.11%
13	Total	Original	25 & 65	29 & 69	10.637858	
14	Total	1% Improvement	25 & 65	29 & 69	10.652288	0.14%
15	Total	Original	25 & 65	30 & 70	10.463678	
16	Total	1% Improvement	25 & 65	30 & 70	10.479380	0.15%

The Longevity Hedge Ratio (LHR)

Change of only 0.15% in PV for a hedged position using LHR

Change of Annuity PV After 5 years (age 70) with no hedge

Change	PV	% Change
Original	8.178181	0.00%
0.50%	8.384289	2.52%
1.00%	8.600211	5.16%
1.50%	8.826754	7.93%
2.00%	9.064788	10.84%
2.50%	9.315151	13.90%
3.00%	9.578427	17.12%
3.50%	9.854629	20.50%
4.00%	10.142902	24.02%

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Varying assumptions and conditions

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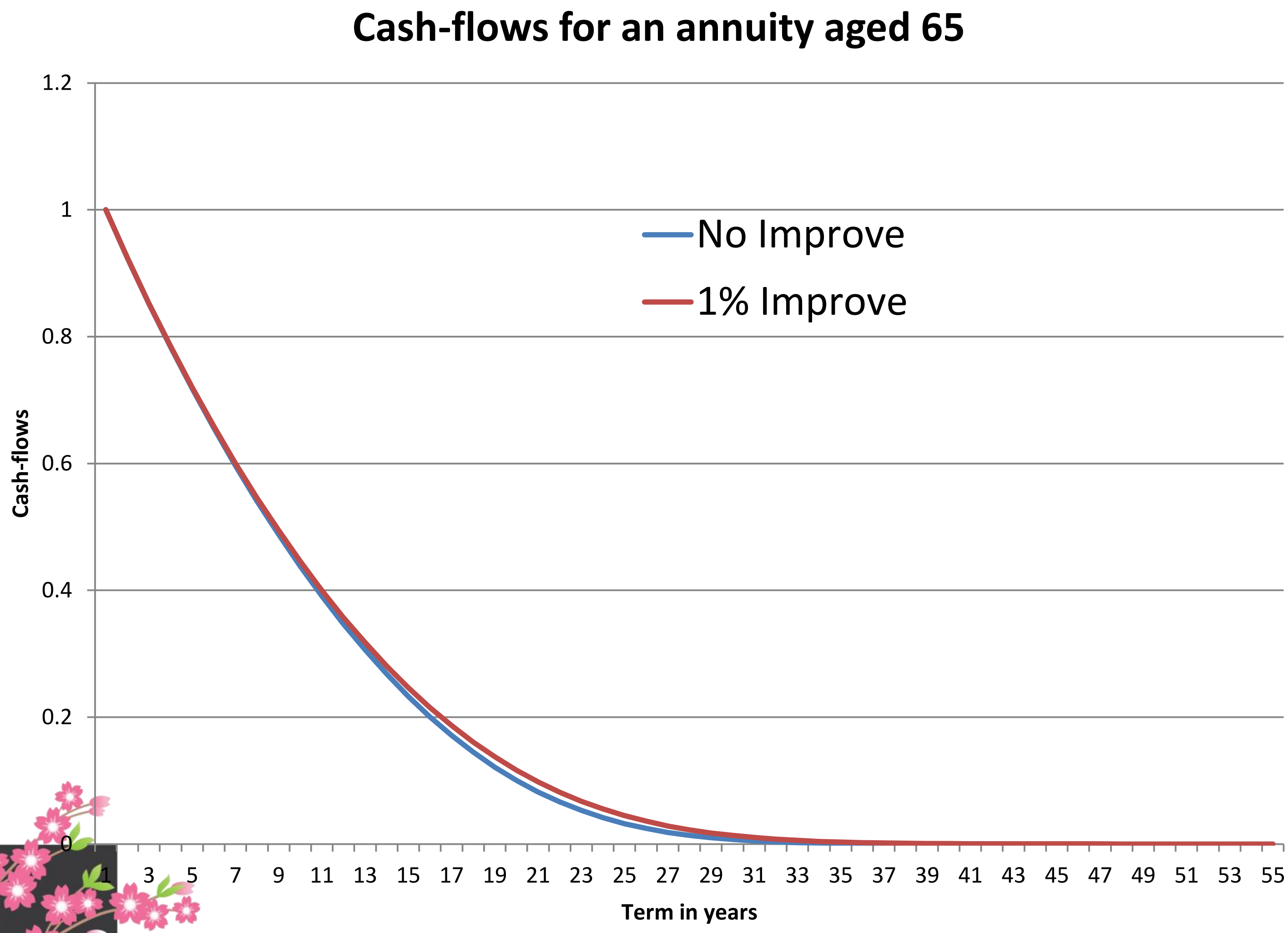
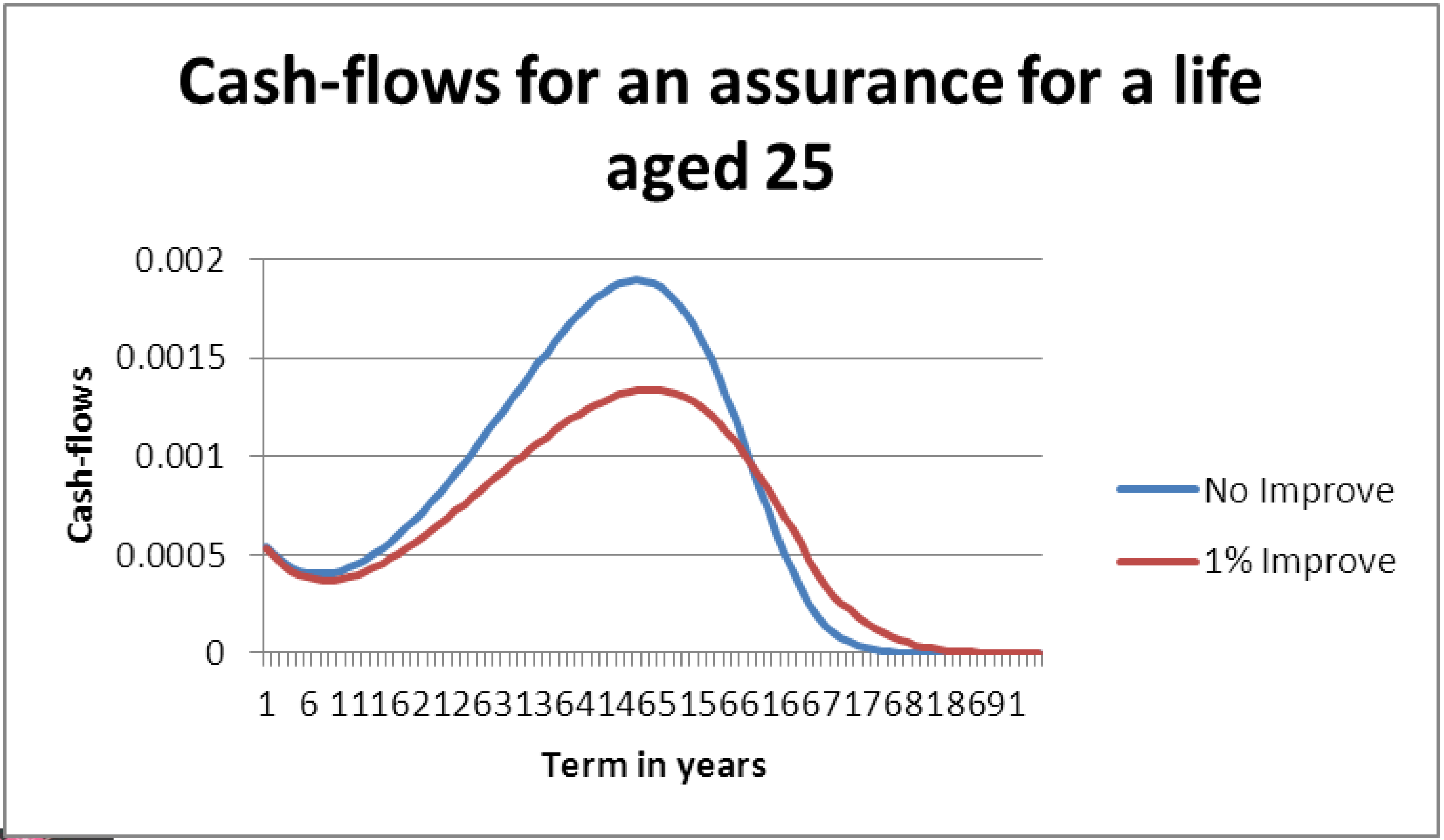
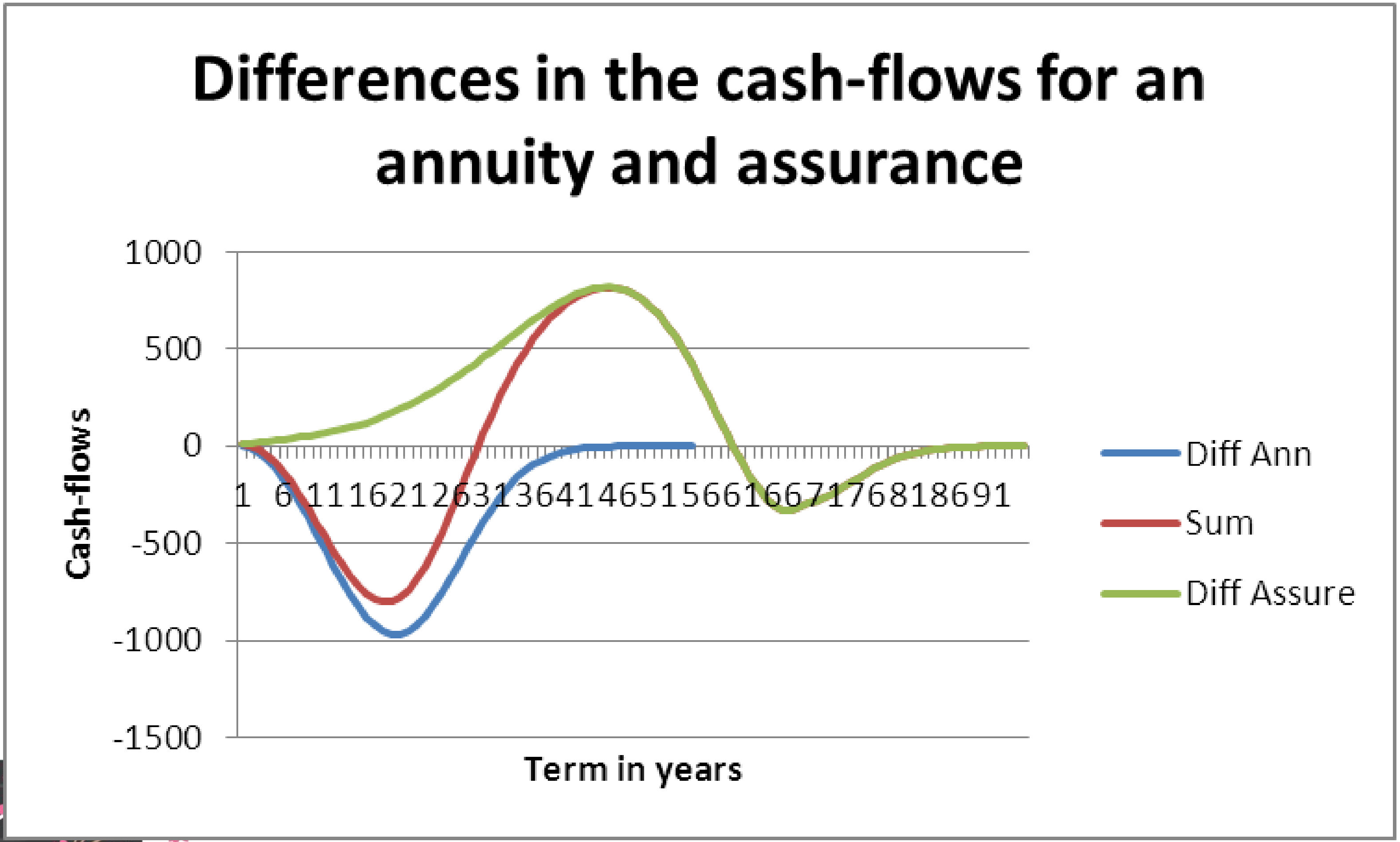
Varying assumptions and conditions



Varying assumptions and conditions

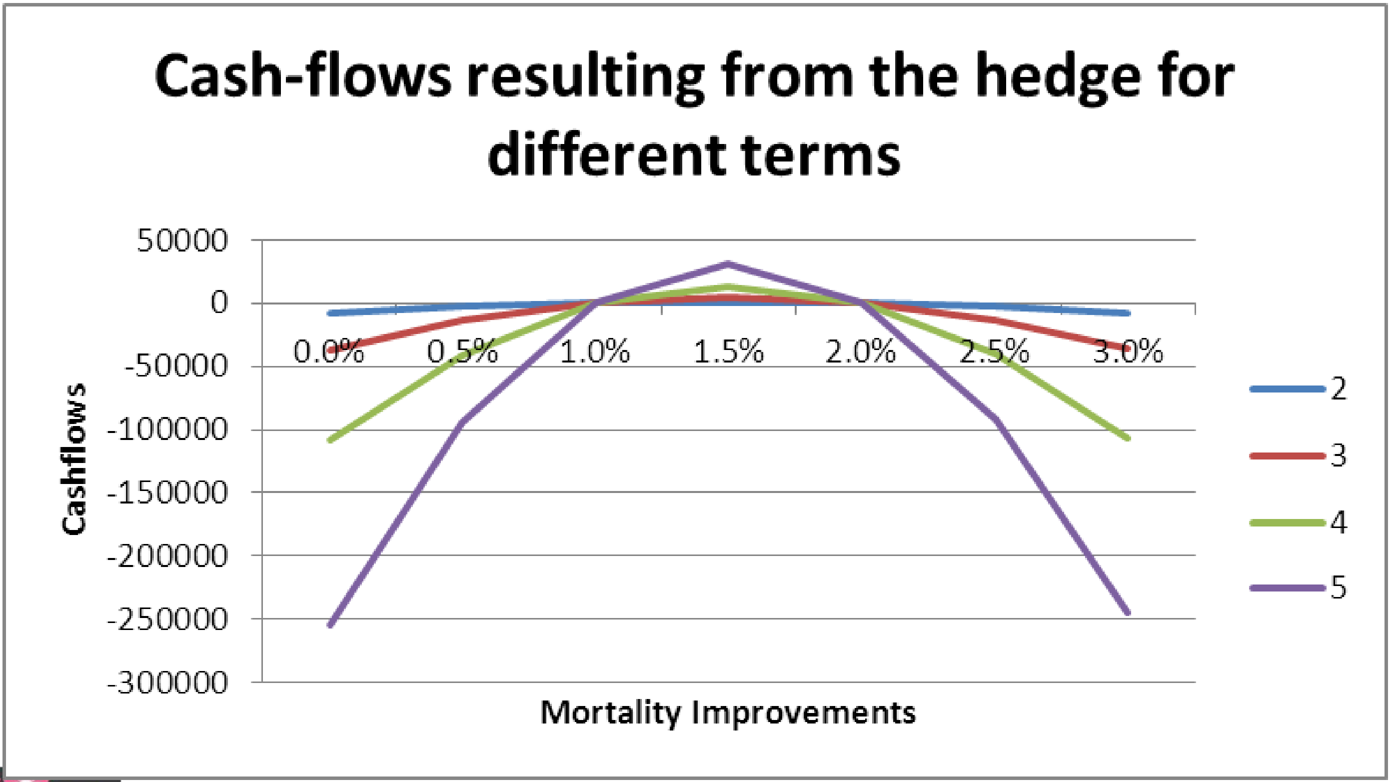


Varying assumptions and conditions



Varying assumptions and conditions

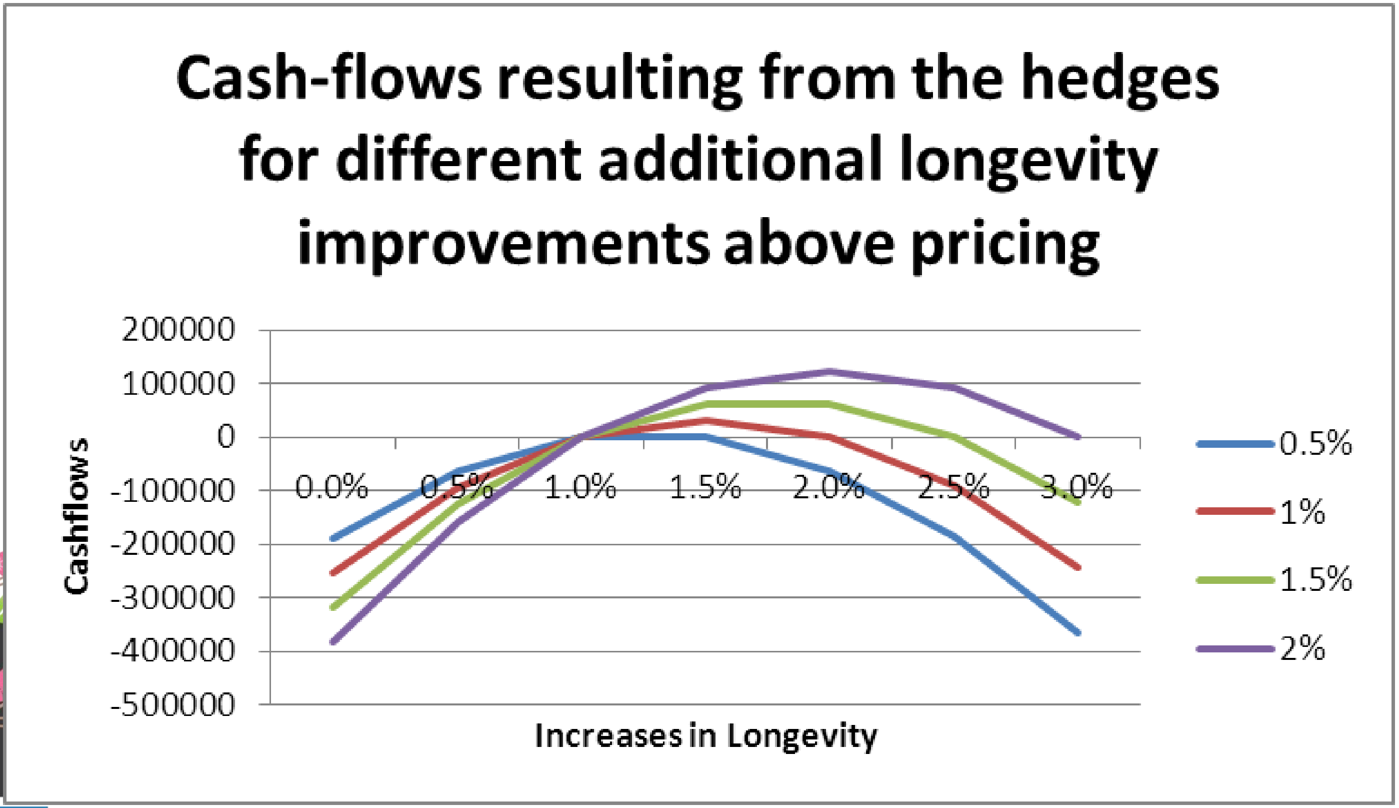
- The long term hedge creates practical problems
- Would like to hedge for a shorter term



We assumed a 1% improvement in longevity in pricing and hedged for another 1% (total 2% improvement).

Varying assumptions and conditions

- Consider annuities and assurances priced for a 1% improvement. If we then hedge for further improvements in steps of 0.5% each, using a term of 5 years, the cash-flows will be as follows



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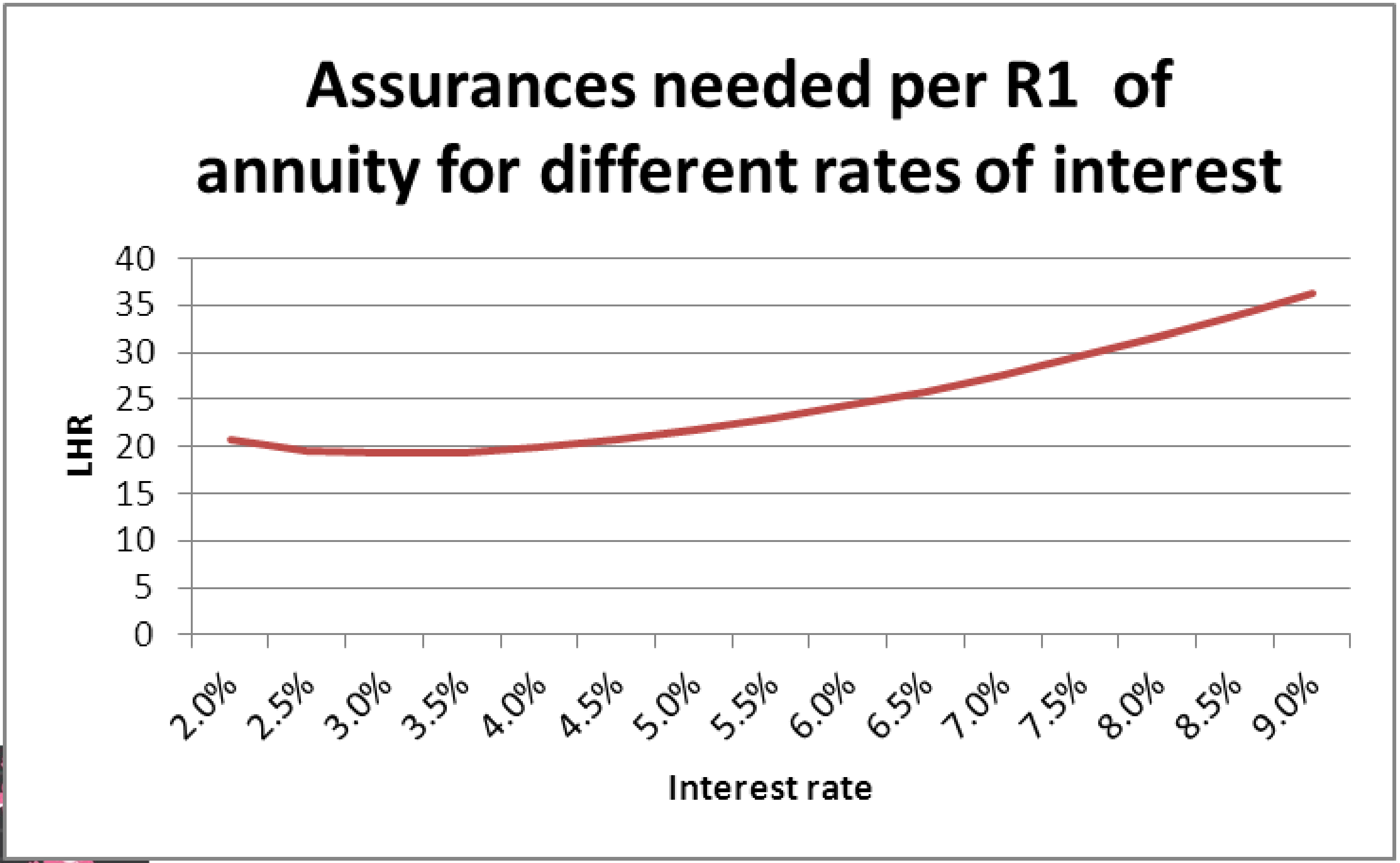
Varying assumptions and conditions

- Consider interest changes affecting LHR



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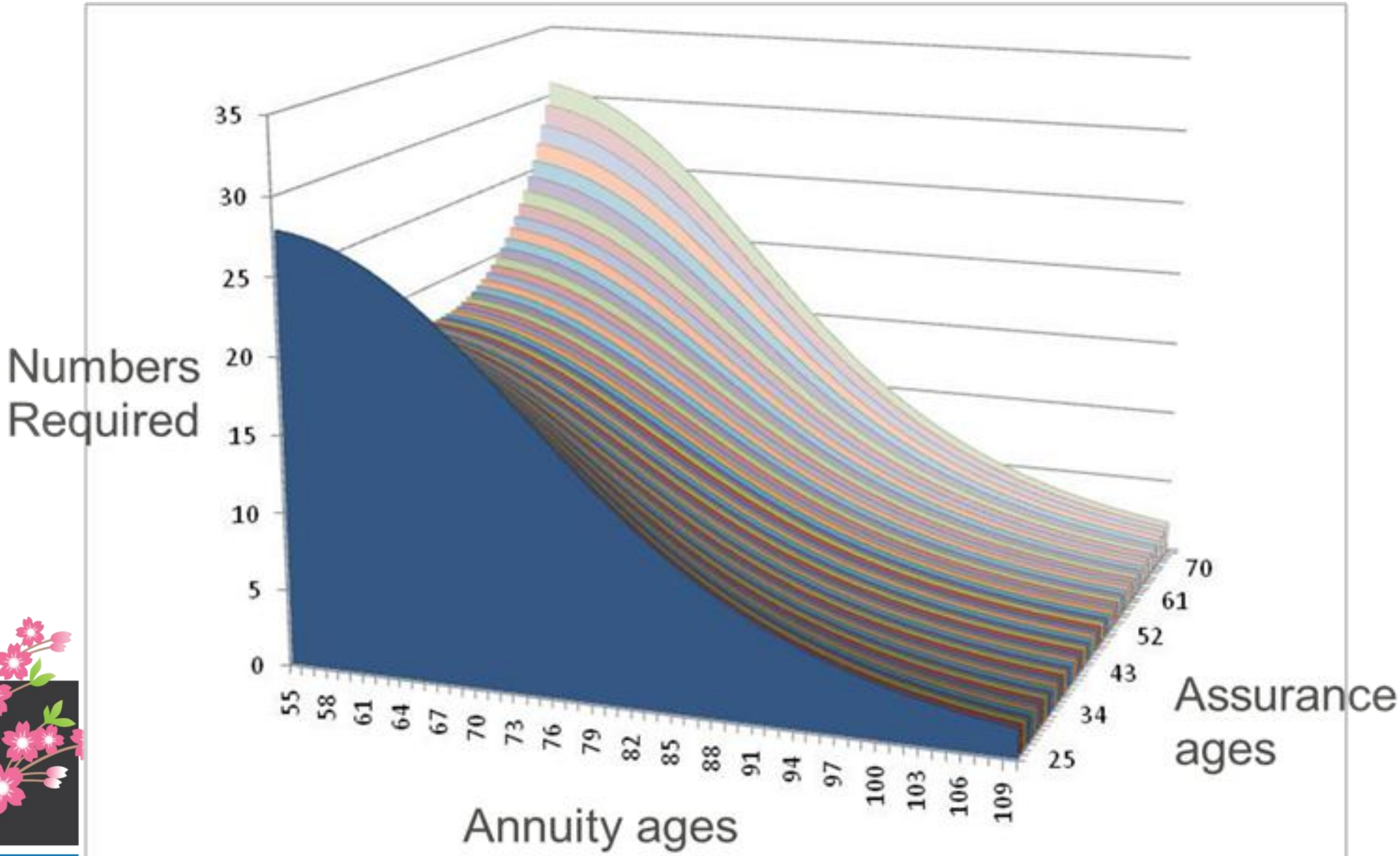
Varying assumptions and conditions

- LHR for different ages for annuitants and assured lives



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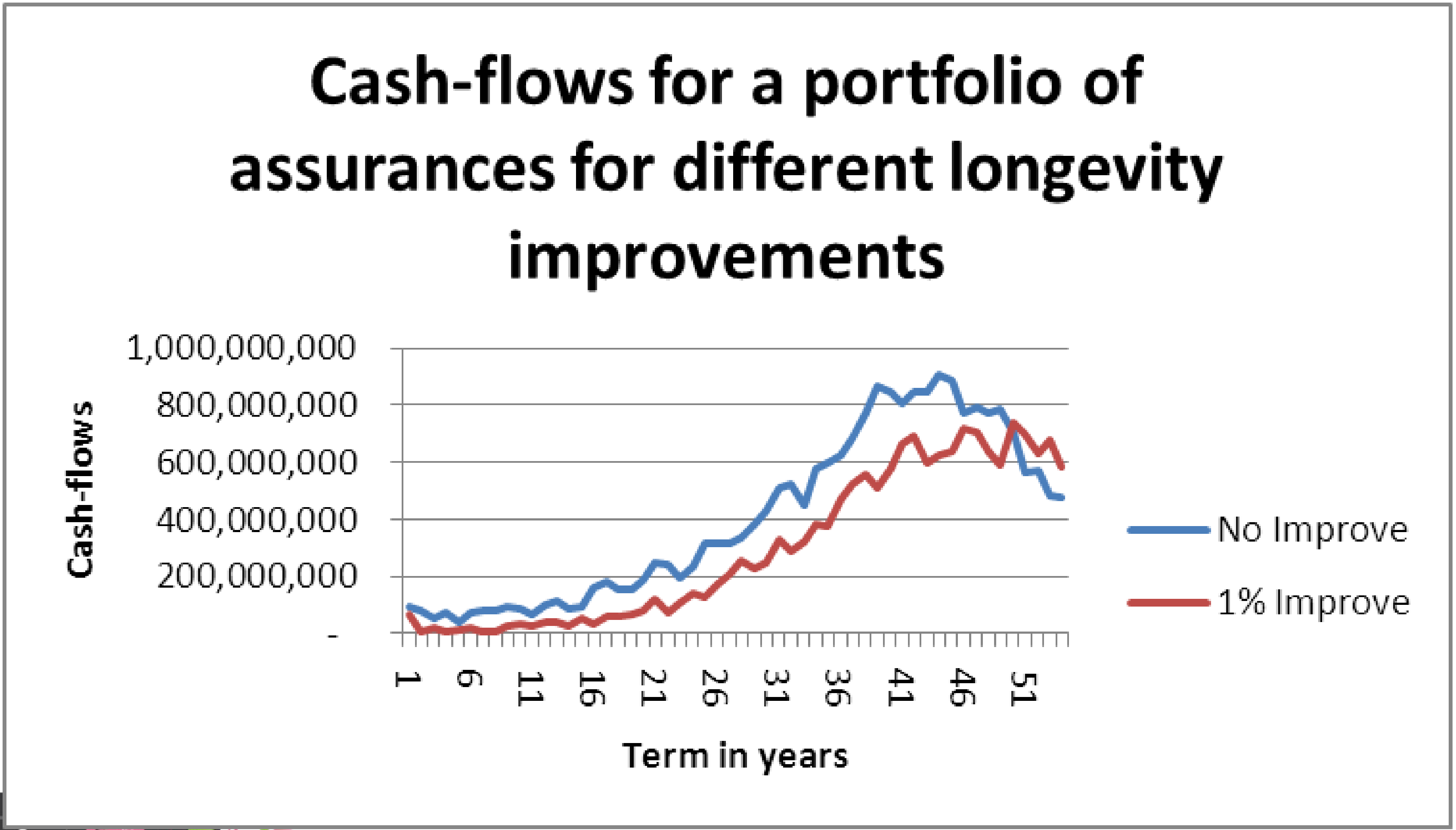
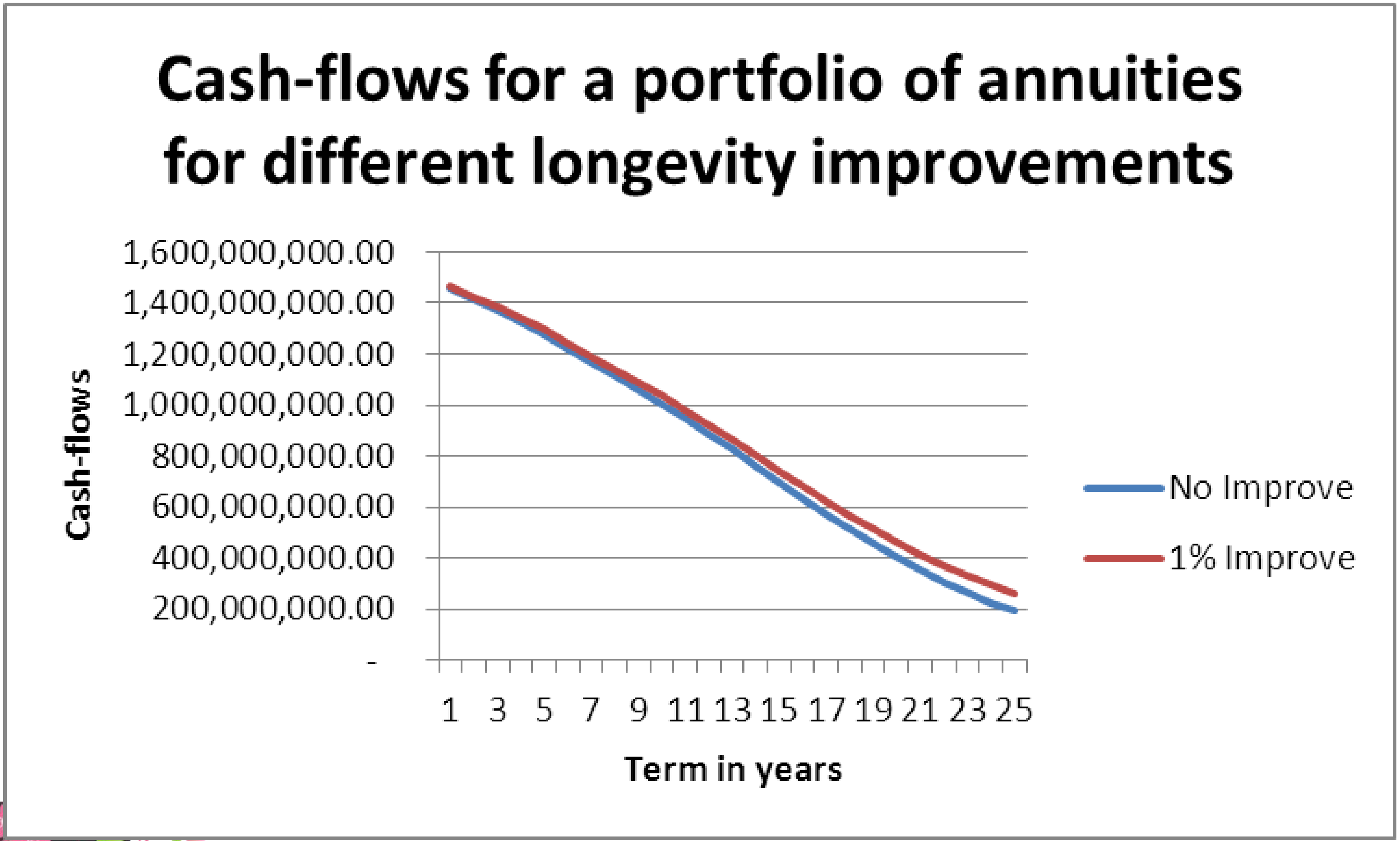
Simulation study

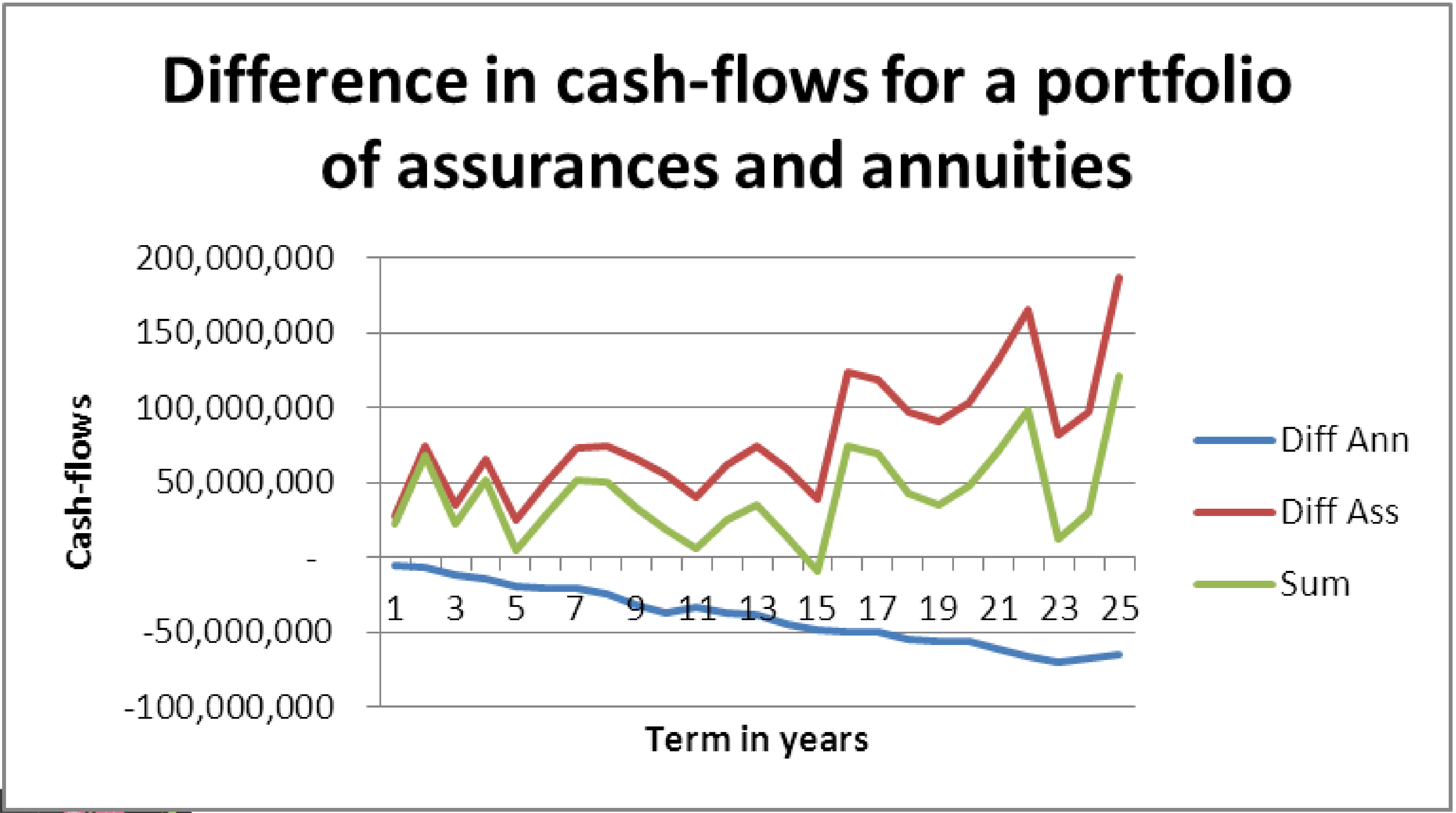
Simulation study



Simulation study



Simulation study



- Conclusion
  - Easy calculation of PV movements
  - LHR for any two particular portfolios
  - NO underlying mortality index required
  - Limited term hedges possible (1 year)
  - Allowance for lapses and other decrements
  - Could effectively smooth cash-flows

- Why not use it?

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Thank you

Questions?