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Incorporating Model Error into the Actuary's Estimate of Uncertainty: A Practical Approach

A presentation to the 30th International Congress of Actuaries

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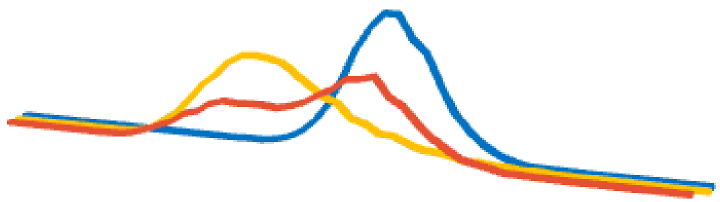
Session Description

- Within the property/casualty insurance industry, increased interest is being placed on understanding the variability inherent in a point estimate of unpaid claims

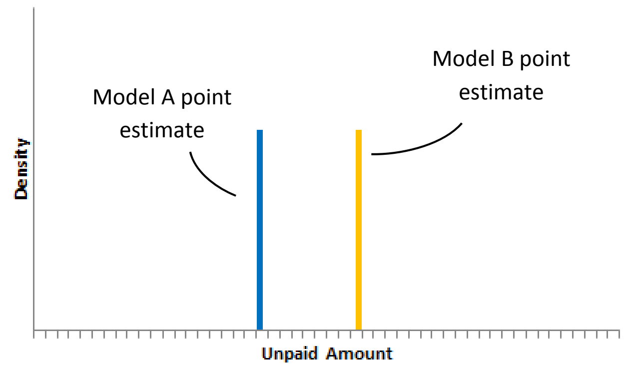


- The session will begin with a dilemma that confronts actuaries when relying upon a *single* model to measure the variability around a central estimate based on *multiple* models
- We will then provide an overview of the basic building blocks to estimating reserve variability and will then address a component of reserve variability that is often overlooked: model uncertainty
- This session will present practical methodologies for incorporating model uncertainty into the actuary's estimate of uncertainty and will use a case study to demonstrate their use

Dilemma

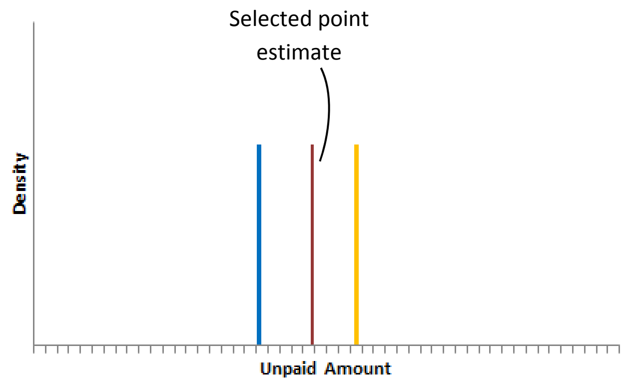
Dilemma

- Consider a situation where we have two models, Model A & Model B, that each produce a point estimate:



Dilemma

- Consider a situation where we have two models, Model A & Model B, that each produce a point estimate:
- Assume the actuary selects the central estimate to be the average of the point estimates from the two models:

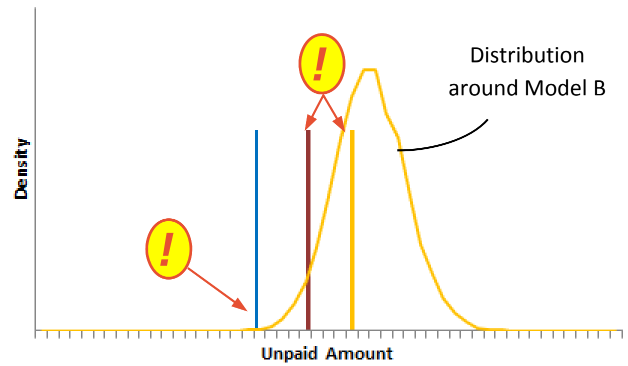


$$\boldsymbol{Central\ Estimate = \frac{(Model\ A + Model\ B)}{2}}$$

- How do we estimate the uncertainty in our central estimate?

Dilemma

- One way might be to estimate uncertainty using one of our underlying models as the basis
  - Using Model B as the basis for estimating uncertainty:
- This raises two issues:
  - ! Central estimate (red) is not "central" within distribution
  - ! Model A point (blue) estimate appears unlikely yet given 50% weight

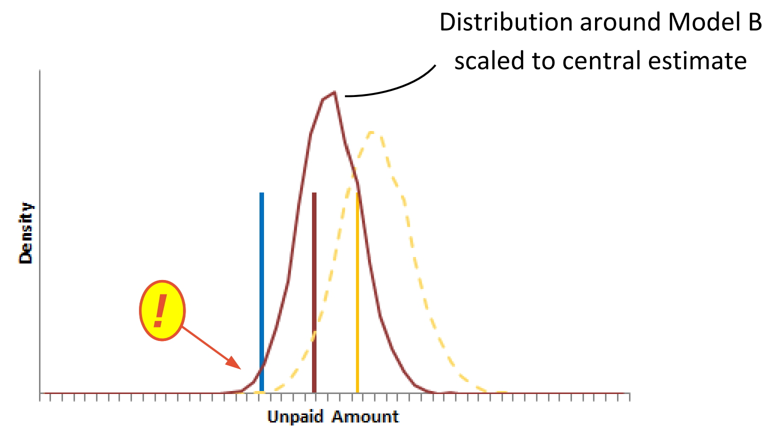


Dilemma

- The first issue can be resolved by scaling:

$$Additive\ Scaling = \hat{y}'_p = \hat{y}_p + [central\ estimate - \bar{y}]$$

$$Multiplicative\ Scaling = \hat{y}'_p = \hat{y}_p \frac{[central\ estimate]}{\bar{y}}$$



- The central estimate (red) is now “central” with distribution
- However, the second issue remains:

! Model A point estimate (blue) still appears unlikely yet given 50% weight

Dilemma

- It is common to estimate unpaid claims using more than one model
- It is rare for different models to produce point estimates that are equivalent
- Current approaches to estimating uncertainty tend to derive variability within the context of a single model
- Central estimate is often not equivalent to any single model.

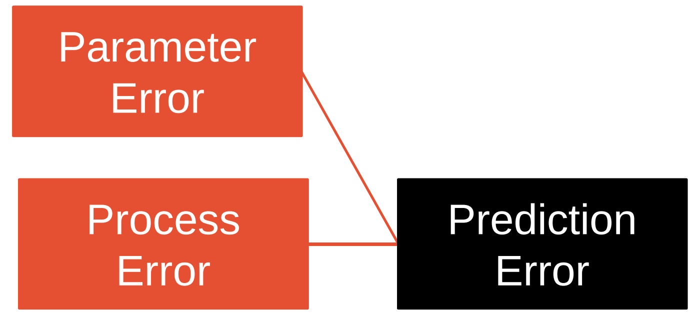
How do we derive a suitable distribution of variability?

Incorporating Model Uncertainty

Overview of Approach

Uncertainty in an Actuarial Central Estimate

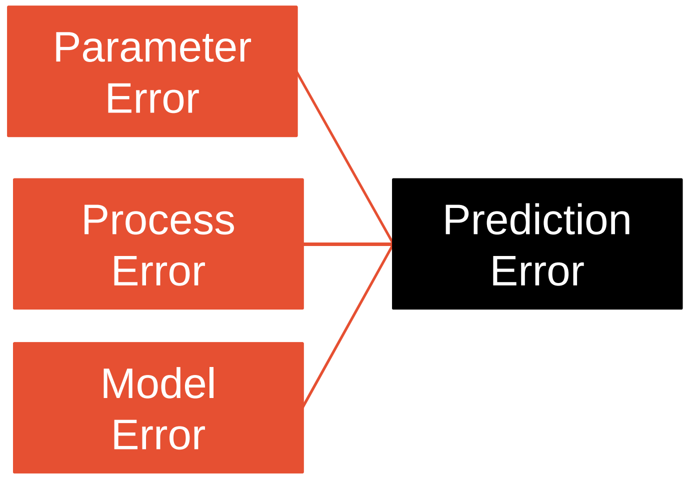
- Measuring uncertainty is a challenge in our profession because the unpaid claim process is unknown and the output from this process is not a repeatable exercise
- Many approaches exist to estimating the uncertainty in an unpaid claim estimate
  - Mack, Bootstrapping, MCMC, practical stochasitic simulation
- Two common themes in these approaches:
  - Prediction error is comprised of parameter error and process error
  - A **single** model is assumed to be representative of the unpaid claim process



Selected Ultimate		Paid CL Method
31,037		31,037
28,187		28,187
27,732		27,732
31,809	=	31,809
32,304		32,304
30,936		30,936
26,141		26,141
25,494		25,494
23,108		23,108

Uncertainty in an Actuarial Central Estimate

- However, this is rarely the case, and actuaries will commonly employ multiple models
- We therefore need some way in which to reflect the additional implied uncertainty among the models



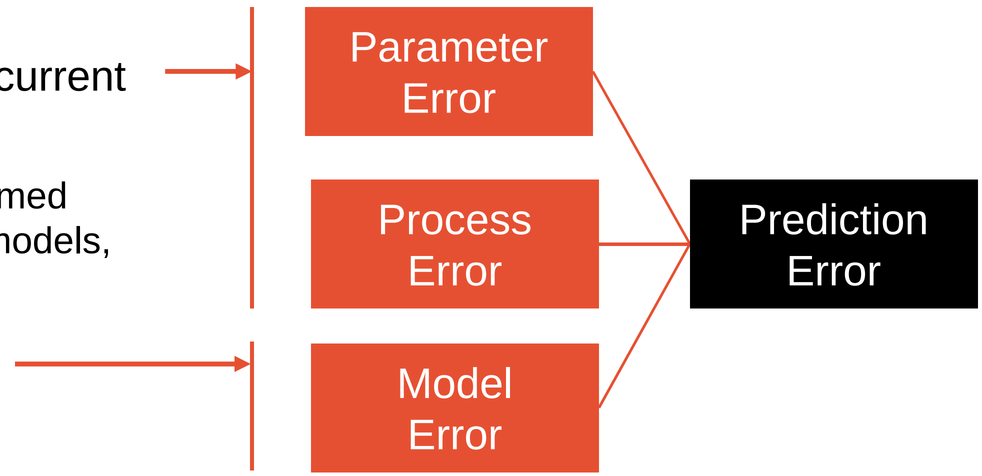
Selected Ultimate		Paid CL Method	Incurred CL Method	Paid BF Method
31,218		31,037	31,400	31,037
28,341		28,187	28,494	28,187
28,181		27,732	28,630	27,734
32,926	=	31,809	34,044	31,832
32,518		32,304	32,732	32,322
31,142		30,936	31,663	30,825
26,298		26,141	26,354	26,400
27,703		25,494	29,261	28,356
25,150		23,108	26,776	25,566

Our Approach

Generate a distribution comprised of simulations about each model using current approaches:

- Bootstrapping; simulation from an assumed distribution; simulation from analytical models, simulating and scaling; etc.

Weighted sample



Our Approach

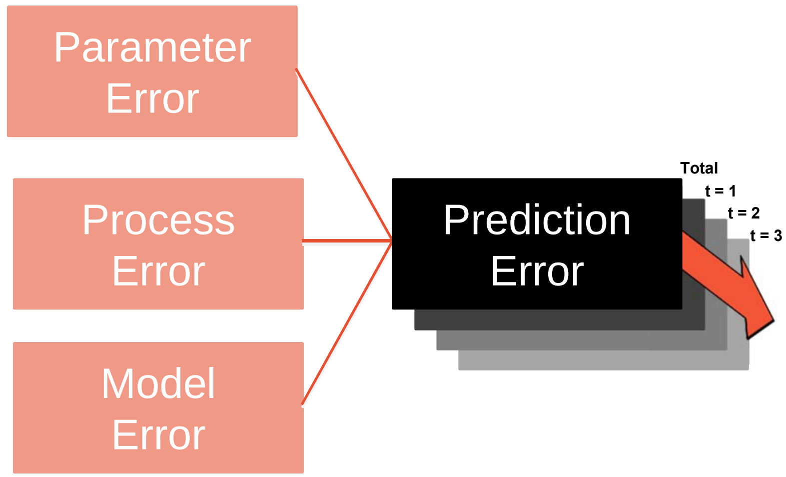
Generate a distribution comprised of simulations about each model using current approaches:

- Bootstrapping; simulation from an assumed distribution; simulation from analytical models; simulating and scaling, etc.

Weighted sample

Aggregating results across multiple years requires additional rigor:

- **Rank Tying** and **Model Tying** approaches are available to generate aggregate distributions

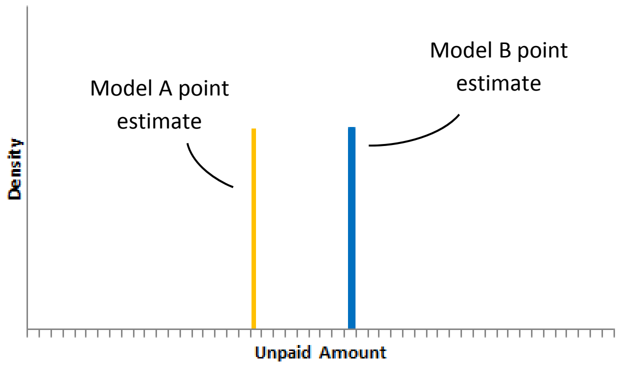


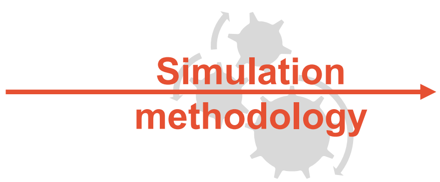
Weighted Sampling

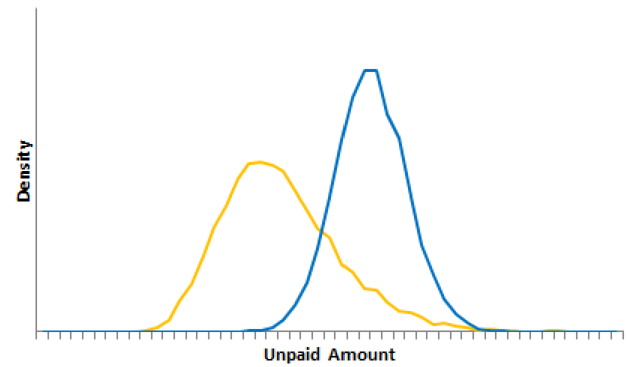
Single Years

Sampling of methods

- Start by creating simulated distributions for each of Model A and B:







Model A Simulations		Model B Simulations	
Sim	Value	Sim	Value
1	3.4	1	3.6
2	2.5	2	4.6
3	1.8	3	5.2
4	3.8	4	4.4
5	4.4	5	3.4
6	3.0	6	3.6
7	2.0	7	4.4
8	6.0	8	3.9
9	3.7	9	3.4
10	6.4	10	3.0

Sampling of methods

- Create a ‘Model Matrix’ based on selected weighting
- In this case, we will use 50-50 weighting between Model A and B
- Simulations are pulled from each model based on this ‘Model Matrix’

Model A Simulations

Sim	Value
1	3.4
2	2.5
3	1.8
4	3.8
5	4.4
6	3.0
7	2.0
8	6.0
9	3.7
10	6.4

Weighting Selection

	Weight
Model A	50%
Model B	50%

Model Matrix

Sim	Value
1	B
2	A
3	A
4	B
5	A
6	A
7	B
8	B
9	A
10	A

Sampled Simulations

Sim	Value
1	3.6
2	2.5
3	1.8
4	4.4
5	4.4
6	3.0
7	4.4
8	3.9
9	3.7
10	6.4

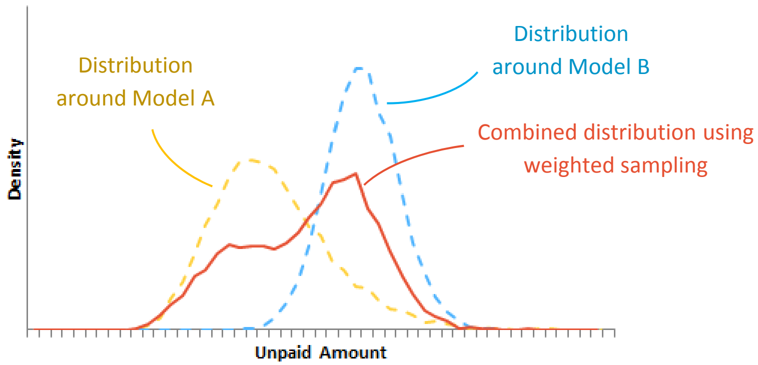
Model B Simulations

Sim	Value
1	3.6
2	4.6
3	5.2
4	4.4
5	3.4
6	3.6
7	4.4
8	3.9
9	3.4
10	3.0

Sampling of methods

- Comparison of results from Weighted Sampling between Model A and Model B:

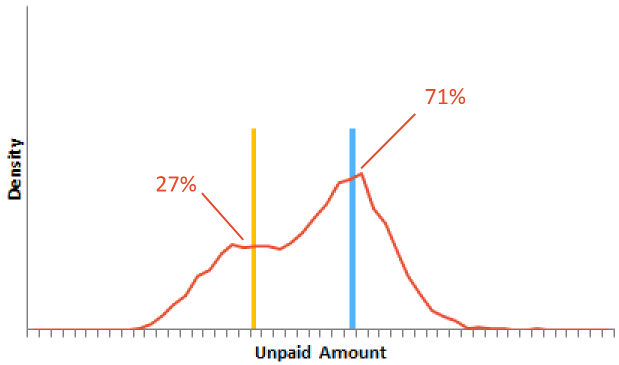
Using weighted sampling



Sampling of methods

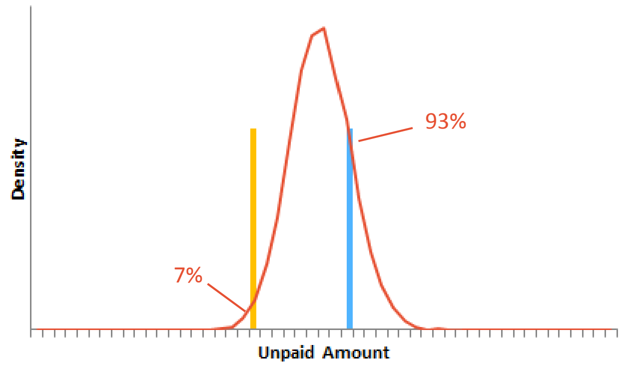
- Comparison of results from Weighted Sampling vs. Scaling:

Using weighted sampling



vs.

Using scaling



Sampling of methods

- Adjusting our underlying weights will shift the resulting distribution accordingly:

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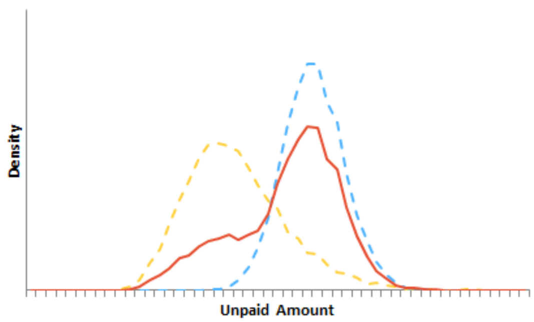

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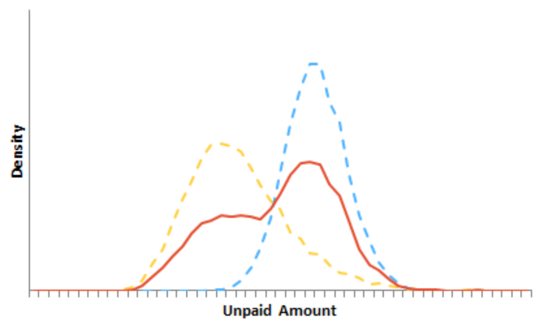

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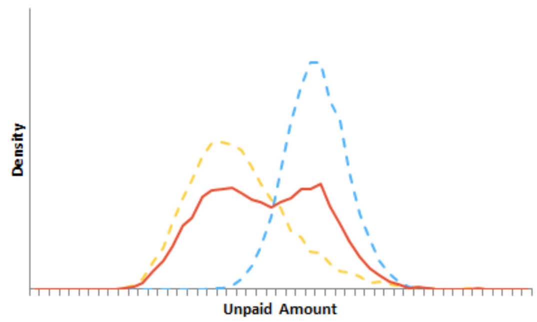
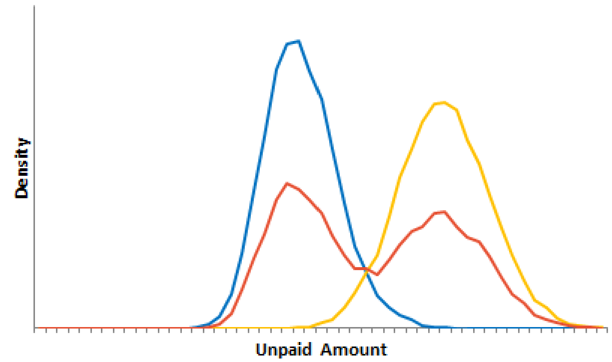
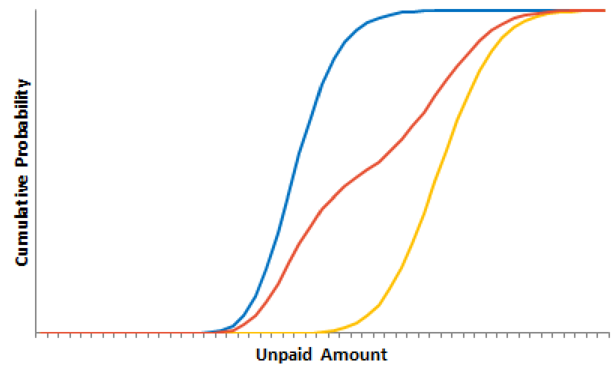


Sampling of methods Multi-modal distributions

- Weighted Sampling may produce ‘lumpy’, or multi-modal, probability density distributions
- However, the probabilities across a range of outcomes may be more easily interpreted using the associated cumulative probabilities graph
- Further adjustments could be made to the simulated results if such an outcome was deemed problematic





Sampling of methods

- So far, we have considered a scenario with just a single set of simulations
- What if we have multiple sets of predictions?
  - Multiple accident years, for example

Model A Simulations

Sim	t = 1	t = 2	t = 3
1	3.4	5.8	28.8
2	2.5	12.5	28.0
3	1.8	6.5	24.0
4	3.8	8.8	20.0
5	4.4	8.7	14.5
6	3.0	10.7	14.0
7	2.0	9.4	16.9
8	6.0	7.6	24.9
9	3.7	9.7	25.0
10	6.4	8.6	29.0

Model B Simulations

Sim	t = 1	t = 2	t = 3
1	3.6	12.0	19.9
2	4.6	13.3	26.9
3	5.2	16.1	27.2
4	4.4	11.3	22.7
5	3.4	17.2	26.9
6	3.6	11.3	15.7
7	4.4	10.7	22.9
8	3.9	13.3	22.6
9	3.4	13.5	20.4
10	3.0	13.2	15.0

Weighted Sampling

Multiple Years

Sampling of methods Multiple Year Aggregations

- Again, for each time period, we can create a 'Model Matrix' based on the selected weighting

Model A Simulations

Sim	t = 1	t = 2	t = 3
1	3.4	5.8	28.8
2	2.5	12.5	28.0
3	1.8	6.5	24.0
4	3.8	8.8	20.0
5	4.4	8.7	14.5
6	3.0	10.7	14.0
7	2.0	9.4	16.9
8	6.0	7.6	24.9
9	3.7	9.7	25.0
10	6.4	8.6	29.0

Weighting Selections

	t = 1	t = 2	t = 3
Model A	50%	50%	50%
Model B	50%	50%	50%

Model Matrix

Sim	t = 1	t = 2	t = 3
1	B	B	B
2	A	B	A
3	A	B	A
4	B	B	A
5	A	A	B
6	A	A	A
7	B	B	A
8	B	A	B
9	A	B	A
10	A	A	B

Sampled Simulations

Sim	t = 1	t = 2	t = 3
1	3.6	12.0	19.9
2	2.5	13.3	28.0
3	1.8	16.1	24.0
4	4.4	11.3	20.0
5	4.4	8.7	26.9
6	3.0	10.7	14.0
7	4.4	10.7	16.9
8	3.9	7.6	22.6
9	3.7	13.5	25.0
10	6.4	8.6	15.0

Model B Simulations

Sim	t = 1	t = 2	t = 3
1	3.6	12.0	19.9
2	4.6	13.3	26.9
3	5.2	16.1	27.2
4	4.4	11.3	22.7
5	3.4	17.2	26.9
6	3.6	11.3	15.7
7	4.4	10.7	22.9
8	3.9	13.3	22.6
9	3.4	13.5	20.4
10	3.0	13.2	15.0

Sampling of methods **Multiple Year Aggregations**

A note on simulation tying

- Typically, the methods that are used to generate the simulations around each of the underlying models do not treat each accident year in isolation, but rather produce year-by-year results that are intrinsically related to each other
- This is reflected in each and every simulation, which we can think of as ‘strings’
- This means that we are able to calculate the total unpaid amount for each simulation by simply summing across each row

Sim	t = 1	t = 2	t = 3	Total
#1	3.4	5.8	28.8	37.9
#2	2.5	12.5	28.0	43.0

- In this manner, any accident year correlation that is inherent to the model can be maintained

Sampling of methods **Multiple Year Aggregations**

A note on simulation tying : Another Dilemma !

- What happens when we mix samples from different models in simulation ‘strings’
- Where a break occurs in a ‘string’, we destroy any correlation that may have been included in our model

Sim	t = 1	t = 2	t = 3	Total
#1	3.4	5.8	19.9	37.9
#2	4.6	12.5	28.0	43.0

Sampling of methods **Multiple Year Aggregations**

A note on simulation tying: Another Dilemma !

- What happens when we mix samples from different models in simulation ‘strings’
- Where a break occurs in a ‘string’, we destroy any correlation that may have been included in our model
- If we simply randomly-arrange our samples across simulations, we essentially destroy any year-by-year correlation in our results and we are no longer able to sum across the rows to get the total (unless this is desired)

Sim	t = 1	t = 2	t = 3	Total
#1	3.4	5.8	19.9	?
#2	4.6	12.5	28.0	?

Sampling of methods Multiple Year Aggregations

- Going back to our sampled simulations - because we sampled independently for each time period, we have broken the links intrinsic to the underlying model(s)

Model A Simulations

Sim	t = 1	t = 2	t = 3	Total
1	3.4	5.8	28.8	37.9
2	2.5	12.5	28.0	43.0
3	1.8	6.5	24.0	32.3
4	3.8	8.8	20.0	32.7
5	4.4	8.7	14.5	27.6
6	3.0	10.7	14.0	27.8
7	2.0	9.4	16.9	28.2
8	6.0	7.6	24.9	38.4
9	3.7	9.7	25.0	38.4
10	6.4	8.6	29.0	44.0

Model B Simulations

Sim	t = 1	t = 2	t = 3	Total
1	3.6	12.0	19.9	35.5
2	4.6	13.3	26.9	44.8
3	5.2	16.1	27.2	48.6
4	4.4	11.3	22.7	38.4
5	3.4	17.2	26.9	47.6
6	3.6	11.3	15.7	30.6
7	4.4	10.7	22.9	38.0
8	3.9	13.3	22.6	39.7
9	3.4	13.5	20.4	37.2
10	3.0	13.2	15.0	31.3

Weighting Selections

	t = 1	t = 2	t = 3
Model A	50%	50%	50%
Model B	50%	50%	50%

Model Matrix

Sim	t = 1	t = 2	t = 3
1	B	B	B
2	A	B	A
3	A	B	A
4	B	B	A
5	A	A	B
6	A	A	A
7	B	B	A
8	B	A	B
9	A	B	A
10	A	A	B

Sampled Simulations

Sim	t = 1	t = 2	t = 3	Total
1	3.6	12.0	19.9	
2	2.5	13.3	28.0	X
3	1.8	16.1	24.0	
4	4.4	11.3	20.0	
5	4.4	8.7	26.9	
6	3.0	10.7	14.0	
7	4.4	10.7	16.9	
8	3.9	7.6	22.6	
9	3.7	13.5	25.0	
10	6.4	8.6	15.0	

Sampling of methods Multiple Year Aggregations

- With this example (equal weighting for each accident period), we can get around the problem by sampling just one time and ensuring that we pick the same simulation for every time period

Model A Simulations

Sim	t = 1	t = 2	t = 3	Total
1	3.4	5.8	28.8	37.9
2	2.5	12.5	28.0	43.0
3	1.8	6.5	24.0	32.3
4	3.8	8.8	20.0	32.7
5	4.4	8.7	14.5	27.6
6	3.0	10.7	14.0	27.8
7	2.0	9.4	16.9	28.2
8	6.0	7.6	24.9	38.4
9	3.7	9.7	25.0	38.4
10	6.4	8.6	29.0	44.0

Model B Simulations

Sim	t = 1	t = 2	t = 3	Total
1	3.6	12.0	19.9	35.5
2	4.6	13.3	26.9	44.8
3	5.2	16.1	27.2	48.6
4	4.4	11.3	22.7	38.4
5	3.4	17.2	26.9	47.6
6	3.6	11.3	15.7	30.6
7	4.4	10.7	22.9	38.0
8	3.9	13.3	22.6	39.7
9	3.4	13.5	20.4	37.2
10	3.0	13.2	15.0	31.3

Weighting Selections

	t = 1	t = 2	t = 3	Total
Model A	50%	50%	50%	50%
Model B	50%	50%	50%	50%

Model Matrix

Sim	t = 1	t = 2	t = 3	Total
1	B	B	B	B
2	A	A	A	A
3	A	A	A	A
4	A	A	A	A
5	B	B	B	B
6	A	A	A	A
7	A	A	A	A
8	B	B	B	B
9	A	A	A	A
10	B	B	B	B

Sampled Simulations

Sim	t = 1	t = 2	t = 3	Total
1	3.6	12.0	19.9	35.5
2	2.5	12.5	28.0	43.0
3	1.8	6.5	24.0	32.3
4	3.8	8.8	20.0	32.7
5	3.4	17.2	26.9	47.6
6	3.0	10.7	14.0	27.8
7	2.0	9.4	16.9	28.2
8	3.9	13.3	22.6	39.7
9	3.7	9.7	25.0	38.4
10	3.0	13.2	15.0	31.3

- This approach achieves the objective in that each individual accident period reflects the desired weighting *and* we maintain the correlation inherent to each individual simulation
- *However…*

Sampling of methods Multiple Year Aggregations

- …what if our selected weightings vary for each origin year?
- In this case, we need to sample independently to maintain appropriate year-by-year representation

Model A Simulations

Sim	t = 1	t = 2	t = 3	Total
1	3.4	5.8	28.8	37.9
2	2.5	12.5	28.0	43.0
3	1.8	6.5	24.0	32.3
4	3.8	8.8	20.0	32.7
5	4.4	8.7	14.5	27.6
6	3.0	10.7	14.0	27.8
7	2.0	9.4	16.9	28.2
8	6.0	7.6	24.9	38.4
9	3.7	9.7	25.0	38.4
10	6.4	8.6	29.0	44.0

Model B Simulations

Sim	t = 1	t = 2	t = 3	Total
1	3.6	12.0	19.9	35.5
2	4.6	13.3	26.9	44.8
3	5.2	16.1	27.2	48.6
4	4.4	11.3	22.7	38.4
5	3.4	17.2	26.9	47.6
6	3.6	11.3	15.7	30.6
7	4.4	10.7	22.9	38.0
8	3.9	13.3	22.6	39.7
9	3.4	13.5	20.4	37.2
10	3.0	13.2	15.0	31.3

Weighting Selections

	t = 1	t = 2	t = 3	Total
Model A	90%	50%	33%	?
Model B	10%	50%	67%	?

Model Matrix

Sim	t = 1	t = 2	t = 3	Total
1	B	B	B	X
2	A	A	A	X
3	A	B	B	X
4	A	B	A	X
5	A	A	B	X
6	A	A	B	X
7	A	B	A	X
8	A	A	B	X
9	A	B	B	X
10	A	A	B	X

No possible weighting at *Total* level would replicate selected year-by-year weights…

…therefore cannot sample at total level

Sim	t = 1	t = 2	t = 3	Total
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

Sampling of methods Multiple Year Aggregations

- We require some manner of rearranging our simulations to reflect underlying correlations
- Going back to our earlier example, sampling individually by years, we suggest 2 ways in which to achieve this….

Model A Simulations

Sim	t = 1	t = 2	t = 3	Total
1	3.4	5.8	28.8	37.9
2	2.5	12.5	28.0	43.0
3	1.8	6.5	24.0	32.3
4	3.8	8.8	20.0	32.7
5	4.4	8.7	14.5	27.6
6	3.0	10.7	14.0	27.8
7	2.0	9.4	16.9	28.2
8	6.0	7.6	24.9	38.4
9	3.7	9.7	25.0	38.4
10	6.4	8.6	29.0	44.0

Weighting Selections

	t = 1	t = 2	t = 3
Model A	50%	50%	50%
Model B	50%	50%	50%

Model Matrix

Sim	t = 1	t = 2	t = 3
1	B	B	B
2	A	B	A
3	A	B	A
4	B	B	A
5	A	A	B
6	A	A	A
7	B	B	A
8	B	A	B
9	A	B	A
10	A	A	B

Sampled Simulations

Sim	t = 1	t = 2	t = 3
1	3.6	12.0	19.9
2	2.5	13.3	28.0
3	1.8	16.1	24.0
4	4.4	11.3	20.0
5	4.4	8.7	26.9
6	3.0	10.7	14.0
7	4.4	10.7	16.9
8	3.9	7.6	22.6
9	3.7	13.5	25.0
10	6.4	8.6	15.0

Model B Simulations

Sim	t = 1	t = 2	t = 3	Total
1	3.6	12.0	19.9	35.5
2	4.6	13.3	26.9	44.8
3	5.2	16.1	27.2	48.6
4	4.4	11.3	22.7	38.4
5	3.4	17.2	26.9	47.6
6	3.6	11.3	15.7	30.6
7	4.4	10.7	22.9	38.0
8	3.9	13.3	22.6	39.7
9	3.4	13.5	20.4	37.2
10	3.0	13.2	15.0	31.3

Sampling of methods Multiple Year Aggregations

1) Rank Tying:

Rearrange the sampled simulations themselves using a ‘borrowed’ correlation matrix

Model A Simulations

Sim	t = 1	t = 2	t = 3	Total
1	3.4	5.8	28.8	37.9
2	2.5	12.5	28.0	43.0
3	1.8	6.5	24.0	32.3
4	3.8	8.8	20.0	32.7
5	4.4	8.7	14.5	27.6
6	3.0	10.7	14.0	27.8
7	2.0	9.4	16.9	28.2
8	6.0	7.6	24.9	38.4
9	3.7	9.7	25.0	38.4
10	6.4	8.6	29.0	44.0

Model B Simulations

Sim	t = 1	t = 2	t = 3	Total
1	3.6	12.0	19.9	35.5
2	4.6	13.3	26.9	44.8
3	5.2	16.1	27.2	48.6
4	4.4	11.3	22.7	38.4
5	3.4	17.2	26.9	47.6
6	3.6	11.3	15.7	30.6
7	4.4	10.7	22.9	38.0
8	3.9	13.3	22.6	39.7
9	3.4	13.5	20.4	37.2
10	3.0	13.2	15.0	31.3

Weighting Selections

	t = 1	t = 2	t = 3
Model A	50%	50%	50%
Model B	50%	50%	50%

Model Matrix

Sim	t = 1	t = 2	t = 3
1	B	B	B
2	A	B	A
3	A	B	A
4	B	B	A
5	A	A	B
6	A	A	A
7	B	B	A
8	B	A	B
9	A	B	A
10	A	A	B

Sampled Simulations

Sim	t = 1	t = 2	t = 3
1	3.6	12.0	19.9
2	2.5	13.3	28.0
3	1.8	16.1	24.0
4	4.4	11.3	20.0
5	4.4	8.7	26.9
6	3.0	10.7	14.0
7	4.4	10.7	16.9
8	3.9	7.6	22.6
9	3.7	13.5	25.0
10	6.4	8.6	15.0

Sampling of methods Multiple Year Aggregations

2) Model Tying:

Rearranging the Weighted Samples 'Model Matrix' prior to pulling through the reserves from the underlying model

Model A Simulations

Sim	t = 1	t = 2	t = 3	Total
1	3.4	5.8	28.8	37.9
2	2.5	12.5	28.0	43.0
3	1.8	6.5	24.0	32.3
4	3.8	8.8	20.0	32.7
5	4.4	8.7	14.5	27.6
6	3.0	10.7	14.0	27.8
7	2.0	9.4	16.9	28.2
8	6.0	7.6	24.9	38.4
9	3.7	9.7	25.0	38.4
10	6.4	8.6	29.0	44.0

Model B Simulations

Sim	t = 1	t = 2	t = 3	Total
1	3.6	12.0	19.9	35.5
2	4.6	13.3	26.9	44.8
3	5.2	16.1	27.2	48.6
4	4.4	11.3	22.7	38.4
5	3.4	17.2	26.9	47.6
6	3.6	11.3	15.7	30.6
7	4.4	10.7	22.9	38.0
8	3.9	13.3	22.6	39.7
9	3.4	13.5	20.4	37.2
10	3.0	13.2	15.0	31.3

Weighting Selections

	t = 1	t = 2	t = 3
Model A	50%	50%	50%
Model B	50%	50%	50%

Model Matrix

Sim	t = 1	t = 2	t = 3
1	B	B	B
2	A	B	A
3	A	B	A
4	B	B	A
5	A	A	B
6	A	A	A
7	B	B	A
8	B	A	B
9	A	B	A
10	A	A	B

Sampled Simulations

Sim	t = 1	t = 2	t = 3	Total
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

Aggregating Results

Rank Tying

Aggregating Results Rank Tying

- This approach involves rearranging the sampled, simulated reserves

Model A Simulations

Sim	t = 1	t = 2	t = 3	Total
1	3.4	5.8	28.8	37.9
2	2.5	12.5	28.0	43.0
3	1.8	6.5	24.0	32.3
4	3.8	8.8	20.0	32.7
5	4.4	8.7	14.5	27.6
6	3.0	10.7	14.0	27.8
7	2.0	9.4	16.9	28.2
8	6.0	7.6	24.9	38.4
9	3.7	9.7	25.0	38.4
10	6.4	8.6	29.0	44.0

Model B Simulations

Sim	t = 1	t = 2	t = 3	Total
1	3.6	12.0	19.9	35.5
2	4.6	13.3	26.9	44.8
3	5.2	16.1	27.2	48.6
4	4.4	11.3	22.7	38.4
5	3.4	17.2	26.9	47.6
6	3.6	11.3	15.7	30.6
7	4.4	10.7	22.9	38.0
8	3.9	13.3	22.6	39.7
9	3.4	13.5	20.4	37.2
10	3.0	13.2	15.0	31.3

Weighting Selections

	t = 1	t = 2	t = 3
Model A	50%	50%	50%
Model B	50%	50%	50%

Model Matrix

Sim	t = 1	t = 2	t = 3
1	B	B	B
2	A	B	A
3	A	B	A
4	B	B	A
5	A	A	B
6	A	A	A
7	B	B	A
8	B	A	B
9	A	B	A
10	A	A	B

Sampled Simulations

Sim	t = 1	t = 2	t = 3
1	3.6	12.0	19.9
2	2.5	13.3	28.0
3	1.8	16.1	24.0
4	4.4	11.3	20.0
5	4.4	8.7	26.9
6	3.0	10.7	14.0
7	4.4	10.7	16.9
8	3.9	7.6	22.6
9	3.7	13.5	25.0
10	6.4	8.6	15.0

Aggregating Results Rank Tying

- We can ‘borrow’ a correlation matrix from one of the underlying models
- We do this by calculating the reserve ranks for each year for the underlying models

Model A Simulations

Sim	t = 1	t = 2	t = 3	Total
1	3.4	5.8	28.8	37.9
2	2.5	12.5	28.0	43.0
3	1.8	6.5	24.0	32.3
4	3.8	8.8	20.0	32.7
5	4.4	8.7	14.5	27.6
6	3.0	10.7	14.0	27.8
7	2.0	9.4	16.9	28.2
8	6.0	7.6	24.9	38.4
9	3.7	9.7	25.0	38.4
10	6.4	8.6	29.0	44.0

Model A Simulation Ranks

Sim	t = 1	t = 2	t = 3
1	6	10	2
2	8	1	3
3	10	9	6
4	4	5	7
5	3	6	9
6	7	2	10
7	9	4	8
8	2	8	5
9	5	3	4
10	1	7	1

Sampled Simulations

Sim	t = 1	t = 2	t = 3
1	3.6	12.0	19.9
2	2.5	13.3	28.0
3	1.8	16.1	24.0
4	4.4	11.3	20.0
5	4.4	8.7	26.9
6	3.0	10.7	14.0
7	4.4	10.7	16.9
8	3.9	7.6	22.6
9	3.7	13.5	25.0
10	6.4	8.6	15.0

Model B Simulations

Sim	t = 1	t = 2	t = 3	Total
1	3.6	12.0	19.9	35.5
2	4.6	13.3	26.9	44.8
3	5.2	16.1	27.2	48.6
4	4.4	11.3	22.7	38.4
5	3.4	17.2	26.9	47.6
6	3.6	11.3	15.7	30.6
7	4.4	10.7	22.9	38.0
8	3.9	13.3	22.6	39.7
9	3.4	13.5	20.4	37.2
10	3.0	13.2	15.0	31.3

Model B Simulation Ranks

Sim	t = 1	t = 2	t = 3
1	7	7	8
2	2	4	3
3	1	2	1
4	4	9	5
5	8	1	2
6	6	8	9
7	3	10	4
8	5	5	6
9	9	3	7
10	10	6	10

Aggregating Results Rank Tying

- We then select which model to use as the basis for our rank-tying (in this case, Model B)…
- …and reorder the sampled simulations accordingly on a year-by-year basis

Model A Simulations

Sim	t = 1	t = 2	t = 3	Total
1	3.4	5.8	28.8	37.9
2	2.5	12.5	28.0	43.0
3	1.8	6.5	24.0	32.3
4	3.8	8.8	20.0	32.7
5	4.4	8.7	14.5	27.6
6	3.0	10.7	14.0	27.8
7	2.0	9.4	16.9	28.2
8	6.0	7.6	24.9	38.4
9	3.7	9.7	25.0	38.4
10	6.4	8.6	29.0	44.0

Model A Simulation Ranks

Sim	t = 1	t = 2	t = 3
1	6	10	2
2	8	1	3
3	10	9	6
4	4	5	7
5	3	6	9
6	7	2	10
7	9	4	8
8	2	8	5
9	5	3	4
10	1	7	1

Sampled Simulations: Re-ordered by Model B

Sim	t = 1	t = 2	t = 3
1	3.6	10.7	16.9
2	4.4	12.0	25.0
3	6.4	13.5	28.0
4	4.4	8.6	22.6
5	3.0	16.1	26.9
6	3.7	8.7	15.0
7	4.4	7.6	24.0
8	3.9	11.3	20.0
9	2.5	13.3	19.9
10	1.8	10.7	14.0

Model B Simulations

Sim	t = 1	t = 2	t = 3	Total
1	3.6	12.0	19.9	35.5
2	4.6	13.3	26.9	44.8
3	5.2	16.1	27.2	48.6
4	4.4	11.3	22.7	38.4
5	3.4	17.2	26.9	47.6
6	3.6	11.3	15.7	30.6
7	4.4	10.7	22.9	38.0
8	3.9	13.3	22.6	39.7
9	3.4	13.5	20.4	37.2
10	3.0	13.2	15.0	31.3

Model B Simulation Ranks

Sim	t = 1	t = 2	t = 3
1	7	7	8
2	2	4	3
3	1	2	1
4	4	9	5
5	8	1	2
6	6	8	9
7	3	10	4
8	5	5	6
9	9	3	7
10	10	6	10

Aggregating Results Rank Tying

- Once rearranged, we can then sum across the rows to calculate a total reserve for each simulation

Model A Simulations

Sim	t = 1	t = 2	t = 3	Total
1	3.4	5.8	28.8	37.9
2	2.5	12.5	28.0	43.0
3	1.8	6.5	24.0	32.3
4	3.8	8.8	20.0	32.7
5	4.4	8.7	14.5	27.6
6	3.0	10.7	14.0	27.8
7	2.0	9.4	16.9	28.2
8	6.0	7.6	24.9	38.4
9	3.7	9.7	25.0	38.4
10	6.4	8.6	29.0	44.0

Model A Simulation Ranks

Sim	t = 1	t = 2	t = 3
1	6	10	2
2	8	1	3
3	10	9	6
4	4	5	7
5	3	6	9
6	7	2	10
7	9	4	8
8	2	8	5
9	5	3	4
10	1	7	1

Sampled Simulations: Re-ordered by Model B

Sim	t = 1	t = 2	t = 3	Total
1	3.6	10.7	16.9	31.2
2	4.4	12.0	25.0	41.4
3	6.4	13.5	28.0	47.8
4	4.4	8.6	22.6	35.6
5	3.0	16.1	26.9	46.1
6	3.7	8.7	15.0	27.5
7	4.4	7.6	24.0	35.9
8	3.9	11.3	20.0	35.2
9	2.5	13.3	19.9	35.7
10	1.8	10.7	14.0	26.5

Model B Simulations

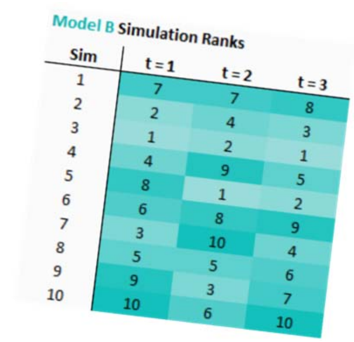
Sim	t = 1	t = 2	t = 3	Total
1	3.6	12.0	19.9	35.5
2	4.6	13.3	26.9	44.8
3	5.2	16.1	27.2	48.6
4	4.4	11.3	22.7	38.4
5	3.4	17.2	26.9	47.6
6	3.6	11.3	15.7	30.6
7	4.4	10.7	22.9	38.0
8	3.9	13.3	22.6	39.7
9	3.4	13.5	20.4	37.2
10	3.0	13.2	15.0	31.3

Model B Simulation Ranks

Sim	t = 1	t = 2	t = 3
1	7	7	8
2	2	4	3
3	1	2	1
4	4	9	5
5	8	1	2
6	6	8	9
7	3	10	4
8	5	5	6
9	9	3	7
10	10	6	10

Aggregating Results Rank Tying: Summary

- Rank Tying is a means of combining simulations across origin periods while maintaining the same parameter variance dependency structure associated with one of the underlying projection models
- In essence, this approach assumes that the introduction of model uncertainty does not produce any dependency across origin periods
- Rank Tying dependencies across accident years:
  - Process Error = None
  - Parameter Error = Select a single model for source
  - Model Error = None
- *Should* there be correlation among accident years for model uncertainty?
  - It may be argued that if a model is biased to overestimate or underestimate then it will likely have a similar bias across all origin periods



Model B Simulation Ranks

Sim	t = 1	t = 2	t = 3
1	7	7	8
2	2	4	3
3	1	2	1
4	4	9	5
5	8	1	2
6	6	8	9
7	3	10	4
8	5	5	6
9	9	3	7
10	10	6	10

Aggregating Results

Model Tying

Aggregating Results Model Tying

- This method also involves reordering the simulations
- However, in this case, we will be rearranging at the 'Model Matrix' stage, prior to pulling through the reserves from the underlying model

Model A Simulations

Sim	t = 1	t = 2	t = 3	Total
1	3.4	5.8	28.8	37.9
2	2.5	12.5	28.0	43.0
3	1.8	6.5	24.0	32.3
4	3.8	8.8	20.0	32.7
5	4.4	8.7	14.5	27.6
6	3.0	10.7	14.0	27.8
7	2.0	9.4	16.9	28.2
8	6.0	7.6	24.9	38.4
9	3.7	9.7	25.0	38.4
10	6.4	8.6	29.0	44.0

Weighting Selections

	t = 1	t = 2	t = 3
Model A	50%	50%	50%
Model B	50%	50%	50%

Model Matrix

Sim	t = 1	t = 2	t = 3
1	B	B	B
2	A	B	A
3	A	B	A
4	B	B	A
5	A	A	B
6	A	A	A
7	B	B	A
8	B	A	B
9	A	B	A
10	A	A	B

Sampled Simulations

Sim	t = 1	t = 2	t = 3	Total
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

Model B Simulations

Sim	t = 1	t = 2	t = 3	Total
1	3.6	12.0	19.9	35.5
2	4.6	13.3	26.9	44.8
3	5.2	16.1	27.2	48.6
4	4.4	11.3	22.7	38.4
5	3.4	17.2	26.9	47.6
6	3.6	11.3	15.7	30.6
7	4.4	10.7	22.9	38.0
8	3.9	13.3	22.6	39.7
9	3.4	13.5	20.4	37.2
10	3.0	13.2	15.0	31.3

Aggregating Results Model Tying

- We wish to reorder the 'Model Matrix' to maximize the degree to which 'A's in one year are grouped with 'A's in other years, and the degree to which 'B's are grouped with 'B's
- We do this to maximize the correlation of the <u>method selected</u> in each of the accident years

Model A Simulations

Sim	t = 1	t = 2	t = 3	Total
1	3.4	5.8	28.8	37.9
2	2.5	12.5	28.0	43.0
3	1.8	6.5	24.0	32.3
4	3.8	8.8	20.0	32.7
5	4.4	8.7	14.5	27.6
6	3.0	10.7	14.0	27.8
7	2.0	9.4	16.9	28.2
8	6.0	7.6	24.9	38.4
9	3.7	9.7	25.0	38.4
10	6.4	8.6	29.0	44.0

Weighting Selections

	t = 1	t = 2	t = 3
Model A	50%	50%	50%
Model B	50%	50%	50%

Model Matrix: Re-ordered

Sim	t = 1	t = 2	t = 3
1	A	B	A
2	A	A	A
3	B	B	B
4	A	A	A
5	A	A	A
6	B	B	B
7	A	A	A
8	B	B	B
9	A	B	A
10	B	B	B

Sampling error may mean that we do not achieve an exact 50/50 split in each year so 'perfect strings' are not always possible

Sim	Total
9	
10	

Model B Simulations

Sim	t = 1	t = 2	t = 3	Total
1	3.6	12.0	19.9	35.5
2	4.6	13.3	26.9	44.8
3	5.2	16.1	27.2	48.6
4	4.4	11.3	22.7	38.4
5	3.4	17.2	26.9	47.6
6	3.6	11.3	15.7	30.6
7	4.4	10.7	22.9	38.0
8	3.9	13.3	22.6	39.7
9	3.4	13.5	20.4	37.2
10	3.0	13.2	15.0	31.3

Aggregating Results Model Tying

- We can now select the samples from our underlying methods using the sampling 'Model Matrix' reordered such that we maximize the 'model correlation'…

Model A Simulations

Sim	t = 1	t = 2	t = 3	Total
1	3.4	5.8	28.8	37.9
2	2.5	12.5	28.0	43.0
3	1.8	6.5	24.0	32.3
4	3.8	8.8	20.0	32.7
5	4.4	8.7	14.5	27.6
6	3.0	10.7	14.0	27.8
7	2.0	9.4	16.9	28.2
8	6.0	7.6	24.9	38.4
9	3.7	9.7	25.0	38.4
10	6.4	8.6	29.0	44.0

Weighting Selections

	t = 1	t = 2	t = 3
Model A	50%	50%	50%
Model B	50%	50%	50%

Model Matrix: Re-ordered

Sim	t = 1	t = 2	t = 3
1	A	B	A
2	A	A	A
3	B	B	B
4	A	A	A
5	A	A	A
6	B	B	B
7	A	A	A
8	B	B	B
9	A	B	A
10	B	B	B

Sampled Simulations

Sim	t = 1	t = 2	t = 3
1	3.4	12.0	28.8
2	2.5	12.5	28.0
3	5.2	16.1	27.2
4	3.8	8.8	20.0
5	4.4	8.7	14.5
6	3.6	11.3	15.7
7	2.0	9.4	16.9
8	3.9	13.3	22.6
9	3.7	13.5	25.0
10	3.0	13.2	15.0

Model B Simulations

Sim	t = 1	t = 2	t = 3	Total
1	3.6	12.0	19.9	35.5
2	4.6	13.3	26.9	44.8
3	5.2	16.1	27.2	48.6
4	4.4	11.3	22.7	38.4
5	3.4	17.2	26.9	47.6
6	3.6	11.3	15.7	30.6
7	4.4	10.7	22.9	38.0
8	3.9	13.3	22.6	39.7
9	3.4	13.5	20.4	37.2
10	3.0	13.2	15.0	31.3

Aggregating Results **Model Tying**

- …allowing us to simply sum across the sampled simulation ‘strings’ to derive our set of total simulated reserves

Model A Simulations

Sim	t = 1	t = 2	t = 3	Total
1	3.4	5.8	28.8	37.9
2	2.5	12.5	28.0	43.0
3	1.8	6.5	24.0	32.3
4	3.8	8.8	20.0	32.7
5	4.4	8.7	14.5	27.6
6	3.0	10.7	14.0	27.8
7	2.0	9.4	16.9	28.2
8	6.0	7.6	24.9	38.4
9	3.7	9.7	25.0	38.4
10	6.4	8.6	29.0	44.0

Model B Simulations

Sim	t = 1	t = 2	t = 3	Total
1	3.6	12.0	19.9	35.5
2	4.6	13.3	26.9	44.8
3	5.2	16.1	27.2	48.6
4	4.4	11.3	22.7	38.4
5	3.4	17.2	26.9	47.6
6	3.6	11.3	15.7	30.6
7	4.4	10.7	22.9	38.0
8	3.9	13.3	22.6	39.7
9	3.4	13.5	20.4	37.2
10	3.0	13.2	15.0	31.3

Weighting Selections

	t = 1	t = 2	t = 3
Model A	50%	50%	50%
Model B	50%	50%	50%

Model Matrix: Re-ordered

Sim	t = 1	t = 2	t = 3
1	A	B	A
2	A	A	A
3	B	B	B
4	A	A	A
5	A	A	A
6	B	B	B
7	A	A	A
8	B	B	B
9	A	B	A
10	B	B	B

Sampled Simulations

Sim	t = 1	t = 2	t = 3	Total
1	3.4	12.0	28.8	44.2
2	2.5	12.5	28.0	43.0
3	5.2	16.1	27.2	48.6
4	3.8	8.8	20.0	32.7
5	4.4	8.7	14.5	27.6
6	3.6	11.3	15.7	30.6
7	2.0	9.4	16.9	28.2
8	3.9	13.3	22.6	39.7
9	3.7	13.5	25.0	42.2
10	3.0	13.2	15.0	31.3

Aggregating Results Model Tying: Summary

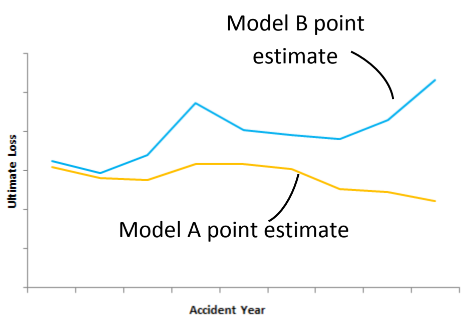
- Using the Method Tying approach ensures that, where possible, the original ‘strings’ of simulations through each year are kept intact, thereby inherently including the dependencies implied by the underlying models
- However, where perfect ‘string’s aren’t possible due to changing weights, we are essentially breaking origin period correlation caused by parameter error within a model, as we are combining simulations from different models randomly.
  - This may be a desirable effect

Model Matrix: Re-ordered

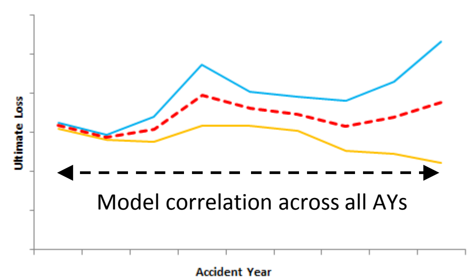
Sim	t = 1	t = 2	t = 3
1	A	B	A
2	A	A	A
3	B	B	B
4	A	A	A
5	A	A	A
6	B	B	B
7	A	A	A
8	B	B	B
9	A	B	A
10	B	B	B

Example

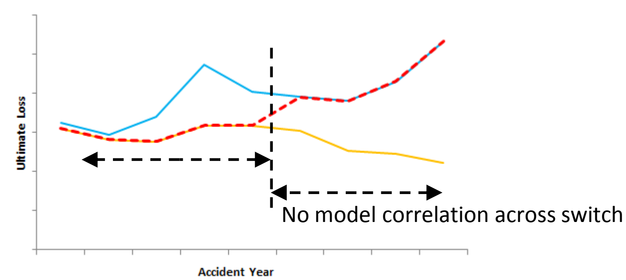
Underlying Models



Equal weights across all AYs

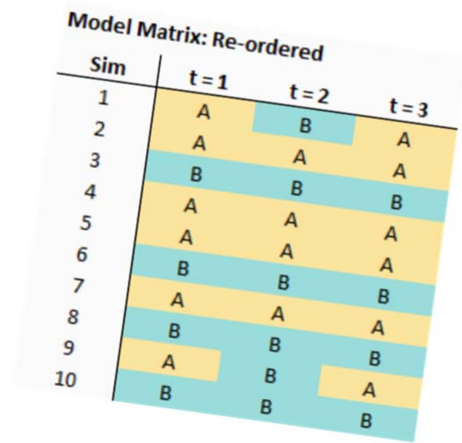


Weight switching from Model A to Model B



Aggregating Results Model Tying: Summary

- Using the Method Tying approach ensures that, where possible, the original ‘strings’ of simulations through each year are kept intact, thereby inherently including the dependencies implied by the underlying models
- However, where perfect ‘string’s aren’t possible due to changing weights, we are essentially breaking origin period correlation caused by parameter error within a model, as we are combining simulations from different models randomly.
  - This may be a desirable effect
  - Pre-sorting the original sets of simulations (prior to sampling) imposes a proxy dependency between models
- Rank Tying dependencies across accident years:
  - Process Error = None
  - Parameter Error = Yes, to the extent selection weights between models implies it should exist
  - Model Error = Yes



Model Matrix: Re-ordered

Sim	t = 1	t = 2	t = 3
1	A	B	A
2	A	A	A
3	B	B	B
4	A	A	A
5	A	A	A
6	B	B	B
7	A	A	A
8	B	B	B
9	A	B	A
10	B	B	B

Aggregating Results Model Tying

- In a situation where equal weights are applied to each accident year, this approach will yield very similar results to the method suggested earlier – i.e. sampling just once and ensuring that the same simulation is picked for each time period:

Weighted sampling at individual years, then Model Tying ≈ **Weighted sampling at total**

Model Matrix: Re-ordered

Sim	t = 1	t = 2	t = 3
1	A	B	A
2	A	A	A
3	B	B	B
4	A	A	A
5	A	A	A
6	B	B	B
7	A	A	A
8	B	B	B
9	A	B	A
10	B	B	B

Model Matrix

Sim	t = 1	t = 2	t = 3	Total
1	B	B	B	B
2	A	A	A	A
3	A	A	A	A
4	A	A	A	A
5	B	B	B	B
6	A	A	A	A
7	A	A	A	A
8	B	B	B	B
9	A	A	A	A
10	B	B	B	B

Aggregating Results

Summary

Aggregating Results **Summary**

- We have outlined three ways in which yearly reserve uncertainty estimates can be aggregated to determine the variability around the **total** (i.e. all year) unpaid loss estimates:
  - Weighted sampling at a **total** level
  - Weighted sampling and re-arranging **sampled simulations** with **Rank Tying**
  - Weighted sampling and re-arranging the **Model Matrix** with **Model Tying**
- It is not always easy to predict how the approaches will compare as it depends on the weightings employed and the results of the respective models across accident years

Model Matrix

Sim	t = 1	t = 2	t = 3	Total
1	B	B	B	B
2	A	A	A	A
3	A	A	A	A
4	A	A	A	A
5	B	B	B	B
6	A	A	A	A
7	A	A	A	A
8	B	B	B	B
9	A	A	A	A
10	B	B	B	B

Sampling the total

Model B Simulation Ranks

Sim	t = 1	t = 2	t = 3
1	7	7	8
2	2	4	3
3	1	2	1
4	4	9	5
5	8	1	2
6	6	8	9
7	3	10	4
8	5	5	6
9	9	3	7
10	[illegible]	6	10

Rank Tying

Model Matrix: Re-ordered

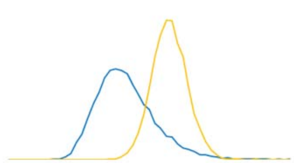
Sim	t = 1	t = 2	t = 3
1	A	B	A
2	A	A	A
3	B	B	B
4	A	A	A
5	A	A	A
6	B	B	B
7	A	A	A
8	B	B	B
9	A	B	A
10	B	B	B

Model Tying

Aggregating Results Summary

- All three approaches are scalable to allow for the incorporation of multiple models and multiple accident years in the estimate of reserve uncertainty
- Furthermore, the Rank Tying and Method Tying approaches involve sampling at the individual year level and therefore also support the ability to apply weights specific to each accident year

2 Methods…



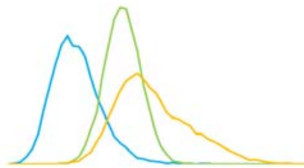
Weighting Selections

	t = 1	t = 2	t = 3
Model A	50%	50%	50%
Model B	50%	50%	50%

Model Matrix

Sim	t = 1	t = 2	t = 3
1	B	B	B
2	A	B	A
3	A	B	A
4	B	B	A
5	A	A	B
6	A	A	A
7	B	B	A
8	B	A	B
9	A	B	A
10	A	A	B

…3 Methods…



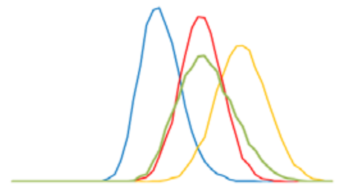
Weighting Selections

	t = 1	t = 2	t = 3
Model A	33%	50%	25%
Model B	33%	25%	25%
Model C	33%	25%	50%

Model Matrix

Sim	t = 1	t = 2	t = 3
1	C	C	C
2	A	B	A
3	A	B	A
4	C	C	A
5	A	A	B
6	A	A	A
7	B	B	A
8	B	A	B
9	A	C	C
10	A	A	B

…4 Methods… …etc.…



Weighting Selections

	t = 1	t = 2	t = 3
Model A	25%	33%	50%
Model B	25%	33%	50%
Model C	25%	33%	0%
Model D	25%	0%	0%

Model Matrix

Sim	t = 1	t = 2	t = 3
1	C	C	A
2	D	B	B
3	A	B	A
4	C	C	A
5	D	A	B
6	A	A	A
7	D	B	A
8	B	A	B
9	C	C	B
10	A	A	B

- This allows actuaries to reflect the same weighting philosophy in their uncertainty estimate as employed in their selection of the central estimate

Case Study

Application of Approach

Case Study Underlying Models

- Three models are investigated
- For the central estimate, each model is given equal weight (for each accident year)

Central Estimate: Selected Weighting

	Model A	Model B	Model C
2009	33.3%	33.3%	33.3%
2010	33.3%	33.3%	33.3%
2011	33.3%	33.3%	33.3%

Central Estimate: Reserves

	Model A	Model B	Model C	Selected
2009	3.6	3.8	3.9	3.8
2010	9.1	12.9	13.5	11.8
2011	19.9	23.5	22.9	22.1

Case Study Variability around individual models

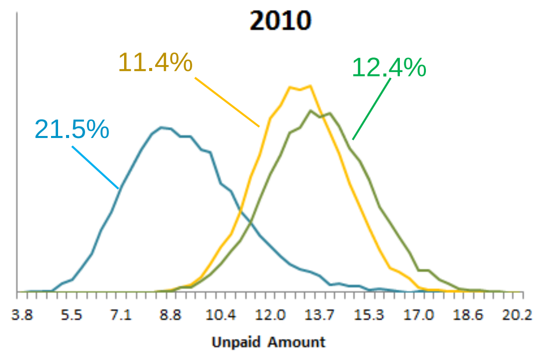
- Three models are investigated
- For the central estimate, each model is given equal weight (for each accident year)
- Traditional methods are used to produce predictive distribution around each model (based on Bootstrap approach)

Central Estimate: Selected Weighting

	Model A	Model B	Model C
2009	33.3%	33.3%	33.3%
2010	33.3%	33.3%	33.3%
2011	33.3%	33.3%	33.3%

Central Estimate: Reserves

	Model A	Model B	Model C	Selected
2009	3.6	3.8	3.9	3.8
2010	9.1	12.9	13.5	11.8
2011	19.9	23.5	22.9	22.1

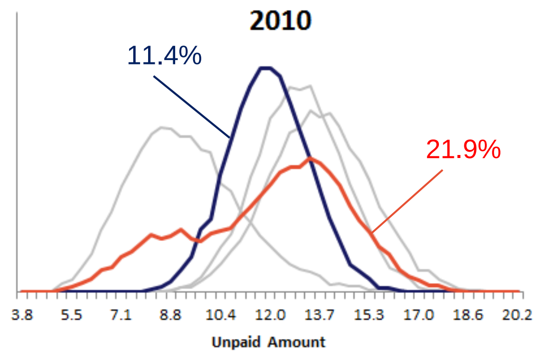


Uncertainty Summary (Coeff. of Var)

	Model A	Model B	Model C
2009	43.2%	11.0%	11.6%
2010	21.5%	11.4%	12.4%
2011	26.7%	17.0%	16.9%

Case Study Variability around multiple models

- We are now faced with the challenge of deriving an estimate of the uncertainty around our prediction, reflecting each model used
- We can employ alternative methods for deriving the uncertainty for individual accident years:
  - Model scaling (using model B)
  - Weighted sampling (using weights)



Central Estimate: Selected Weighting

	Model A	Model B	Model C
2009	33.3%	33.3%	33.3%
2010	33.3%	33.3%	33.3%
2011	33.3%	33.3%	33.3%

Central Estimate: Reserves

	Model A	Model B	Model C	Selected
2009	3.6	3.8	3.9	3.8
2010	9.1	12.9	13.5	11.8
2011	19.9	23.5	22.9	22.1

Uncertainty Summary (Coeff. of Var)

	Model A	Model B	Model C	Selected: Scaled (Model B)	Selected: Wtd Sample
2009	43.2%	11.0%	11.6%	11.0%	25.9%
2010	21.5%	11.4%	12.4%	11.4%	21.9%
2011	26.7%	17.0%	16.9%	17.0%	21.5%

Case Study Aggregating the results

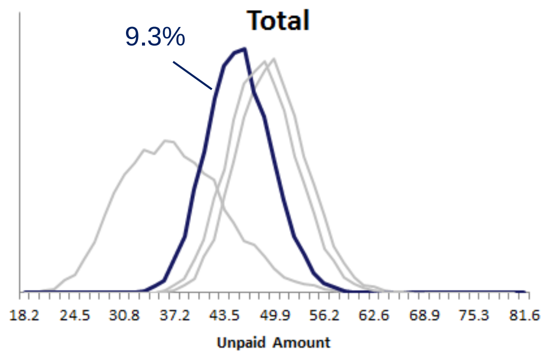
- Finally, we must aggregate the individual accident year results to calculate the total variability estimate
- With the Model Scaling approach, the total estimate is relatively easily to derive as we are utilizing the simulation strings from a single underlying model

Central Estimate: Selected Weighting

	Model A	Model B	Model C
2009	33.3%	33.3%	33.3%
2010	33.3%	33.3%	33.3%
2011	33.3%	33.3%	33.3%

Central Estimate: Reserves

	Model A	Model B	Model C	Selected
2009	3.6	3.8	3.9	3.8
2010	9.1	12.9	13.5	11.8
2011	19.9	23.5	22.9	22.1



Uncertainty Summary (Coeff. of Var)

	Model A	Model B	Model C	Selected: Scaled (Model B)	Selected: Wtd Sample
2009	43.2%	11.0%	11.6%	11.0%	25.9%
2010	21.5%	11.4%	12.4%	11.4%	21.9%
2011	26.7%	17.0%	16.9%	17.0%	21.5%
Total	17.9%	9.1%	9.0%	9.3%	?

Case Study Aggregating the results

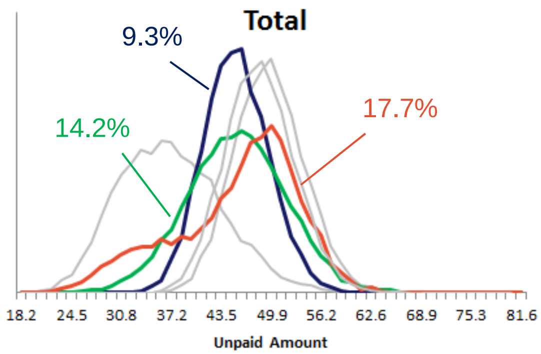
- Similarly, we can utilize the Rank Tying and Model Tying approaches to derive the total variability estimate for our weighted samples:

Model Weighting

	Model A	Model B	Model C
2009	33.3%	33.3%	33.3%
2010	33.3%	33.3%	33.3%
2011	33.3%	33.3%	33.3%

Central Estimate: Reserves

	Model A	Model B	Model C	Selected
2009	3.6	3.8	3.9	3.8
2010	9.1	12.9	13.5	11.8
2011	19.9	23.5	22.9	22.1



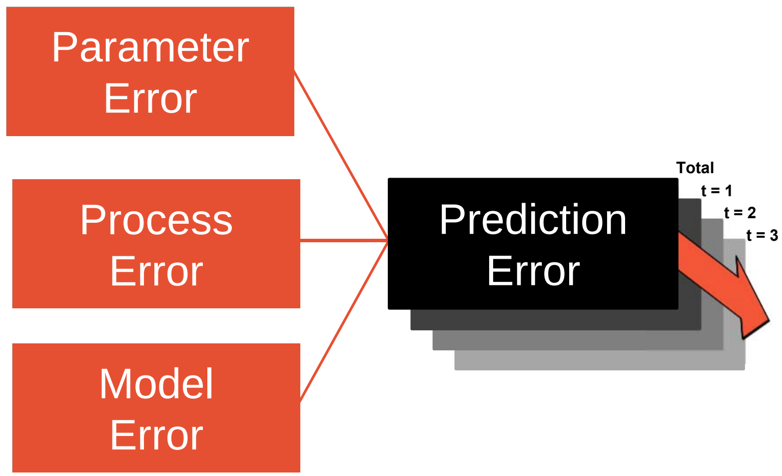
Uncertainty Summary (Coeff. of Var)

	Model A	Model B	Model C	Selected: Scaled (Model B)	Selected: Wtd Sample Rank Tying	Selected: Wtd Sample Model Tying
2009	43.2%	11.0%	11.6%	11.0%	25.9%	25.9%
2010	21.5%	11.4%	12.4%	11.4%	21.9%	21.9%
2011	26.7%	17.0%	16.9%	17.0%	21.5%	21.5%
Total	17.9%	9.1%	9.0%	9.3%	14.2%	17.7%

Incorporating Model Error into Actuary's Estimate of Uncertainty

Summary

Summary



- The uncertainty of a prediction is comprised of three components:
- A number of commonly-employed approaches compute uncertainty under the assumption that a single model is representative of the phenomenon
- **Model error** is evident when the actuary places reliance on multiple models as being instructive of their central estimate of unpaid amounts
- **Weighted sampling** is an approach that can be used to incorporate model uncertainty around a central prediction
- **Rank Tying** and **Model Tying** are practical approaches that can be used to incorporate model uncertainty into an aggregation of multiple predictions (e.g. multiple accident years)
- What we produce is a predictive distribution (or a range around our predictions)
- Such approaches allow the actuary to tackle their analysis of uncertainty in an intuitively similar manner to how they derive their central estimate – i.e. with the use of multiple models and application of weights