Uncertainty

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** Swiss Finance Institute

First of all: the “Eidgenössische Technische Hochschule Zürich”

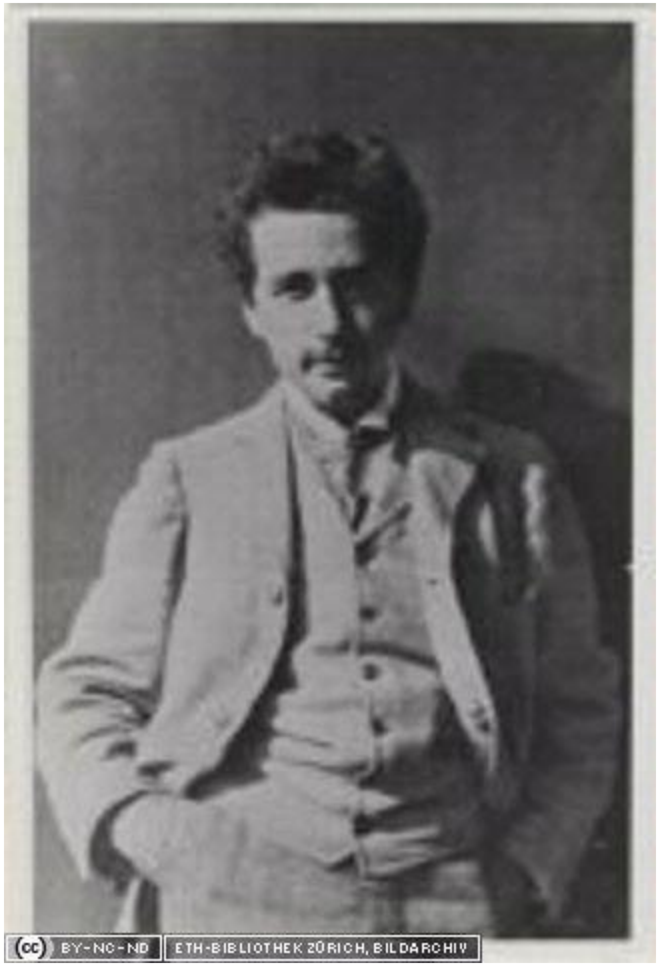
The Building

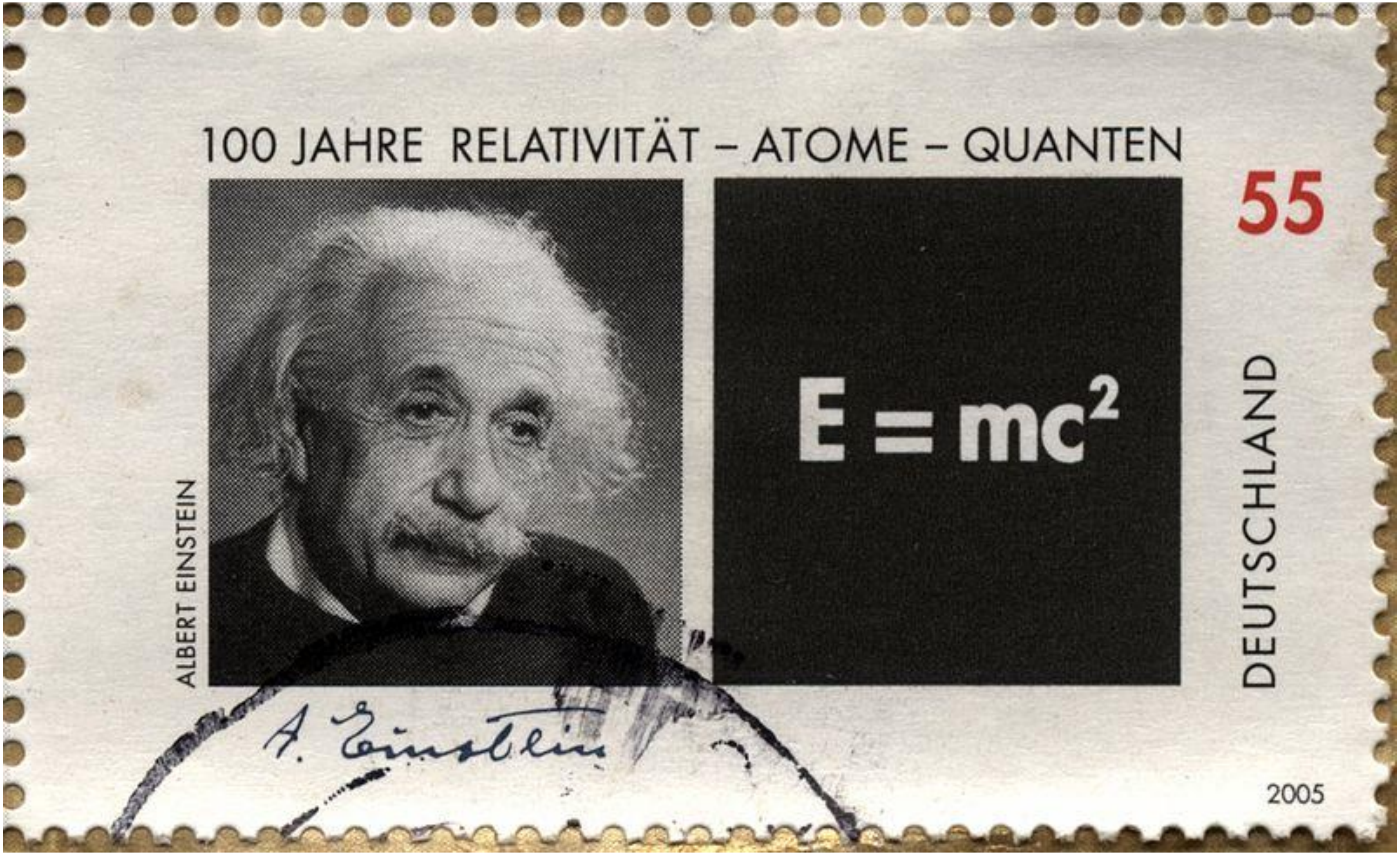
And logo

ETH

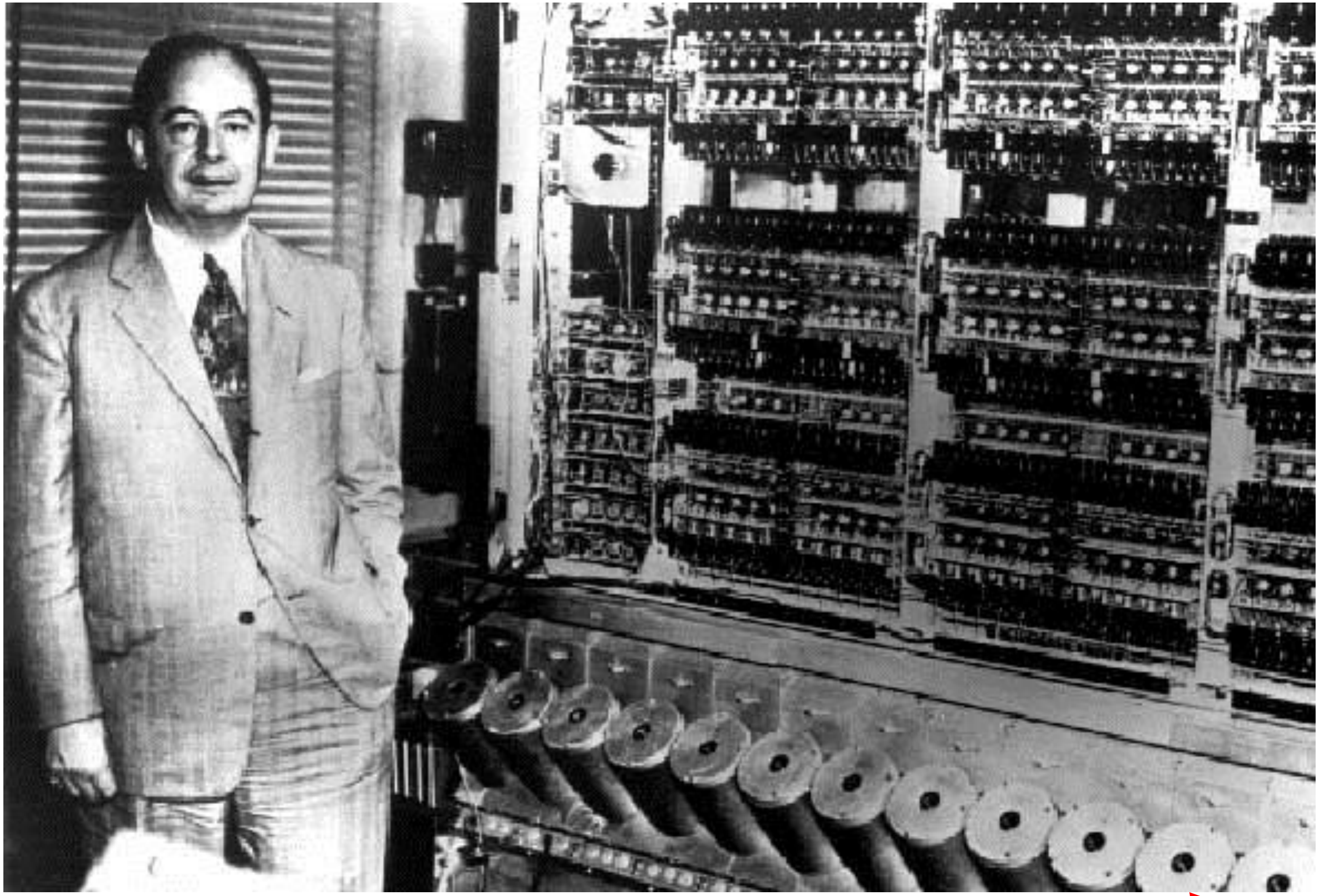
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Two of its former students:





Albert Einstein



John von Neumann

ENIAC

and their(/a) link to Uncertainty:

Albert Einstein:

Relativity, (…), and

“God does not play dice with the universe”

John von Neumann:

Cybernetics, Game Theory, Utility Theory, and

“With four parameters I can fit an elephant,

and with five I can make him wiggle his trunk”

On the role of Mathematics!

"Mathematics is an experimental science. It matters little that the mathematician experiments with pencil and paper while the chemist uses testtube and retort, or the biologist stains and the microscope. The only great point of divergence between mathematics and the other sciences lies in the circumstance that experience only whispers 'yes' or 'no' in reply to our questions, while logic **shouts**."

Norbert Wiener

Two giants of
“shouting“ uncertainty-logic

JACOBI BERNOULLI,

ARS CONJECTANDI,

OPUS POSTHUMUM.

Accedit

TRACTATUS

DE SERIEBUS INFINITIS,

DE LUDO PILÆ RETICULARIS.

BASILEÆ,

Impensis THURNISIORUM, Fratrum.

(1713) + 300 = 2013

Jakob I. Bernoulli, 1655 - 1705

$$(\Omega, \mathscr{F}, P)$$

Independence, conditional probability

A.N. Kolmogorov, 1903 - 1987

ERGEBNISSE DER MATHEMATIK UND IHRER GRENZGEBIETE

GRUNDBEGRIFFE DER WAHRSCHEINLICHKEITS-RECHNUNG

VON

A. KOLMOGOROFF

BERLIN

VERLAG VON JULIUS SPRINGER

1933

(1933)

A historical anecdote/puzzle/quiz

Original German version: 1933

Herrn A. Khintchine, der das ganze Manuskript sorgfältig durchgelesen und dabei mehrere Verbesserungen vorgeschlagen hat, danke ich an dieser Stelle herzlich.

Kljasma bei Moskau, Ostern 1933.

A. Kolmogoroff.

English translation(s): 1950, 1956:

Kljasma near Moscow, Easter 1933.

Russian translation(s): 1936, 1974, 1998

Приношу свою сердечную благодарность А. Я. Хинчину, внимательно прочитавшему всю рукопись и предложившему целый ряд улучшений.

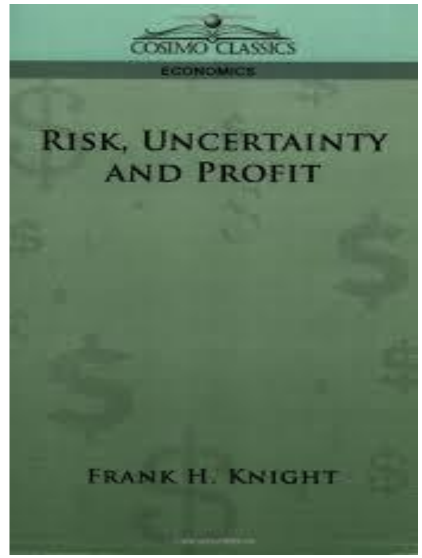
Клязьма близ Москвы, 1 мая 1933 г.

А. Колмогоров

May 1, 1933!

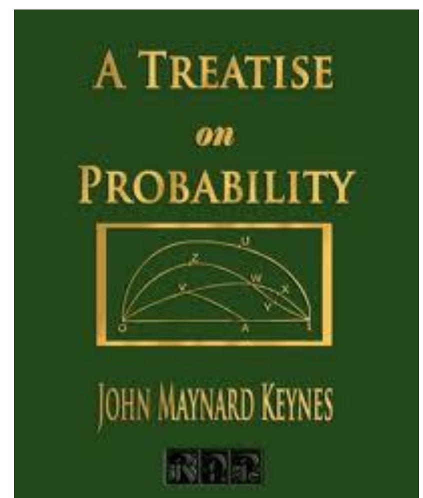
And two giants from economics of the "whispering" kind

Frank H. Knight
(1885 – 1972)



Knightian
Uncertainty

John Maynard Keynes
(1883 – 1946)



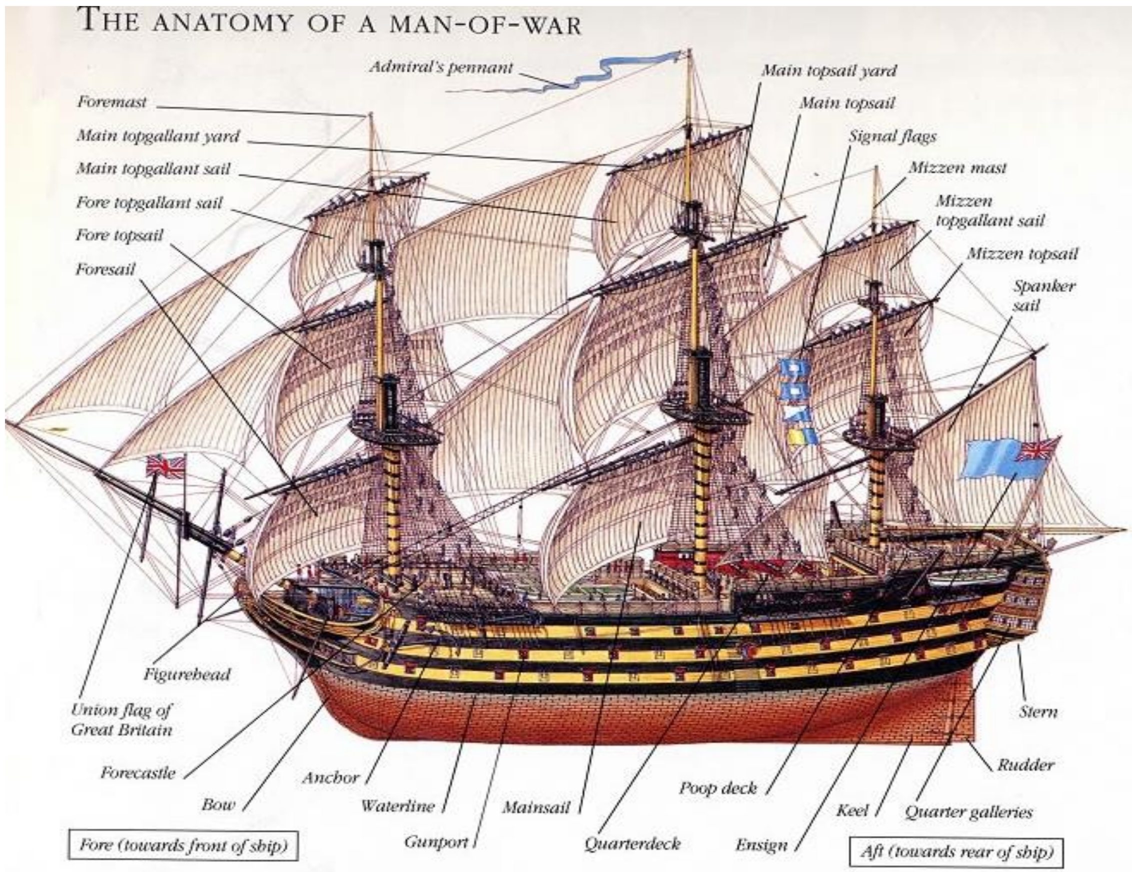
(Recent) economic events and financial crises added various variations

- Against the Gods (Bernstein)
- Fooled by Randomness (Taleb)
- Black Swans (Taleb)
- Red Dragons (Sornette)
- The Known, the Unknown and the Unknowable (Rumsfeld et al.)
- …
- A comment on Knightian Uncertainty:

A ‘BEAUFORT SCALE’ OF PREDICTABILITY

(M.H.A. Davis, 2013)

The Beaufort wind scale was devised by Francis Beaufort (later, Rear-Admiral Sir Francis) in 1805 for use by the Royal Navy, expressed in terms of wind in the sails of a ‘man of war’.



The Beaufort scale of wind intensity

<table>
<tr><th>No.</th><th>Description</th><th colspan="2">Beaufort's criterion</th></tr>
<tr><td>0</td><td>Calm</td><td colspan="2">Calm</td></tr>
<tr><td>1</td><td>Light Air</td><td colspan="2">Just sufficient to give steerage way</td></tr>
<tr><td>2</td><td>Light breeze</td><td rowspan="3">With which a well-conditioned man of war, under all sail, and 'clean full', would go in smooth water from ...</td><td>1 to 2 knots</td></tr>
<tr><td>3</td><td>Gentle breeze</td><td>3 to 4 knots</td></tr>
<tr><td>4</td><td>Moderate breeze</td><td>5 to 6 knots</td></tr>
<tr><td>5</td><td>Fresh breeze</td><td rowspan="5">In which a well-conditioned man of war, under all sail, and 'clean full', could just carry close-hauled ...</td><td>royals</td></tr>
<tr><td>6</td><td>Strong breeze</td><td>single-reefs and top-gallant sails</td></tr>
<tr><td>7</td><td>Moderate gale</td><td>double-reefs, jib, etc.</td></tr>
<tr><td>8</td><td>Fresh gale</td><td>triple-reefs, courses, etc.</td></tr>
<tr><td>9</td><td>Strong gale</td><td>close-reefs and courses</td></tr>
<tr><td>10</td><td>Whole gale</td><td colspan="2">With which she could only bear close-reefed maintop-sail and reefed fore-sail</td></tr>
<tr><td>11</td><td>Storm</td><td colspan="2">With which she would be reduced to storm staysails</td></tr>
<tr><td>12</td><td>Hurricane</td><td colspan="2">To which she could show no canvass</td></tr>
</table>

(M.H.A. Davis)

A Beaufort scale of predictability

No.	Description	Criterion
0	IID	Probability given axiomatically: no modelling required
1	⋮	↘
⋮	⋮	Decreasing reliability in probability forecasting
⋮	⋮	↘
n	Knightian Uncertainty	Insufficient data/predictability for any probability forecasting

For a particular problem, the classification would depend on

- The data available
- The prediction horizon

In our example, we're at Beaufort 1 or 2 for a 1-week predictor, but what about a 3-month predictor?

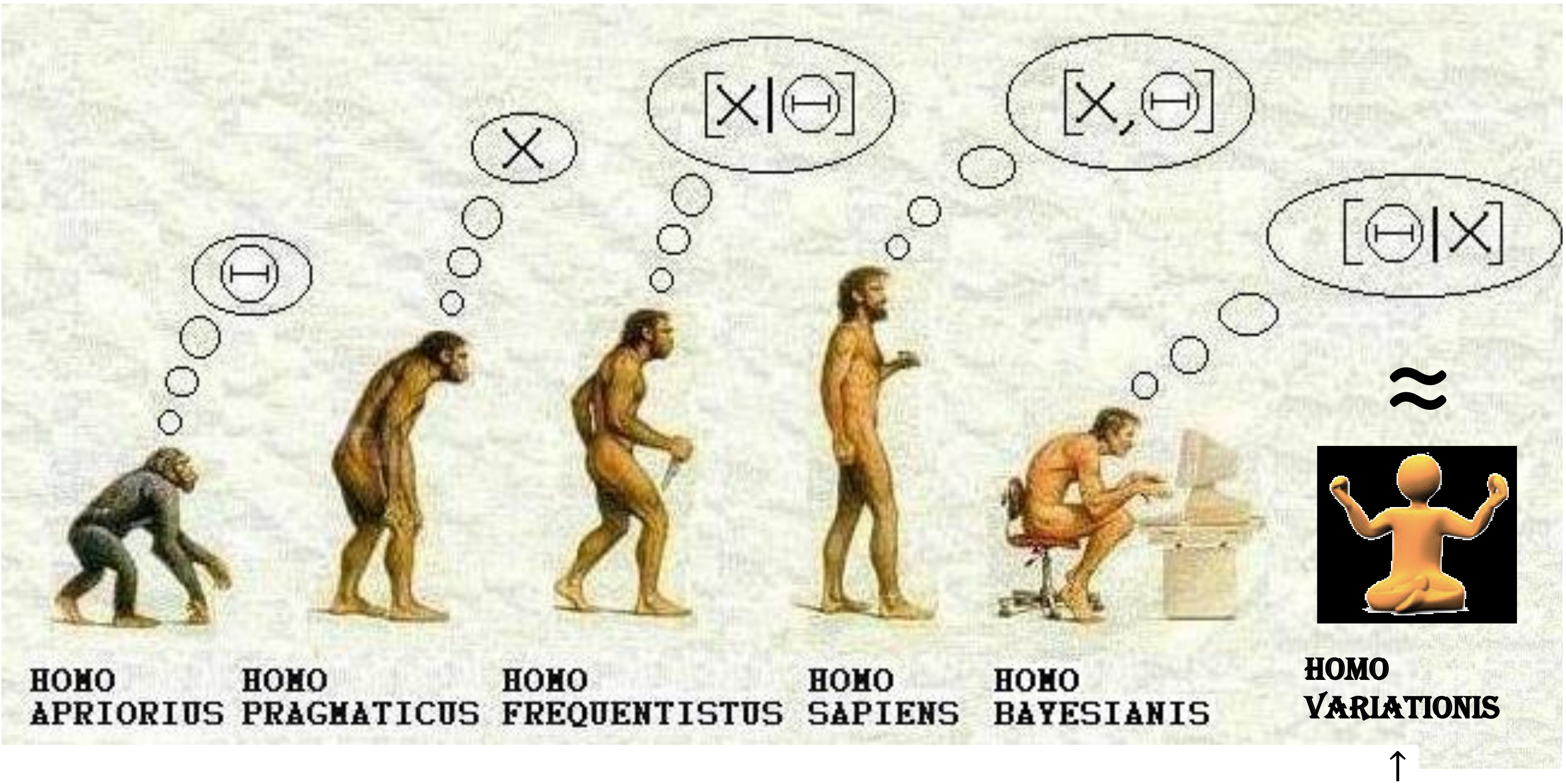
The classification could be based on the best calibration achievable using a standard set of data-driven algorithms.

(M.H.A. Davis)

We face interesting technological as well as methodological chalenges:

- IT → Bayes becomes numerically feasible
- Big Data (data size (!) ↔ information content (?))
- n (sample size) small versus p (# variables) large
- machine learning, causality, ...
- “I keep saying the sexy job in the next ten years will be statisticians. People think I'm joking, but who would've guessed that computer engineers would've been the sexy job of the 1990s?” (2009, Hal Varian, Chief Economist, Google)

Concerning Statistical Uncertainty



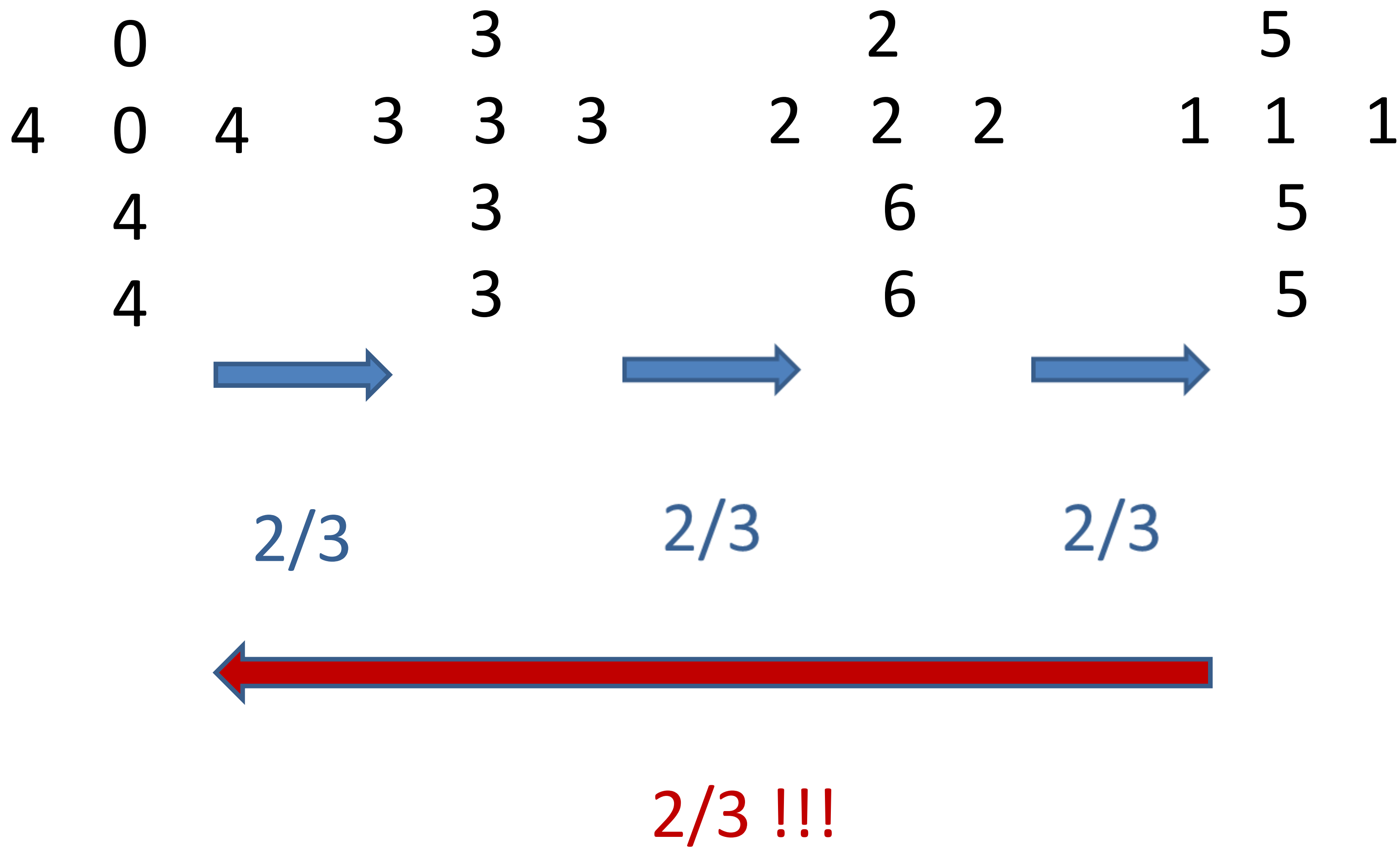
"An approximate answer to the right problem
is worth a good deal more than
an exact answer to an approximate problem."
John W. Tukey, 1915 – 2000

}

Variational Bayesian inference

Kay H. Brodersen

Interludium: a disturbing example



Three uncertainty examples from industry:

(1) "We actually got an external advisor [to assess how frequently a particular event might happen] and they came out with one in 100,000 years and we said "no", and I think we submitted one in 10,000 years. But that was a year and a half before it happened. It doesn't mean to say it was wrong: it was just unfortunate that the 10,000th year was so near."

UK House of Lords/House of Commons, June 12, 2013,
Changing banking for good, Volumes I and II (⍰ HBOS)

(2) "It is hard for us, and without being flippant, to even see a scenario within any realm of reason that would see us losing USD 1 in any of these [CDS, Credit Protection] transactions"

August 2007, Joseph J. Cassano, AIG-FP

(Also: NYT, 31/10/08: "Behind AIG's fall, risk models failed to pass real-world test.")

"… To complete this [proposed CDS transaction] review, Professor Gorton [then University of Pennsylvania, now Yale School of Management] used a sophisticated actuarial model to make sure that the proposed deal was fundamentally sound and to determine an appropriate attachment point. The process was designed to minimize risk to AIG-FP."

J. Cassano, hearing in front of the Financial Crisis Inquiry Commission, June 30, 2010

(3) A (very) broad brush definition of Solvency or (Risk-)Capital Adequacy:

Solvency = Capital/RWAs

where Capital = … and RWA = Risk Weighted Assets

”$7 billion, or more than 50% of the total $13 billion RWA reduction, could be achieved by modifying risk related models.” … ”The change in VaR [risk measure] methodology effectively masked the significant changes in the portfolio.”

(United States Senate, March 15, 2013, JPMorgan Chase Whale trades: a case history of derivatives risks and abuses)

An interesting regulatory discussion:

BCBS-Consultative Documents, May 2012 (**R1**), October 2013, Fundamental Review of the Trading Book:

From **R1**: Page 41, Question 8: «What are the likely constraints with moving from VaR to ES, including any challenges in delivering robust backtesting, and how might these be best overcome?»

Value-at-Risk: frequency measure of rare event

Expected Shortfall: severity measure of rare event

VaR = If whereas ES = What If

Lifting a methodological tip of the (model) uncertainty veil through an example from Operational Risk

VaR Aggregation

Consider:

- One-period risk positions $X_1, \ldots, X_d$ with known distribution functions (dfs) F_i, $i = 1, \ldots, d$;
- Portfolio position $X_d^+ = X_1 + \cdots + X_d$;
- $\mathrm{VaR}_\alpha(X_i)$, $i = 1, \ldots, d$, the marginal VaR's at the common confidence level $\alpha \in (0,1)$.

Task:

$$\text{Calculate } \mathrm{VaR}_\alpha(X_d^+)$$

Problem:

- We need a *joint* model for the random vector $\mathbf{X} = (X_1, \ldots, X_d)'$

VaR Aggregation

- **X** elliptical

$$\mathrm{VaR}_\alpha(X_d^+) \leq \sum_{1=1}^{d} \mathrm{VaR}_\alpha(X_i)$$

  Examples: multivariate Gaussian, multivariate Student t.

- **X** comonotone i.e. there exist increasing functions ψ_i, $i = 1, \ldots, d$ and a random variable Z so that

$$X_i = \psi_i(Z)$$

  then

$$\mathrm{VaR}_\alpha(X_d^+) = \sum_{i=1}^{d} \mathrm{VaR}_\alpha(X_i)$$

  i.e. VaR_α (like ES_α) is comonotone additive.

- Diversification benefit: one often uses

$$(1-\delta) \sum_{i=1}^{d} \mathrm{VaR}_\alpha(X_i), \; 0 < \delta < 1.$$

VaR Bounds

The Fréchet (unconstrained) problem

$$\underline{\mathrm{VaR}}_\alpha(X_d^+) = \inf_F\{\mathrm{VaR}_\alpha(X_1^F + \cdots + X_d^F) : X_i \overset{d}{\sim} F_i, i = 1, \ldots, d\}$$

$$\overline{\mathrm{VaR}}_\alpha(X_d^+) = \sup_F\{\mathrm{VaR}_\alpha(X_1^F + \cdots + X_d^F) : X_i \overset{d}{\sim} F_i, i = 1, \ldots, d\}$$

Dependence Uncertainty

Two important measures

Measure 1 Superadditivity ratio

$$\overline{\triangle}_{\alpha,d}(X_d^+) = \frac{\overline{\mathrm{VaR}}_\alpha(X_d^+)}{\sum_{i=1}^d \mathrm{VaR}_\alpha(X_i)}.$$

Measure 2 Ratio between worst-ES and worst-VaR

$$\mathcal{B}_{\alpha,d}(X_d^+) = \frac{\overline{\mathrm{ES}}_\alpha(X_d^+)}{\overline{\mathrm{VaR}}_\alpha(X_d^+)} = \frac{\sum_{i=1}^d \mathrm{ES}_\alpha(X_i)}{\overline{\mathrm{VaR}}_\alpha(X_d^+)}.$$

Dependence Uncertainty

Superadditivity ratio: some examples

- Short tailed risks
  - LogNormal(2,1)-distributed risks $\Rightarrow \overline{\triangle}_{0.999,d}(X_d^+) \approx 1.4.$
  - Gamma(3,1)-distributed risks $\Rightarrow \overline{\triangle}_{0.999,d}(X_d^+) \approx 1.1.$
- Heavy tailed risks
  - Pareto(2)-distributed risks $\Rightarrow \overline{\triangle}_{0.999,d}(X_d^+) \approx 2.$

In QRM applications often Pareto(θ) with $\theta \in [0.5, 5]$:

- $[0.5, 1]$ catastrophe insurance,
- $[3, 5]$ market return data,
- $\theta \geq 0.5$ for operational risk.

VaR versus ES: Dependence Uncertainty

Asymptotic equivalence for large dimensions of the risk portfolio, under some general conditions:

$$\lim_{d\to\infty} \frac{\overline{\mathrm{ES}}_\alpha(X_d^+)}{\overline{\mathrm{VaR}}_\alpha(X_d^+)} = 1$$

details

- In the case of F_i being identical:

$$\overline{\triangle}_{\alpha,d}(X_d^+) \approx \frac{\mathrm{ES}_\alpha(X_1)}{\mathrm{VaR}_\alpha(X_1)}.$$

Application: Operational Risk

Definition

Operational risk is the risk of losses resulting from inadequate or failed internal processes, people and systems, or external events.

Remark: This definition includes legal risk but excludes reputational and strategic risk.

Application: Operational Risk

The LDA Operational risk capital calculation under Basel II

The ingredients:

- Risk measure VaR_α
- Holding period: 1 year
- Confidence level: 99.9%, $\alpha = 0.999$
- The data 7×8 matrix; 8 Business lines, 7 Loss types
- Often: aggregate column-wise $\Rightarrow \mathrm{VaR}_\alpha^{(1)}, \ldots, \mathrm{VaR}_\alpha^{(8)}$

$$\text{Aggregate:} \quad \sum_{i=1}^{8} \mathrm{VaR}_\alpha^{(i)} = \mathrm{VaR}_\alpha^{+}.$$

Example: Pareto(2) risks

Sharp bounds on VaR and ES for the sum of d Pareto(2) distributed rvs for $\alpha = 0.999$; VaR_α^+ corresponds to the comonotonic case.

	$d = 8$	$d = 56$
$\underline{\text{VaR}}_\alpha$	31	53
$\underline{\text{ES}}_\alpha$	178	472
VaR_α^+	245	1715
$\overline{\text{VaR}}_\alpha$	465	3454
$\overline{\text{ES}}_\alpha$	498	3486
$\overline{\triangle}_\alpha(X_d^+)$	1.898	2.014
$\mathcal{B}_\alpha(X_d^+)$	1.071	1.009

An inhomogeneous Portfolio

Dependence-uncertainty spreads of VaR and ES for an inhomogeneous portfolio $X_d^+ = X_1 + \cdots + X_d$, where $X_i \sim \text{Pareto}(2 + 0.1i),\ i = 1, \ldots, 5$; $X_i \sim \text{Exp}(i - 5), i = 6, \ldots, 10$; $X_i \sim \text{Log–Normal}(0, (0.1(i - 10))^2),\ i = 11, \ldots, 20$.

	$n = 5$			$n = 20$		
	best	worst	spread	best	worst	spread
$\text{ES}_{0.975}$	22.48	44.88	22.40	29.15	102.35	73.20
$\text{VaR}_{0.975}$	9.79	41.46	31.67	21.44	100.65	79.21
$\text{VaR}_{0.9875}$	12.06	56.21	44.16	22.12	126.63	104.51
$\text{VaR}_{0.99}$	12.96	62.01	49.05	22.29	136.30	114.01
$\frac{\overline{\text{ES}}_{0.975}}{\overline{\text{VaR}}_{0.975}}$		1.08			1.02	

Generally, $\text{VaR}_\alpha(X_d^+)$ has a larger DU-spread compared to $\text{ES}_\beta(X_d^+)$ for $\alpha \geq \beta$; see Embrechts, Wang and Wang (2014).

Conclusion

- We have discussed a bit of “history of uncertainty”
- Of course, there is much, much more out there …
- For (actuarial) applications, model and dependence uncertainty are very important
- Mathematics is useful in setting the shouting boundaries between which whispering reality evolves
- Operational risk offers an interesting example
- Best-worst case scenarios are relevant for stress testing
- Include “realistic” scenarios
- More details on Friday, April 4: “Uncertainty, a continuing discussion” Room 115-A, 8:00 a.m. – 9:30 a.m., see perhaps some of you there!

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THANK YOU!