Optimal Investment, Consumption and Insurance with Recursive Utility

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Product design and advice is not all about tax

- it is also about time, risk, and preferences

Consumption-investment-insurance under time-additive utility

$$dX(t) = \overbrace{X(t)\left(\left(r+\pi(t)(\alpha-r)\right)dt+\sigma\pi(t)\right)dW(t)}^{\text{capital gains}} + \underbrace{w(t)\,dt}_{\text{labor income}} - \underbrace{c(t)\,dt}_{\text{consumption}} - \underbrace{\widehat{\mu}(t)\,b(t)\,dt}_{\text{life insurance premium}}$$

$$V(t,x) = E_{t,x}\left[\int_t^n \left(\underbrace{u(c(s))\,I(s)\,ds}_{\text{utility from consumption}} + \underbrace{u(X(s)+b(s))\,dN(s)}_{\text{utility from bequest}}\right)\right]$$

$$u(c)+\mu(t)\left(u(x+b)-V(t,x)\right)$$

$u \sim$ power

$$
\begin{aligned}
c^{*}(t,x) &= \underbrace{f(t)}_{\text{consumption-to-wealth ratio}} \left(x + \underbrace{h(t)}_{\text{human capital}} \right) \\
\pi^{*}(t,x)\, x &= \underbrace{k}_{\text{Merton's proportion}} \qquad (x + h(t)) \\
b^{*}(t,x) + x &= \underbrace{g(t)}_{\text{level of protection}} \qquad (x + h(t))
\end{aligned}
$$

Consumption-investment under separation time and risk preferences

$$V(t,x) = \int_t^n v\left(\underbrace{u^{-1}\left(E_{t,x}\left[u\left(c\left(s\right)\right)\right]\right)}_{\text{certainty equivalent}}\right) ds$$

$$u \sim v \sim \text{power}$$

$$\underbrace{a\left(c, V\left(t,x\right)\right)}_{\text{normalized aggregator}}$$

$$c^*\left(t,x\right) = f\left(t\right)\left(x + h\left(t\right)\right)$$

$$a\left(c, V\left(t,x\right)\right) + \mu\left(t\right)\left(u\left(x+b\right) - V\left(t,x\right)\right) ?$$

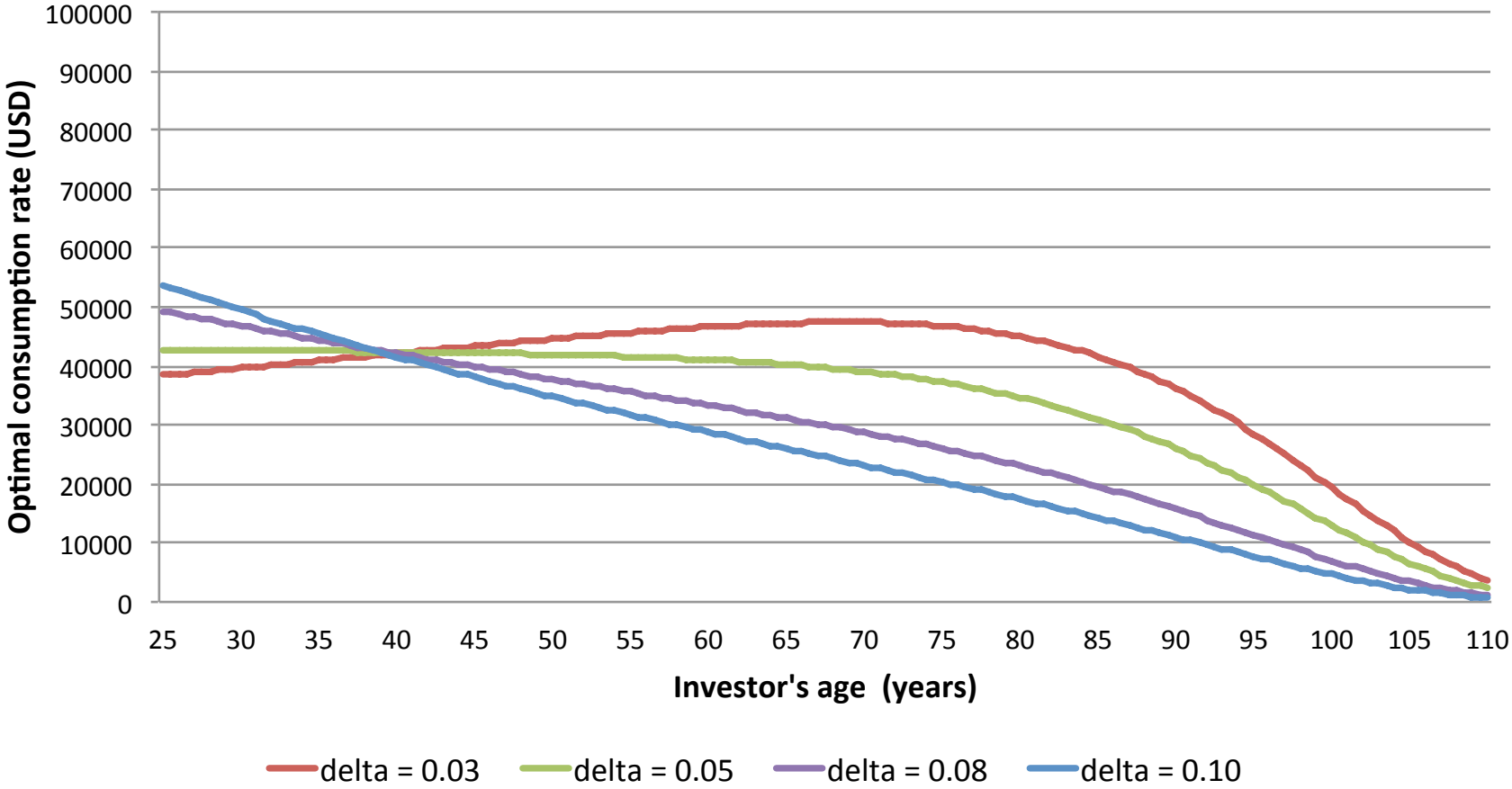
Consumption-investment-insurance under separation time and risk preferences

$$V(t,x)=\int_t^n w\left(v^{-1}\left(\begin{array}{c} v\left(\underbrace{u^{-1}\left(E_{t,x}\left[u\left(c\left(s\right)\right)I\left(s\right)\right]\right)}_{\text{certainty equivalent}}\right) \\ +v\left(\underbrace{u^{-1}\left(E_{t,x}\left[u\left(X\left(s\right)+b\left(s\right)\right)\frac{dN\left(s\right)}{ds}\right]\right)}_{\text{certainty equivalent}}\right)\end{array}\right)\right)ds$$

$$u \sim v \sim w \sim \text{power}$$

$$a\left(c, x+b, V\left(t, x\right)\right)$$

$$\begin{aligned} c^{*}\left(t, x\right) &= f\left(t\right)\left(x+h\left(t\right)\right) \\ b^{*}\left(t, x\right)+x &= g\left(t\right)\left(x+h\left(t\right)\right) \end{aligned}$$


Optimal consumption rate (USD)
100000
90000
80000
70000
60000
50000
40000
30000
20000
10000
0
25
30
35
40
45
50
55
60
65
70
75
80
85
90
95
100
105
110
Investor's age (years)
delta = 0.03
delta = 0.05
delta = 0.08
delta = 0.10