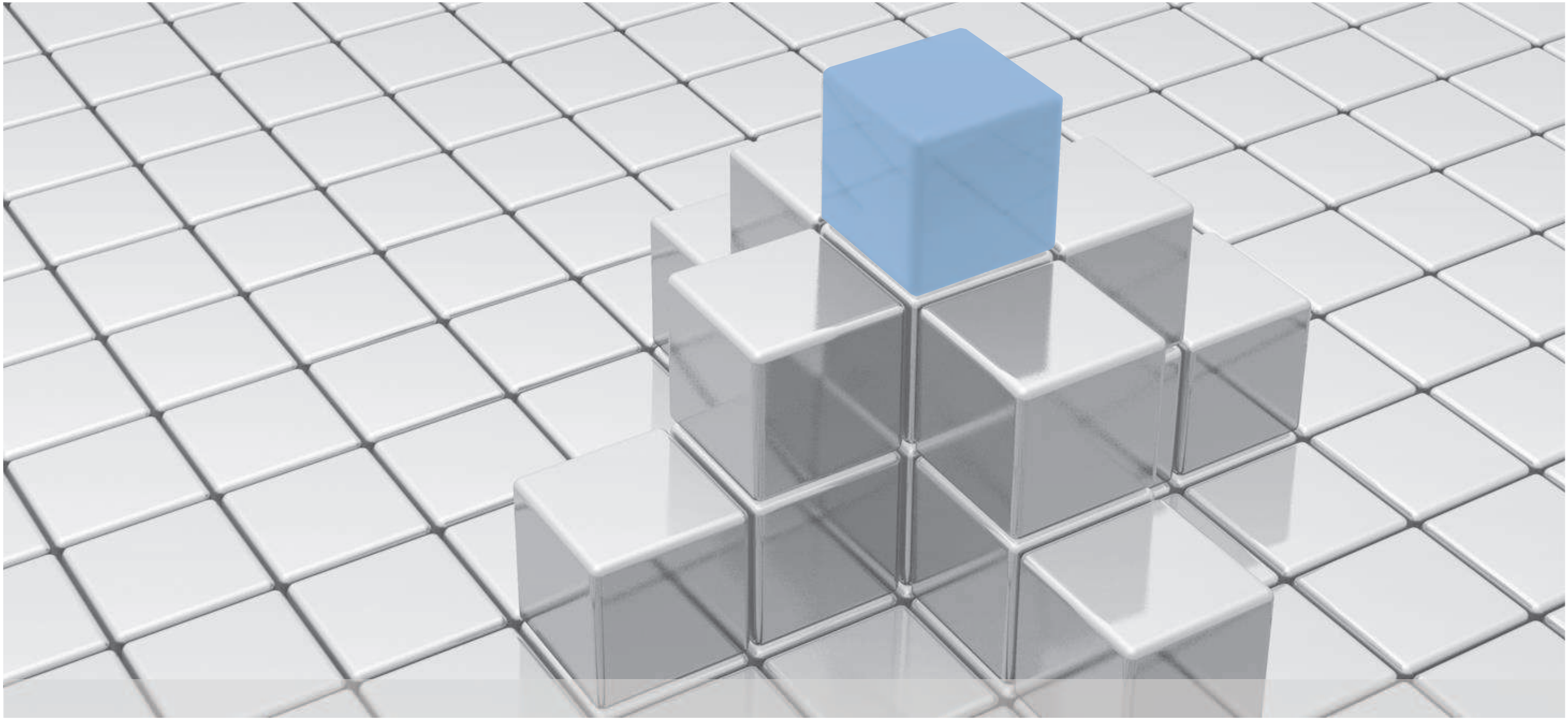


UNCERTAINTY IN CATASTROPHE MODELLING

KEY ISSUES LEADING TO INTERNAL CAPITAL MODEL MISSPECIFICATION AND INSTABILITY

ICA 2014 – Washington DC, USA // *by Yuriy Krvavych (PwC)*

Why Uncertainty? Why Cat Modelling? ...

Cats as the key driver of capital requirements: cat losses are first in line to outcompete other material risks for capital consumption and threaten company's solvency.

Uncertainty is particularly large for modelled cats: due to infrequent nature of cat events and limited historical data, e.g., 50% – 230% of PML estimate of 1-in-100 year US hurricane loss produced by physical models (*J. Major, Guy Carpenter (2011)*).

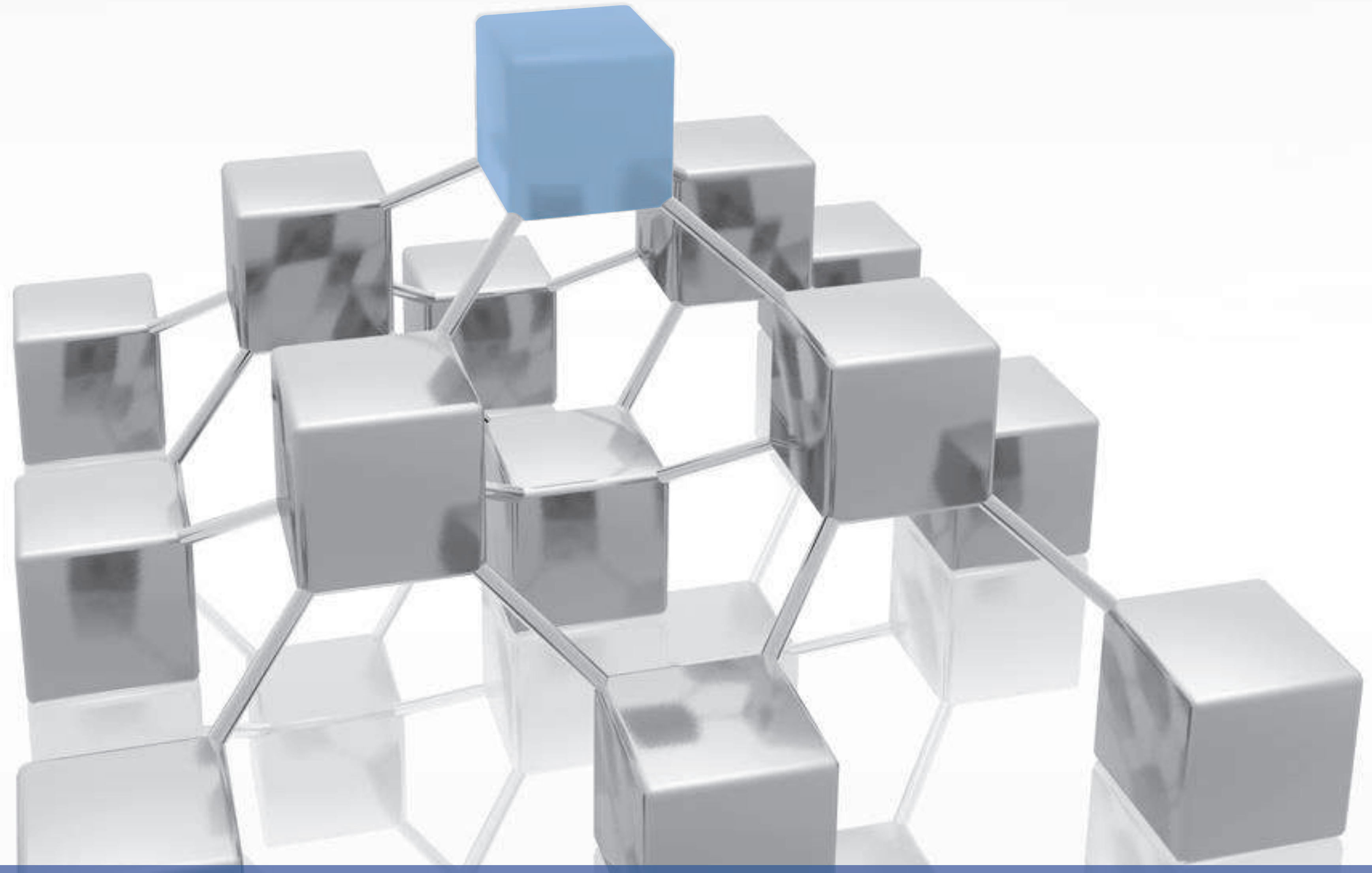
Reliance on a 'black box' - an ideal place to hide uncertainty: neither 'physical cat models' nor 'internal capital models' allow for uncertainty.

Failure to acknowledge uncertainty leads to internal capital model mis-specification and instability known as **'the tail wagging the dog'** syndrome:

Generally growing interest in quantifying uncertainty: reinsurance pricing; Solvency II Binary Events and Events Not In Data (ENID).

Outline

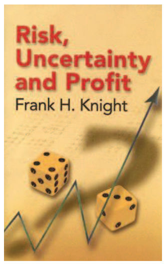
1) Uncertainty in catastrophe modelling
   - what is it?
   - where does it come from?
   - can (shall) it be quantified?
2) Quantifying and managing uncertainty
   - reduced sampling error in 'actuarial modelling'
     - ◇ *use of better statistical techniques and smarter technology*
   - reduced uncertainty of the science underlying physical catastrophe models
     - ◇ *multi-model approach - model blending, fusion, etc.*
   - Other aspects - event clustering, dependence;
3) Conclusions



- Uncertainty in catastrophe modelling -

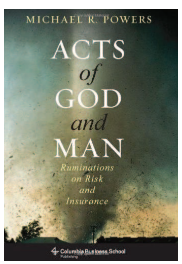
Uncertainty - what is it? (1)

Frank H. Knight (1921) distinguishes 'uncertainty' from 'risk' as follows:



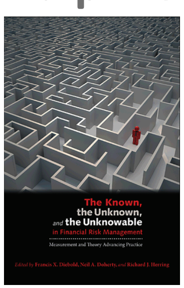
- ◇ **'risk'** can be predicted from empirical data using formal statistical methods;
- ◇ **'uncertainty'** cannot be predicted because it has no historical precedent.

Michael R. Powers' (2013) Ruminations on Risk and Insurance:



... from a quantitative point of view the difference between 'risk' and 'uncertainty' is anything more than a simple distinction between *"lesser risk"* and *"greater risk"*

Ralph Gomory's (1995) KuU framework [in F. Diebold et al. 2010] ...



risk and uncertainty are part of much broader conceptual framework of modern risk management, called "Known, unknown and unknowable"(KuU). KuU can simply be described as *"risk, uncertainty and ignorance"*

Uncertainty - what is it? (2)

Knightian *risk* and *uncertainty* are sometimes treated as different kinds of uncertainty:

- *aleatoric* - irreducible uncertainty representing pure probabilistic variability; and respectively
- *epistemic* - the kind of uncertainty that can be reduced by gaining more information.

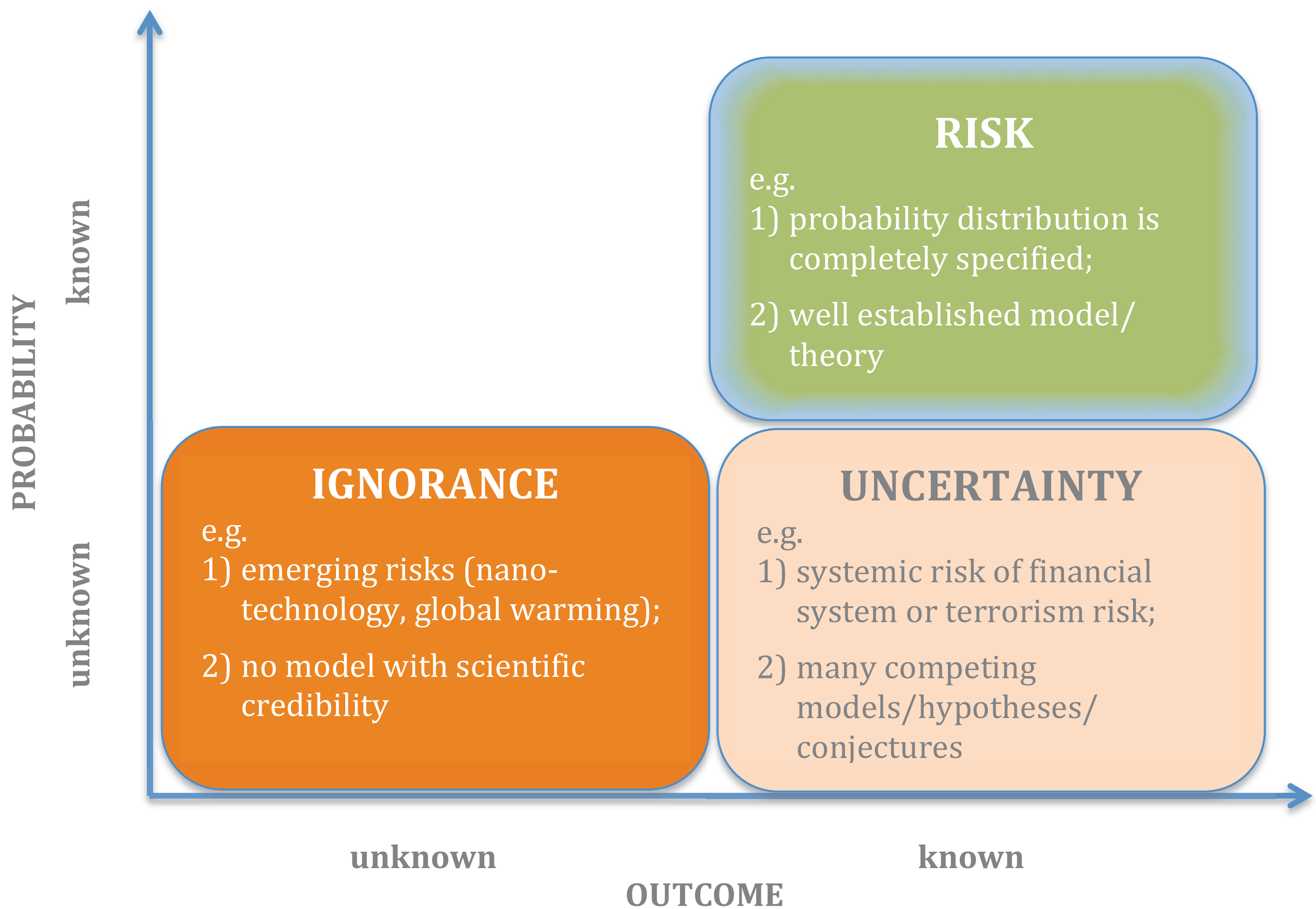
Simple example ...

Throwing the dice ...

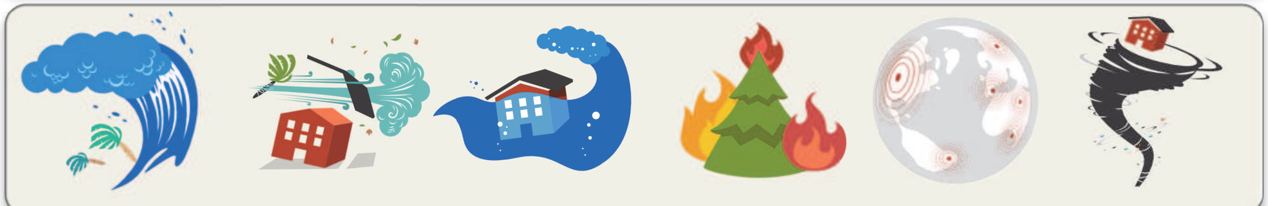
Fair dice:
both outcomes and odds are known → *Risk*

Biased dice:
known outcomes but imperfect knowledge of odds
→ *Risk + Uncertainty*

More examples within KuU framework...

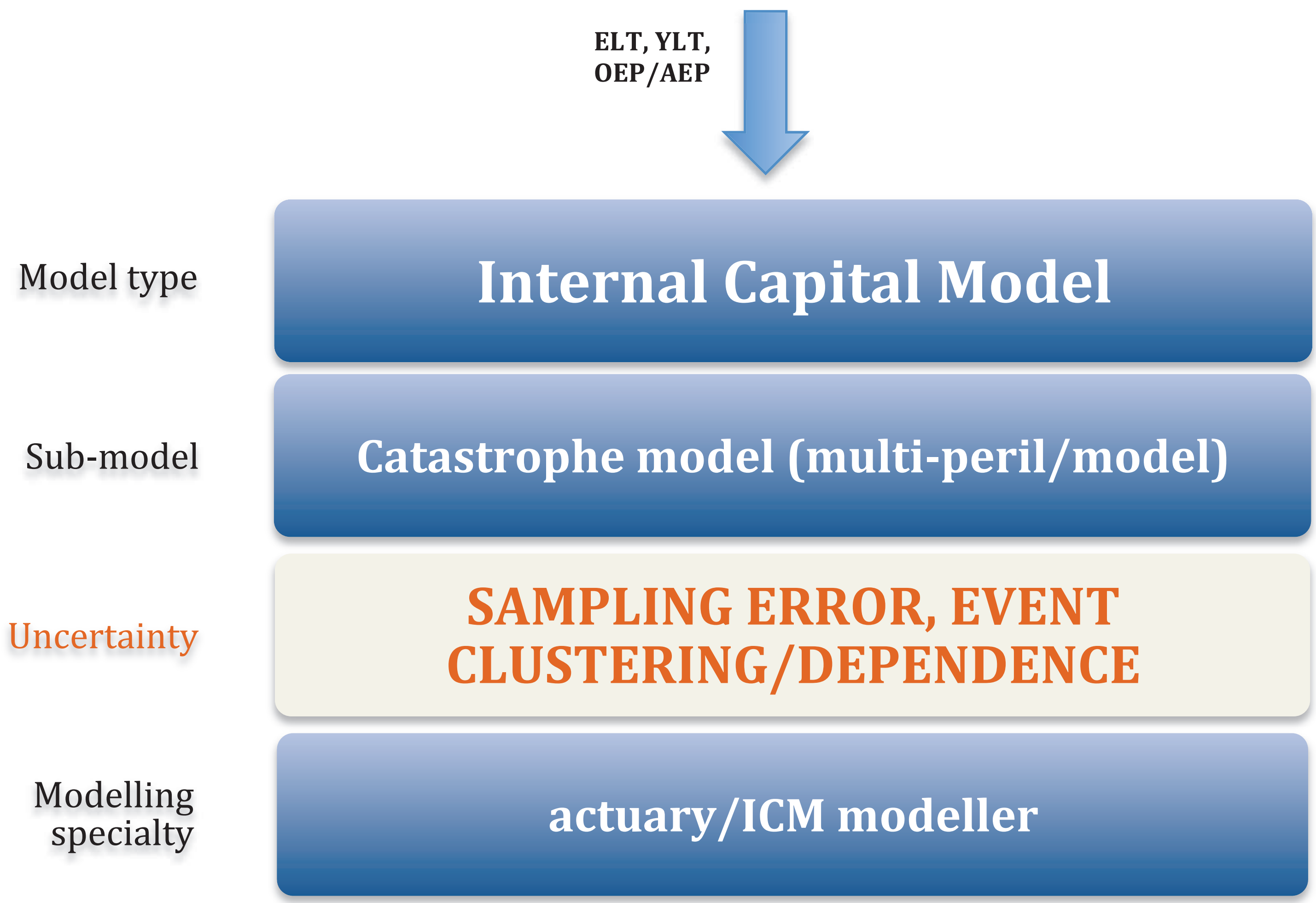


What kind and where does it come from? - Physical Cat Modelling:

Model type	Physical Cat Models (e.g. RMS, AIR, EQECAT, etc)		
Perils			
Sub-model	HAZARD (where, when and how many)	IMPACT (Damage)	INSURED LOSS
Uncertainty	SPATIAL & TEMPORAL; LMITED HISTORICAL DATA	VULNARABILITY; LMITED HISTORICAL DATA	ACCURACY OF INSURANCE/EXPOSURE DATA
Modelling specialty	scientists	engineers	brokers/risk exposure managers

ELT, YLT, OEP/AEP

What kind and where does it come from? - Actuarial Cat Modelling:

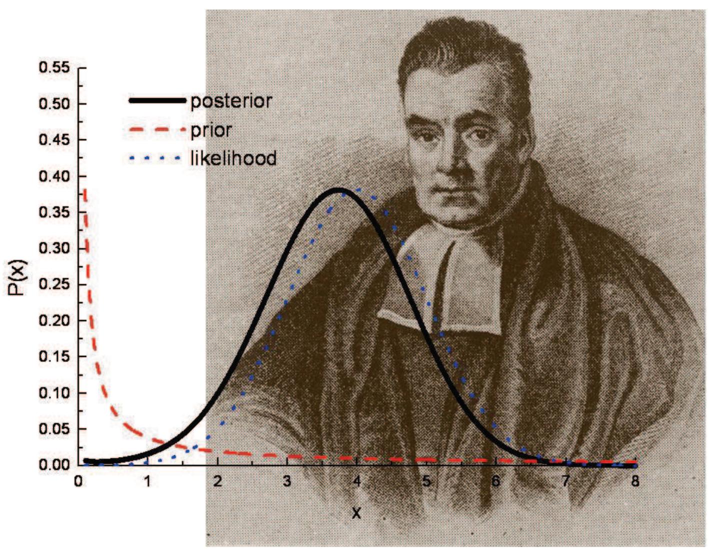


Uncertainty should be quantified ...

How can we quantify uncertainty?

Key solution: Bayesian approach

“Presbyter (Bishop) Takes Knight” (*M. Powers*)



Thomas Bayes, 1702-61

Frank H. Knight, 1885-1972

… quantified and managed

Important types of uncertainty (ranked in descending order):

1) **Limited Historical Data** – especially high for infrequent natural perils:

   - **quantification** – via bootstrapping;
   - **management** – can be significantly reduced via using multi-model blending approach.

2) **Sampling Error** – significant when simulating cat losses of low-frequency and high-severity cat events:

   - **quantification** – via stress testing;
   - **management** – can be significantly reduced/avoided via using variance reduction techniques.

3) *Physical Model Specification* – moderate, taking into account increased research and physical model builder's experience.
4) *Unknown Physical Factors/Phenomena* – could be significant but hard to quantify, e.g., long-term weather cycles, global warming.

Focus is on 1) and 2).

- Quantifying and managing uncertainty -

Uncertainty in actuarial cat modelling (1)

Sampling error - problem formulation

Event Loss Table (ELT) - key output of physical cat models (e.g. RMS, AIR)

Event ID	Event Rate	Loss Amount	Exposure Value	STDI	STDC
...	...	...	...	...	...
0689231	0.0001	9,832,721	31,037,161	3,471,528	4,539,270
...	...	...	...	...	...

- a database of all possible independent events for a given peril;
- i^{th} entry - an event specific Compound Poisson, $\mathrm{CP}(N_i(\lambda_i), X_i)$, with event frequency λ_i ('Event Rate') and individual event severity X_i;
- 'first (λ) and second (X) aleatoric uncertainty' – RMS type vs. AIR type.

Problem

$\boxed{\text{Standard modelling platform}} + \boxed{\text{MC simulation}} =$

$\boxed{\text{Sampling error of 1-in-200 Probable Maximum Loss (PML) could exceed 12\%}}$

MC Sim = 'toss the coin' and pick an event, then 'toss the coin' again to pick severity for a given event ⇒ YLT(YET)

Can we increase the number of simulations? No, not practical, as it could kill the ICM run!

Uncertainty in actuarial cat modelling (2)

Sampling error - solution (1)

Occurrence Exceedance Probability (OEP) - another important output of physical cat models. It carries a 'statistical DNA' of ELT - a distribution of maxima (i.e. Survival Function of PML).

Actuaries/modellers could significantly improve cat simulation result, when constructing YLT/YET, by fully utilising statistical properties of both ELT and OEP.

ELT = set of independent event specific $\mathrm{CP}(N_i, X_i)$ = one big $\mathrm{CP}(N, X)$

$$N_{(i)} \sim \mathrm{Poi}\left(\lambda_{(i)}\right); \quad X_i \sim \mathrm{CDF}_{X_i}(x); \quad \lambda = \sum_{i \in \mathrm{ELT}} \lambda_i; \quad X \sim \mathrm{CDF}_X(x) = \sum_{i \in \mathrm{ELT}} \frac{\lambda_i}{\lambda} \mathrm{CDF}_{X_i}(x).$$

ELT and OEP are functionally related

$$\mathrm{CDF}_{PML}(x) = \sum_{n \geq 0} \mathbb{P}\left(PML \leq x | N = n\right) \times \mathbb{P}(N = n) = \sum_{n \geq 0} \left(\mathrm{CDF}_X(x)\right)^n \times \mathbb{P}(N = n)$$

$$\boxed{\mathrm{CDF}_{PML}(x) = \mathrm{P}_N\left(\mathrm{CDF}_X(x)\right) \overset{\text{Poi}}{=} e^{-\lambda \times (1 - \mathrm{CDF}_X(x))}}$$

Uncertainty in actuarial cat modelling (3)

Sampling error - solution (2)

Quantification

- **use of stress testing** – stressing random seeds;
- **reconciliation** – distribution of event frequency, event severity, aggregate loss and PML.

Management: procedures leading to significant reduction in Sampling Error ($< 1\%$ at 1-in-200 PML)

1) **Draw event losses from one big $\mathrm{CP}(N, X)$ – ELT or inverse OEP**

2) **Stratify $\mathrm{CP}(N, X)$ on flattened Latin hypercube**

3) **Use of alternative modelling platforms**

- **Examples of alternative modelling platforms** – Matlab and Wolfram Mathematica, allow for variance reduction techniques;
- **Parallelisation** – multi-core CPU or many-core GPU (CUDA), e.g. GPU computing is multiple times faster (*A. Rau-Chaplin, (2012)*);
- **HadoopLink** – useful when dealing with big data.

Uncertainty in physical cat modelling (1)

Limited historical data (1)

Quantification: uncertainty band for frequency and severity

✓ Might have already been quantified. RMS? ...

✓ ... and if not then it can be done separately for frequency and severity and then combined into PML:

- **Frequency** – $\boxed{\text{s.e.}(\widehat{\lambda}) = \sqrt{\frac{1}{m(m-1)} \sum_{i=1}^{m} \left(k_i - \widehat{\lambda}\right)^2}; \quad \widehat{\lambda} = \sqrt{\frac{1}{m} \sum_{i=1}^{m} k_i}}$,

  e.g. European WS: $m = 114$ years of historical data; annual event frequency estimate $\widehat{\lambda} = 0.55652$ and $\text{s.e.}(\widehat{\lambda}) = 0.06008$.

- **Severity** – using bootstrapping (two alternatives):

  1) Resampling and replicating historical data and rerun physical cat model – lengthy process and not practical;
  2) Parametric bootstrapping of ELT:

     a) Draw m data points from severity distribution $\text{CDF}_X(x)$ in ELT;
     b) Resample and replicate m points, and then fit the new distribution;
     c) Repeat b) many times and derive the confidence interval.

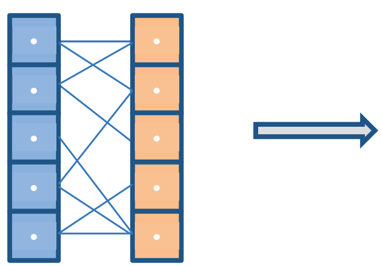
Limited historical data (2)

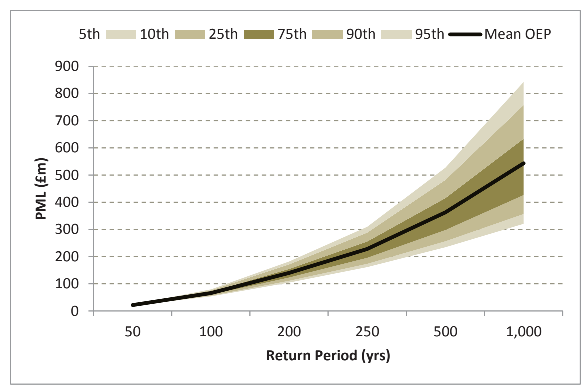
Quantification: focusing on uncertainty band for OEP curve

Compound uncertainty in mean annual frequency $\widehat{\lambda}$ and severity distribution $\widehat{\mathrm{CDF}}_X$ using

$$\boxed{\mathrm{CDF}_{PML}(x) = e^{-\lambda \times (1-\mathrm{CDF}_X(x))}}$$

- k uncertainty quantiles of $\widehat{\lambda}$ times k uncertainty quantiles of $\widehat{\mathrm{CDF}}_X(x)$ given loss x;
- for each x sort k^2 combinations - values of $\mathrm{CDF}_{PML}(x)$, and derive confidence intervals.



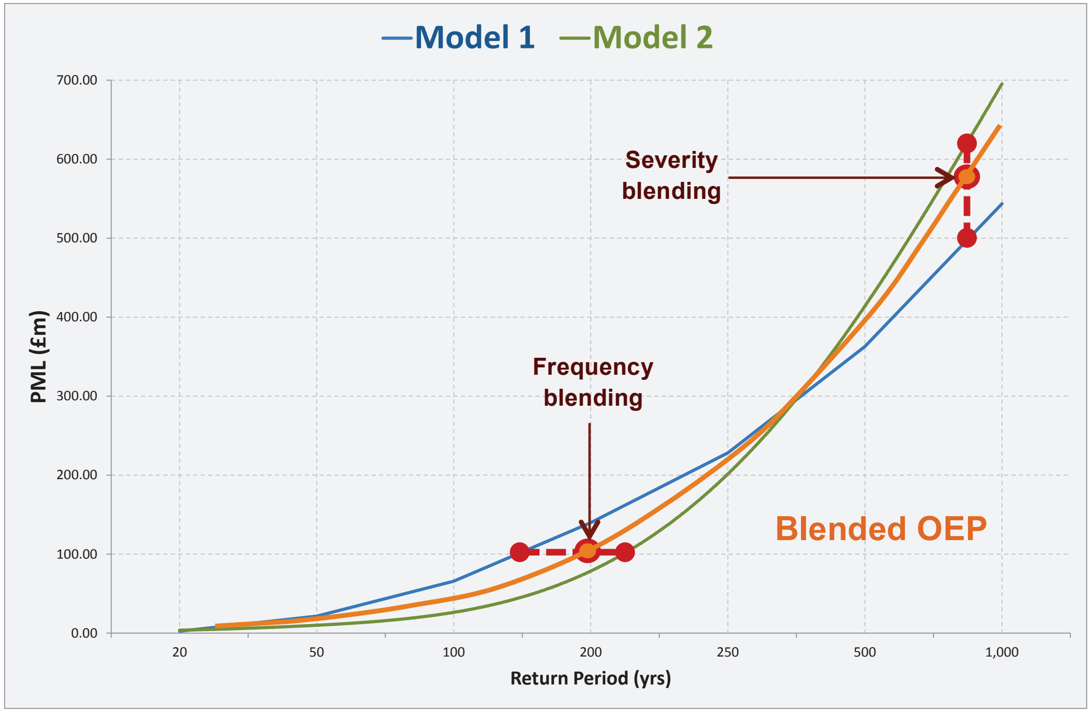


Example: Bootstrapping analysis of uncertainty of OEP estimate for US hurricane (*D. Miller, GC (1999)*): 90% confidence interval ranges from 0.5 to 2.5 times central estimate.

Managing uncertainty (1)

Use of multiple cat models

- model blending;
    - frequency blending;
    - severity blending;
    - arithmetic vs. geometric weighting;
- model fusion (more complex model blending).



Example: Severity blending			
Return period (yrs)	Model 1 PML (£m)	Model 2 PML (£m)	Blended PML (£m)
10	1.5	1.4	1.5
20	2.3	1.9	2.2
50	21.4	12.9	19.7
100	65.7	26.3	45.1
200	139.9	83.9	102.3
250	228.1	200.0	212.9
500	362.8	435.4	413.4
1000	543.8	698.0	649.5

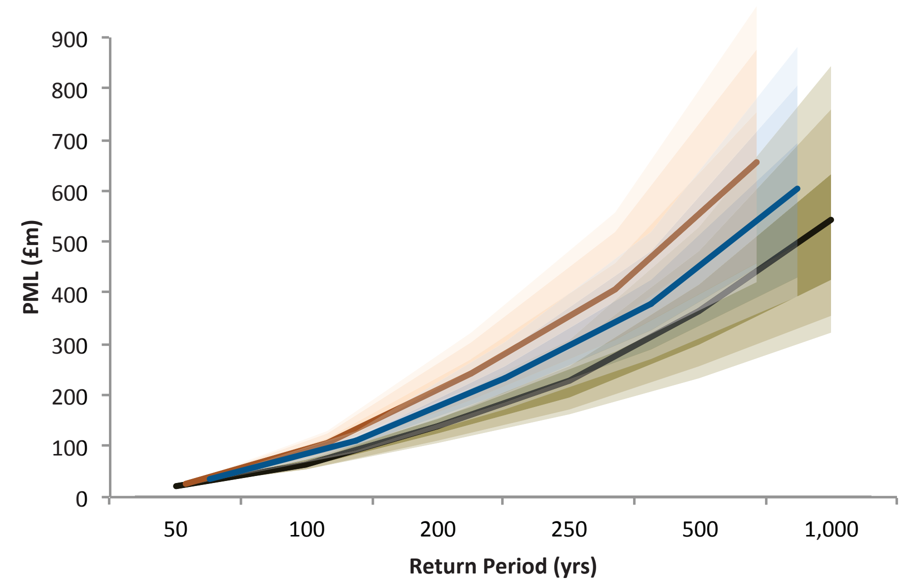
Example: Frequency blending			
PML (£m)	Model 1 Return period (yrs)	Model 2 Return period (yrs)	Blended OEP Return period (yrs)
100	148.7	211.3	199.2
150	210.9	232.1	221.0
200	235.3	250.0	241.5
250	260.1	262.4	266.7
300	377.0	374.8	375.2
350	476.2	448.9	451.3
400	701.6	489.1	502.4
450	750.3	610.2	700.1

Uncertainty in physical cat modelling (4)

Managing uncertainty (2)

Advantage of using multiple cat models:

model blending allows independent ‘imperfections’ to diversify away and thus **may lead to reduction in uncertainty**;



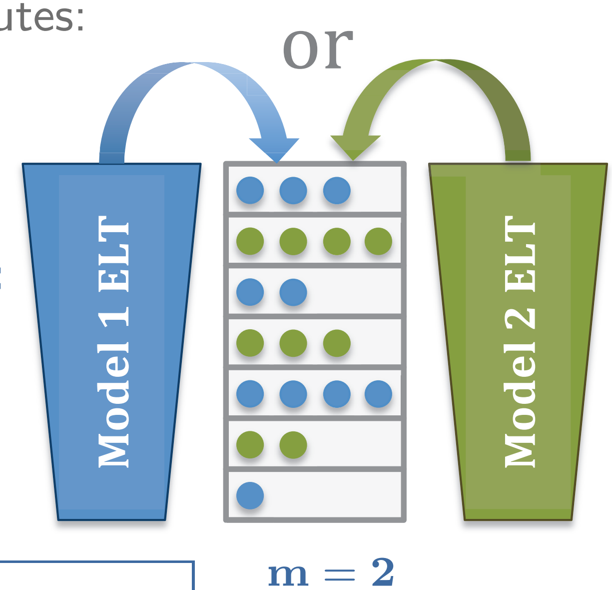
Challenges:

- some blending approaches come at the expense of loosing ELT information, e.g. severity blending;
- selecting model blending weights is rather challenging:
  - ‘knowledge’ of physical cat models;
  - judgement expertise.

Frequency blending: analytical structure of arithmetic averaging

Blending m models with weights w_i. The i-th model's attributes: OEP_i curve and ELT_i table, i.e. $S_i \sim \mathrm{CP}(N_i, X_i)$.

Sampling procedure uses 'mixed distribution structure', i.e. for each simulation year it randomly picks i-th model with probability w_i from which multiple events are drawn:



$N = \sum_{i=1}^{m} N_i \cdot I_i$ and $S = \sum_{i=1}^{m} S_i \cdot I_i$,
$I = (I_1, I_2, ..., I_m)$ is a mixing indicator:
$I_i = 1 \bigwedge I_j = 0,\ j \neq i$ with probability w_i.

$$\boxed{\mathrm{CDF}_{PML}(x) = \mathbb{E}_I \left[\mathbb{P}\left[PML \leq x \mid I\right]\right] = \sum_{i=1}^{m} w_i \cdot \mathrm{CDF}_{PML_i}(x).}$$

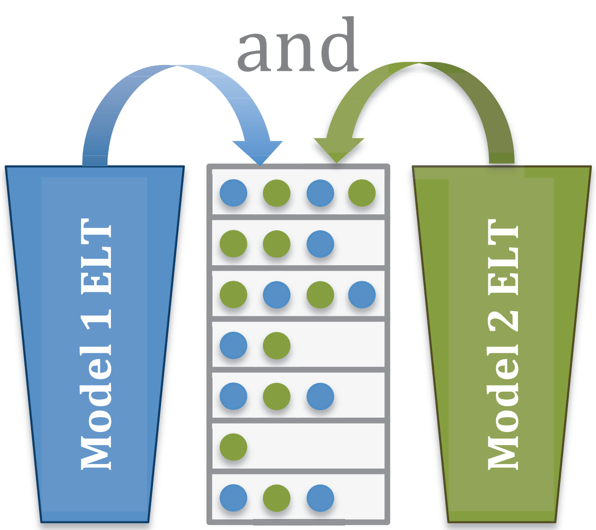
$\mathrm{Var}[N] \overset{\text{cond Var}}{=} \sum_{i=1}^{m} w_i \cdot \lambda_i + \sum_{i=1}^{m-1} \sum_{j>i} w_i w_j \cdot (\lambda_i - \lambda_j)^2 \geq \mathbb{E}[N] \longleftarrow$ **overdispersion**;

$m_k[S] = \sum_{l=1}^{k} \binom{k}{l} \cdot \mathbb{E}_I \left[m_{k-l}[S \mid I] \cdot (\mathbb{E}[S \mid I] - \mathbb{E}[S])^l\right] \longleftarrow$ **central moments of aggregate loss for reconciliation procedure**, where $m_0(S) = 1$ and $m_1(S) = 0$.

Using a multi-model approach (2)

Frequency blending: analytical structure of geometric averaging

Within each simulation year sampling procedure uses 'mixture distribution structure', i.e. each event is drawn from a separate i-th model that is randomly picked with probability w_i:



$N = \sum_{i=1}^{m} N_i(w_i \cdot \lambda_i)$ and $S = \sum_{i=1}^{m} S_i^*$, $S_i^* \sim \mathrm{CP}(N_i(w_i \cdot \lambda_i), X_i)$

$S \sim \mathrm{CP}\left(N(\lambda), Z\right)$, where $\lambda = \sum_{i=1}^{m} w_i \cdot \lambda_i$

$$\boxed{\Rightarrow \quad \mathrm{CDF}_Z(x) = \sum_{i=1}^{m} \frac{w_i \cdot \lambda_i}{\lambda} \cdot \mathrm{CDF}_{X_i}(x)}$$

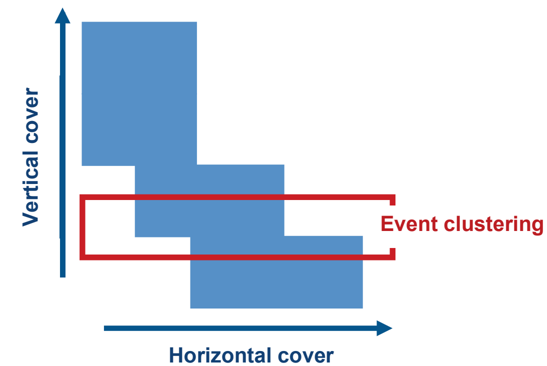
$$\boxed{\mathrm{CDF}_{PML}(x) = e^{-\lambda(1-\mathrm{CDF}_Z(x))} = \prod_{i=1}^{m} \left[\mathrm{CDF}_{PML_i}(x)\right]^{w_i}}$$

$$\mathrm{Var}[N] = \mathbb{E}[N] = \sum_{i=1}^{m} w_i \cdot \lambda_i; \quad m_k[S] = \sum_{i=1}^{m} w_i \cdot m_k[S_i^*].$$

Other issues

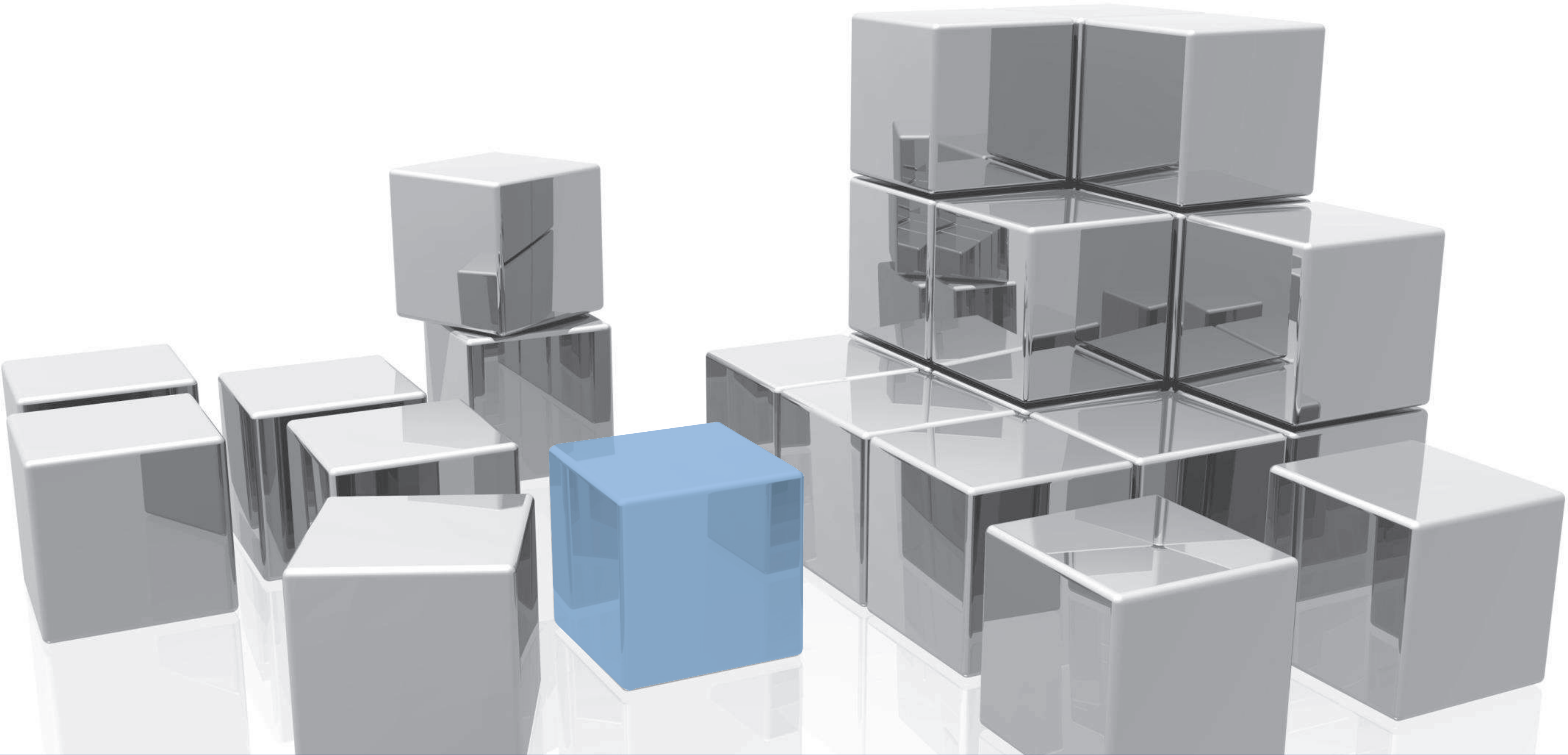
Event clustering/dependency...

- In an ideal world the key assumption of ELT: all events are independent;
- In reality, though, medium-/small-sized cat events tend to patch together affecting the volatility of earnings and also reinsurance purchasing decision making.



What can we do about this?

- Event dependence means overall event frequency in ELT is overdispersed – back to stratified simulation of CP;
- Lévy copula – powerful tool to model dependency between event specific CPs, e.g. for events triggering certain loss layers. Please refer to *B. Avanzi et al (2011)* and references therein.



- Conclusion -

Conclusions

Key takeaway points

- **Knowing ‘unknowns’**
  - catastrophe modelling is associated with high uncertainty;
  - failure to recognise uncertainty – understand, quantify and manage it, could result in misleading management information.
- **Challenging the ‘black box’** used in modelling cats
  - *physical cat modelling* – uncertainty due to limited historical data of infrequent peril events;
  - *actuarial cat modelling* – sampling error, unnecessary and can be reduced via variance reduction techniques.
- **Being model-agnostic**
  - use of alternative modelling platforms (if necessary);
  - use of multi-model approach (i.e. model blending/fusion).

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Thank You