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Multi-State Microeconomic Model for Pricing and Reserving a disability insurance policy over an arbitrary period

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MERCER

Some key disability statistics:

- *One disabling accident per second: US*
- *One disabling illness per 2 seconds: UK, Canada, France*

Motivation and Setting

- The universal trigger event for Disability Insurance = the inability to work
- Compensation systems:
  - ➢ Public health insurance
  - ➢ Private health care coverage:
    - ✓ Group insurance
    - ✓ Individually purchased
- Group insurance « paradox »

Many different risk profiles

But Often Uniform Premium = Risks averaged

Covered Risk = Aggregate Risk

- Key idea to simplify risk assessment

Make as if many identical individuals: Representative Insured (RI) Assumption

Multistate Modeling

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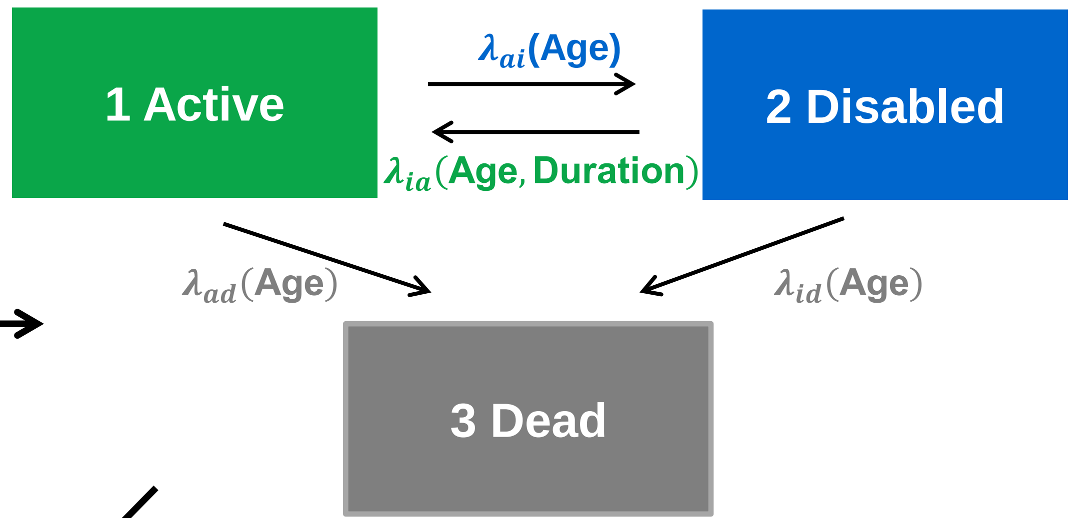
Overview of the Multi-State Model

Transition	Depends on
Disablement	Age
Recovery	Age and Duration
Death of an active life	Age
Death of a disabled life	Age



Transition	Modeling	Resulting Process
Disablement	Poisson	Locally Time-Homogeneous Markov renewal
Recovery	Cox MPH	Locally Time-Homogeneous Semi-Markov
Death of an active life	Mortality Tables	Locally Time-Homogeneous Markov renewal
Death of an disabled life	Mortality Tables	Locally Time-Homogeneous Markov renewal

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Estimation and Graduation of Transition Intensities

- **Disablement**

Poisson coefficients $\hat{\theta}$ → $\hat{\lambda}_{ai}(t|Z=z) = exp(\hat{\theta}'z)$

- **Recovery**

Cox Coefficients $\hat{\beta}$
Baseline Hazard $\hat{\lambda}_{0,ia}(\cdot)$
Frailty v

→ $\hat{\lambda}_{ia}(t|Z=z) = \hat{\lambda}_{0,ia}(t)\, exp(z'\hat{\beta})v$

- **Mortality**

Annual Mortality rates q_x → $\hat{\lambda}_{(i,a);d}(x \times 365.25) = -\frac{\ln(1-\hat{q}_x)}{365.25}$

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Application

- **Representative insured**
  - **Male**
  - **25**
  - **Large City**
  - **Finance & Insurance**
  - **$ 65,000**

- **Main conditions**

Parameter	Value
Deferred Period	91 Days
Targeted replacement rate a	85 %
Maximum Benefit Amount	a x $450,000
State-guaranteed minimum replacement rate	50%

- In the simplest case

$$B(s,t) = a \times \text{Salary} \times (t - s)$$

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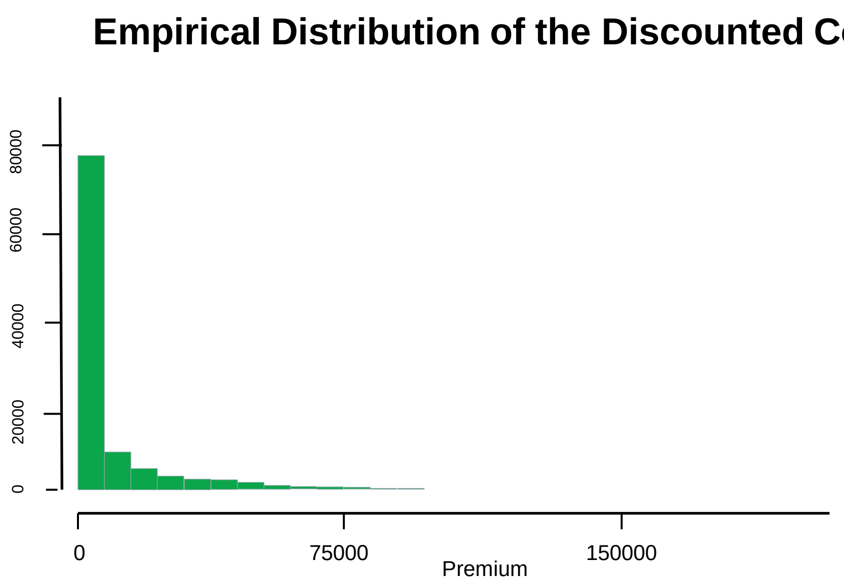
Simulation Results: Summary



Statistic	Variable: Aggregate Duration of Disability Spells	Variable: Total Discounted Cost of DI
Mean	247.2	6,582.62
Std Dev.	205.4	14,446.32
Skewness	2.22	3.79
Maximum	2,684.3	258,342.07

Deferred Period

Coefficient of Variation > 2

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Towards a simple technical account

- Modified Standard Deviation Principle (MSDP) consistent with the assumption (RI)
  - Aggregate Cost $\mathbf{S_n}$

$$\Pi(S_n) = \mathbb{E}[S_n] + \xi\sigma(S_n) \qquad \text{(MSDP)}$$

- The following convergence holds

$$S_n - \Pi(S_n) \xrightarrow{d} \mathcal{N}(-\xi, 1)$$

Scenario :

- **No waiver of premiums**
- **No disability > deferred period the first 2 years**
- **Risk horizon: retirement**
- **99.5% solvency constraint**

Time	y = 0	y =1^-	y =2^-
Assets	**271.25**	**279.39**	**567.16**
Reserves	**(117.63)**	**55.53**	**235.41**
Claims paid	**0**	**0**	**0**
Profit	**388.88**	**223.86**	**331.75**

Conclusions and extensions

(RI) assumption, although apparently rough, simplifies the Multi-State Model and facilitates risk management.
We get more accurate and consistent pricing and reserving.

- **Extensions**
  - ➢ Deviations from the rescaled limit distribution
  - ➢ Optimal Representative Insured
  - ➢ Heterogeneous insured models

References

- Cordeiro, I.M.F., A multiple state model for the analysis of permanent health insurance claims by cause of disability. *Insurance : Mathematics & Economics* **30**, pp.167-186, Elsevier, 2002.
- Möller, T., Numerical evaluation of Markov transition probabilities based on discretized product integral. *Scandinavian Actuarial Journal*, pp.76-87, 1992.
- Pitacco, E., Actuarial models for pricing disability benefits: Towards a unifying approach. *Insurance: Mathematics & Economics* **16**, pp.39-62, Elsevier, 1994.
- Renshaw, A., & Haberman, S., Modelling the recent time trends in UK permanent health insurance recovery, mortality and claim inception transition intensities. *Insurance :Mathematics & Economics*, **27**, pp.365-396, Elsevier, 2000.
- Stenberg, F., Manca, R., & Silvestrov, D., An Algorithmic Approach to Discrete Time Non-homogeneous Backward Semi-Markov Reward Processes with an Application to Disability Insurance. *Methodol Comput Appl Probab* **9** pp. 497-519, 2007.
- Waters, H.R., A multiple state model for permanent health insurance. *CMIR* **12**, pp.5-20, 1991.
- Wolthuis, H., & Hoem, J.M., The retrospective premium reserves. *Insurance: Mathematics & Economics*, **5**, pp.217-254, Elsevier, 1986.

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Thank you for your attention!

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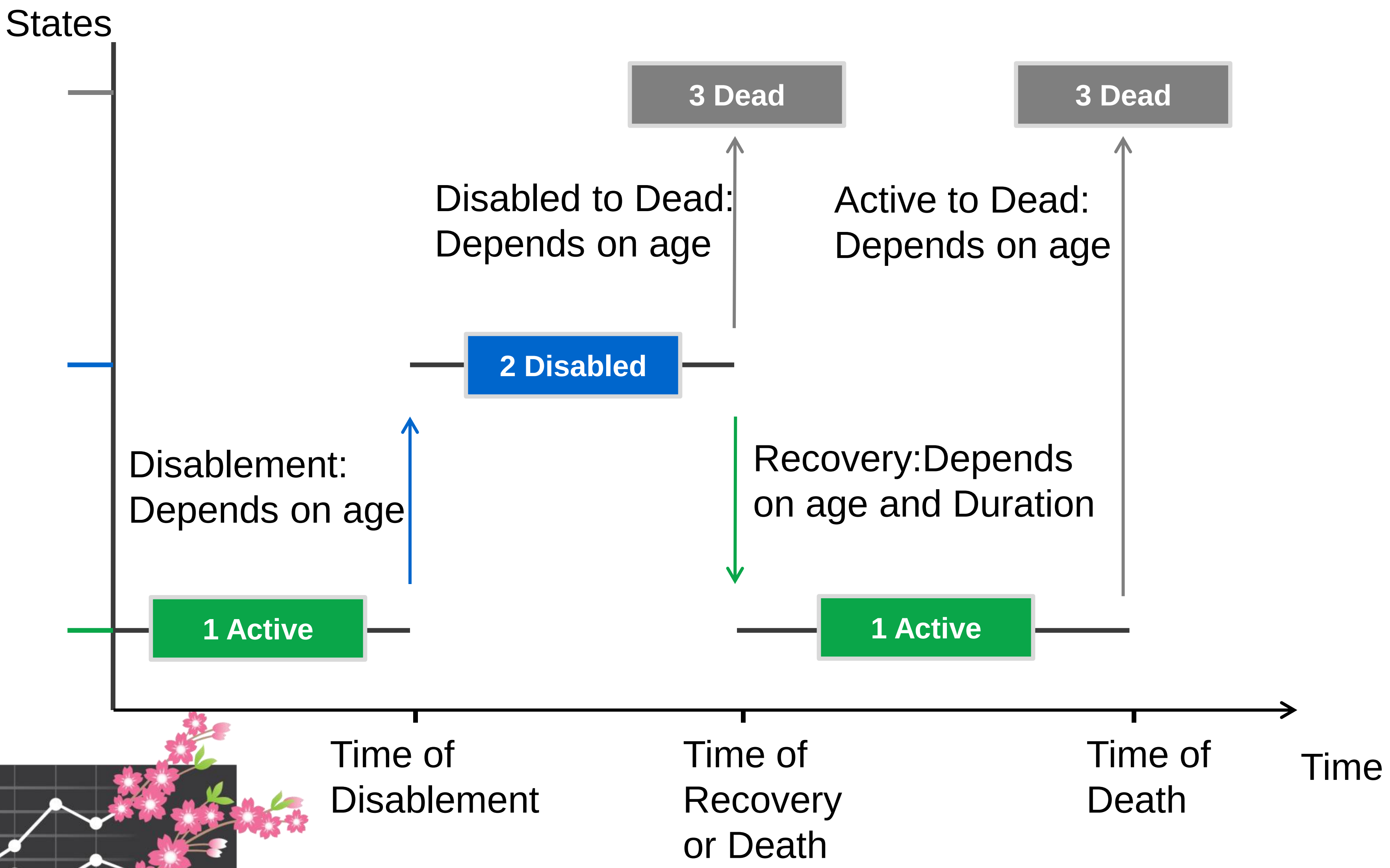
Appendix

Multi-State Model: a trajectory example



Probabilistic Framework

- Let $\{(X_t, D_t), t \geq 0\}$ be a bivariate process where X_t is the state occupied at time t (right-continuous paths) and D_t is the duration of stay in this state.
- **Markov disablement process:** the instantaneous transition rate from the 'active' state to the 'disabled' state depends only on the age

$$\lambda_{\mathrm{ai}}(\mathrm{x}+\mathrm{t}) = \lim_{\Delta \mathrm{t} \downarrow 0} \frac{\Pr[X_{t+\Delta t} = i \mid X_t = a]}{\Delta t}$$

- **Semi-Markov Recovery process:** depends both on the age and the duration of the current instance of disability

$$\lambda_{\mathrm{ia}}(\mathrm{x}+\mathrm{t}; \mathrm{s}) = \lim_{\Delta \mathrm{t} \downarrow 0} \frac{\Pr[X_{t+\Delta t} = a \mid X_t = i, D_t = s]}{\Delta t}$$

- **Mortality intensities from the disability state and from the 'active' state : equal and Markov**

$$\lambda_{\mathrm{id}}(\mathrm{x}+\mathrm{t}) = \lambda_{\mathrm{ad}}(\mathrm{x}+\mathrm{t})$$

Different assumptions drive transitions and require a modeling specific to each type of transition.

Reserves Dynamics

- ***Thiele's differential equation for the Active Prospective Reserve***

  For an insured aged x at policy issue, we have at time t $\notin \mathrm{Disc}(\Pi_a) = \{t_{a,0}, t_{a,1}, \dots, t_{a,q}\}$

  $$dV_a(t) = r(t)\, V_a(t)\, dt + \pi_a(t)\, dt - \lambda_{ai}(x+t)\, dt\, (c_{ai}(t) + V_i(t,0) - V_a(t)) + \lambda_{ad}(x+t)\, dt\, V_a(t)$$

  where $t \mapsto \Pi_a(t)$ *is a right-continuous and non-decreasing premium process,*
  and $t \mapsto c_{ai}(t)$ *is a lump sum in case of transition to disability.*
  The solution is uniquely determined with the conditions

  $$V_a(t_{a,j}) = V_a(t_{a,j}^{-}) + \Delta\Pi_a(t_{a,j}), \qquad j = 0,1,\dots,q$$

- ***Thiele's differential equation for the Disabled Prospective Reserve***

  For an insured aged x at policy issue, disabled at time t $\notin \mathrm{Disc}_s(B_i) = \{t_{i,s,0}, t_{i,s,1}, \dots, t_{i,s,q_s}\}$ *with duration s since the disability onset*

  $$d_t V_i(t,s) = r(t)\, V_i(t,s)\, dt - d_s V_i(t,s) - b_i(t, t-s)\, ds - \lambda_{ia}(x+t,s)\, dt\, (V_a(t) - V_i(t,s)) + \lambda_{id}(x+t,s)\, dt\, V_i(t,s)$$

  where $(t, t-s) \mapsto B_i(t, t-s)$ *is a right-continuous and non-decreasing benefit process, well-defined for t* $\geq s$.
  Again, the solution is uniquely determined with the conditions

  $$V_i(t_{i,s,j}) = V_i(t_{i,s,j}^{-}) - \Delta B_i(t_{i,s,j}, t_{i,s,j} - s), \qquad j = 0,1,\dots,q_s$$

Thiele's equations exhibit positive and negative contributions to the reserve, which make intuitive sense.