Lee-Carter mortality forecasting for Uzbekistan

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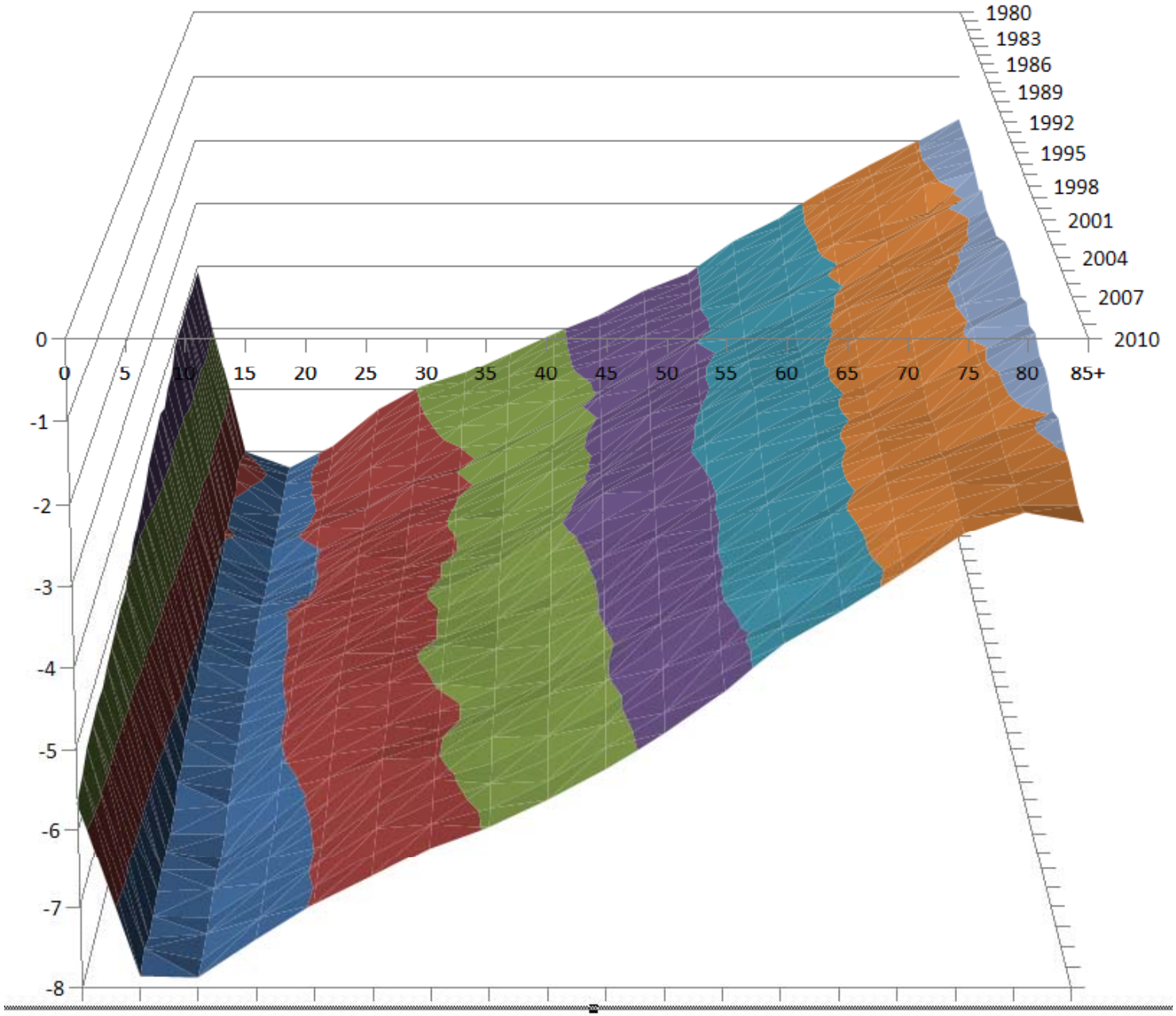
Population projections:

- **Deterministic models:**
  - Cohort-component method (low, medium, high scenarios);
  - Ratio method (low, medium, high scenarios).
- **Stochastic models:**
  - Time series model (Lee R., Carter L., 1992, USA, Sweden, Japan, Germany, Hungary, Lithuania, China et al.)
  - Expert judgement model(Lutz W., Sanderson R., Sherbov, 1997, Austria, UNPD, The World Bank, et el.);
  - Extrapolation of empirical errors (Alho, Keilman, De Beer, 1997, Norway, Finland et al.).

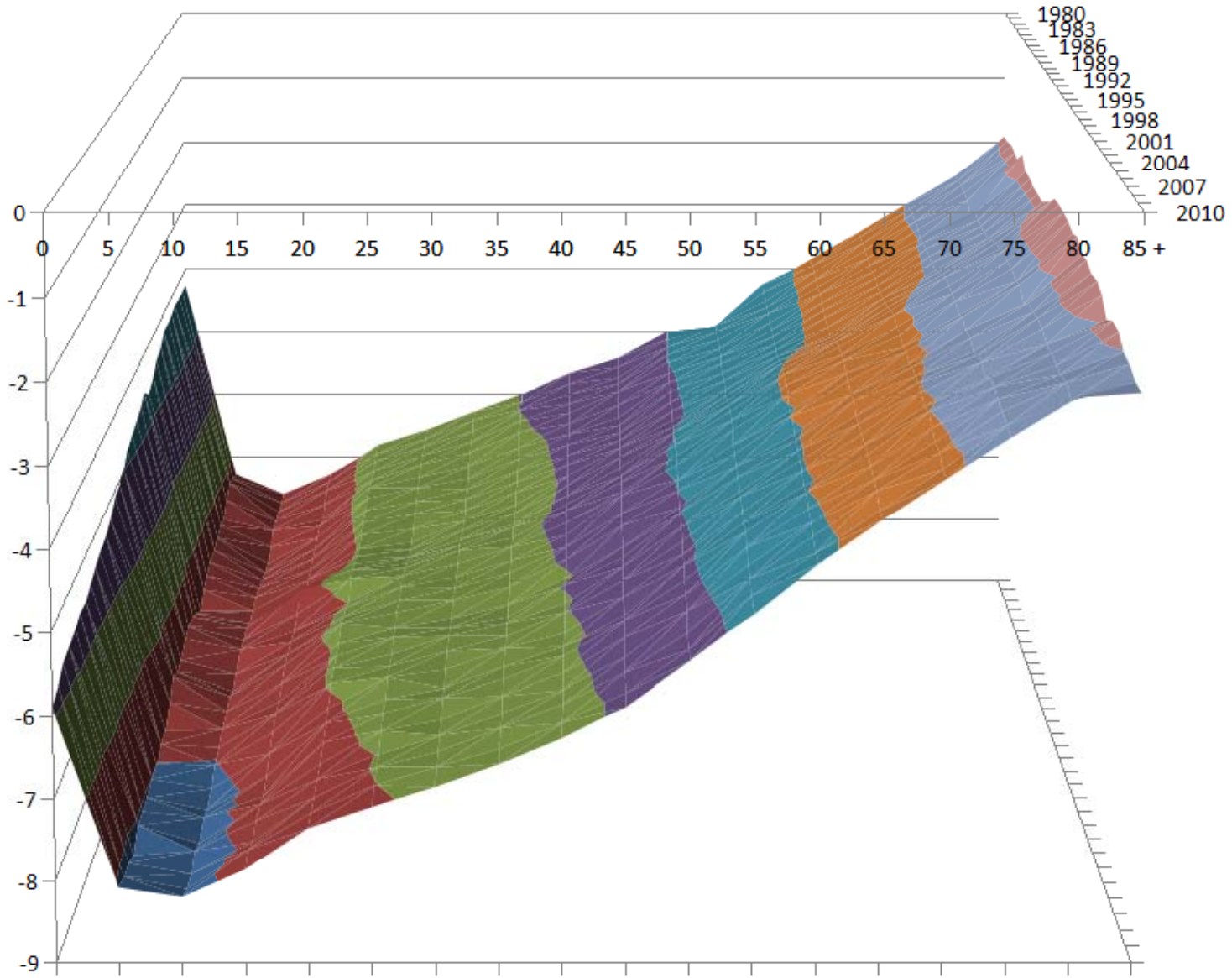
Projections – Outputs:

- Population total size;
- Population age-sex structure;
- Life expec-tancy;
- Dependency ratio.
- …

$\ln(m_{x,t})$ for years 1980-2010. Uzbekistan, male



$\ln(m_{x,t})$ for years 1980-2010. Uzbekistan, female



The Lee-Carter method

$$\ln(m_{x,t}) = a_x + b_x k_t + e_{x,t}$$

- $m_{x,t}$ – *the* central death rate for age x and time t (the ratio of deaths at age *x* to the mean population alive at that age)*;*
- a_x - the age pattern of death rates;
- b_x - age-specific sensitivity to the death rate;
- k_t - time-varying mortality index;
- $e_{x,t}$ – residual error.

I step: Estimating parameters a_x, b_x and k_t.

The normalization conditions:

$$\sum_{t=1}^{31} k_t = 0\,, \qquad \sum_{1}^{18} b_x = 1$$

Then obtain:

$$a_x = \sum_t \ln(m_{x,t})$$

Apply the singular value decomposition (SVD) method to $\ln(m_{x,t}) - a_x$, estimate b_x and k_t .

Estimates of parameters a_x and b_x:

Age group	female		male	
	ax	*bx*	*ax*	*bx*
0	-4,95	0,68	-4,73	0,67
5	-7,69	0,39	-7,34	0,43
10	-7,88	0,28	-7,46	0,32
15	-7,50	0,23	-7,04	0,22
20	-6,97	0,20	-6,61	0,20
25	-6,74	0,13	-6,25	0,14
30	-6,54	0,16	-5,97	0,12
35	-6,28	0,17	-5,70	0,15
40	-5,95	0,20	-5,33	0,19
45	-5,57	0,14	-4,92	0,16
50	-5,07	0,10	-4,47	0,12
55	-4,61	-0,03	-4,07	0,04
60	-4,12	-0,06	-3,63	0,00
65	-3,67	-0,12	-3,24	-0,03
70	-3,21	-0,14	-2,83	-0,04
75	-2,77	-0,15	-2,45	-0,04
80	-2,35	-0,12	-2,10	0,04
85+	-1,82	-0,06	-1,78	0,17

Estimates parameter k_t

Year	kt (female)	kt(male)	Year	kt (female)	kt(male)
1980	1,109703	0,933423	1996	0,095366	0,280313
1981	1,11633	0,994483	1997	0,007357	0,132776
1982	1,121461	1,049145	1998	0,008028	0,059127
1983	1,034853	0,939661	1999	-0,24338	-0,17249
1984	0,93831	0,82199	2000	-0,42446	-0,27632
1985	0,916449	0,649117	2001	-0,47837	-0,39319
1986	0,76612	0,562138	2002	-0,61979	-0,54896
1987	0,66295	0,795386	2003	-0,80165	-0,77298
1988	0,551626	0,314156	2004	-0,9605	-0,93118
1989	0,43059	0,39054	2005	-0,96692	-0,90924
1990	0,297716	0,315995	2006	-1,04991	-0,96967
1991	0,27294	0,390568	2007	-1,18308	-1,15503
1992	0,323122	0,430129	2008	-1,25738	-1,23819
1993	0,397681	0,38324	2009	-1,30489	-1,35073
1994	0,351109	0,407069	2010	-1,35697	-1,47925
1995	0,245593	0,347965			

II step: k_t forecasting

k_t parameter is modeled with a random walk with drift:

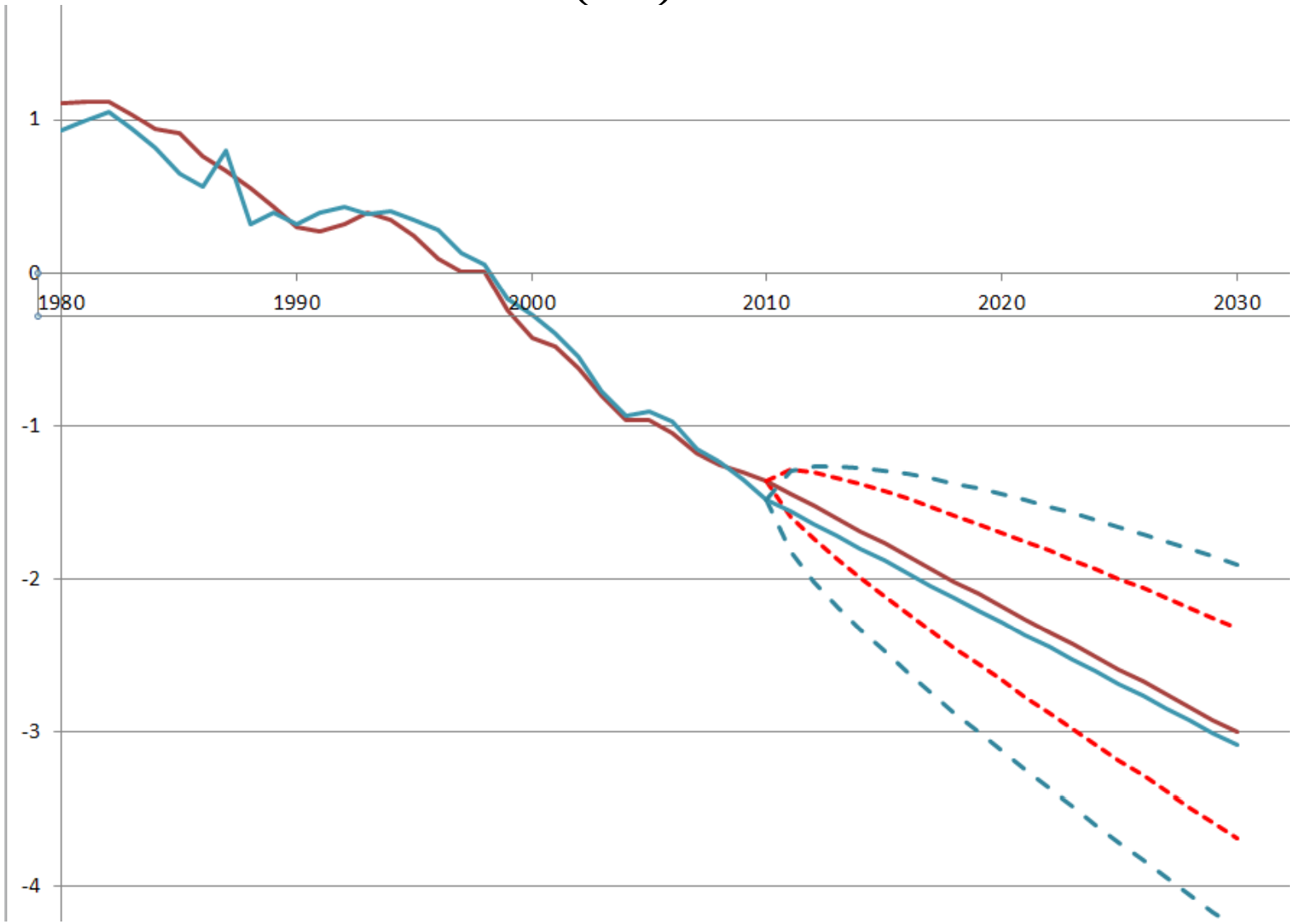
$$k_t = c + k_{t-1} + \varepsilon_t$$

where c - the drift term, and $\varepsilon_t \sim N(0,\sigma^2)$.

The estimates: $c_{male} = -0.804$, $c_{female} = -0.0822$,

$\sigma_{male} = 0.1295$, $\sigma_{female} = 0.0748$

Time-varying index k_t from 1980 – 2010 and the forecasts to 2030 with 95%-confidence intervals for males (blue) and female (red).



Mortality forecasting

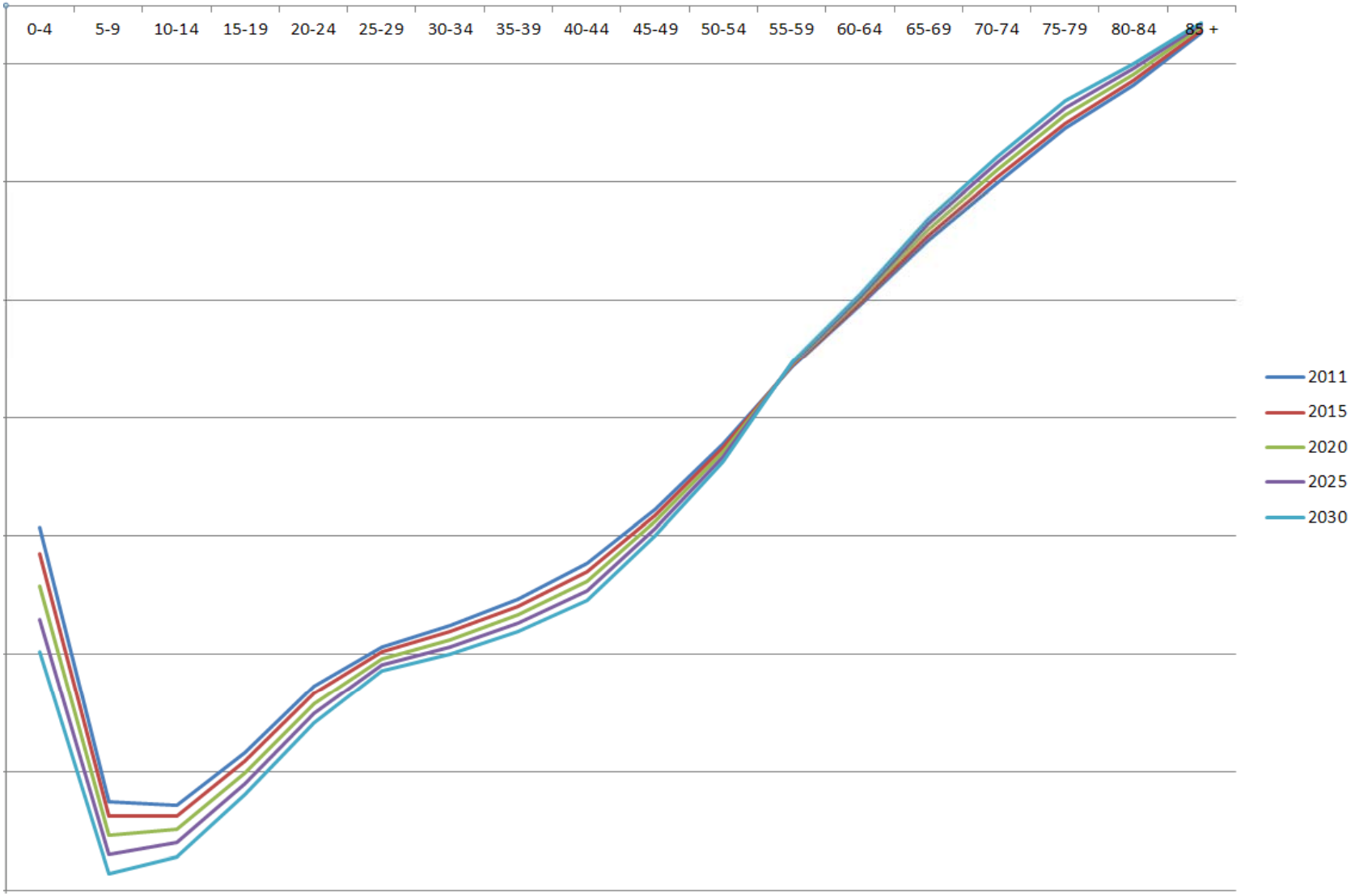
$$m_{x,t+h} = \exp(a_x + b_x k_{t+h} + e_{x,t+h}),$$

$h = 1, 2, ..., 20;$

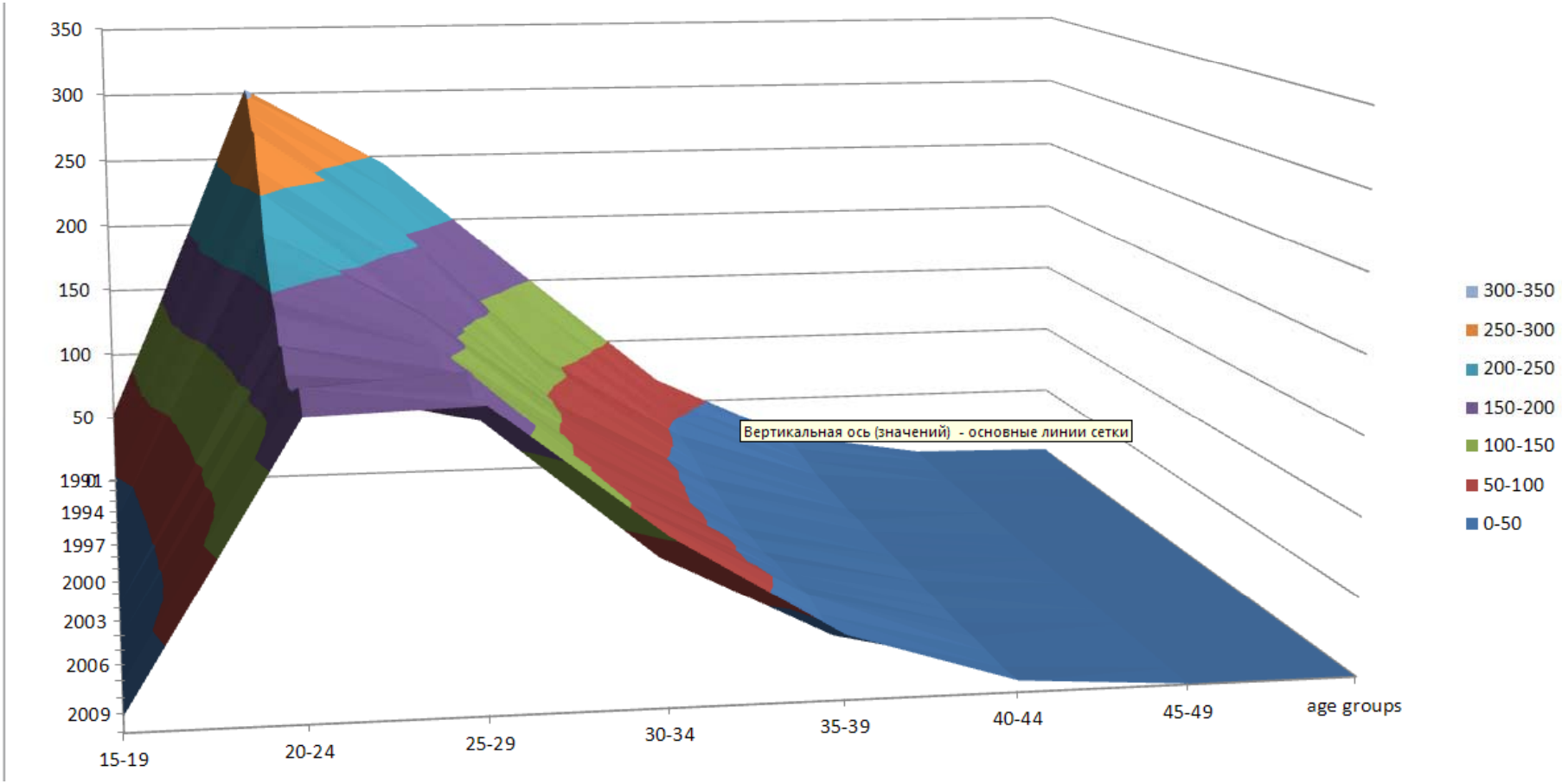
$\mathrm{t} = 2010,$

$\mathrm{x} = 0\text{-}4, 5\text{-}9, 10\text{-}14, ..., 80\text{-}84, 85+$

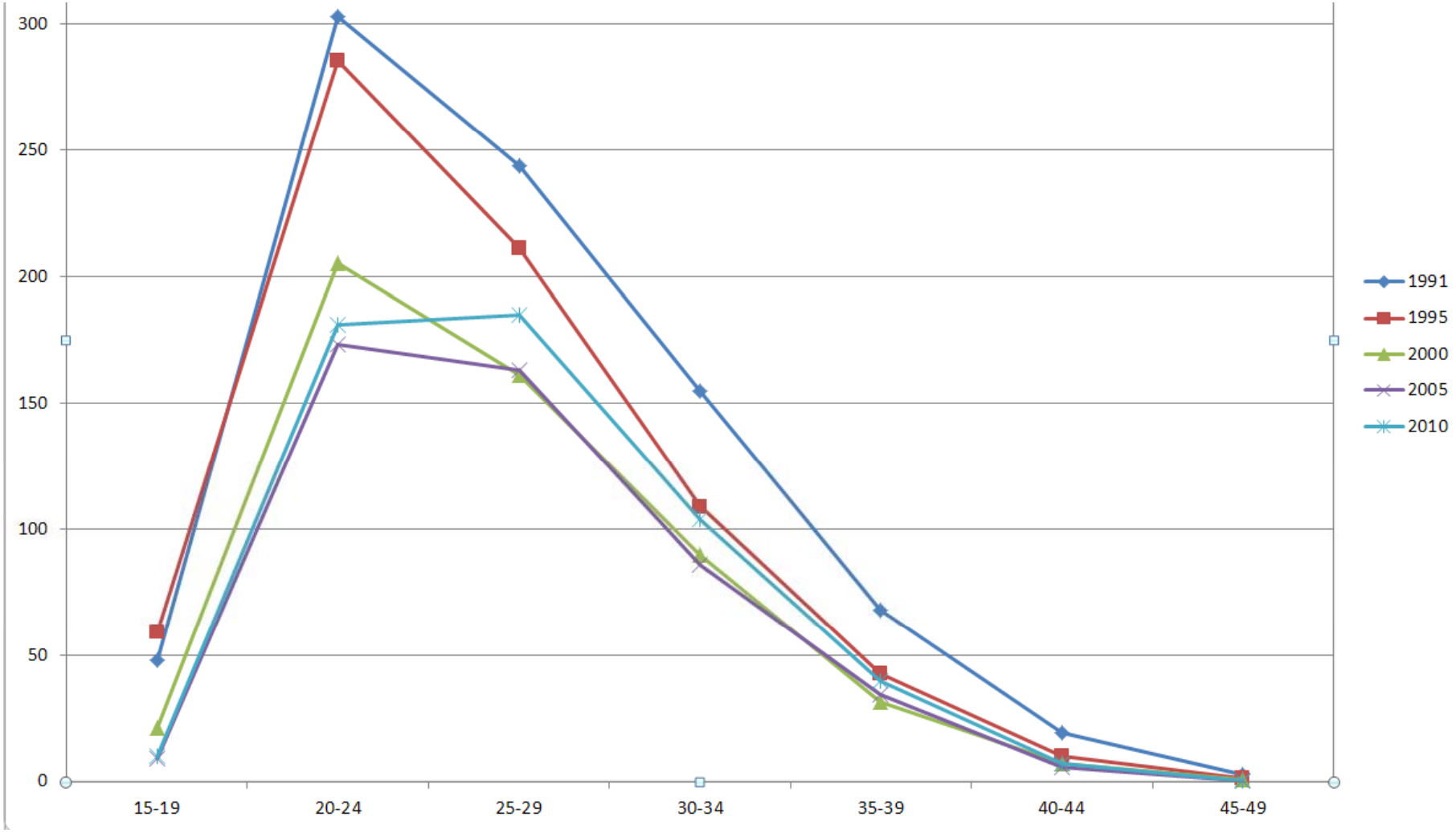
Central age death rate projections

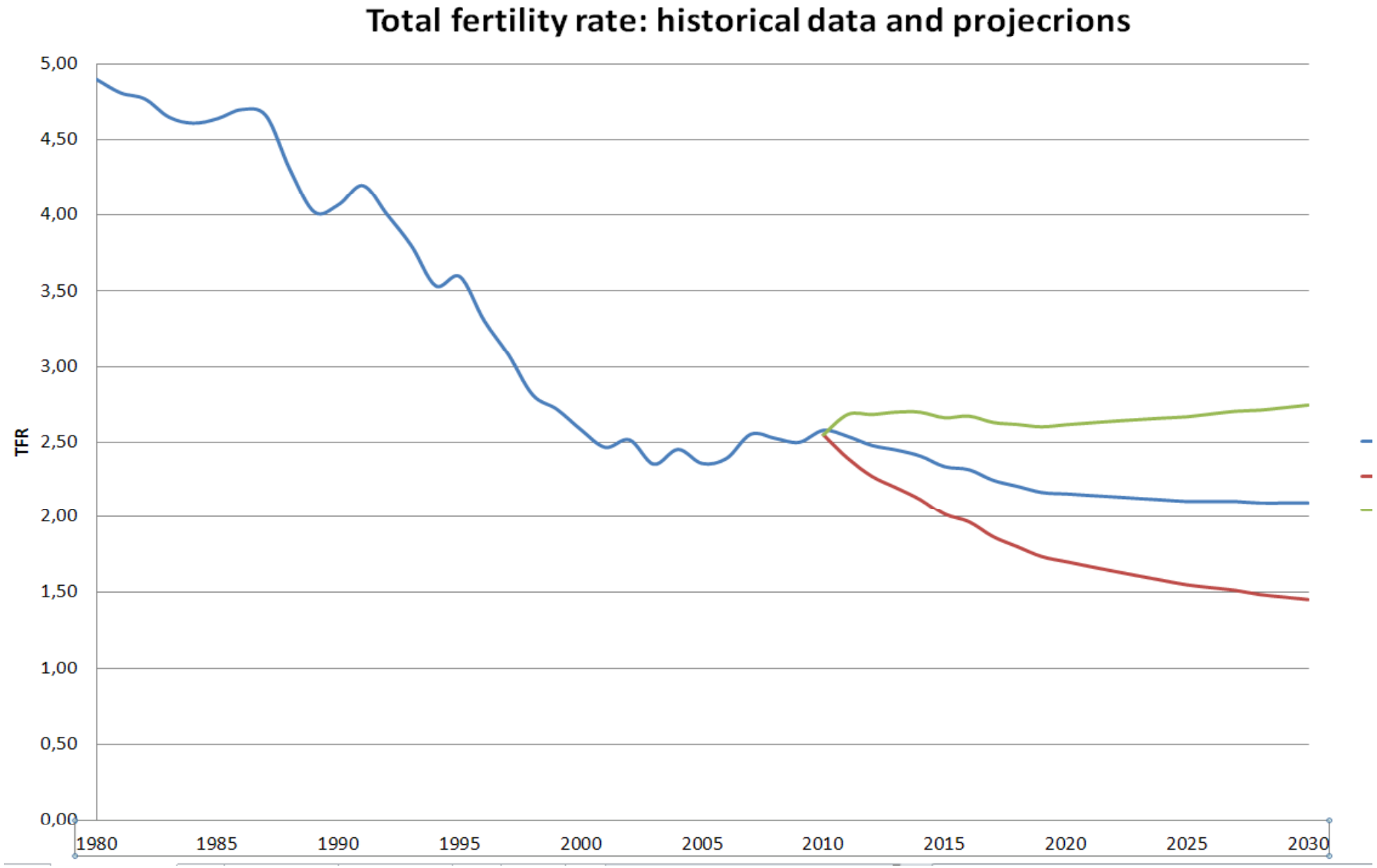


Age specific fertility rates (ASFR) for period 1991-2010

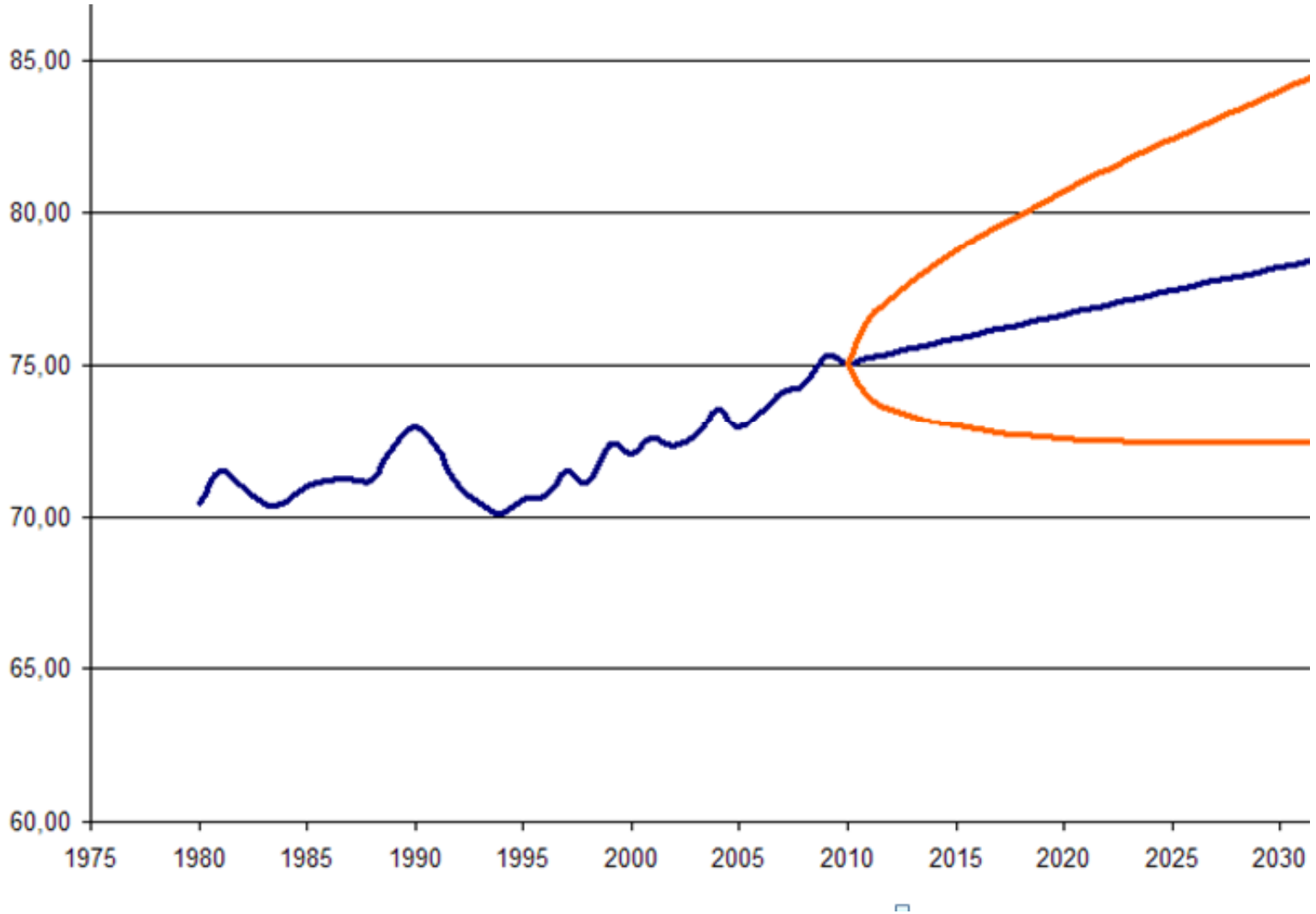


ASFR distribution dynamics:

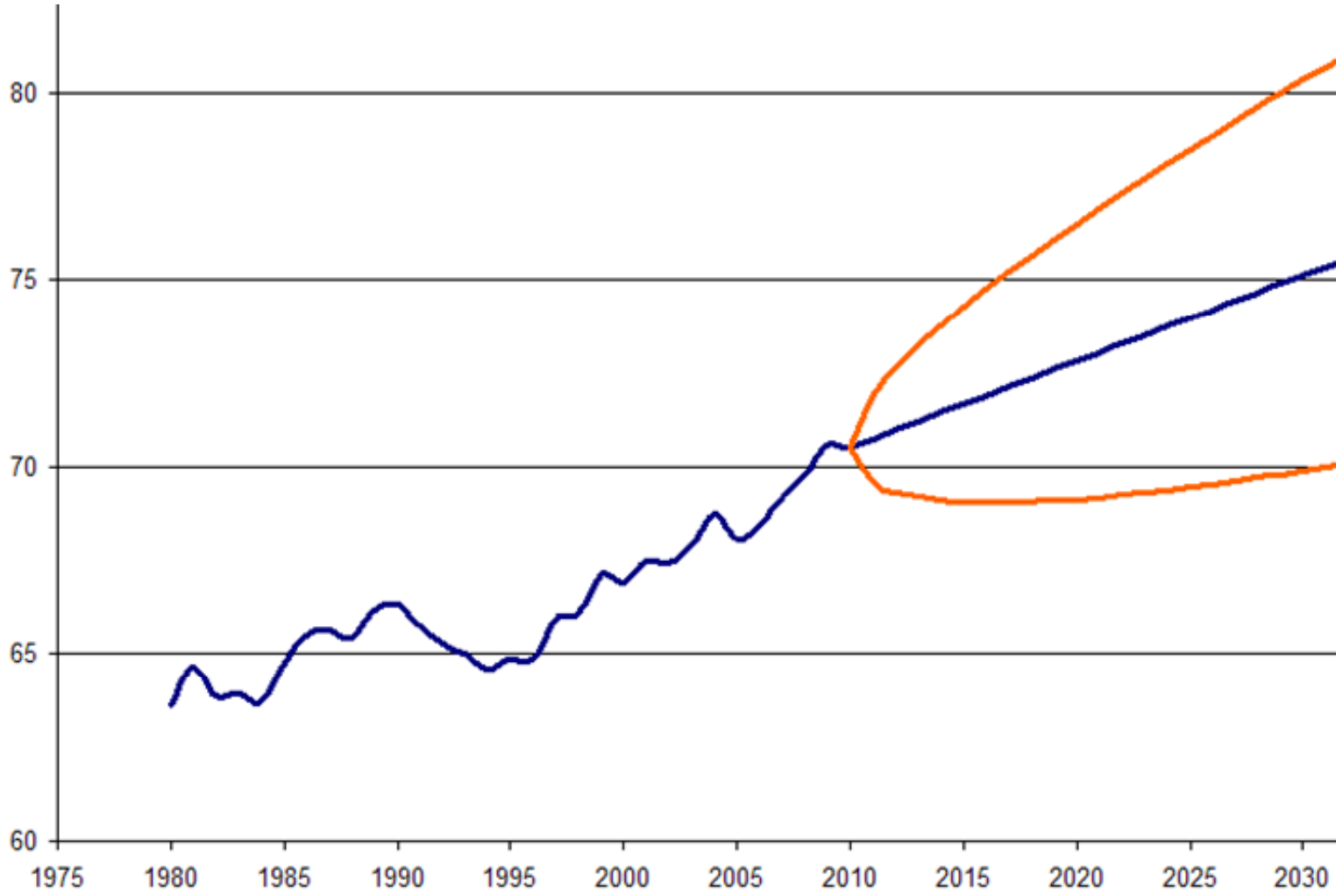



Total fertility rate: historical data and projecrions
TFR
5,00
4,50
4,00
3,50
3,00
2,50
2,00
1,50
1,00
0,50
0,00
1980
1985
1990
1995
2000
2005
2010
2015
2020
2025
2030

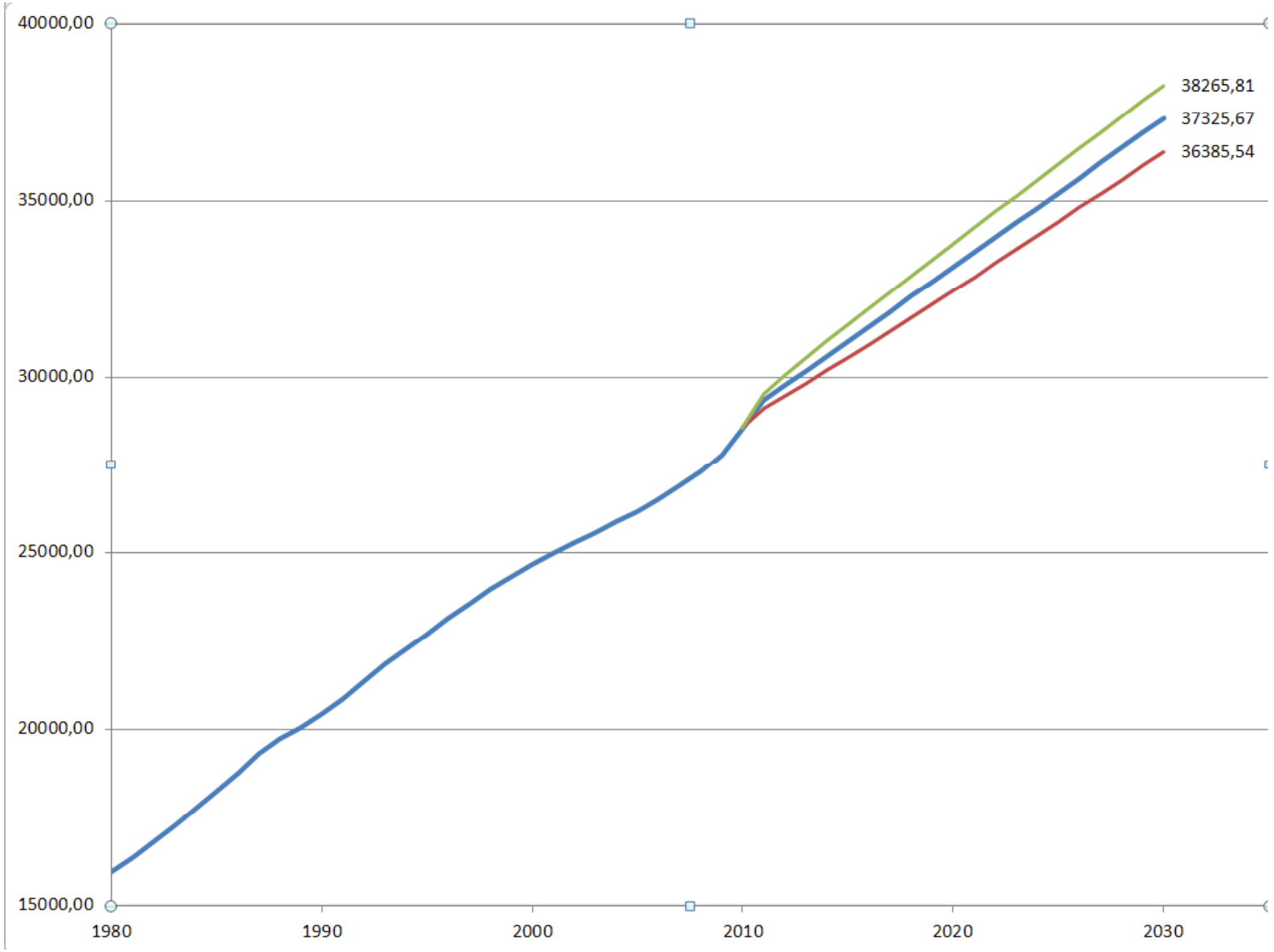
Life expectancy, female



Life expectancy, male



Population of Uzbekistan: historical data and projections (000)



References

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