Non Gaussian yields: which impact on default options retirement plans?

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An empirical study on American and French financial yields on the long run

- Yields must be considered « dividends included » and in real terms
- Duration lowers risk: equities appear as the less risky asset in the LR, the risk decreasing quicker than in the Gaussian case
- Equities prices (DS) are characterised by mean reversion and strong synchronisation with economic growth.
- This calls for a inter temporal diversification and a life cycle allocation for default options retirement plans.

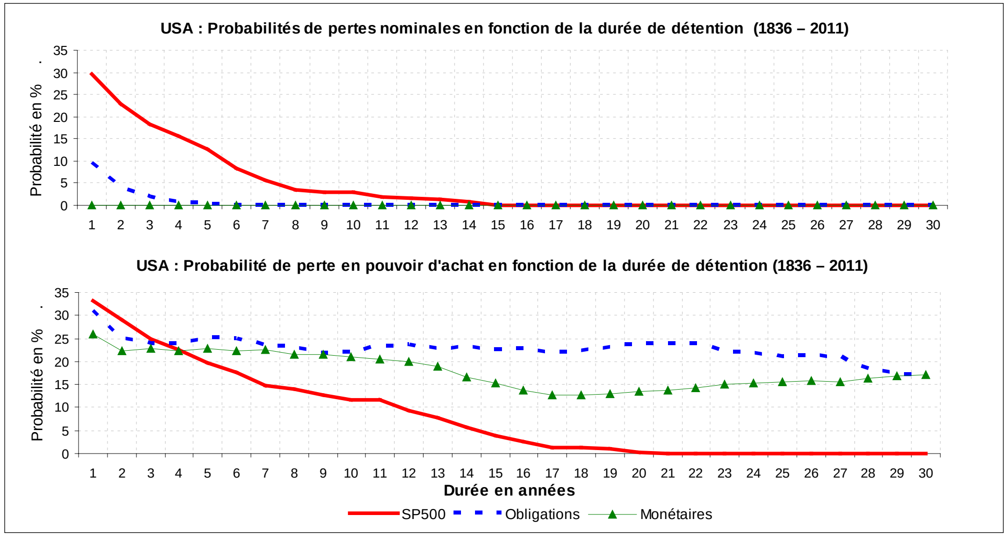
Yields must be considered « dividends included » and in real terms

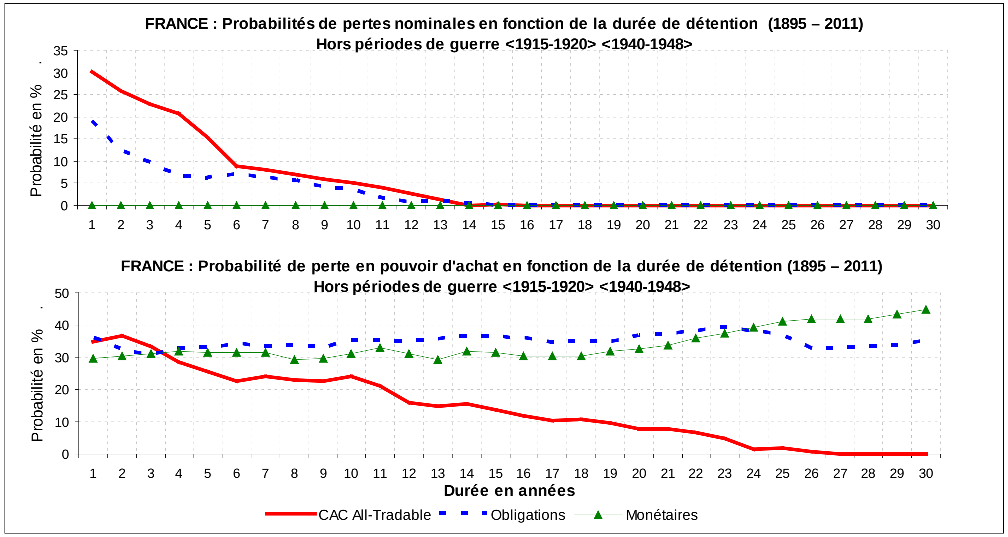
If nominal yields show similar performances in both the countries …

	1895-1914		1914-1950		1950-2011		1995-2011		1895-2011	
	United-States	France	United-States	France	United-States	France	United-States	France	United-States	France
Money	4.3%	2.5%	2.2%	3.1%	4.6%	5.9%	3.3%	3.1%	3.8%	4.5%
Bonds	3.1%	2.2%	3.5%	3.4%	6.0%	7.3%	6.9%	6.5%	4.8%	5.3%
equities	7.7%	6.7%	7.1%	10.8%	10.6%	10.5%	7.9%	6.8%	9.1%	10.0%
Inflation rate	1.9%	0.5%	2.5%	13.5%	3.6%	4.7%	2.5%	1.6%	3.0%	6.8%

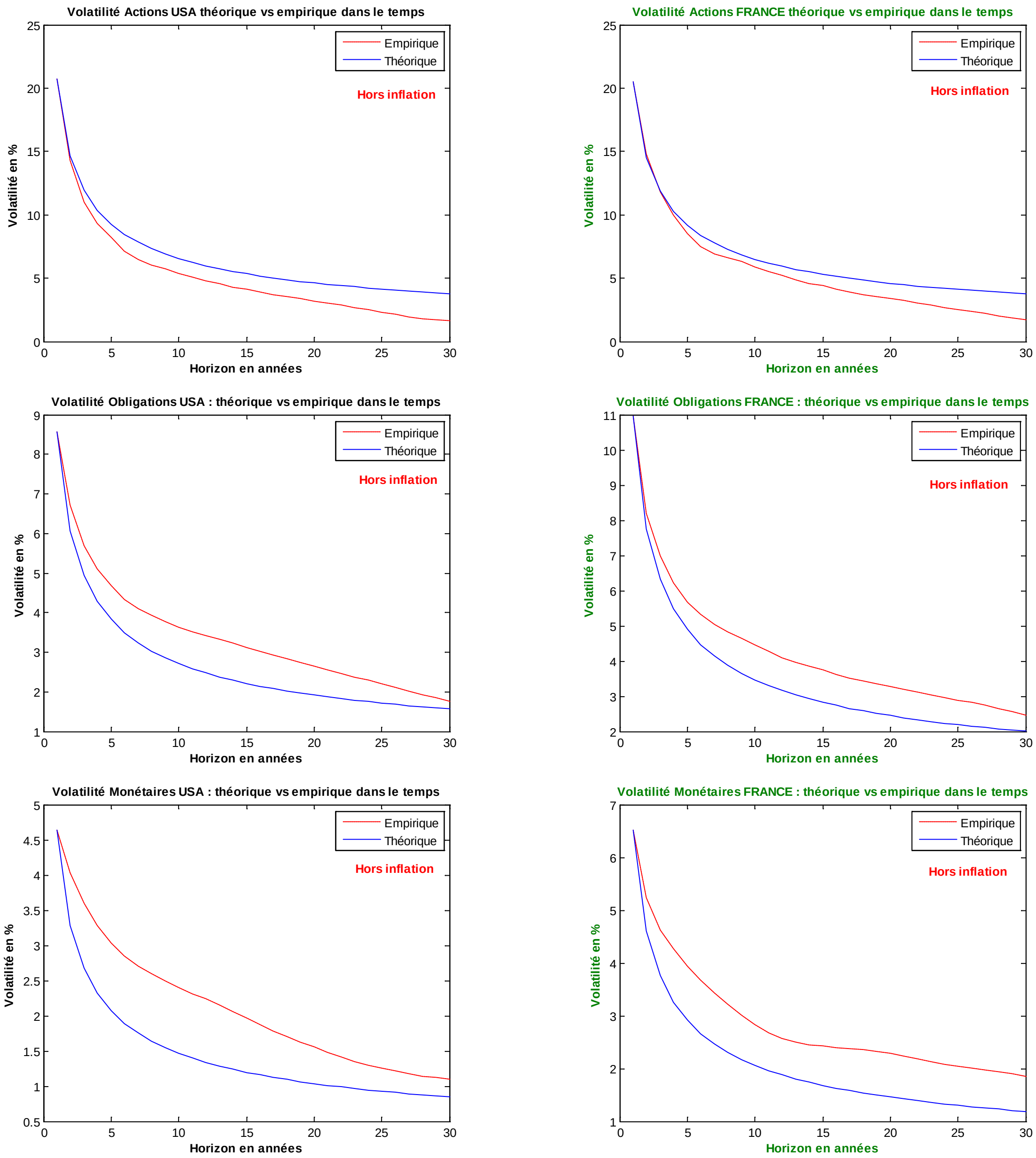
Real yields are much lower in France, which drives us to cancel war periods.

	1895-1914		1914-1950		1950-2011		1995-2011		1895-2011		France 1885-2011 war periods excluded*
	United-States	France	United-States	France	United-States	France	United-States	France	United-States	France	
Money	2.4%	2.1%	-0.3%	-10.4%	1.0%	1.2%	0.8%	1.5%	0.8%	-2.2%	0.9%
Bonds	1.2%	1.8%	1.1%	-10.1%	2.3%	2.6%	4.5%	4.9%	1.8%	-1.5%	2.1%
equities	5.8%	6.3%	4.6%	-2.8%	7.0%	5.8%	5.4%	5.2%	6.1%	3.2%	5.3%



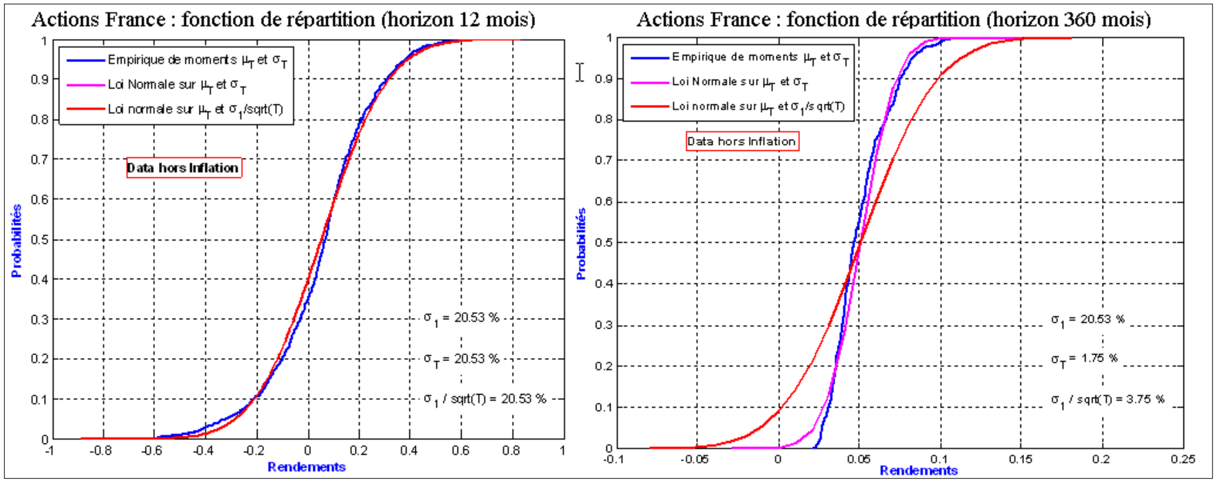


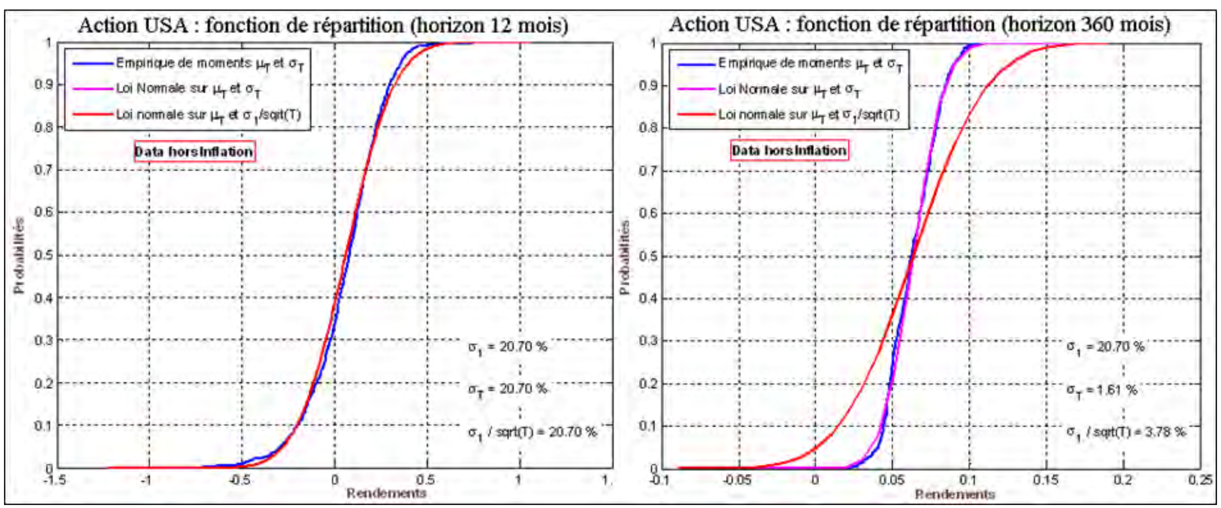
Duration lowers risk: equities appear as the less risky asset in the long run



Duration lowers equities risk: it decreases quicker than in the Gaussian case

After 30 years, equities yields show a risk decrease which is quicker than in the Gaussian case





Stationarity tests show that equities prices (DS) are characterised by mean reversion

	Augmented Dickey Fuller			KPSS Test		
Log-Returns USA	stat.	cValue	verdict	stat.	cValue	verdict
Equities	-41.5286	-1.9416	Stationarity	0.0252	0.1460	Stationarity
Bonds	-39.7891	-1.9416	Stationarity	0.3734	0.1460	Non Stationarity
Money	-38.1088	-1.9416	Stationarity	0.9038	0.1460	Non Stationarity
	Dickey Fuller Augmenté			KPSS Test		
Log-Returns France	stat.	cValue	verdict	stat.	cValue	verdict
Equities	-30.2137	-1.9416	Stationarity	0.0507	0.1460	Stationarity
Bonds	-31.6050	-1.9416	Stationarity	0.1884	0.1460	Non Stationarity
Money	-21.5008	-1.9416	Stationarity	0.5440	0.1460	Non Stationarity

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➢ *This calls for a inter temporal diversification and a life cycle allocation for default options retirement plans.*

2 – Which performances with a semi parametric allocation strategy?

- The non Gaussian distribution pleads for a CF VaR as the risk to be minimized in a life allocation
- A higher place for equities with the CF VaR criterium
- Higher replacement rates as well

Conclusion

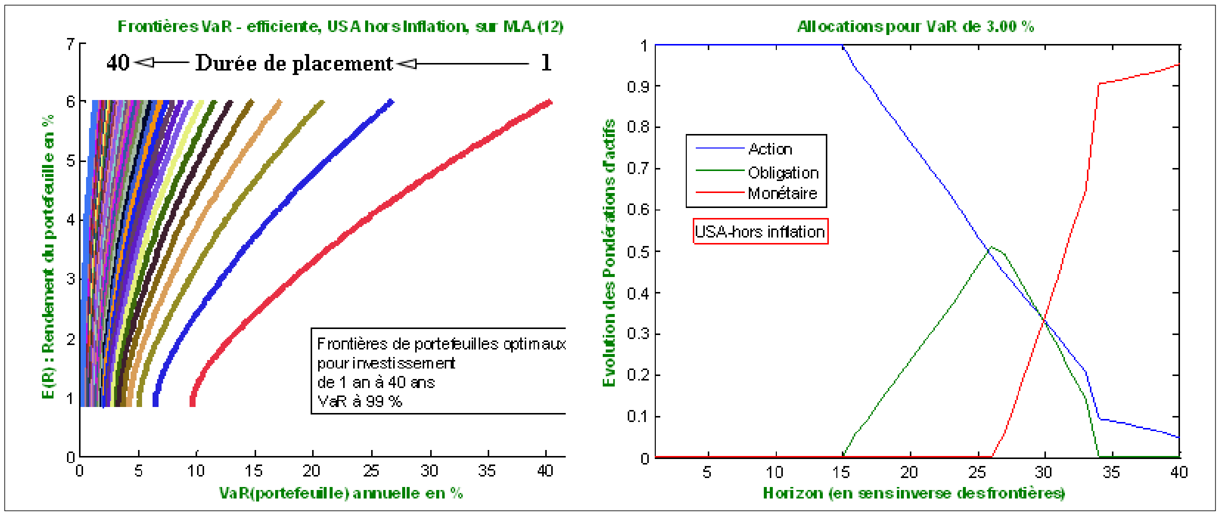
The non Gaussian distribution pleads for a CF VaR as the risk to be minimized in a life allocation strategy

- Usual problem:
  - **Max μ- $\lambda_1\sigma^2$**
    - with: **μ, expected return**
    - **λ_1 , risk aversion parameter**
    - **σ^2 , return variance**
- Non gaussian distribution problem:
  - **Max μ- $\lambda_1\sigma^2$+λ_2S-λ_3K**
    - with: **λ_1 , λ_2 , λ_3 , respectively: variance aversion, asymetry preference, kurtosis aversion.**

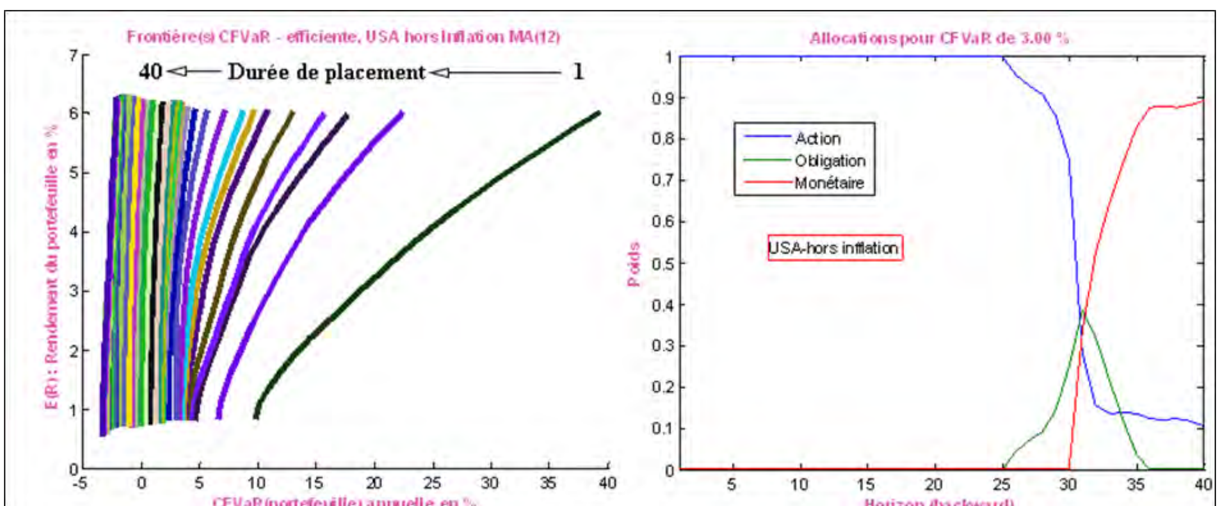
The non Gaussian distribution pleads for a CF VaR as the risk to be minimized in a life allocation strategy

Optimisation programme for a parametric VaR	Optimisation programme for a Cornish-Fisher (CF) VaR
$(\mathcal{P})\begin{cases}\min\limits_{w}(VaR_\alpha(w)) \\ S.C: \begin{cases} w' * \mu = \mu_P \\ \sum_{i=1}^{3} w_i = 1 \\ 0 \le w_i \le 1 \end{cases}\end{cases}$	$(\mathcal{P})\begin{cases}\min\limits_{w}(\widetilde{VaR}_\alpha(w)) \\ S.C: \begin{cases} w' * \mu = \mu_P \\ \sum_{i=1}^{3} w_i = 1 \\ 0 \le w_i \le 1 \end{cases}\end{cases}$
With: $VaR_\alpha = w' * \mu + z_\alpha * \sigma$	With: $\widetilde{VaR}_\alpha = w' * \mu + Z_\alpha^{Cornish-Fisher} * \sigma$ $Z_\alpha^{Cornish-Fisher} \approx Z_\alpha + \frac{1}{6}({Z_\alpha}^2 - 1) * S + \frac{1}{24}({Z_\alpha}^3 - 3Z_\alpha) * K - \frac{1}{36}(2{Z_\alpha}^3 - 5Z_\alpha) * S^2$ $S(X) = \frac{\mu_3}{\sigma^3}$: Skewness $\widetilde{K}(X) = \frac{\mu_4}{\sigma^4}$: Kurtosis $K = \widetilde{K} - 3$: *excess Kurtosis*

A higher place for equities with the CF VaR strategy: simulations with a 3% VaR in the US case …

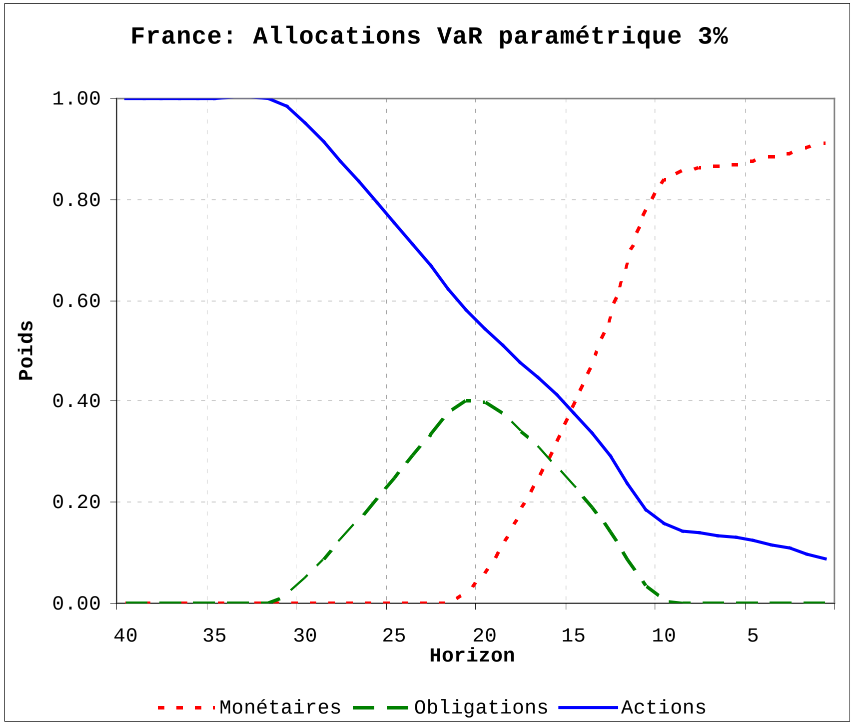


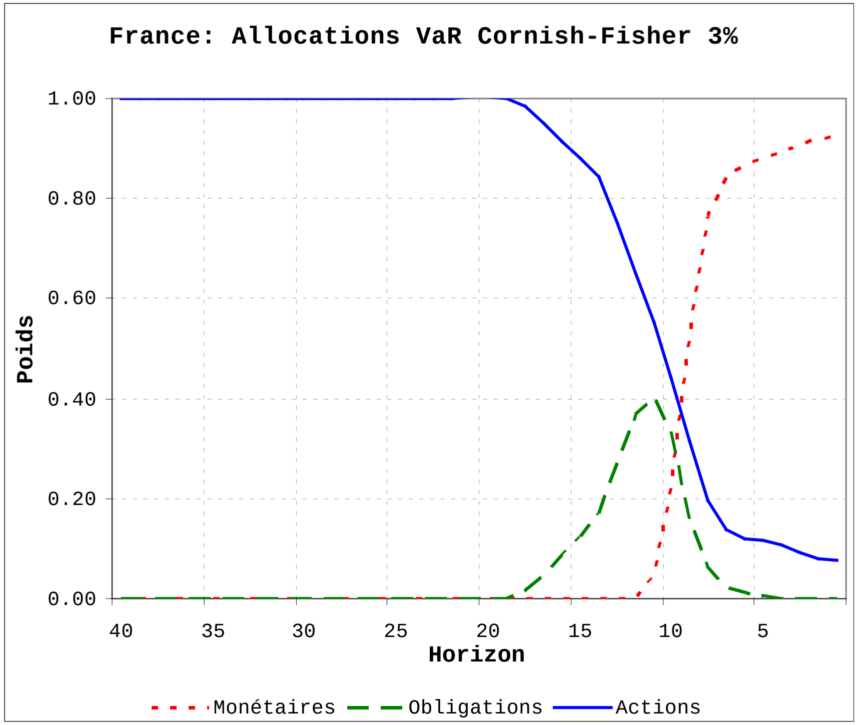
Parametric VaR



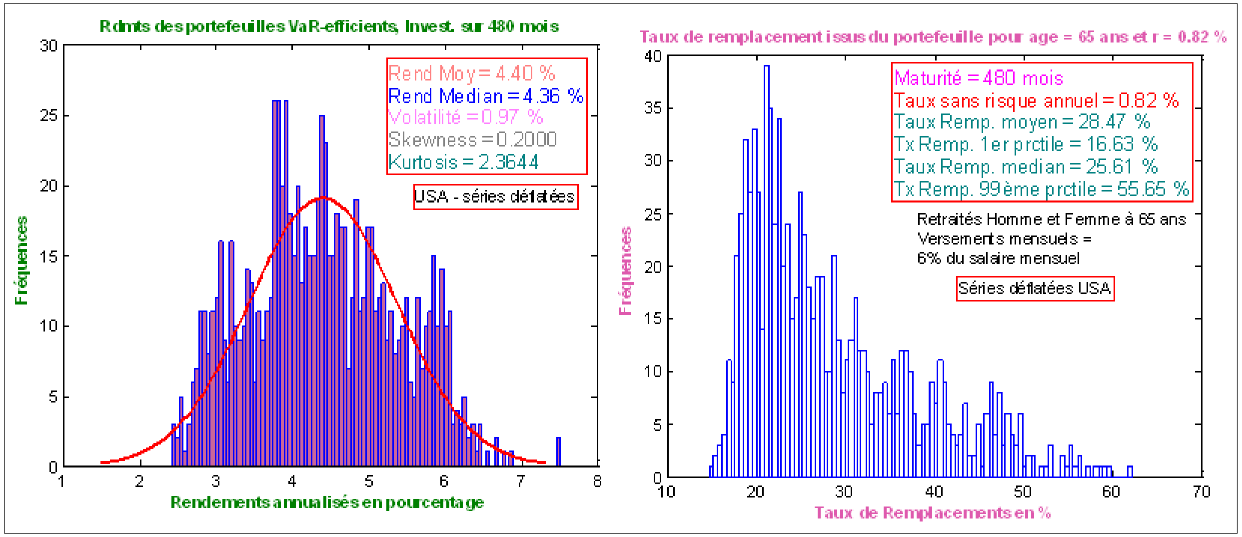
CF VaR

… and in the French case

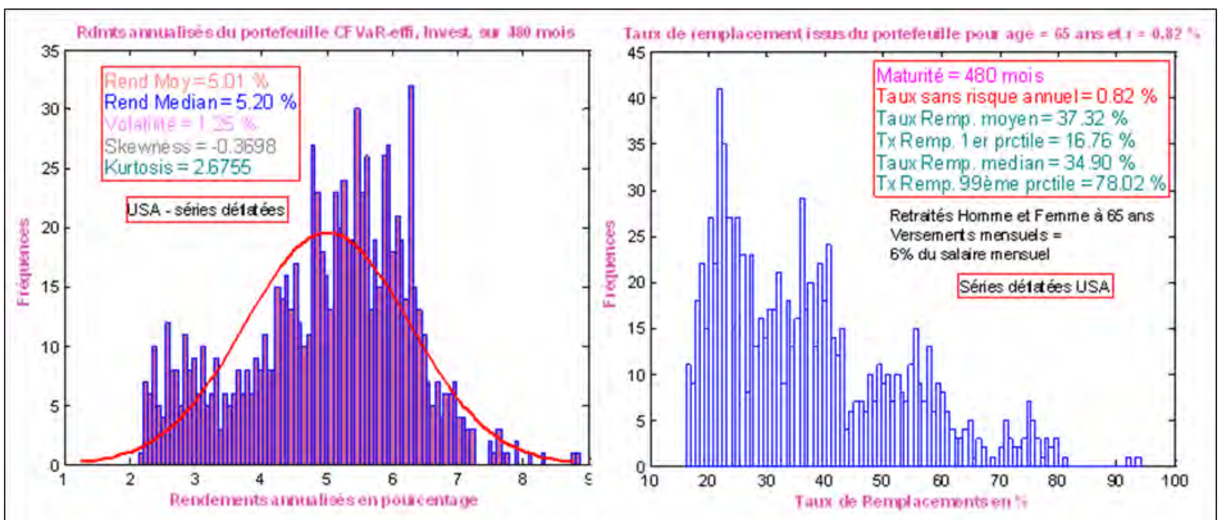




A strategy which provides better replacement rates: US case



Parametric VaR



CF VaR

Conclusion

- It’s worth taking into account the non Gaussian characteristics of equity distribution
- Non Gaussian yields deserve semi parametric VaR strategies
- Monte Carlo simulations must be avoided