Guarantee valuation in Notional Defined Contribution pension systems

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Outline

Aim of this talk

The aim of this presentation is to:

- describe the notional defined contribution (NDC) pension scheme with its advantages and shortocomings;
- develop a pricing framework for guarantees in NDC first pillar pension systems;
- and compare it to the insurance and financial markets framework.

Overview of pension systems

- Basic financing techniques
  - Pay as you go (PAYG): current contributors pay current pensioners (Unfunded schemes)
  - Funding: contributions are accumulated in a fund which earns a market return (Funded schemes)
- Benefit formulae
  - Defined Benefit: Pension is calculated according to a fixed formula which usually depends on the members salary and the number of contribution years.
  - Defined Contribution: Pension is dependent on the amount of money contributed each year and their return.

$\Rightarrow$ Notional Accounts: Mix of PAYG **and** Defined Contribution !

Notional Defined Contribution

What's NDC?

- A pension scheme managed by the State,
- with Pay-as-you-go (PAYG) as financing technique: '*current contributors pay current pensioners*'.
- The pension formula depends on the amount contributed and the return which equal to the notional rate.
- At retirement age: Accumulated capital $\Rightarrow$ Annuity which takes into account:
  - Life expectancy of the individual
  - The indexation of pensions and technical interest rate
- We assume that the notional rate is equal to the rate of increase of the total contributions.

Notional Defined Contribution (C'td)

Mathematically, the pension for someone retiring at age x_r at time t can be stated as:

$$P(x_r, t) = \frac{NDC(x_r, t)}{a(x_r, t)} \tag{1}$$

where:

$$NDC(x_r, t) = \sum_{x=x_0}^{x_r-1} \underbrace{C(x, t - x_r + x)}_{\text{yearly contributions}} \prod_{i=t-x_r+x}^{t} \left(\underbrace{1 + nr_i}_{\text{notional rate}} \right)$$

and $a(x_r, t)$ is the whole time annuity at time t for age x_r.
$\Rightarrow$ It is crucial to have an **appropriate capital** in order to have an adequate pension!

Advantages of NDC

- It's *more or less* actuarially fair (takes into account life expectancy and contributions)
- Portability of pension rights between jobs, occupations and sectors is permitted.
- It promises to deal with the effects of population ageing more or less automatically.
- Arbitrariness in benefit indexation rules and adjustment factors is avoided.

Challenge faced

Disadvantages of a pure NDC system (w/out guarantees):

- The return risk is fully borne by the participants.
- The amount of pension at retirement is unknown!
- The notional rate depends on demography's and salary's rate of increase: it can be close to 0% or negative!

Why should we consider guarantees?

- We offer a minimum value for each contribution at retirement.
- **However**, which is the price for the guarantee provider?
- ...and how can we price the option on an asset which is not traded in the market?

Answer: pricing in incomplete markets techniques!

Utility indifferent pricing

Definition

The indifference price on an option is defined as the function price which makes these two situations equivalent:

- *hold a financial portfolio (which maximizes capital and hedges the option)* **and** *hold (resp. sell) the written option at time t and receive (resp. pay) the payoff at maturity;*
- **only** *hold the financial portfolio.*

We are in an incomplete market, so the price will depend on

- the utility function used
- and the risk aversion γ.

In our case, we use the exponential utility:

$$U(x) = -e^{-\gamma x}, \gamma > 0 \tag{2}$$

The pension system

- The working population pays a contribution rate $\pi \in (0, 1)$ of their income to the pension system;
- each contribution earns a rate of return (notional rate) based on the total contribution base at time t.

The total contribution base is then: $Y(t) = \pi P(t) L(t)$, where:

- $P(t)$ represents the working population
- and $L(t)$ is the mean salary earned by the workers.
- Their evolution is driven by a Geometric Brownian Motion ($\rightarrow$ lognormally distributed)

The pension system (C'td)

For the **pension system provider** or state, the guarantee scheme entails:

- holding the contribution base which represents the returns of contribution,
- a loss of $K - \frac{Y(T)}{Y(t)}$ if the contribution base performed badly

Mathematically, it becomes the put option:

$$g(Y(T)) = \left(K - \frac{Y(T)}{Y(t)}\right)^+ \tag{3}$$

with

$$\begin{aligned} dY(t) &= d(\pi P(t)L(t)) \\ &= \underbrace{(g + \alpha_P + \rho_{L,P}\sigma_L\sigma_P)\, Y(t)}_{\text{drift}}\, dt + \underbrace{\sigma_L Y(t) dW_L(t)}_{\text{salary risk}} + \underbrace{\sigma_r Y(t) dW_P(t)}_{\text{demographic risk}} \end{aligned} \tag{4}$$

Incomplete markets

- **Reminder**: The salary and population risks are not traded in the market

$\rightarrow$ the market is incomplete!

- We can't use the classical models! (e.g. Black & Scholes)
- However, the risks linked to the contribution are **correlated** to those which are traded!
- We can then price the underlying non-hedgeable risks, by using the traded risks as a **proxy**.

Financial portfolio

We create a self-financing portfolio with the assets available in the market:

$$\begin{aligned}
\frac{dX(s)}{X(s)} &= \theta_0(s)\frac{dS_0(s)}{S_0(s)} + \theta_P(s)\frac{dP(s,T)}{P(s,T)} + \theta_S(s)\frac{dS(s)}{S(s)} \\
&= (r(s) + \theta_S(s)\lambda_S\sigma_S - \theta_P(s)q\sigma(s,T))dt \\
&+ \theta_S(s)\lambda_S dW_S(s) - \theta_P(s)q\sigma(s,T)dW_r(s) \qquad (5) \\
X_t &= x \; 0 \leq t \leq s \leq T
\end{aligned}$$

with:

- $\theta_0(s)$ proportion invested in the bank account $S_0(s)$;
- $\theta_S(s)$ proportion invested in the risky asset $S(s)$;
- $\theta_P(s)$ proportion invested in the zero-coupon bond $P(s,T)$.
- **Constraint**: $\theta_0(s) + \theta_S(s) + \theta_P(s) = 1$

The price

Proposition

The price of a European option $G = g\left(\frac{Y(T)}{Y(t)}\right)$ *under the exponential utility in a market like the one presented is given by*

$$p(x, y, r, t) = P_Q(t, T)\frac{\delta}{\gamma}\log\left(\mathrm{E}_{\mathcal{Q}^T}\left[e^{\frac{\gamma}{\delta}g\left(\frac{Y(T)}{Y(t)}\right)}|Y(t) = y\right]\right) \tag{6}$$

where:
δ *constant which depends on the level of correlation between the pension risks and the traded risks;*
γ *is the risk aversion;*
$\frac{d\mathcal{Q}^T}{d\mathcal{P}}$ *is the forward measure;*

Particular cases: Independent & Complete markets

- If the notional index is totally independent we find a zero-utility insurance premium:

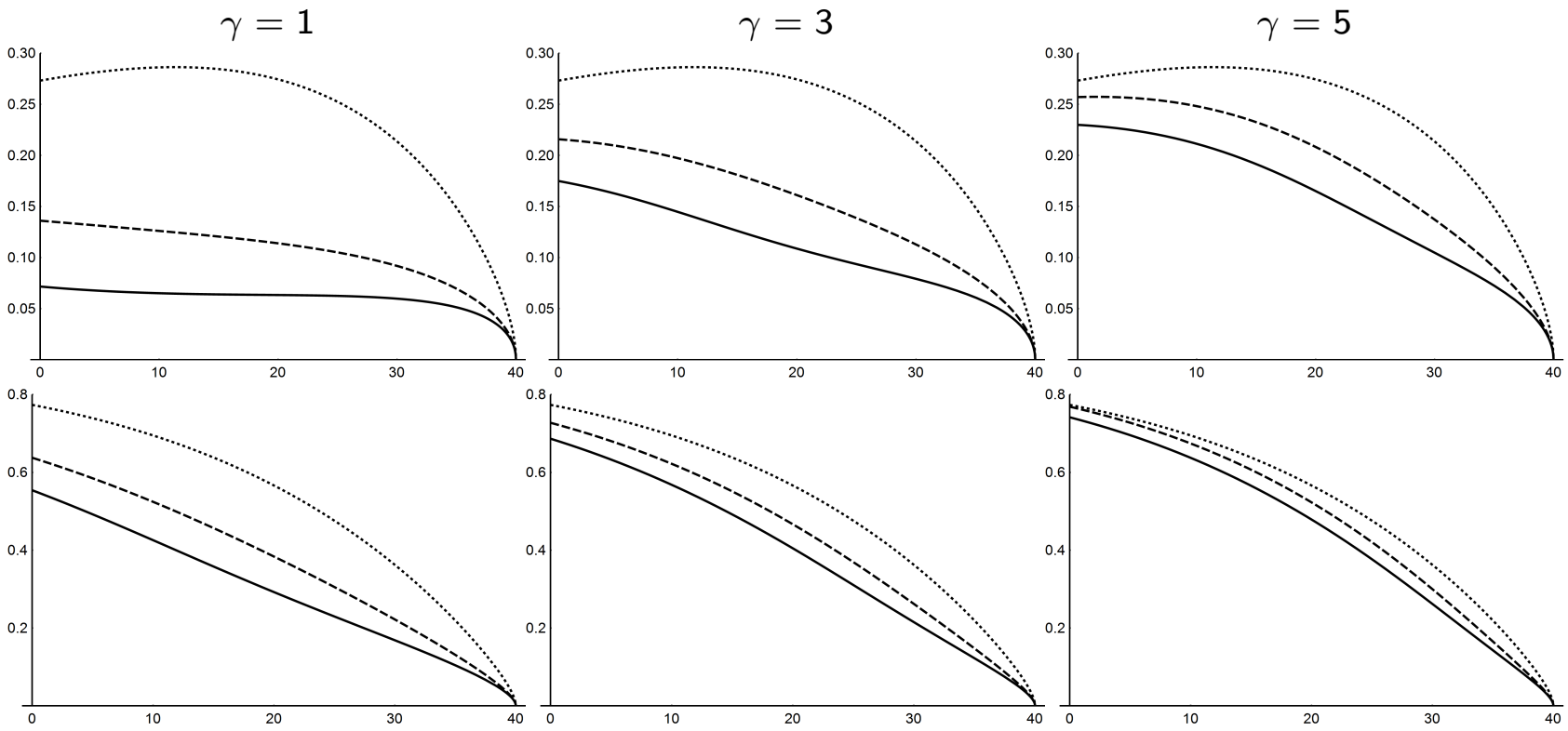
$$p(x, y, r, t) = P_Q(t, T)\frac{1}{\gamma}\log\left(\mathrm{E}_{\mathcal{P}}\left[e^{\gamma g\left(\frac{Y(T)}{Y(t)}\right)}|Y(t) = y\right]\right) \tag{7}$$

- If the notional index is traded we find a complete markets price:

$$\begin{aligned} p(x, y, r, t) &= P_Q(t, T)\mathrm{E}_{\mathcal{Q}^T}\left[g\left(\frac{Y(T)}{Y(t)}\right)|Y(t) = y\right] \\ &= \text{Black \& Scholes Formula!} \end{aligned} \tag{8}$$

Comparison between the intermediate price, the insurance and B&S Price

Figure : The value of a put option guaranteeing $i_G = 4\%$ (first row), $i_G = 6\%$ (second row): un-correlated exponential price (continuous line), imperfect correlation exponential price (discontinuous line) and perfect correlation B&S case (pointed line) *Source:* the authors.



Interpretation

Link between pure insurance, intermediate price and Black & Scholes:

- For some risk aversions and guarantees we can have lower prices than Black & Scholes.
  $\Rightarrow$ I'm ready to pay 'more' for a better hedge.

For the other graphics:

- Price $\nearrow$ when risk aversion $\nearrow$.
  $\Rightarrow$ I'm ready to pay a higher price to insure me against losses when I'm more 'afraid' of them.
- Price $\nearrow$ with guaranteed rate $\nearrow$.
  $\Rightarrow$ I'm ready to pay a higher price linked to higher guarantees.

Cost of the guarantee: An example

Assume the following economy:

Salary (Gross - Annual)	39.532,00
Salary rate of increase	1%
Contribution rate	16,86%
Life expectancy at 65	20,02

Then the Monthly Minimum Pensions (MMP) and the Total Guarantee Cost (TGC) for different minimum returns are:

	0%	1%	2%	3%	4%
TGC (% of capital)	2,26%	4,14%	6,69%	9,27%	11,53%
MMP - State	1.505,80	1.822,50	2.235,42	2.764,12	3.427,86
MMP - Contributors	1.471,70	1.732,05	2.002,60	2.252,07	2.420,45

Conclusion

- We obtained a closed-form formula which prices different options written on the notional index;
- The notional index is the rate of return on the contributions and is stochastic;
- This price doesn't depend on the initial capital!
- This tool allows to put a price on promises to the participants for a better risk management.
- **Issue**: How can we finance this guarantees? Who should pay? Contributors? The State?

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Thank you for your attention!