AFIR-ERM Finance, Investment & ERM

ASTIN Non-Life Insurance

Terms of Reference

AFIR/ASTIN Working Party on Risk Aggregation with correlation matrices

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Background

In many standardized risk quantification frameworks risks are aggregated using correlation matrices. The formula is valid for aggregating standard deviations but not Value at Risk (VaR) or shortfall (Tail VaR) risk measures where the VaR to standard deviation ratio decreases with the aggregation level. (See appendix)

Specific Aims

The aim of the AFIR/ASTIN Working Party on Risk Aggregation with correlation matrices (RaWCM) is to develop and apply techniques to correct for the fat tail thinning effect when aggregating risks.

The research will include:

- Gather different sets of standard approaches where the method is used:
  - EIOPA Solvency II standard formula
  - FINMA SST standard model
  - ICS model
  - S&P new model
- Assume distributions of underlying risks
  - Aggregate risks with standard correlation method
  - Aggregate risks with analytical, monte carlo method using various marginal distributions
  - Derive adjustment factor for standard correlation method
- Develop an adjustment method with a reduced set of inputs
  - VaR/std
  - Correlation, tail dependence

Timeline / Milestones

1. Q1 2023
   Recruit volunteer ASTIN / AFIR Working Party members to gather data and perform the research. The candidates are kindly requested to send an email with a short resume to the following address: iaasections@actuaries.org

2. Q2 2023:
   a. Review standard frameworks, correlation matrices quantile levels, aggregation granularity

3. Q3 2023
   a. Quantify the Develop correction method
   b. Develop a prototype software application to aggregate risks

4. Q4 2023:
   Disseminate findings:
   a. Publication, possibly in the ASTIN Bulletin.
   b. Presentation at the ASTIN/AFIR Colloquium in Brussels (2024).
   c. ASTIN/AFIR webinar.
   d. Possibly posting of the software on the ASTIN/AFIR website.

ASTIN Working Party Members

Working Party Champion: Mr Peter Middelkamp.
For more information, please feel free to contact: pmiddelkamp@gmail.com

The Working Party members should have experience with aggregating risk in internal and standard models, basic to advanced knowledge in stochastic calculus and a no fear attitude towards experimenting with random numbers.

Volunteers for contributing to any of the above tasks are kindly requested to send an email with a short resume to the following address: iaasections@actuaries.org

Appendix

Derivation of formula and adjustment (draft)

Quick recap on VaR, derive ICS aggregation formula

- Assume X_i are random variables representing unexpected losses of risk $X_i = (loss_{i,1} \quad \dots \quad \text{loss}_{i,1,000,000}), i = 1 \dots n$
- **Total loss**

$$X_t = \sum_{i=1}^{n} X_i$$

- **Variance of the totals**:

$$\sigma_t^2 = \sum_{i=1}^{n} {\sigma_i}^2 + 2\sum_{i,j:i<j}^{n} \text{cov}(X_i, X_j)$$

σ: standard deviaton of X_i, cov: covariance

- Using matrix notation and correlation or aggregation coefficient ρ:

$$\boldsymbol{\sigma_t^2} = (\boldsymbol{\sigma_1} \quad \cdots \quad \boldsymbol{\sigma_n}) \begin{pmatrix} \boldsymbol{\rho_{1,1}} & \cdots & \boldsymbol{\rho_{1,n}} \\ \vdots & \ddots & \vdots \\ \boldsymbol{\rho_{n,1}} & \cdots & \boldsymbol{\rho_{n,n}} \end{pmatrix} \begin{pmatrix} \boldsymbol{\sigma_1} \\ \vdots \\ \boldsymbol{\sigma_n} \end{pmatrix}$$

ICS aggregation formula

Table 38: Aggregation matrix between risks

	Life	Non-Life	Catastrophe	Market	Credit
Life	100%	0%	25%	25%	25%
Non-Life	0%	100%	25%	25%	25%
Catastrophe	25%	25%	100%	25%	25%
Market	25%	25%	25%	100%	25%
Credit	25%	25%	25%	25%	100%

Aggregating Value At Risks
ICS approach

- If the Value at Risk (VaR) is always **a constant** times the standard deviation:

$$\mathbf{VaR}(X_i) = \boldsymbol{c}\ \boldsymbol{\sigma_i}$$

e.g.: for normal $X_i, c = N^{-1}(99.5\%) = 2.6$

$$(c\sigma_t)^2 = (c\sigma_1 \quad \cdots \quad c\sigma_n) \begin{pmatrix} \rho_{1,1} & \cdots & \rho_{1,n} \\ \vdots & \ddots & \vdots \\ \rho_{n,1} & \cdots & \rho_{n,n} \end{pmatrix} \begin{pmatrix} c\sigma_1 \\ \vdots \\ c\sigma_n \end{pmatrix}$$

- Then we can also aggregate VaRs:

$$\boldsymbol{VaR^2(X_t)} = (\mathbf{VaR_1} \quad \cdots \quad \mathbf{VaR_n}) \begin{pmatrix} \boldsymbol{\rho_{1,1}} & \cdots & \boldsymbol{\rho_{1,n}} \\ \vdots & \ddots & \vdots \\ \boldsymbol{\rho_{n,1}} & \cdots & \boldsymbol{\rho_{n,n}} \end{pmatrix} \begin{pmatrix} \mathbf{VaR_1} \\ \vdots \\ \mathbf{VaR_n} \end{pmatrix}$$

What if c is not a constant?

Aggregating Value At Risks
Unadjusted ICS aggregation method punishes IAIGs with diverse tail fatness

- If the Value at risk is **not a constant** times the standard deviation:

$$\mathbf{VaR}(X_i) = \boldsymbol{c_i}\ \boldsymbol{\sigma}(\boldsymbol{X_i})$$

$$(c_t\sigma(X_t))^2 < (c_1\sigma_1 \quad \cdots \quad c_n\sigma_n) \begin{pmatrix} \rho_{1,1} & \cdots & \rho_{1,n} \\ \vdots & \ddots & \vdots \\ \rho_{n,1} & \cdots & \rho_{n,n} \end{pmatrix} \begin{pmatrix} c_1\sigma_1 \\ \vdots \\ c_n\sigma_n \end{pmatrix}, c_t < c_i$$

- For Swiss Re:

$$\mathbf{VaR}(X_t) = \mathbf{0.9} \sqrt{(\mathbf{VaR_1} \quad \cdots \quad \mathbf{VaR_n}) \begin{pmatrix} \boldsymbol{\rho_{1,1}} & \cdots & \boldsymbol{\rho_{1,n}} \\ \vdots & \ddots & \vdots \\ \boldsymbol{\rho_{n,1}} & \cdots & \boldsymbol{\rho_{n,n}} \end{pmatrix} \begin{pmatrix} \mathbf{VaR_1} \\ \vdots \\ \mathbf{VaR_n} \end{pmatrix}}$$

- If there are X_i not normally distributed, the aggregation with **tail dependence should be done using Monte Carlo simulations methods** with sufficient number of realisations

– Reinsurance risk profiles expose fat tails, thin bellies and have tail dependence
– The aggregation result would need to be adjusted to reflect it accordingly