Economic IRR and Its Application

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Abstract

Integrated management of capital, risk and return is essential for the contemporary Enterprise Risk Management (ERM). The relationships between three elements are measured by three metrics: Economic Solvency Ratio (ESR), Return on Equity (ROE) and Return on Risk Capital (RORC). The risk-adjusted return metrics represented by RORC have an important role in decision making because they are inextricably linked to strategic planning and its analysis. RORC is popularly used in practice, but has limitations in that RORC itself gives little information about ESR.

In this paper, we introduce a risk-adjusted return metric called "Economic Internal Rate of Return" (Economic IRR) as a complement to RORC. Economic IRR is a form of internal rate of return that includes initial required capital as part of initial investment. After explaining the development of the models for Economic IRR, we show how we apply them to life insurance and annuity. The derived equations are acceptable not only theoretically but also intuitively. Then we explore the pros and cons of Economic IRR from the perspective of effectiveness of practice. We find that Economic IRR excels in effectiveness and efficiency and is very useful in decision making. After showing an example of practical use in Fukoku Mutual Life, we conclude that Economic IRR, when it is effectively incorporated into the decision making process, will enhance the integrated management of capital, risk and return, and consequently will strengthen the ability of risk-taking.

Keywords

ESR, ROE, RORC, IRR, Economic IRR, Risk premium for required capital, Risk-return efficiency, Partial economic solvency ratio, Risk-release rate, Discrete model, Continuous model, Closed model, Semi closed model, Open model, Stationary model, Approximation for term insurance, Decision making, Pricing strategy, Sales strategy, ALM strategy, Capital strategy, Dividend strategy, Management cycle, Efficient frontier, Optimal portfolio.

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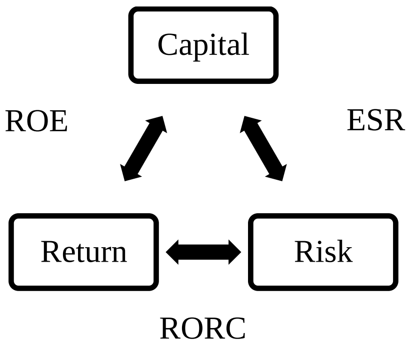
1. Introduction

Integrated management of capital, risk and return is essential for the contemporary ERM. The interrelations between three elements are measured by three metrics: ESR, ROE and RORC as illustrated in Figure 1.1.



$$\frac{\text{Return}}{\text{Risk}} = \frac{\text{Capital}}{\text{Risk}} \times \frac{\text{Return}}{\text{Capital}}$$

$$\text{RORC} = \text{ESR} \times \text{ROE}$$

Figure 1.1

The risk-adjusted return metrics represented by RORC have an important role in decision making because they are inextricably linked to strategic planning and its analysis. RORC is popularly used in practice, but has limitations in that RORC itself only stands for the return on required capital, thus giving little information about ESR. To complement such limitations, we introduce a risk-adjusted return metric called “Economic IRR”.

2. Economic IRR and Its Feature

Economic IRR is a form of internal rate of return that includes initial required capital as part of the initial investment. While IRR can be defined as a discount rate that equates present value of future cash flows with initial investment, Economic IRR can be defined as a discount rate that equates present value of the future cash flows with the sum of initial investment and initial required capital.

$$\sum \frac{\text{Future Cash Flow}_t}{(1+\text{IRR})^t} = (\text{Initial Investment}) \tag{2-1}$$

$$\sum \frac{\text{Future Cash Flow}_t}{(1+\text{Economic IRR})^t} = (\text{Initial Investment}) + (\text{Initial Required Capital}) \tag{2-2}$$

The most notable feature of Economic IRR is that it can clarify the difference in size between present value of future profits and initial required capital.

Let R_0 denote the required capital at time 0. From Equation (2-2) we have

$$\mathrm{R}_0 \quad = \quad -(\text{Initial Investment}) \ + \ \sum \frac{\text{Future Cash Flow}_t}{(1+\text{Economic IRR})^t}. \tag{2-3}$$

Let i denote the risk-free rate. The present value of future profits at time 0, denoted PVFP_0 is given by

$$\mathrm{PVFP}_0 \ = \ -(\text{Initial Investment}) \ + \ \sum \frac{\text{Future Cash Flow}_t}{(1+i)^t} \ . \tag{2-4}$$

Thus, we obtain the following relationships between Economic IRR, PVFP_0 and R_0.

$$\begin{aligned}
&\text{Economic IRR} < i \quad \leftrightarrow \quad \mathrm{PVFP}_0 < \mathrm{R}_0 \quad \leftrightarrow \quad \frac{\mathrm{PVFP}_0}{\mathrm{R}_0} < 1 \\
&\text{Economic IRR} = i \quad \leftrightarrow \quad \mathrm{PVFP}_0 = \mathrm{R}_0 \quad \leftrightarrow \quad \frac{\mathrm{PVFP}_0}{\mathrm{R}_0} = 1 \\
&\text{Economic IRR} > i \quad \leftrightarrow \quad \mathrm{PVFP}_0 > \mathrm{R}_0 \quad \leftrightarrow \quad \frac{\mathrm{PVFP}_0}{\mathrm{R}_0} > 1
\end{aligned} \tag{2-5}$$

Hereinafter we refer to the portion of Economic IRR exceeding the risk-free rate as "risk premium for required capital", and we refer to the ratio of the present value of future profits to required capital as "partial economic solvency ratio" respectively.

2.1 Discrete Model for Economic IRR

In this section, we develop a discrete model for Economic IRR. The concept of "risk release rate" is the key for the modeling. Figure 2.1 is a diagram of the discrete model.

Notation	Explanation
R_t	Required capital at time t
i	Risk-free rate
r	Return rate on required capital (RORC)
l	Risk-release rate (RRR)
$i\mathrm{R}_t$	Risk-free return on R_t
$r\mathrm{R}_t$	Return on R_t
$l\mathrm{R}_t$	Released risk from R_t
PVFR_t	Present value of future required capitals at time t
PVFP_t	Present value of future profits at time t
ESR_t	Partial economic solvency ratio at time t

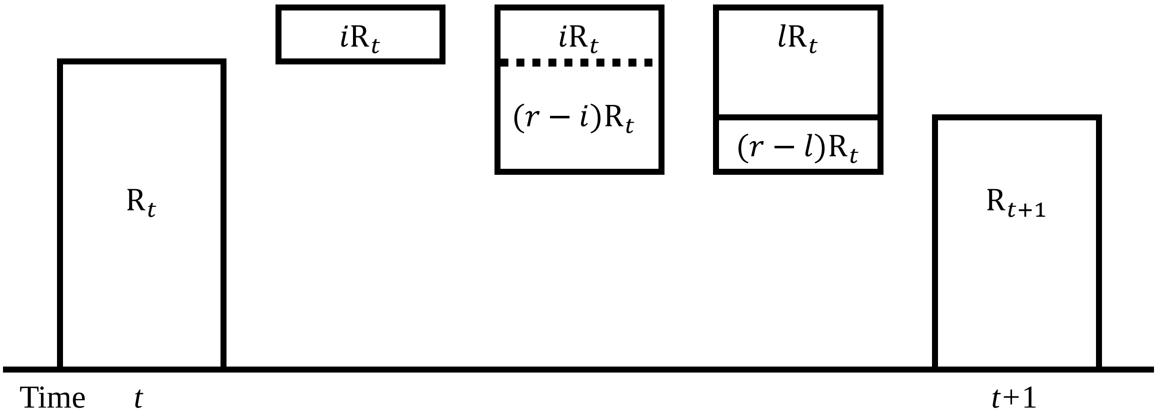


Figure 2.1

Assume the required capital declines exponentially, then R_t becomes

$$\mathrm{R}_t = (1+i-l)\ \mathrm{R}_{t-1} = (1+i-l)^t\ \mathrm{R}_0. \tag{2-6}$$

Then PVFR_0 becomes

$$\mathrm{PVFR}_0 = \sum_{t=0}^{\infty}(1+i)^{-t}\mathrm{R}_t = \sum_{t=0}^{\infty}(1+i)^{-t}(1+i-l)^t\mathrm{R}_0 = \frac{1+i}{l}\ \mathrm{R}_0. \tag{2-7}$$

The present value of future risk-free return on required capitals at time 0 becomes

$$\sum_{t=1}^{\infty}(1+i)^{-t}i\mathrm{R}_{t-1} = \frac{i}{1+i}\ \sum_{t=0}^{\infty}(1+i)^{-t}(1+i-l)^t\mathrm{R}_0 = \frac{i}{l}\ \mathrm{R}_0. \tag{2-8}$$

The present value of future profits excess of risk-free on required capitals at time 0 becomes

$$\sum_{t=1}^{\infty}(1+i)^{-t}(r-i)\mathrm{R}_{t-1} = \frac{r-i}{l}\ \mathrm{R}_0\ . \tag{2-9}$$

Thus, PVFP_0 becomes

$$\mathrm{PVFP}_0 = \frac{i}{l}\ \mathrm{R}_0 + \frac{r-i}{l}\ \mathrm{R}_0 = \frac{r}{l}\ \mathrm{R}_0\ . \tag{2-10}$$

Consequently, ESR_0 becomes

$$\mathrm{ESR}_0 = \frac{\mathrm{PVFP}_0}{\mathrm{R}_0} = \frac{r}{l}\ , \tag{2-11}$$

and PVFP/PVFR Ratio at time 0 becomes

$$\frac{\mathrm{PVFP}_0}{\mathrm{PVFR}_0} = \frac{r}{1+i}\ . \tag{2-12}$$

Note that Equation (2-12) does not become r because risk-free return is recognized at the end of the period, and note that Equation (2-11) and (2-12) are true for any time t

because the outcomes are independent of time.

$$\mathrm{ESR}_t = \frac{\mathrm{PVFP}_t}{\mathrm{R}_t} = \frac{r}{l} \tag{2-13}$$

$$\frac{\mathrm{PVFP}_t}{\mathrm{PVFR}_t} = \frac{r}{1+i} \tag{2-14}$$

To identify Economic IRR and risk premium for required capital, we must find a discount rate that equates present value of future profits with R_0. Let j be a discount rate such that $j > i - l$. The present value of future profits becomes

$$\sum_{t=1}^{\infty} r \ \mathrm{R}_{t-1} \times (1+j)^{-t}$$

$$= r \ (1+j)^{-1} \sum_{t=0}^{\infty} (1+j)^{-t} (1+i-l)^t \mathrm{R}_0 = \frac{r}{j-i+l} \mathrm{R}_0 \ . \tag{2-15}$$

Find j such that

$$\frac{r}{j-i+l} \mathrm{R}_0 = \mathrm{R}_0 \ . \tag{2-16}$$

Finally, we can identify Economic IRR as $r - l + i$ and the risk premium for required capital as $r - l,$ respectively.

2.2 Continuous Model for Economic IRR

In this section, we develop a continuous model for Economic IRR. After considering a model without new business, we develop models that include new business. Our goal is to develop a model in a stationary state. If there is a stationary state in our practice, the model will give us much information.

Notation	Explanation
R_t	Required capital at time *t*
δ	Force of risk-free rate
ρ	Force of RORC
θ	Force of RRR
PVFR_t	Present value of future required capitals at time *t*
PVFP_t	Present value of future profits at time *t*
ESR_t	Partial economic solvency ratio at time *t*

Assume the required capital declines exponentially, then R_t becomes

$$\mathrm{R}_t = e^{(\delta - \theta)t} \ \mathrm{R}_0. \tag{2-17}$$

Then PVFR_0 becomes

$$\mathrm{PVFR}_0 = \int_0^{\infty} e^{(\delta-\theta)t} \mathrm{R}_0 e^{-\delta t}\, dt = \int_0^{\infty} e^{-\theta t}\, \mathrm{R}_0\, dt = \frac{1}{\theta}\, \mathrm{R}_0. \tag{2-18}$$

The present value of future risk-free return on required capitals at time 0 becomes

$$\int_0^{\infty} \delta e^{(\delta-\theta)t} \mathrm{R}_0 e^{-\delta t} dt = \int_0^{\infty} \delta e^{-\theta t}\, \mathrm{R}_0\, dt = \frac{\delta}{\theta}\, \mathrm{R}_0. \tag{2-19}$$

The present value of future profits excess of risk-free on required capitals at time 0 becomes

$$\int_0^{\infty} (\rho-\delta) e^{(\delta-\theta)t} \mathrm{R}_0 e^{-\delta t} dt = \int_0^{\infty} (\rho-\delta) e^{-\theta t}\, \mathrm{R}_0\, dt = \frac{(\rho-\delta)}{\theta}\, \mathrm{R}_0. \tag{2-20}$$

Thus, PVFP_0 becomes

$$\mathrm{PVFP}_0 = \frac{\delta}{\theta}\, \mathrm{R}_0 + \frac{(\rho-\delta)}{\theta}\, \mathrm{R}_0 = \frac{\rho}{\theta}\, \mathrm{R}_0\,. \tag{2-21}$$

Consequently, ESR_0 becomes

$$\mathrm{ESR}_0 = \frac{\mathrm{PVFP}_0}{\mathrm{R}_0} = \frac{\rho}{\theta}, \tag{2-22}$$

and PVFP/PVFR Ratio at time 0 becomes

$$\frac{\mathrm{PVFP}_0}{\mathrm{PVFR}_0} = \rho. \tag{2-23}$$

Note that PVFP/PVFR Ratio at time 0 becomes equal to RORC, which differs from Equation (2-12), and note that Equation (2-22) and (2-23) are true for any time *t* because the outcomes are independent of time. Equation (2-24) is illustrated in Appendix A.

$$\mathrm{ESR}_t = \frac{\mathrm{PVFP}_t}{\mathrm{R}_t} = \frac{\rho}{\theta} \tag{2-24}$$

$$\frac{\mathrm{PVFP}_t}{\mathrm{PVFR}_t} = \rho \tag{2-25}$$

To identify Economic IRR, we must find the discount rate that equates the present value of future profits with R_0. Let ε be a discount rate such that $\varepsilon > \delta - \theta$. The present value of future profits becomes

$$\int_0^{\infty} \rho\, \mathrm{R}_t\, e^{-\varepsilon t} dt = \int_0^{\infty} \rho\, \mathrm{R}_0\, e^{(\delta-\theta)t} e^{-\varepsilon t} dt = \frac{\rho}{\theta+\varepsilon-\delta}\, \mathrm{R}_0\,. \tag{2-26}$$

Find ε such that

$$\frac{\rho}{\theta+\varepsilon-\delta}\, \mathrm{R}_0 = \mathrm{R}_0. \tag{2-27}$$

Finally, we identify Economic IRR as $\rho - \theta + \delta$ and the risk premium for required capital as $\rho - \theta$, respectively.

The relationships between Economic IRR, RORC, RRR and ESR_0 are shown in the following table.

Table 2.1

Economic IRR	RORC vs. RRR	ESR_0
Economic IRR< δ	RORC<RRR	ESR_0<100%
Economic IRR= δ	RORC=RRR	ESR_0=100%
Economic IRR> δ	RORC>RRR	ESR_0>100%

2.3 Closed Model

In this section, we apply the continuous model to a closed segment. We refer to the model as “Closed Model”, focusing applying it to in-force business, illustrated in Figure 2.2.

Notation	Explanation
R_0^{inf}	Required capital for in-force at time 0
R_T^{infc}	Required capital for in-force at time *T* when R_0^{inf} becomes closed at time 0

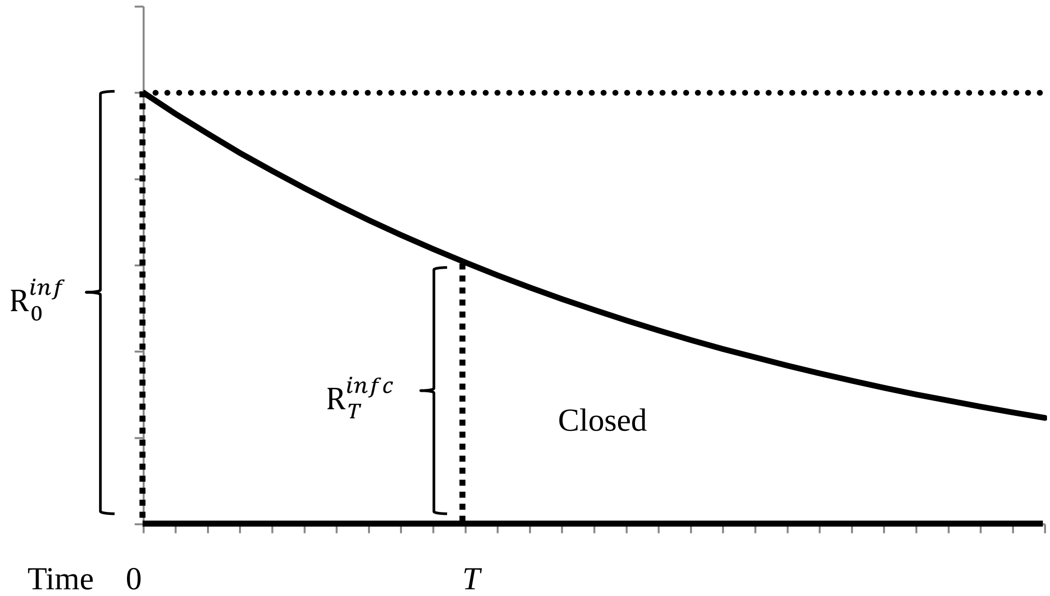


Figure 2.2

R_T^{infc} becomes

$$\mathrm{R}_T^{infc} = e^{(\delta-\theta)T}\mathrm{R}_0^{inf}. \quad (2\text{-}28)$$

Then PVFR_T becomes

$$\mathrm{PVFR}_T = \int_0^{\infty} \mathrm{R}_{T+t}^{infc}\, e^{-\delta t}\, dt = \int_0^{\infty} e^{(\delta-\theta)(T+t)}\, \mathrm{R}_0^{inf}\, e^{-\delta t}\, dt$$

$$= e^{(\delta-\theta)T} \mathrm{R}_0^{inf} \frac{1}{\theta} = \frac{1}{\theta} \mathrm{R}_T^{infc}. \qquad (2\text{-}29)$$

PVFP_T becomes

$$\mathrm{PVFP}_T = \frac{\rho}{\theta}\, \mathrm{R}_T^{infc}. \qquad (2\text{-}30)$$

Consequently, ESR_T becomes

$$\mathrm{ESR}_T = \frac{\mathrm{PVFP}_T}{\mathrm{R}_T^{infc}} = \frac{\rho}{\theta}\ . \qquad (2\text{-}31)$$

and PVFP/PVFR Ratio at time T becomes

$$\frac{\mathrm{PVFP}_T}{\mathrm{PVFR}_T} = \rho\ . \qquad (2\text{-}32)$$

To identify Economic IRR and risk premium for required capital, we must find a discount rate that equates present value of future profits with R_T^{infc}. Let ε be a discount rate such that $\varepsilon > \delta - \theta$. The present value of future profits becomes

$$\int_0^{\infty} \rho\, \mathrm{R}_{T+t}^{infc} e^{-\varepsilon t} dt = \int_0^{\infty} \rho\, e^{(\delta-\theta)(T+t)} e^{-\varepsilon t} \mathrm{R}_T^{infc} dt$$

$$= \frac{\rho}{\theta+\varepsilon-\delta}\ \mathrm{R}_T^{infc}. \qquad (2\text{-}33)$$

Find ε such that

$$\frac{\rho}{\theta+\varepsilon-\delta}\ \mathrm{R}_T^{infc} = \mathrm{R}_T^{infc}. \qquad (2\text{-}34)$$

Finally, we identify Economic IRR as $\rho - \theta + \delta$ and the risk premium for required capital as $\rho - \theta$, respectively.

2.4 Semi-Closed Model

Before we develop a model for an open segment, we consider a new business block that was acquired before time *T* and has been closed just after time *T*. We refer to the model as “Semi-Closed Model”, illustrated in Figure 2.3.

Notation	Explanation
R_T^{new}	Required capital for new business acquired before time *T* evaluated at *T*.

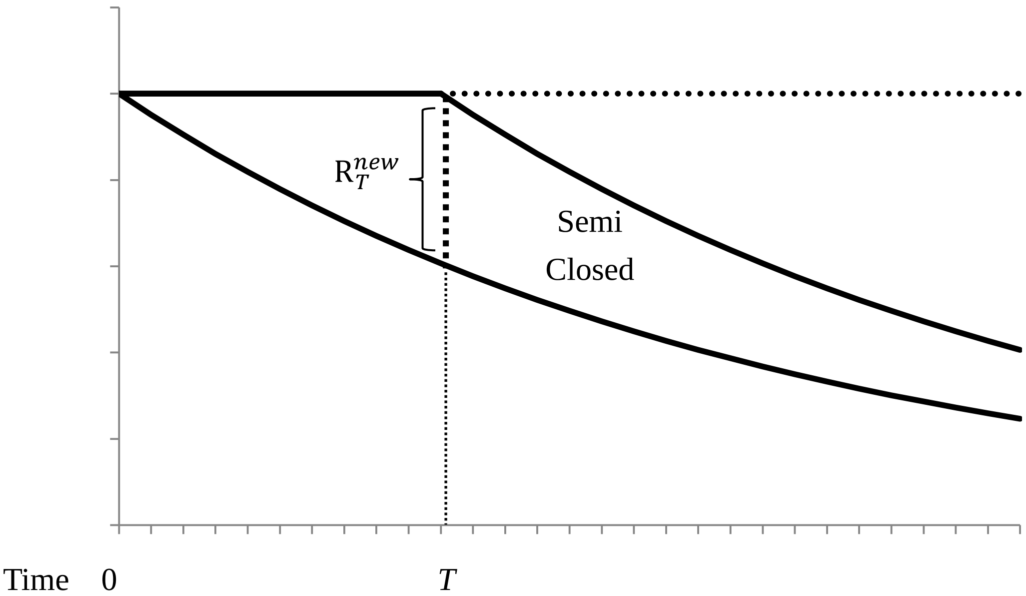


Figure 2.3

R_T^{new} becomes

$$R_T^{new} = \int_0^T R_{T-t}\, dt = \int_0^T e^{(\delta-\theta)(T-t)} R_0 dt = \frac{1}{\theta-\delta} (R_0 - R_T). \qquad (2\text{-}35)$$

Then $PVFR_T$ becomes

$$PVFR_T = \int_0^\infty R_T^{new} e^{(\delta-\theta)t} e^{-\delta t}\, dt = \frac{1}{\theta} R_T^{new}. \qquad (2\text{-}36)$$

$PVFP_T$ becomes

$$PVFP_T = \int_0^\infty \rho R_T^{new} e^{(\delta-\theta)t} e^{-\delta t}\, dt = \frac{\rho}{\theta} R_T^{new}. \qquad (2\text{-}37)$$

Consequently, ESR_T becomes

$$ESR_T = \frac{PVFP_T}{R_T^{new}} = \frac{\rho}{\theta} , \qquad (2\text{-}38)$$

and PVFP/PVFR Ratio at time *T* becomes

$$\frac{PVFP_T}{PVFR_T} = \rho . \qquad (2\text{-}39)$$

To identify Economic IRR and the risk premium for required capital, we must find a discount rate that equates present value of future profits with R_T^{new}. Let ε be a discount rate such that $\varepsilon > \delta - \theta$. The present value of future profits becomes

$$\int_0^{\infty} \rho \mathrm{R}_T^{new}\ e^{(\delta-\theta)t} e^{-\varepsilon t} dt = \frac{\rho}{\theta+\varepsilon-\delta}\ \mathrm{R}_T^{new}. \tag{2-40}$$

Find ε such that

$$\frac{\rho}{\theta+\varepsilon-\delta}\ \mathrm{R}_T^{new} = \mathrm{R}_T^{new}. \tag{2-41}$$

Finally, we identify Economic IRR as $\rho - \theta + \delta$ and the risk premium for required capital as $\rho - \theta$, respectively.

2.5 Open Model

In this section, we develop a model for the open segment. We refer to the model as "Open Model", illustrated in Figure 2.4. We assume that new business will be continuously acquired in a same quality and in a same quantity after time *T*.

Notation	Explanation
$\mathrm{PVFR}_T^{new(T)}$	Present value of future required capitals for new business, which will be acquired from time *T*, evaluated at *T*.
$\mathrm{PVFP}_T^{new(T)}$	Present value of future profits for new business, which will be acquired from time *T*, evaluated at *T*.

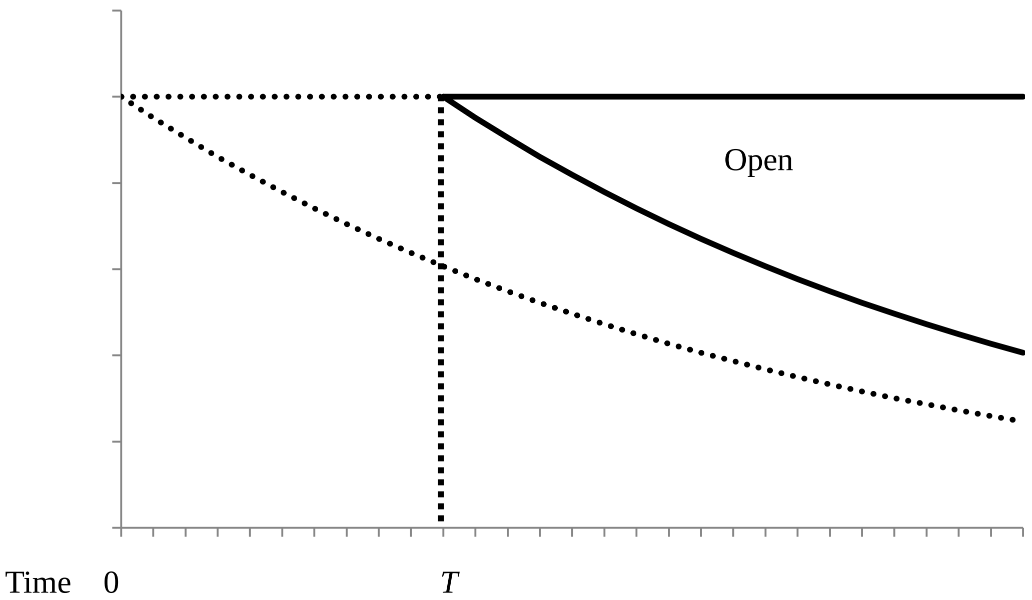


Figure 2.4

Let us consider a moment just after time *T*. The required capital for new business acquired at time *T*+Δt becomes $\mathrm{R}_0\Delta t$, which is independent of *T*. $\mathrm{R}_0\Delta t$ changes to $e^{(\delta-\theta)\Delta u}\mathrm{R}_0\Delta t$ at time *T*+Δt+Δu. The present value of $e^{(\delta-\theta)\Delta u}\mathrm{R}_0\Delta t$ at time *T* becomes

$$e^{-\delta(\Delta u+\Delta t)}e^{(\delta-\theta)\Delta u}\mathrm{R}_0\Delta t \quad = \quad e^{-\Delta\delta t}e^{-\theta\Delta u}\mathrm{R}_0\Delta t\,. \tag{2-42}$$

Thus, $\mathrm{PVFR}_T^{new(T)}$ becomes

$$\mathrm{PVFR}_T^{new(T)} \;=\; \iint_0^{\infty} e^{-\delta t}e^{-\theta u}\mathrm{R}_0\,dudt \quad = \frac{1}{\theta\delta}\mathrm{R}_0. \tag{2-43}$$

$\mathrm{PVFP}_T^{new(T)}$ becomes

$$\mathrm{PVFP}_T^{new(T)} \;=\; \iint_0^{\infty} \rho e^{-\delta t}e^{-\theta u}\mathrm{R}_0\,dudt \quad = \frac{\rho}{\theta\delta}\mathrm{R}_0. \tag{2-44}$$

Note that we cannot identify ESR_T in Open Model because the required capital at time *T* becomes 0. But PVFP/PVFR ratio at time *T* can be identified as

$$\frac{\mathrm{PVFP}_T^{new(T)}}{\mathrm{PVFR}_T^{new(T)}} \;=\; \rho. \tag{2-45}$$

To identify Economic IRR and risk premium for required capital, we must find the discount rate that equates present value of future profits with required capital at time *T*. Let *ε* be a discount rate such that $\varepsilon > \delta - \theta$. The present value of future profits becomes

$$\iint_0^{\infty} \rho e^{-\varepsilon t}\, e^{-\varepsilon u}e^{(\delta-\theta)u}\mathrm{R}_0 du \quad = \frac{\rho}{(\theta+\varepsilon-\delta)\varepsilon}\;\mathrm{R}_0\;. \tag{2-46}$$

The required capital at time *T* must be

$$\int_0^{\infty}\mathrm{R}_0\, e^{-\varepsilon t}\;\,dt \quad = \frac{1}{\varepsilon}\,\mathrm{R}_0. \tag{2-47}$$

Find ε such that

$$\frac{\rho}{(\theta+\varepsilon-\delta)\varepsilon}\;\mathrm{R}_0 \;=\; \frac{1}{\varepsilon}\,\mathrm{R}_0. \tag{2-48}$$

Finally, we identify Economic IRR as $\rho - \theta + \delta$ and the risk premium for required capital as $\rho - \theta$, respectively.

2.6 Assumption for Stationary Model

Before we consider the model in a stationary state, we must clarify the assumptions needed for the modeling. Let R_0^{inf} be the ultimate R_T^{new} after a long period of time.

The R_0^{inf} becomes

$$\mathrm{R}_0^{inf} \;=\; \lim_{\mathrm{T}\to\infty}\mathrm{R}_T^{new} \;= \frac{1}{\theta-\delta}\;\mathrm{R}_0. \tag{2-49}$$

Let R^{new} be the required capital for new business. R_0^{inf} in a moment Δt becomes

$$
\begin{aligned}
R_{0+\Delta t}^{inf} &= R_0^{inf} - \theta R_0^{inf}\,\Delta t + \delta R_0^{inf}\,\Delta t + R^{new}\Delta t \\
&= \frac{1}{\theta-\delta}\,R_0 - (\theta-\delta)\,R_0\,\frac{1}{\theta-\delta}\,\Delta t + R^{new}\Delta t \qquad (2\text{-}50) \\
&= R_0^{inf} - R_0\,\Delta t + R^{new}\,\Delta t\ .
\end{aligned}
$$

In a stationary state, $R_{0+\Delta t}^{inf}$ must become R_0^{inf}. Thus, R^{new} must be R_0.

2.7 Stationary Model

We refer to the model in a stationary state as "Stationary Model". We can develop Stationary Model as a combination of above three models.

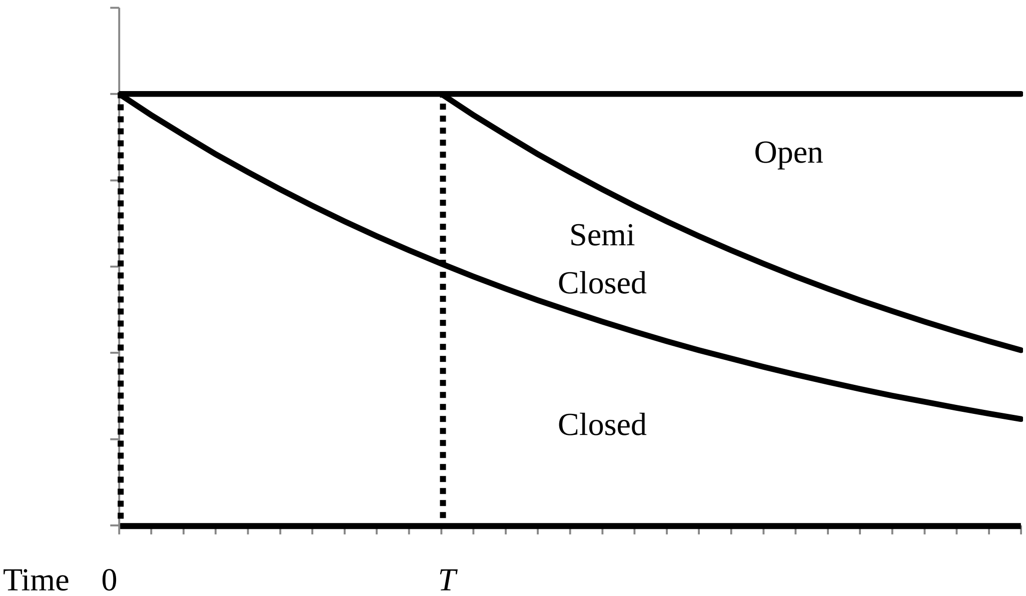


Figure 2.5

R_T^{inf} becomes

$$R_T^{inf} = \frac{1}{\theta-\delta}R_T + \frac{1}{\theta-\delta}(R_0 - R_T) + 0 = \frac{1}{\theta-\delta}\,R_0\ . \qquad (2\text{-}51)$$

Then $PVFR_T$ becomes

$$PVFR_T = \frac{1}{\theta}\,\frac{1}{\theta-\delta}R_T + \frac{1}{\theta}\,\frac{1}{\theta-\delta}(R_0 - R_T) + \frac{1}{\theta\delta}R_0 = \frac{1}{\delta}\,\frac{1}{\theta-\delta}R_0\ . \qquad (2\text{-}52)$$

$PVFP_T$ becomes

$$PVFP_T = \frac{\rho}{\theta}\,\frac{1}{\theta-\delta}R_T + \frac{\rho}{\theta}\,\frac{1}{\theta-\delta}(R_0 - R_T) + \frac{\rho}{\theta\delta}R_0 = \frac{\rho}{\delta}\,\frac{1}{\theta-\delta}R_0\ . \qquad (2\text{-}53)$$

Consequently, ESR_T becomes

$$ESR_T = \frac{PVFP_T}{R_T^{inf}} = \frac{\rho}{\delta}\ . \qquad (2\text{-}54)$$

and PVFP/PVFR Ratio at time T becomes

$$\frac{\text{PVFP}_T}{\text{PVFR}_T} = \rho \ . \tag{2-55}$$

To identify Economic IRR and the risk premium for required capital, we must find a discount rate that equates present value of future profits with required capital at time T. Let ε be a discount rate such that $\varepsilon > \delta - \theta$. Present value of future profits and required capital at time T are derived from the above outcomes.

Table 2.2

	PVFP at time T by ε	Required capital at time T
Closed	$\frac{\rho}{\theta + \varepsilon - \delta} \frac{1}{\theta - \delta} \text{R}_T$	$\frac{1}{\theta - \delta} \text{R}_T$
Semi Closed	$\frac{\rho}{\theta+\varepsilon-\delta} \ \frac{1}{\theta-\delta} \ (\text{R}_0 - \text{R}_T)$	$\frac{1}{\theta-\delta} \ (\text{R}_0 - \text{R}_T)$
Open	$\frac{\rho}{(\theta+\varepsilon-\delta)\varepsilon} \ \text{R}_0$	$\frac{1}{\varepsilon} \text{R}_0$
Total	$\frac{\rho}{\theta+\varepsilon-\delta} (\frac{1}{\theta-\delta} + \frac{1}{\varepsilon})\text{R}_0$	$(\frac{1}{\theta-\delta} + \frac{1}{\varepsilon}) \ \text{R}_0$

The present value of future profits becomes

$$\frac{\rho}{\theta+\varepsilon-\delta} (\frac{1}{\theta-\delta} + \frac{1}{\varepsilon})\text{R}_0. \tag{2-56}$$

The required capital at time T becomes

$$(\frac{1}{\theta-\delta} + \frac{1}{\varepsilon}) \ \text{R}_0. \tag{2-57}$$

Find ε such that

$$\frac{\rho}{\theta+\varepsilon-\delta} (\frac{1}{\theta-\delta} + \frac{1}{\varepsilon})\text{R}_0 = (\frac{1}{\theta-\delta} + \frac{1}{\varepsilon}) \ \text{R}_0. \tag{2-58}$$

Finally, we identify Economic IRR as $\rho - \theta + \delta$ and the risk premium for required capital as $\rho - \theta$ respectively.

We can derive the same outcomes in a direct way. See Appendix B.

The summary of the outcomes is shown in the following table.

Table 2.3

Model	Closed	Semi Closed	Open	Stationary
Required capital at time T	$\frac{1}{\theta-\delta}\mathrm{R_T}$	$\frac{1}{\theta-\delta}(\mathrm{R_0}-\mathrm{R_T})$	0	$\frac{1}{\theta-\delta}\mathrm{R_0}$
PVFR_T	$\frac{1}{\theta}\frac{1}{\theta-\delta}\mathrm{R_T}$	$\frac{1}{\theta}\frac{1}{\theta-\delta}(\mathrm{R_0}-\mathrm{R_T})$	$\frac{1}{\theta\delta}\mathrm{R_0}$	$\frac{1}{\delta}\frac{1}{\theta-\delta}\mathrm{R_0}$
PVFP_T	$\frac{\rho}{\theta}\frac{1}{\theta-\delta}\mathrm{R_T}$	$\frac{\rho}{\theta}\frac{1}{\theta-\delta}(\mathrm{R_0}-\mathrm{R_T})$	$\frac{\rho}{\theta\delta}\mathrm{R_0}$	$\frac{\rho}{\delta}\frac{1}{\theta-\delta}\mathrm{R_0}$
$\frac{\mathrm{PVFP}_T}{\mathrm{PVFR}_T}$	ρ	ρ	ρ	ρ
ESR_T	$\frac{\rho}{\theta}$	$\frac{\rho}{\theta}$	$-$	$\frac{\rho}{\delta}$
Economic IRR	$\rho-\theta+\delta$	$\rho-\theta+\delta$	$\rho-\theta+\delta$	$\rho-\theta+\delta$
Risk Premium	$\rho-\theta$	$\rho-\theta$	$\rho-\theta$	$\rho-\theta$

Note that Open Model and Stationary Model include profits from future new business, thus ESR_T of Stationary Model does not precisely stand for economic solvency ratio.

The case studies are shown in Appendix C and Appendix D.

3. Application to Life Insurance and Annuity

Both IRR and stress testing are generally implemented in pricing practice. When we calculate IRR under the stress scenario where severity is consistent with the required capital in Solvency II, it can be regarded as Economic IRR.

3.1 Single Payment Whole Life

In this section, we consider a single payment whole life. To simplify the argument, we assume that the force of death is constant for any age and the expected interest rate is equal to the risk-free rate. According to the general practice in Japan, the expected mortality rate will be developed by adding a safety margin to the crude mortality, which excludes the effect of selection. Thus, we can usually get mortality gain larger than the safety margin. We assume that the safety margin, based on Solvency II, will cover the 99.5% confidence level of severity.

Notation	Explanation
x	Entry age
l_x^E	Expected number of the insured at age *x*
l_x^A	Actual number of the insured at age *x*
A_x	Statutory reserve for single payment whole life at age *x*
SM_x	Present value of the safety margin embedded in A_x
R_x	Required capital for l_x^A
μ	Force of death expected in pricing
α	Crude/Expected mortality ratio
β	Actual/Expected mortality ratio

At time 0, we assume that l_x^A is equal to l_x^E. It is clear that $(1-\alpha)\mu$ stands for the safety margin and $(1-\beta)\mu$ stands for the mortality gain respectively as shown in Figure 3.1.

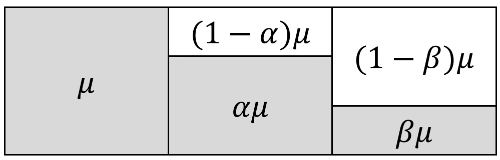


Figure 3.1

The expected number of the insured at age *x* becomes

$$l_{x+t}^{E} = e^{-\mu t} l_x^E \quad , \tag{3-1}$$

and the probability of surviving to age *x*+*t* becomes

$$_t p_x^E = \frac{l_{x+t}^E}{l_x^E} = e^{-\mu t}. \tag{3-2}$$

The actual number of the insured at age *x* becomes

$$l_{x+t}^{A} = e^{-\beta\mu t} l_x^A \quad , \tag{3-3}$$

and the probability of surviving to age *x*+*t* become

$$_t p_x^A = \frac{l_{x+t}^A}{l_x^A} = e^{-\beta\mu t}. \tag{3-4}$$

Statutory Reserve at *x* becomes

$$\mathrm{A}_x = \int_0^\infty \mu\ _t p_x^E\ e^{-\delta t}\ dt = \int_0^\infty \mu\, e^{-\mu t} e^{-\delta t}\ dt = \frac{\mu}{\mu+\delta}. \tag{3-5}$$

Thus, the sum at risk becomes

$$1 - \mathrm{A}_x = \frac{\delta}{\mu+\delta}. \tag{3-6}$$

Because the released safety margin from l_{x+t}^A in a moment Δt becomes

$$\text{(Sum at Risk)} \times (1-\alpha)\ \mu \times l_{x+t}^A \Delta t \quad , \tag{3-7}$$

SM_x becomes

$$\begin{aligned} \mathrm{SM}_x &= \frac{\delta}{\mu+\delta} \int_0^\infty (1-\alpha)\mu\ l_{x+t}^A\ e^{-\delta t}\ dt \\ &= \frac{\delta}{\mu+\delta} \int_0^\infty (1-\alpha)\mu\, e^{-\beta\mu t}\ e^{-\delta t}\ l_x^A\ dt \\ &= \frac{\delta}{\mu+\delta}\ (1-\alpha)\ \mu\ \frac{1}{\delta+\beta\mu}\ l_x^A, \end{aligned} \tag{3-8}$$

SM_{x+t} declines exponentially.

$$\mathrm{SM}_{x+t} = e^{-\delta t}\ \frac{\delta}{\mu+\delta}\ (1-\alpha)\ \mu\ \frac{1}{\delta+\beta\mu}\ l_{x+t}^A = e^{-\beta\mu t}\ \mathrm{SM}_x \ . \tag{3-9}$$

R_x must be SM_x by definition. Then we get

$$\mathrm{R}_{x+t} = e^{-\beta\mu t}\ \mathrm{R}_x \ . \tag{3-10}$$

Thus, the risk release rate (θ) can be identified as $\delta + \beta\mu$.

$$\theta = \delta + \beta\mu \tag{3-11}$$

Consequently, PVFR_x becomes

$$\mathrm{PVFR}_x = \int_0^\infty e^{-\delta t} \mathrm{R}_{x+t}\ dt = \int_0^\infty e^{-(\delta+\beta\mu)t}\ \mathrm{R}_x\ dt = \frac{1}{\delta+\beta\mu}\ \mathrm{R}_x \ . \tag{3-12}$$

Because the mortality gain from l_{x+t}^{A} in a moment Δt becomes

$$\text{(Sum at Risk)} \times (1-\beta)\,\mu \times l_{x+t}^{A}\Delta t \ , \qquad (3\text{-}13)$$

PVFP_x becomes

$$\begin{aligned}\text{PVFP}_x &= \frac{\delta}{\mu+\delta} \int_0^{\infty}(1-\beta)\mu\ l_{x+t}^{A}e^{-\delta t}dt \\ &= \frac{\delta}{\mu+\delta}\ (1-\beta)\mu\ \frac{1}{\delta+\beta\mu}\ l_x^{A} \\ &= \left(\frac{1-\beta}{1-\alpha}\right)\ \text{R}_x\ .\end{aligned} \qquad (3\text{-}14)$$

Consequently, ESR_x becomes

$$\text{ESR}_x = \frac{\text{PVFP}_x}{\text{R}_x} = \frac{1-\beta}{1-\alpha}. \qquad (3\text{-}15)$$

We identify RORC (ρ) as PVFP/PVFR Ratio at age *x*, which becomes

$$\rho = \frac{\text{PVFP}_x}{\text{PVFR}_x} = \left(\frac{1-\beta}{1-\alpha}\right)(\delta+\beta\mu)\ . \qquad (3\text{-}16)$$

Finally, we identify Economic IRR as $\rho-\theta+\delta$ and the risk premium for required capital as $\rho-\theta$, respectively.

$$\rho-\theta+\delta = \left(\frac{\alpha-\beta}{1-\alpha}\right)(\delta+\beta\mu) + \delta = \left(\frac{1-\beta}{1-\alpha}\right)\delta + \left(\frac{\alpha-\beta}{1-\alpha}\right)\beta\mu \qquad (3\text{-}17)$$

$$\rho-\theta = \left(\frac{\alpha-\beta}{1-\alpha}\right)(\delta+\beta\mu) \qquad (3\text{-}18)$$

Note that ρ and θ include δ because δ is assumed to be the expected interest rate.

When α becomes equal to β, we get the following outcome. The outcome will be acceptable not only theoretically but also intuitively.

$$\theta = \delta+\beta\mu \qquad (3\text{-}19)$$

$$\rho = \delta+\beta\mu \qquad (3\text{-}20)$$

$$\text{ESR}_x = 1 \qquad (3\text{-}21)$$

$$\rho-\theta = 0 \qquad (3\text{-}22)$$

3.2 Level Premium Whole Life

In this section, we model the level premium whole life.

Notation	Explanation
π_x	Net level premium of whole life for entry age *x*

The present value of earned net premium becomes

$$\int_0^{\infty} \pi_x \, {}_t p_x^E \; e^{-\delta t} \; dt = \int_0^{\infty} \pi_x \, e^{-\mu t} e^{-\delta t} \; dt = \frac{1}{\mu+\delta} \pi_x . \tag{3-23}$$

Thus, π_x must be μ because $\frac{1}{\mu+\delta}\pi_x$ is equal to $\mathrm{A}_x = \frac{\mu}{\mu+\delta}$.

Statutory reserve must become 0 when the net level premium becomes equal to the risk premium. Consequently, the sum at risk becomes 1.

Therefore, R_x becomes

$$\mathrm{R}_x \;= 1 \; \int_0^{\infty} (1-\alpha)\mu \; l_{x+t}^A \; e^{-\delta t} \; dt = \; (1-\alpha)\, \mu \, \frac{1}{\delta+\beta\mu} \; l_x^A \;, \tag{3-24}$$

and R_{x+t} becomes

$$\mathrm{R}_{x+t} \; = \; (1-\alpha)\, \mu \, \frac{1}{\delta+\beta\mu} \; l_{x+t}^A \; = e^{-\beta\mu t}\, \mathrm{R}_x \; . \tag{3-25}$$

Thus, the risk release rate (θ) can be identified as $\delta + \beta\mu$.

$$\theta \; = \; \delta + \beta\mu \tag{3-26}$$

PVFR_x becomes

$$\mathrm{PVFR}_x \; = \int_0^{\infty} e^{-\delta t} \mathrm{R}_{x+t} \; dt = \; \int_0^{\infty} e^{-(\delta+\beta\mu)t}\, \mathrm{R}_x \; dt \; = \; \frac{1}{\delta+\beta\mu} \; \mathrm{R}_x \; . \tag{3-27}$$

PVFP_x becomes

$$\mathrm{PVFP}_x \; = \; 1 \; \int_0^{\infty} (1-\beta)\mu \; l_{x+t}^A \; e^{-\delta t} \, dt \; = \; \left(\frac{1-\beta}{1-\alpha}\right) \; \mathrm{R}_x \; . \tag{3-28}$$

Consequently, ESR_x becomes

$$\mathrm{ESR}_x = \; \frac{\mathrm{PVFP}_x}{\mathrm{R}_x} \; = \; \frac{1-\beta}{1-\alpha} \; . \tag{3-29}$$

We identify RORC (ρ) as PVFP/PVFR Ratio at age *x*, which becomes

$$\rho = \; \frac{\mathrm{PVFP}_x}{\mathrm{PVFR}_x} \; = \left(\frac{1-\beta}{1-\alpha}\right) \; (\delta + \beta\mu). \tag{3-30}$$

Finally, we identify Economic IRR as $\rho - \theta + \delta$ and the risk premium for required capital as $\rho - \theta$, respectively.

$$\rho - \theta + \delta = \left(\frac{\alpha-\beta}{1-\alpha}\right)(\delta + \beta\mu) + \delta = \left(\frac{1-\beta}{1-\alpha}\right)\delta + \left(\frac{\alpha-\beta}{1-\alpha}\right)\beta\mu \qquad (3\text{-}31)$$

$$\rho - \theta = \left(\frac{\alpha-\beta}{1-\alpha}\right)(\delta + \beta\mu) \qquad (3\text{-}32)$$

3.3 Term Insurance

Next, we develop the model for term insurance using the level premium whole life model as shown in Figure 3.2.

Notation	Explanation
$\mathrm{R}_x^{(n)}$	Required capital of *n*-year term insurance for l_x^A

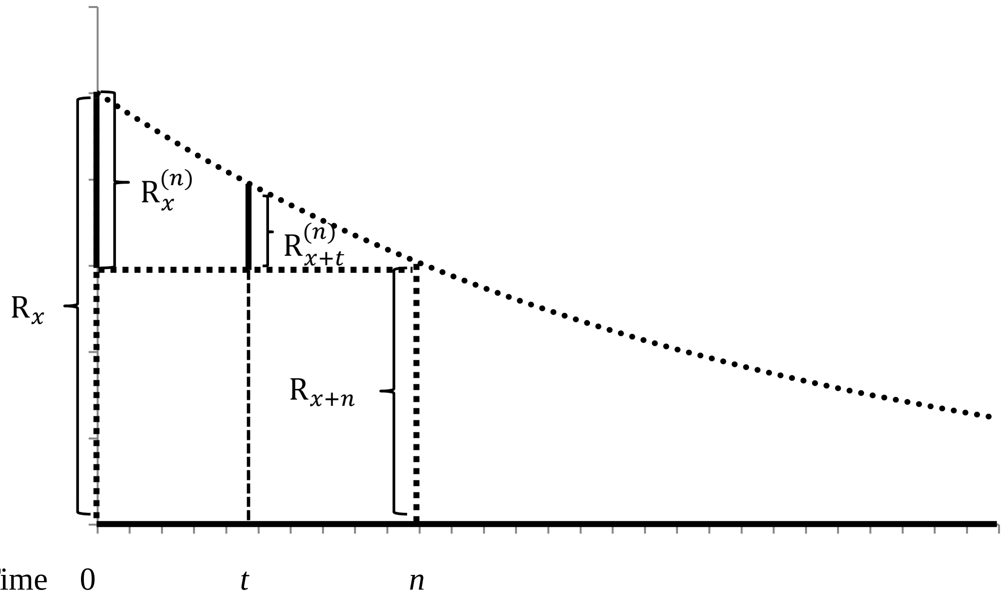


Figure 3.2

$\mathrm{R}_{x+t}^{(n)}$ can be expressed by the required capital for level premium whole Life.

For any *t* such that $0 \le t \le n$, $\mathrm{R}_{x+t}^{(n)}$ becomes

$$\begin{aligned}\mathrm{R}_{x+t}^{(n)} &= \mathrm{R}_{x+t} - e^{-\delta(n-t)}\,\mathrm{R}_{x+n} \\ &= e^{-\beta\mu t}\,\mathrm{R}_x - e^{-\delta(n-t)}\,e^{-\beta\mu n}\,\mathrm{R}_x \\ &= (e^{-\beta\mu t} - e^{-\delta(n-t)}\,e^{-\beta\mu n})\,\mathrm{R}_x\ . \end{aligned} \qquad (3\text{-}33)$$

Thus, $\mathrm{R}_x^{(n)}$ becomes

$$\mathrm{R}_x^{(n)} = (1 - e^{-(\delta+\beta\mu)n})\,\mathrm{R}_x\ . \qquad (3\text{-}34)$$

PVFR_x becomes

$$\mathrm{PVFR}_x = \int_0^n e^{-\delta t}\mathrm{R}_{x+t}^{(n)}\ dt = \int_0^n \left(e^{-(\delta+\beta\mu)t} - e^{-(\delta+\beta\mu)n}\right)\mathrm{R}_x\ dt$$

$$= \frac{\left\{1 - e^{-(\delta+\beta\mu)n} - (\delta+\beta\mu)n\, e^{-(\delta+\beta\mu)n}\right\}}{\delta+\beta\mu}\ \mathrm{R}_x\ . \tag{3-35}$$

PVFP_x becomes

$$\mathrm{PVFP}_x = \left(\frac{1-\beta}{1-\alpha}\right)\ \mathrm{R}_x - e^{-\delta n}\left(\frac{1-\beta}{1-\alpha}\right)\ \mathrm{R}_{x+n}$$

$$= \left(\frac{1-\beta}{1-\alpha}\right)\left(1 - e^{-(\delta+\beta\mu)n}\right)\ \mathrm{R}_x$$

$$= \left(\frac{1-\beta}{1-\alpha}\right)\mathrm{R}_x^{(n)}\ . \tag{3-36}$$

Consequently, ESR_x becomes

$$\mathrm{ESR}_x = \frac{\mathrm{PVFP}_x}{\mathrm{R}_x^{(n)}} = \frac{1-\beta}{1-\alpha}\ . \tag{3-37}$$

We identify RORC (ρ) as PVFP/PVFR Ratio at age x, which becomes

$$\rho = \left(\frac{1-\beta}{1-\alpha}\right)\frac{(\delta+\beta\mu)\left(1 - e^{-(\delta+\beta\mu)n}\right)}{\left\{1 - e^{-(\delta+\beta\mu)n} - (\delta+\beta\mu)n\, \mathrm{e}^{-(\delta+\beta\mu)n}\right\}}\ , \tag{3-38}$$

and identify the risk release rate (θ) as

$$\theta = \frac{\rho}{\mathrm{ESR}_x} = \frac{(\delta+\beta\mu)\left(1 - e^{-(\delta+\beta\mu)n}\right)}{\left\{1 - e^{-(\delta+\beta\mu)n} - (\delta+\beta\mu)n\, \mathrm{e}^{-(\delta+\beta\mu)n}\right\}}\ . \tag{3-39}$$

Finally, we identify Economic IRR as $\rho - \theta + \delta$ and the risk premium for required capital as $\rho - \theta$, respectively.

$$\rho - \theta + \delta = \left(\frac{\alpha-\beta}{1-\alpha}\right)\frac{(\delta+\beta\mu)\left(1 - e^{-(\delta+\beta\mu)n}\right)}{\left\{1 - e^{-(\delta+\beta\mu)n} - (\delta+\beta\mu)n\, \mathrm{e}^{-(\delta+\beta\mu)n}\right\}} + \delta \tag{3-40}$$

$$\rho - \theta = \left(\frac{\alpha-\beta}{1-\alpha}\right)\frac{(\delta+\beta\mu)\left(1 - e^{-(\delta+\beta\mu)n}\right)}{\left\{1 - e^{-(\delta+\beta\mu)n} - (\delta+\beta\mu)n\, \mathrm{e}^{-(\delta+\beta\mu)n}\right\}} \tag{3-41}$$

When n becomes large enough, the θ, ρ and $\rho - \theta$ converge to the equations used to calculate level premium whole life.

$$\theta \rightarrow \delta + \beta\mu \tag{3-42}$$

$$\rho \rightarrow \left(\frac{1-\beta}{1-\alpha}\right)(\delta + \beta\mu) \tag{3-43}$$

$$\rho - \theta \rightarrow \left(\frac{\alpha-\beta}{1-\alpha}\right)(\delta + \beta\mu) \tag{3-44}$$

3.4 Approximation for Term Insurance

In this section, we consider an approximation of the equations used to calculate term insurance. It is very useful for our practice. Denote $(\delta + \beta\mu)n$ as z, then Equation (3-39) becomes

$$\theta = \left(\frac{1}{n}\right) \frac{z(1-e^{-z})}{1-e^{-z}-ze^{-z}} \quad . \qquad (3\text{-}45)$$

When z is small enough, using the Maclaurin series expansion, θ converges to $\frac{2}{n}$.

$$\left(\frac{1}{n}\right) \frac{z(1-e^{-z})}{1-e^{-z}-ze^{-z}} = \left(\frac{1}{n}\right) \frac{z(e^{z}-1)}{e^{z}-1-z}$$

$$= \left(\frac{1}{n}\right) \frac{z\left(z+\frac{1}{2!}z^2+\frac{1}{3!}z^3+\cdots\right)}{\left\{\left(1+z+\frac{1}{2!}z^2+\frac{1}{3!}z^3+\cdots\right)-(1+z)\right\}}$$

$$= \left(\frac{1}{n}\right) \frac{\left(1+\frac{1}{2!}z+\frac{1}{3!}z^2+\cdots\right)}{\left(\frac{1}{2!}+\frac{1}{3!}z+\cdots\right)} \rightarrow \frac{2}{n} \quad . \qquad (3\text{-}46)$$

Consequently, ρ converges to

$$\left(\frac{1-\beta}{1-\alpha}\right)\left(\frac{2}{n}\right), \qquad (3\text{-}47)$$

and $\rho - \theta$ converges to

$$\left(\frac{\alpha-\beta}{1-\alpha}\right)\left(\frac{2}{n}\right). \qquad (3\text{-}48)$$

The approximation works well. It can be confirmed from the following example, which is typical in Japan.

Let $n = 10, \delta = 0.0025, \mu = 0.001$ and $\beta = 0.70$.

$$z = (\delta + \beta\mu)n = (0.0025+0.0007)\times 10 = 0.032, \quad e^{-0.032}=0.968507,$$

$$\theta = \left(\frac{1}{n}\right) \frac{z(1-e^{-z})}{1-e^{-z}-ze^{-z}}$$

$$= \left(\frac{1}{10}\right) \frac{(0.032)(1-0.968507)}{1-0.968507-0.032\times 0.968507} = 0.2012 \approx \frac{2}{10} \quad . \qquad (3\text{-}49)$$

Because the pricing methodology of medical insurance products in Japan is basically the same as term insurance, the approximation can be used widely in practice. Equation (3-49) will be acceptable not only theoretically but also intuitively as is shown in Appendix F.

3.5 Single Payment Immediate Annuity

In this section, we consider a single payment immediate annuity, which pays benefits for a lifetime. In accordance with the pricing practice in Japan, the mortality rate for an annuitant is developed by subtracting a safety margin from the crude mortality.

Notation	Explanation
a_x	Statutory reserve for single payment immediate annuity at *x*

We assume $1 \leq \alpha \leq \beta$. It is clear that $(\alpha - 1)\mu$ stands for the safety margin and $(\beta - 1)\mu$ stands for the survival gain respectively as shown in Figure 3.3.

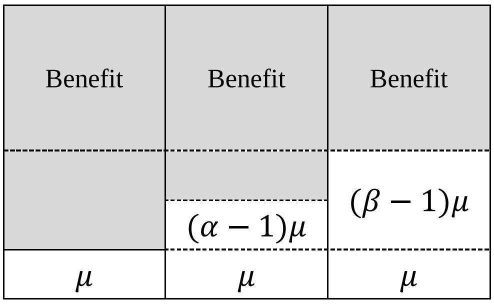


Figure 3.3

Statutory reserve becomes

$$a_x = \int_0^\infty 1 \ {}_t p_x^E \ e^{-\delta t} \ dt = \int_0^\infty 1 \, e^{-\mu t} e^{-\delta t} \ dt = \frac{1}{\mu + \delta}. \tag{3-50}$$

Thus, the sum at risk becomes

$$a_x = \frac{1}{\mu + \delta} \ . \tag{3-51}$$

Released safety margin from l_{x+t}^A in a moment Δt becomes

$$\text{(Sum at Risk)} \times (\alpha - 1) \, \mu \times l_{x+t}^A \Delta t. \tag{3-52}$$

Then, R_x becomes

$$\begin{aligned} \mathrm{R}_x &= \frac{1}{\mu + \delta} \int_0^\infty (\alpha - 1)\mu \ l_{x+t}^A \ e^{-\delta t} \, dt \\ &= \frac{1}{\mu + \delta} \ (\alpha - 1) \, \mu \ \frac{1}{\delta + \beta \mu} \ l_x^A \ , \end{aligned} \tag{3-53}$$

and R_{x+t} becomes

$$\mathrm{R}_{x+t} = \frac{1}{\mu + \delta} \ (\alpha - 1) \, \mu \ \frac{1}{\delta + \beta \mu} \ l_{x+t}^A = e^{-\beta \mu t} \ \mathrm{R}_x \ . \tag{3-54}$$

Thus, the risk release rate (θ) can be identified as $\delta + \beta\mu$.

$$\theta \ = \ \delta + \beta\mu \tag{3-55}$$

Consequently, PVFR_x becomes

$$\mathrm{PVFR}_x \ = \int_0^{\infty} e^{-\delta t}\mathrm{R}_{x+t} \ dt = \int_0^{\infty} e^{-(\delta+\beta\mu)t}\,\mathrm{R}_x \ dt = \frac{1}{\delta+\beta\mu} \ \mathrm{R}_x. \tag{3-56}$$

Because the mortality gain from l_{x+t}^A in a moment Δt becomes

$$(\text{Sum at Risk}) \ \times (\beta - 1)\,\mu \times l_{x+t}^A \Delta t \ , \tag{3-57}$$

PVFP_x becomes

$$\mathrm{PVFP}_x \ = \ \frac{1}{\mu+\delta} \ \int_0^{\infty} (\beta - 1)\mu \ l_{x+t}^A \ e^{-\delta t}\, dt$$

$$= \ \frac{1}{\mu+\delta} \ (\beta - 1)\mu \ \frac{1}{\delta+\beta\mu} \ l_x^A \ = \ \left(\frac{\beta-1}{\alpha-1}\right) \ \mathrm{R}_x \ . \tag{3-58}$$

Consequently, ESR_x becomes

$$\mathrm{ESR}_x = \ \frac{\mathrm{PVFP}_x}{\mathrm{R}_x} \ = \ \frac{\rho}{\theta} \ = \ \frac{\beta-1}{\alpha-1} \ . \tag{3-59}$$

We identify RORC (ρ) as PVFP/PVFR Ratio at age *x*, which becomes

$$\rho = \frac{\mathrm{PVFP}_x}{\mathrm{PVFR}_x} = \left(\frac{\beta-1}{\alpha-1}\right) (\delta + \beta\mu) \ . \tag{3-60}$$

Finally, we identify Economic IRR as $\rho - \theta + \delta$ and the risk premium for required capital as $\rho - \theta$ respectively.

$$\rho - \theta + \delta = \left(\frac{\beta-\alpha}{\alpha-1}\right) (\delta + \beta\mu) \ + \delta \ = \ \left(\frac{\beta-1}{\alpha-1}\right) \delta + \left(\frac{\beta-\alpha}{\alpha-1}\right)\beta\mu \tag{3-61}$$

$$\rho - \theta \qquad = \left(\frac{\beta-\alpha}{\alpha-1}\right) (\delta + \beta\mu) \tag{3-62}$$

Appendix E gives an application example of these equations.

4. Pros and Cons of Economic IRR

In this section, we explore the pros and cons of Economic IRR from the viewpoint of effectiveness in decision making. We discuss the following five management issues.

4.1 Pricing Strategy

As is shown in the section 3, Economic IRR has a close relationship with pricing. This is the most notable feature of Economic IRR that cannot be substituted by other risk-adjusted return metrics. In addition, Economic IRR enables us to measure not only the risk-return efficiency but also the effect on ESR, whereas RORC only measures the risk-return efficiency.

4.2 Sales Strategy

Investment-type products with guarantees are popular in Japan, reflecting the national character that loves savings. But they are less profitable for life insurers under the current low interest rate environment. The balance between profitability and ease-of-sale is an eternal challenge in considering the sales strategy. Because “profitability” can be regarded as “risk-return effectiveness” in many cases, the risk adjusted return metrics can be used to assess the balance. Economic IRR is unique in that meaning as it associates the balance with ESR.

4.3 ALM Strategy

As is described in ICP 16.5.6, many financial markets throughout the world do not have long duration fixed-income assets to back long duration liabilities. Therefore, duration mismatch risk would be unavoidable in many cases. Duration matching is effective in preparing for future interest rate decline risk, but it may increase the losses caused by a dynamic surrender if an interest rate hike occurs in the future. Considering the low plausibility of further decline of interest rate, we should rather prepare for the future interest rate hike risk. The risk-adjusted return metrics including Economic IRR help us to solve such issues.

4.4 Capital Strategy

Many insurers set the ESR target in their management plan. To achieve the target, a planned enhancement of surplus must be implemented. When future retained earnings and required capitals are included in the plan, feasibility of enhancement can be significantly improved. Economic IRR, using with ESR, can be used to formulate the plan and to evaluate the progress, whereas RORC only evaluates the cost of capital.

4.5 Dividend Strategy

The policyholder dividend is the most important issue for mutual life insurance companies. In considering the dividend strategy, the balance between the increase of dividend and the enhancement of surplus should be critical because they cause conflict with each other. Economic IRR enables us to evaluate the balance.

In summary, Economic IRR excels in effectiveness and efficiency and complements other metrics. It is very useful in decision making.

5. Example of Practical Use

In this section, we show an example of practical use in Fukoku Mutual Life. Economic IRR and other metrics are widely used in our management cycle, which is shown in Appendix G. For background context, we briefly introduce the company.

Fukoku Mutual Life, a mutual life insurance company with a history of over 90 years in Japan, is known for its unique "Customer-Centric" approach. According to its corporate philosophy, the company acts in the best interests of policyholders and values quality over quantity. The company believes that maintaining financial soundness regardless of the operating environment is the most important objective as a life insurer. The goal requires the company to ensure a stable earnings base and to establish a robust financial platform. Along with those background values and objectives, the company emphasizes both solvency metric and risk-adjusted return metrics in decision making.

5.1 Pricing Strategy

The pricing strategy of Fukoku Mutual Life is basically subject to its risk appetite that includes the 230% ESR target. The pricing policy is "conservative but competitive" and an exquisite balance between the two is the key to successful pricing. Economic IRR is useful to adjust the balance. Table 5.1 shows an example.

Table 5.1

	Pricing A	Pricing B	Pricing C	Pricing D
Economic IRR*	Over 8.3%	7.0%	0	Below 0
$PVFP_0$ vs. R_0	$PVFP_0 > R_0$	$PVFP_0 > R_0$	$PVFP_0 = R_0$	$PVFP_0 < R_0$
ESR_0	$ESR_0 > 230\%$	$ESR_0 = 200\%$	$ESR_0 = 100\%$	$ESR_0 < 100\%$

Note that the 230% ESR target is equivalent to "8.3% risk premium for required capital". The risk premium for required capital is hereinafter denoted as "Economic IRR*".

Pricing A, which Economic IRR* satisfies the target, is sufficiently conservative. But if the price becomes more expensive than customer expectations, then it implies that the pricing is less competitive. A very unique, hit product would make the pricing possible. "Product Differentiation" is the key in this case. Pricing B, which Economic IRR* becomes slightly less than the target, remains conservative. Taking the market competitiveness into consideration, the price will become preferable. Pricing C, which Economic IRR* becomes zero, needs further consideration. In this case, we always discuss the possibility of ancillary sales. If Economic IRR* would be improved to a satisfactory level by the synergy effect, the price would be barely acceptable. Pricing D, which Economic IRR* becomes below zero, is inappropriate because the profitability is not sufficient to cover its own required capital.

A pricing strategy is subject to the company's risk appetite, which may vary depending on the product. Economic IRR, when efficiently and effectively incorporated into the process, will sophisticate the pricing strategy.

5.2 Sales Strategy

As described in the section 4.2, the balance between profitability and ease-of-sale is a challenge when considering the sales strategy. One way to meet this challenge is to assess the efficient frontier of a planned new business portfolio. The diversification effect between Investment-type products and Protection-type products will help us to consider the balance. Figure 5.1 is an illustration of the efficient frontier and Table 5.2 is its key metrics. Note that examples are provided for illustrative purposes only and should not be considered definitive figures. The table indicates that, to satisfy the ESR target, the portion of Investment-type products must be within the range of 0%~50%.

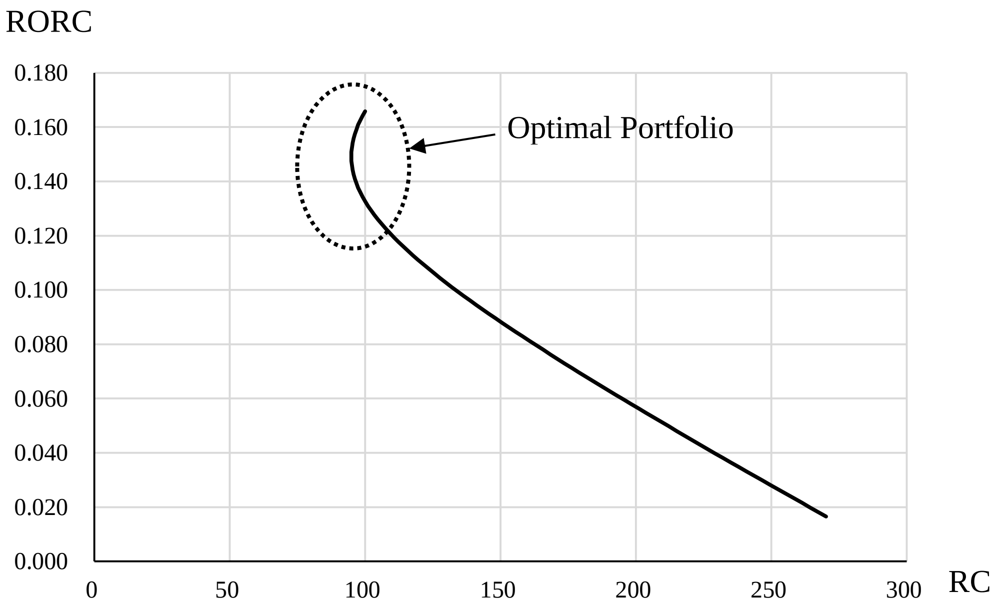


Figure 5.1

Table 5.2

Portion	0%	10%	20%	30%	40%	50%	60%
RORC	16.6%	14.9%	13.3%	11.6%	9.9%	8.3%	6.6%
RC	100	95	100	114	134	158	184
RORC×RC	16.6	14.2	13.3	13.2	13.3	13.1	12.2
Economic IRR*	8.3%	10.4%	11.1%	10.7%	9.7%	8.6%	7.7%
ESR_0	232%	276%	290%	280%	260%	239%	221%

In this table, "Portion" stands for the ratio of Investment-type products to overall new business, "RORC×RC" stands for the annualized average return on RC and "Economic IRR*" stands for the risk premium for required capital respectively.

To find the optimal portfolio, we must assess RORC×RC and ESR_0. Figure 5.2 is a graph showing RORC×RC and ESR_0. The solid line is RORC×RC and the broken line is ESR_0. A portion satisfying both the larger RORC×RC and the higher ESR_0 will be optimal. The graph shows that 10%~30% will be optimal for risk-return efficiency. Note that the result may vary depending on the constraints about capital. For example, when we intend to minimize the required capital, 10% will become optimal. And when we intend to maximize ESR and there are no constraints about capital, 20% will become optimal, which can be directly identified by Economic IRR*.

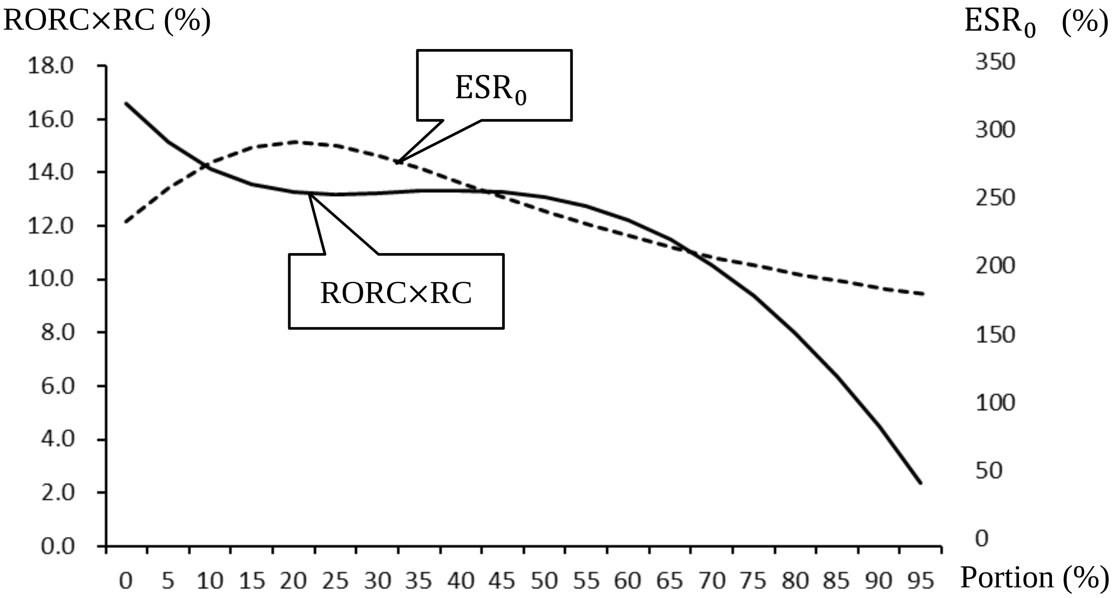


Figure 5.2

As is described in section 5.1, when ancillary sales are reasonably expected, the synergy effect can be also measured by Economic IRR*. The tactic of using Investment-type products as the door opener is effective to meet customers' needs while maintaining profitability. The tactic works well in Fukoku Mutual Life's marketing.

5.3 ALM Strategy

Because of the low plausibility of further decline of interest rate, we should prepare for the future interest rate hike risk. Reducing the existing duration gap would rather increase the losses caused by a dynamic surrender. Fukoku manages the risk very carefully and gives explicit attention within its ALM policy, ensuring that the risk is effectively managed, not only by holding adequate capital but also by maintaining stable profits. In fact, as the example in Appendix D shows, negative spread was sufficiently absorbed by the stable mortality gains. The risk-adjusted return metrics tell us that maintaining stable profits is still an effective way to fortify against the long duration of insurance liabilities.

5.4 Capital Strategy

Capital strategy is the key to maintaining financial soundness. As is mentioned above, our target for Fukoku is 230% ESR. Taking future soundness and risk-return efficiency into consideration, the enhancement plan to achieve the target is made. Consequently, it includes prospects of retained earnings and required capitals. ESR, Economic IRR and other metrics are used to evaluate the soundness and risk-return efficiency. These metrics are also used to assess both quality and quantity of capital. Based on the assessment, we have increased the retained earnings continuously and have issued subordinated bonds whenever appropriate. As a result, in the fiscal year 2016, the Tier 1 ratio has become 87% and ESR has become over 200%, which can cover the extraordinary risks, such as recurrence of financial crisis, great earthquakes and their consequences. We are steadily making progress towards the target.

5.5 Dividend Strategy

Our dividend policy at Fukoku is to maintain a stable dividend level taking the policyholders' expectation into consideration. Furthermore, we intend to increase the dividend, maintaining balance with the enhancement of surplus.

In deciding the fiscal years' dividend plan, we take the following five steps:

- Step 1. Assess the profit performance and analyze the source of profit.
- Step 2. Project future soundness and risk-return efficiency; assess the future progress of enhancement of surplus and future available earnings for dividend.
- Step 3. Develop a dividend plan, which is reviewed by the appointed actuary.
- Step 4. Discuss the plan at the Board Meeting.

Step 5. Submit the plan to the Annual Meeting of Representatives.

In assessing the future available earnings for dividend, Economic IRR* is very useful in determining whether or not the policy has enough capacity to increase the dividend, because the earnings exceeding 8.3% Economic IRR* can be regarded as available earnings for dividend.

6. Conclusion

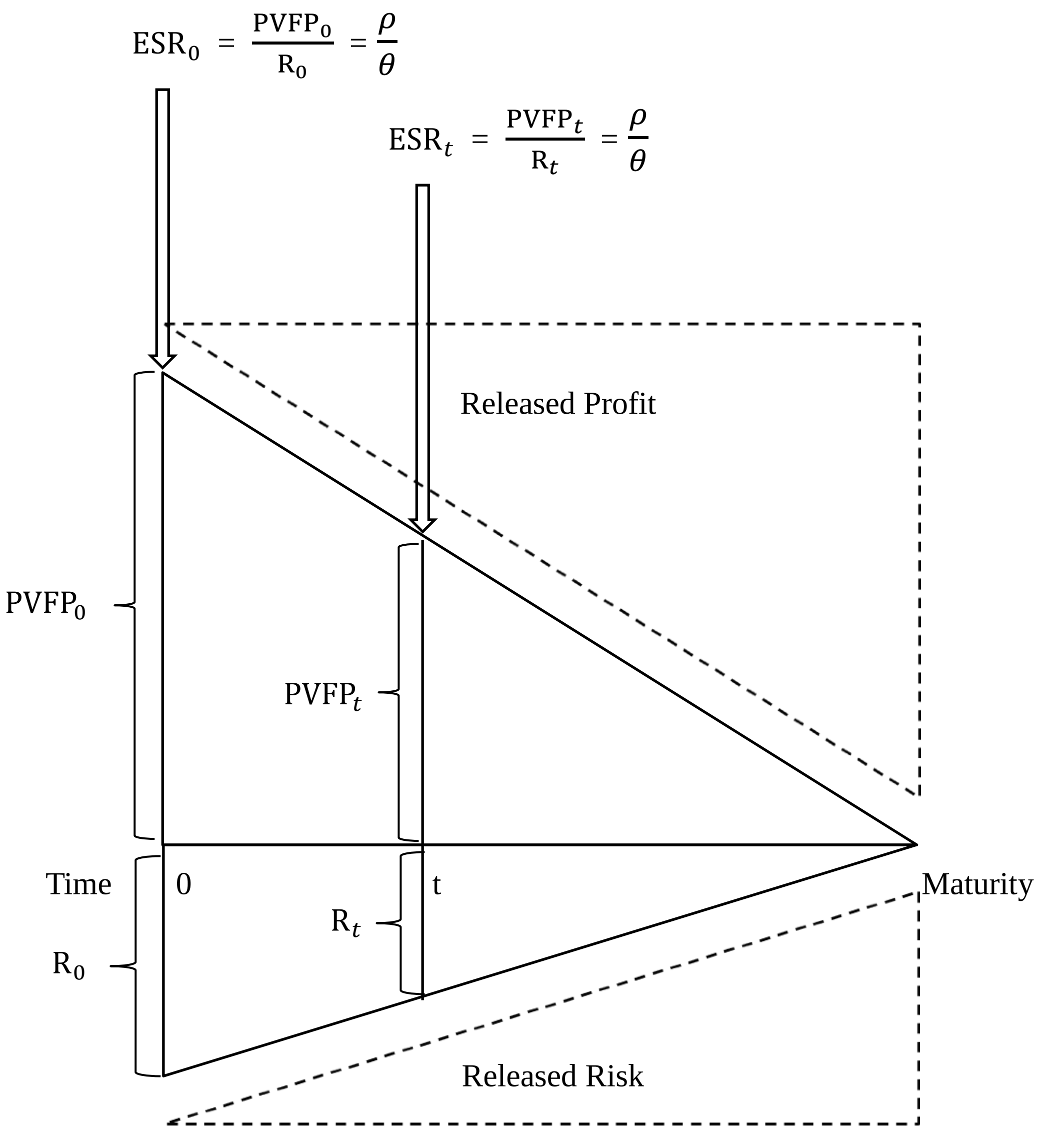
In Japan life insurers are preparing for the coming economic value-based solvency regulation. Reviewing global trends and discussions, we are developing the process.

We conclude that Economic IRR, when it is effectively incorporated into the decision making process, will enhance the integrated management of capital, risk and return, and consequently will strengthen the ability of risk-taking.

References

1. James Lam (2014) “Enterprise Risk Management: From Incentives to Controls”, Second Edition. WILEY.
2. Robin J. Cunningham, Thomas N. Herzog, Richard L. London (2012) “Models for Quantifying Risk” ACTEX Publications, Inc.
3. Venelin A. Yanakiev (2016) “Capital Steering for Shareholder Value.” SOA Annual Meeting, Session 77, P.9.
4. James Crispin, Sean McGuire (2011) “Risk-adjusted performance measures for Solvency II” Life Actuarial Conference 2011.
5. International Actuarial Association, “Actuarial Aspects of ERM for Insurance Companies”, January 2016.
6. International Actuarial Association, “Practice Note On Enterprise Risk Management For Capital And Solvency Purposes In The Insurance Industry”, August 2008
7. Tobias Bear, Amit Mehta and Hamid Samandari, “The use of economic capital in performance management for banks: A perspective”. McKinsey Working Papers on Risk, Number 24. January 2011.
8. Donna Megregian, Rob Stone, Wing Wong, Oliver Gillespie, Margaret O’Connor, Milliman, Inc. “Survey of Reflecting Risk in Pricing”, September 2010.
9. International Association of Insurance Supervisors(IAIS), “Insurance core principles” , Updated November 2015.

Appendix A. Illustration of ESR_t



Risk and profit are two sides of the same coin. Risk is released as time goes by, accompanied by the corresponding profit. Profit is retained in free surplus or is paid out as policyholders' dividend.

Profit, when retained in free surplus contributes to further risk-taking.

Appendix B. Stationary Model Equations Derived in a Direct Way

Required capital at time T becomes

$$\mathrm{R}_T^{inf} = \frac{1}{\theta-\delta}\ \mathrm{R}_0. \tag{B-1}$$

Then, PVFR_T becomes

$$\int_0^{\infty} \mathrm{R}_T^{inf} e^{-\delta t}\ dt = \frac{1}{\delta}\ \frac{1}{\theta-\delta}\, \mathrm{R}_0. \tag{B-2}$$

PVFP_T becomes

$$\int_0^{\infty} \rho \mathrm{R}_T^{inf} e^{-\delta t}\ dt = \frac{\rho}{\delta}\ \frac{1}{\theta-\delta}\, \mathrm{R}_0. \tag{B-3}$$

Consequently, ESR_T becomes

$$\frac{\mathrm{PVFP}_T}{\mathrm{R}_T^{inf}} = \frac{\rho}{\delta}, \tag{B-4}$$

and PVFP/PVFR Ratio at time T becomes

$$\frac{\mathrm{PVFP}_T}{\mathrm{PVFR}_T} = \rho. \tag{B-5}$$

To identify Economic IRR and the risk premium for required capital, we must find the discount rate which equates present value of future profits with required capital at time T. Let ε be an interest force such that $\varepsilon > \delta - \theta$. The present value of future profits by ε becomes

$$\int_0^{\infty} \rho\ \mathrm{R}_T^{inf}\ e^{-\varepsilon t}\ dt = \frac{\rho}{\varepsilon}\ \mathrm{R}_T^{inf} = \frac{\rho}{\varepsilon}\ \frac{1}{\theta-\delta}\ \mathrm{R}_0. \tag{B-6}$$

The required capital at time T becomes

$$\mathrm{R}_T^{inf} + \frac{1}{\varepsilon} \mathrm{R}_0 = \left(\frac{1}{\theta-\delta} + \frac{1}{\varepsilon}\right) \mathrm{R}_0\ . \tag{B-7}$$

Find ε such that

$$\frac{\rho}{\varepsilon}\ \frac{1}{\theta-\delta}\ \mathrm{R}_0 = \left(\frac{1}{\theta-\delta} + \frac{1}{\varepsilon}\right) \mathrm{R}_0. \tag{B-8}$$

Finally, we identify Economic IRR as $\rho - \theta + \delta$ and the risk premium for required capital as $\rho - \theta$ respectively.

Appendix C. Case Study 1. Improve the Profitability of New Business

If we improve the profitability of new business, how it will affect ESR of future in-force business? The outcome from Closed and Semi Closed Model can be used for this case study. Assume that required risk is unchanged. Let ρ be RORC of current in-force business and ρ_* be RORC of future new business respectively.

While PVFR_T is $\frac{1}{\theta}\frac{1}{\theta-\delta}\mathrm{R}_0$, unchanged, PVFP_T becomes

$$\frac{\rho}{\theta}\frac{1}{\theta-\delta}\mathrm{R}_T + \frac{\rho_*}{\theta}\frac{1}{\theta-\delta}(\mathrm{R}_0 - \mathrm{R}_T) = \frac{\rho-\rho_*}{\theta}\frac{1}{\theta-\delta}\mathrm{R}_T + \frac{\rho_*}{\theta}\frac{1}{\theta-\delta}\mathrm{R}_0, \tag{C-1}$$

and ESR_T becomes

$$\frac{\mathrm{PVFP}_T}{\mathrm{R}_T^{inf}} = \frac{\rho-\rho_*}{\theta}\frac{\mathrm{R}_T}{\mathrm{R}_0} + \frac{\rho_*}{\theta}. \tag{C-2}$$

Because R_T converges to 0 when T goes to infinity, PVFP_T converge to

$$\frac{\rho_*}{\theta}\frac{1}{\theta-\delta}\mathrm{R}_0. \tag{C-3}$$

Consequently, ESR_T converges to

$$\frac{\rho_*}{\theta}, \tag{C-4}$$

and PVFP/PVFR Ratio converges to

$$\frac{\mathrm{PVFP}_\infty}{\mathrm{PVFR}_\infty} = \rho_*. \tag{C-5}$$

The outcome indicates that ρ will be replaced by ρ_* in the end. It is useful when we intend to improve the profitability of products.

Appendix D. Case Study 2. Mitigate the Impact of Duration Mismatch Risk

If required capital includes risk which has little contribution to return, what should we do to maintain risk-return efficiency? In fact, we often face this situation, typically as duration mismatch risk. The duration mismatch risk itself has little contribution to the return. The risk indicates losses caused by future negative spread. As is described in ICP 16.5.6, many financial markets throughout the world do not have long fixed-income assets to back long duration liabilities. Thus, the duration mismatch risk would be unavoidable in many cases. In Japan, life insurers have focused to sell products which produce stable mortality and/or morbidity gains and have covered the negative spread successfully.

Assume that required risk R_* is added to R_0, where R_* itself has little contribution to the return, thus has little impact on PVFP_T. And assume that correlation between R_0 and R_* is 0. Denote RORC for in-force business as ρ and RORC for new business as ρ_*.

PVFR_T becomes

$$\frac{1}{\delta}\,\frac{1}{\theta-\delta}\sqrt{(\mathrm{R}_0^2+\mathrm{R}_*^2)}\,. \qquad \text{(D-1)}$$

PVFP_T converges to

$$\frac{\rho_*}{\delta}\,\frac{1}{\theta-\delta}\,\mathrm{R}_0 \qquad \text{(D-2)}$$

then PVFP/PVFR Ratio converges to

$$\frac{\mathrm{PVFP}_\infty}{\mathrm{PVFR}_\infty} = \rho_*\,\frac{\mathrm{R}_0}{\sqrt{(\mathrm{R}_0^2+\mathrm{R}_*^2)}}\,. \qquad \text{(D-3)}$$

To maintain risk-return efficiency, the profitability must be improved to meet

$$\rho_* \geq \rho\,\frac{\sqrt{(\mathrm{R}_0^2+\mathrm{R}_*^2)}}{\mathrm{R}_0}\,. \qquad \text{(D-4)}$$

As is shown in this appendix, focus on selling products with larger profitability which produce stable profits is, we believe, still an effective way to mitigate the impact of duration mismatch risk.

Appendix E. Statutory Reserve, PVFP and Best Estimate of Technical Provision

In this appendix, we show statutory reserve less the best estimate of technical provision becomes PVFP_x.

(1) Single Payment Whole Life

Statutory reserve for *x* is

$$\frac{\mu}{\mu+\delta}\ l_x^A\ . \tag{E-1}$$

Because the best estimate of technical provision for *x* becomes

$$\frac{\beta\mu}{\beta\mu+\delta}\ l_x^A\ , \tag{E-2}$$

statutory reserve less the best estimate of technical provision becomes

$$\frac{\mu}{\mu+\delta}\ l_x^A - \frac{\beta\mu}{\beta\mu+\delta}\ l_x^A\ =\ \frac{\mu(\beta\mu+\delta)-\beta\mu(\mu+\delta)}{(\mu+\delta)(\ \beta\mu+\delta)}\ l_x^A\ =\ \mathrm{PVFP}_x\ . \tag{E-3}$$

(2) Level Premium Whole Life

Statutory Reserve for *x* is 0.

Because the best estimate of technical provision for *x* becomes

$$\frac{\beta\mu}{\beta\mu+\delta}\ l_x^A\ -\ \frac{1}{\beta\mu+\delta}\pi_x l_x^A\ =\left(\frac{\beta\mu}{\beta\mu+\delta}\ -\ \frac{\mu}{\beta\mu+\delta}\right) l_x^A\ \ =\ -\mathrm{PVFP}_x\ , \tag{E-4}$$

statutory reserve less the best estimate of technical provision becomes

$$0-(-\mathrm{PVFP}_x) =\ \mathrm{PVFP}_x\ . \tag{E-5}$$

(3) Single Payment Immediate Annuity

Statutory Reserve for *x* is

$$\frac{1}{\mu+\delta}\ l_x^A\ . \tag{E-6}$$

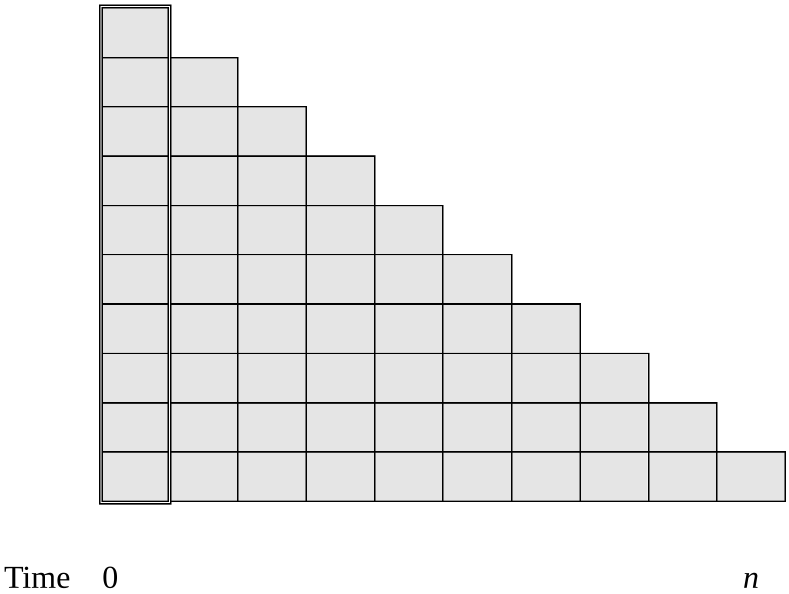
Because the best estimate of technical provision for *x* becomes

$$\frac{1}{\beta\mu+\delta}\ l_x^A\ , \tag{E-7}$$

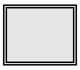
statutory reserve less the best estimate of technical provision becomes

$$\frac{1}{\mu+\delta}\ l_x^A - \frac{1}{\beta\mu+\delta}\ l_x^A\ =\ \frac{(\beta-1)\mu}{(\mu+\delta)(\ \beta\mu+\delta)}\ l_x^A\ =\ \mathrm{PVFP}_x\ . \tag{E-8}$$

Appendix F. Intuitive Understanding of Approximation for Term Insurance



Assume that θ is equal to ρ.

Square  stands for R_x, the required capital at time 0.

Total of  becomes $\frac{1}{2}n(n+1)$.

Single  is released from the required capital every year.

$\text{PVFR}_x \approx \frac{1}{2}n(n+1)$ and $\text{PVFP}_x \approx n$.

Thus, $\theta = \dfrac{\text{PVFP}_x}{\text{PVFR}_x} = \dfrac{n}{\frac{1}{2}n(n+1)} \approx \dfrac{2}{n}$.

Appendix G. Management Cycle in Fukoku Mutual Life

Action ✓ Pricing & Sales Planning - Revise Product Strategy - Revise Sales Strategy ✓ Capital Enhancement by: - Retaining Earnings - Issuing Subordinated Bonds ✓ Dividend - Increase Dividend - Balance with Capital Enhancement	Plan ✓ Products & Pricing - Conservative but Competitive Pricing - Consistent with ESR Target ✓ Sales Planning - Focus on Protection-type Selling - Use of Door Opener ✓ ALM - Focus on Risk-return Efficiency - Prepare for Interest Rate Hike Risk
Check ✓ Assessment of KPIs & KRIs ✓ Source of Profit Analysis ✓ Projection of: - Future Cash Flows and Soundness - Future Risk-return Efficiency ✓ Stress Testing ✓ Assessment of Capital Adequacy - Focus on Quantity & Quality ✓ Implementation of ORSA	Do ✓ Implementation of Business Plan ✓ Monitoring of KPIs - New Business, In-force Business, etc. ✓ Monitoring of KRIs - ESR, A/E Ratios, Var, etc. ✓ Monitoring of Risk Control Methods - Hedging of Foreign Exchange Risk ✓ Monitoring of Risk-return efficiency - RORC, Economic IRR, etc.

In this diagram, "KPIs" stands for the Key Performance Indicators and "KRIs" stands for the Key Risk Indicators, respectively.