A third-order factor for immunization: implications in solvency II

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About the speaker

- **J. Iñaki De La Peña**
- Full Professor at University of the Basque Country
- He currently works at the Financial Economics I Department. Joseba Iñaki does research in Private and Public Pensions, Social Security and ALM.

- **University of the Basque Country**
- More than 50.000 pupils
- More than 6.500 Professors
- 31 Faculties
- …

Aim

- to analyse the **effects of adding more terms** of Taylor expansion **to an immunization strategy**, in order **to free up** the part of the insurance firm’s capital requirement for solvency **due to interest risk**

Structure

- Section 1: Duration and other terms for immunization of liabilities.
- Section 2: Modelling framework.
- Section 3: Linear program and the key rules for testing the models.
- Section 4: Results.
- Section 5: Comments and conclusions.

The problem

- Immunization has been closely linked to duration - Macaulay (1938)-

"average period" -Hicks (1939)-

"weighted average time period" -Samuelson (1945) -

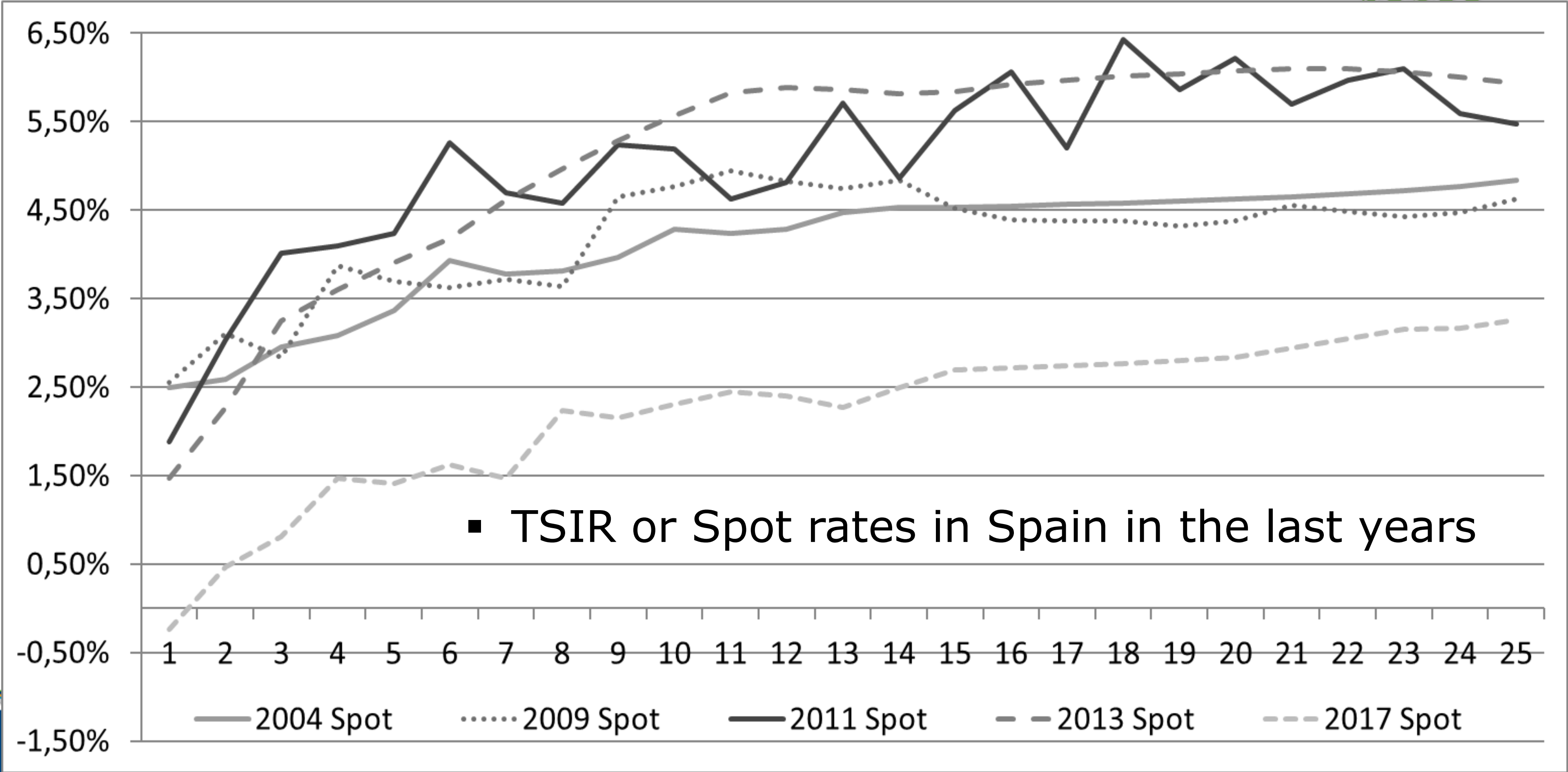
- if duration of assets and liabilities is equal, the portfolio is protected against a local parallel movement in the yield curve.

Shocks

Fisher and Weil (1971) From One to Multiple liabilities

¿parallel shifts?

The problem


6,50%
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1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
▪ TSIR or Spot rates in Spain in the last years
2004 Spot
2009 Spot
2011 Spot
2013 Spot
2017 Spot

The problem

The literature indicates that the model for both assets and liabilities that uses duration is sufficient to immunize against interest rate (movements=risk)

Yes?

Three models will be developed to capture part of the influence of interest rate changes on assets and liabilities

- Model 1. Only Duration
- Model 2. Plus Convexity
- Model 3. Plus the fourth term in Taylor series (Expansion 1)
- Model 4. Plus the fifth term in Taylor series (Expansion 2).

These terms must help effectively to monitor the risk of immunization (Hürlimann, 2016).

Methodology: The origin

- Let us consider a stream of non-negative random liability flows, $\{L_1, L_2, \dots, L_n\}$, expected to occur at a time $\{1,2,\dots,n\}$, and a portfolio of bond flows $\{A_1, A_2, \dots, A_n\}$, at the same dates.
- Let $S(i)_0$ be the surplus of this block of business or net worth.

$$S(i)_0 = A(i)_0 - L(i)_0 = \sum_{h=1}^{n} a_h - \sum_{h=1}^{n} l_h$$

- Let $S(i)_0^*$ be the present value of the balance (assets minus liabilities)when the basic TSIR changes.

$$S(i)_0^* = A(i)_0^* - L(i)_0^* = \sum_{h=1}^{n} a_h^* - \sum_{h=1}^{n} l_h^* = S(i)_0 + \sum_{h=1}^{n} (a_h - l_h) \cdot \left(\frac{P_h^*}{P_h} - 1 \right)$$

This is the new value due to an interest rate shock

Methodology: Taylor Series

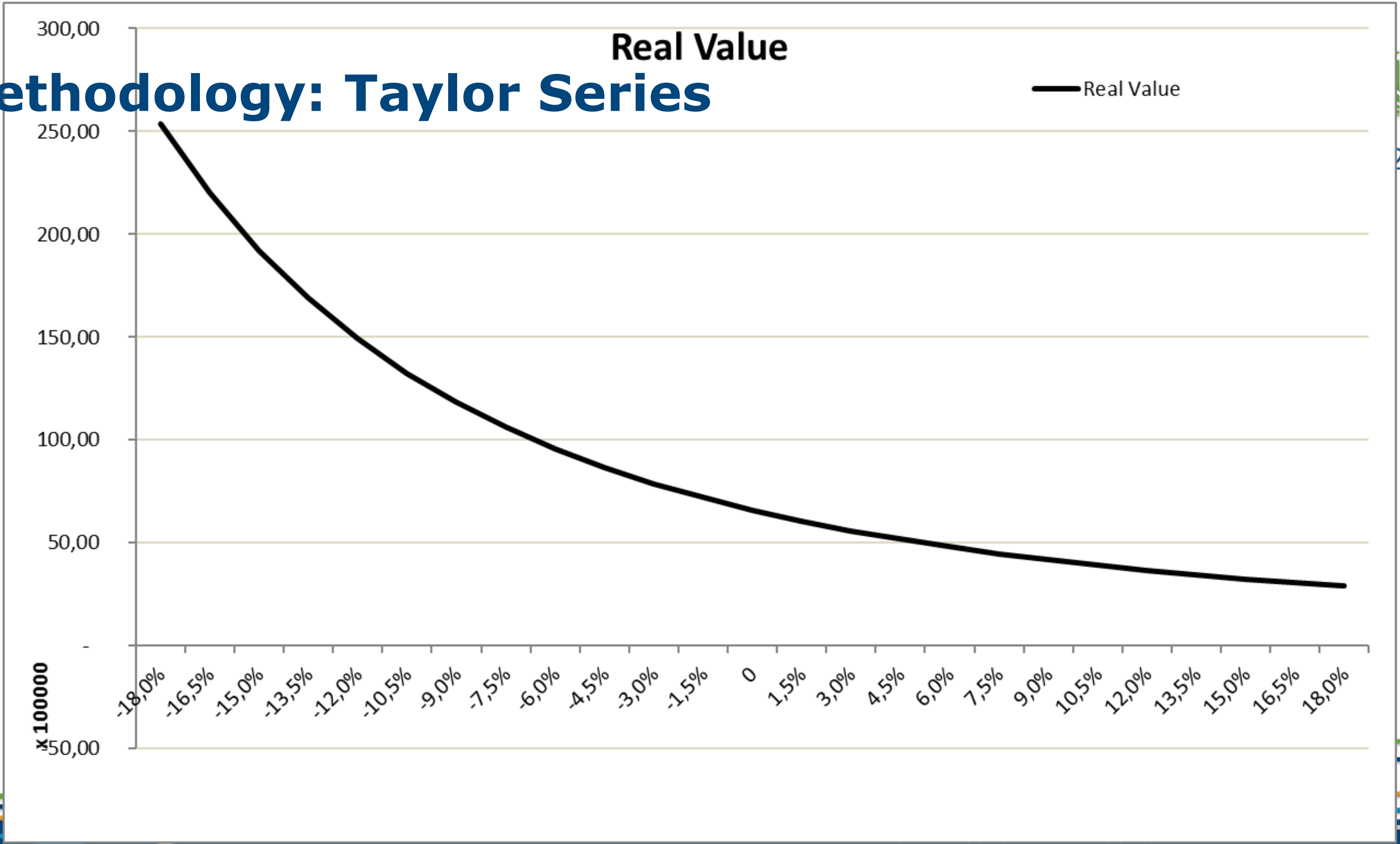
- Using the Taylor series, an expression can be approximated that enables the new value to be obtained on the basis of the previous value and the interest rate variation (ε).

$$f(i)_0^* = f(i)_0 + \frac{1}{1!} \cdot \frac{df(i)_0}{di} \cdot \varepsilon + \frac{1}{2!} \cdot \frac{d^2 f(i)_0}{di^2} \cdot \varepsilon^2 + \cdots + R(i)_n$$

- For a non-flat TSIR

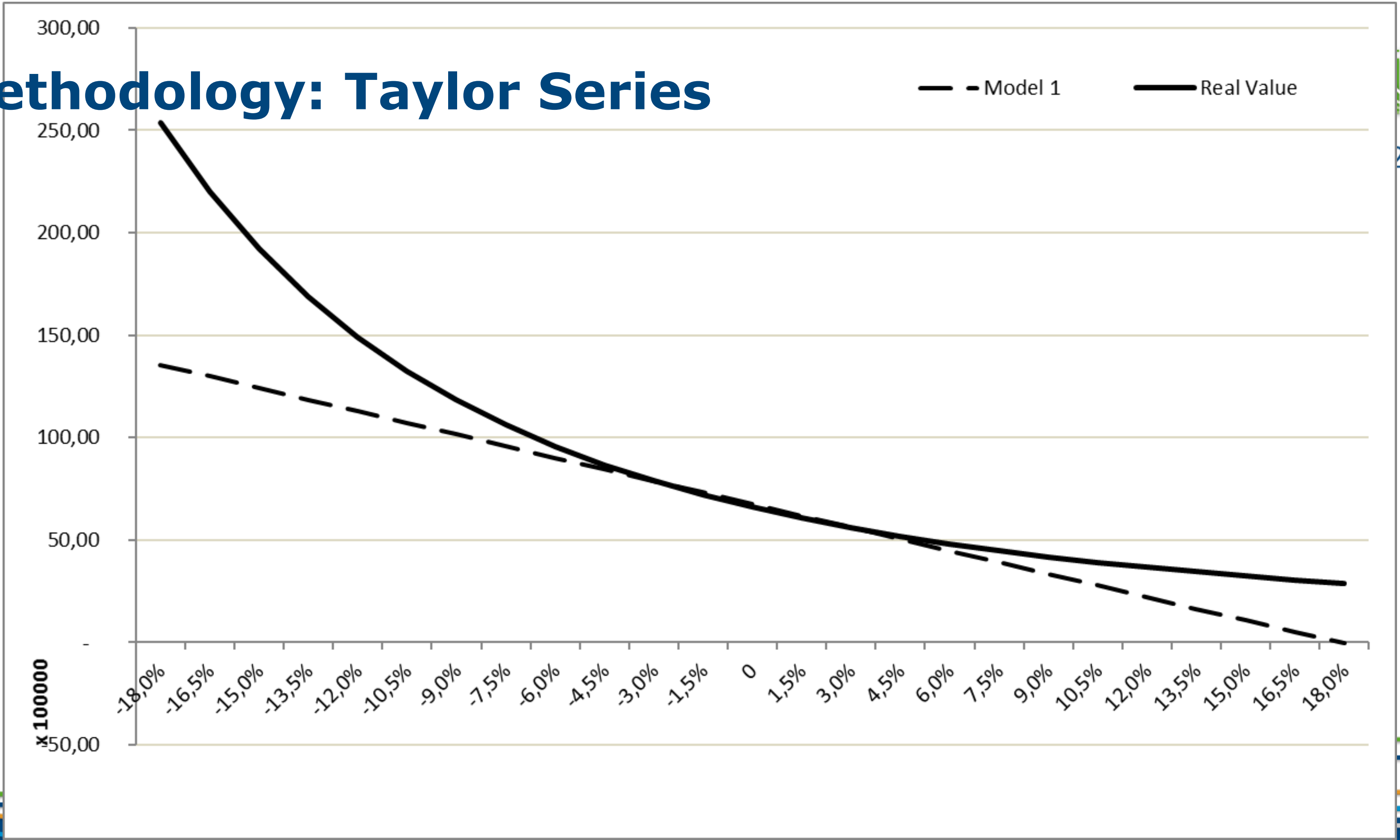
$$\{ {}_1i_0 \neq {}_2i_0 \neq \cdots \neq {}_hi_0 \}$$

Methodology: Taylor Series

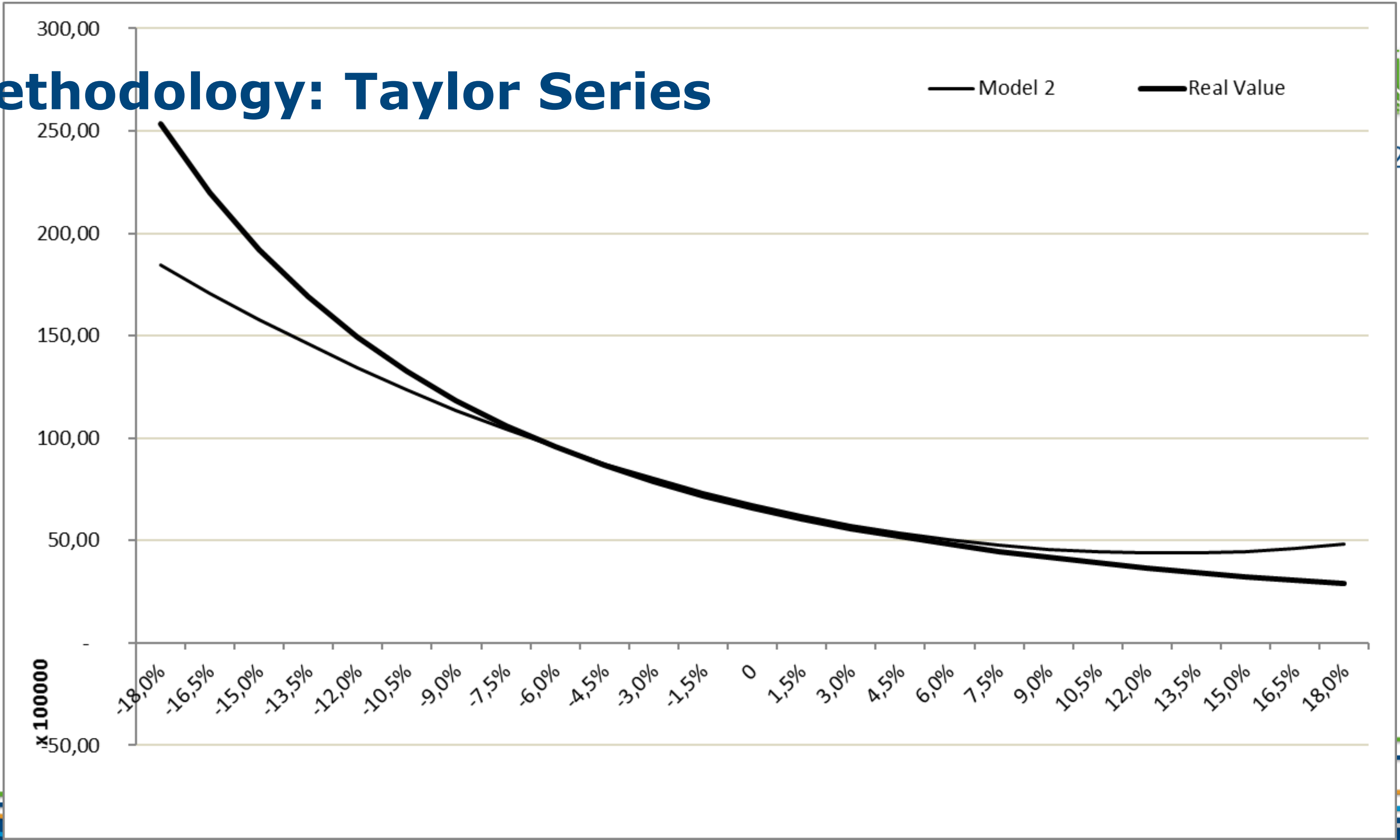


2018

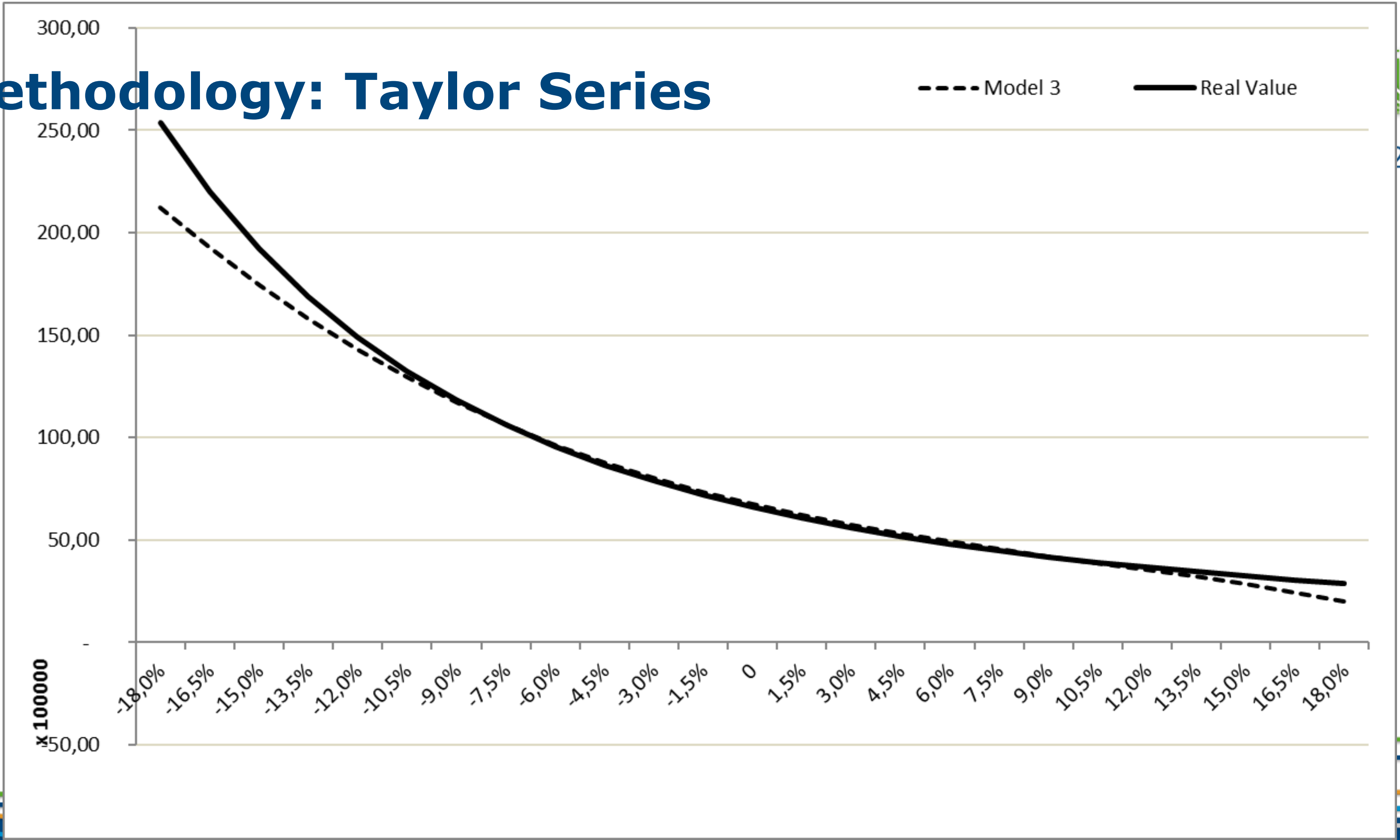
Methodology: Taylor Series



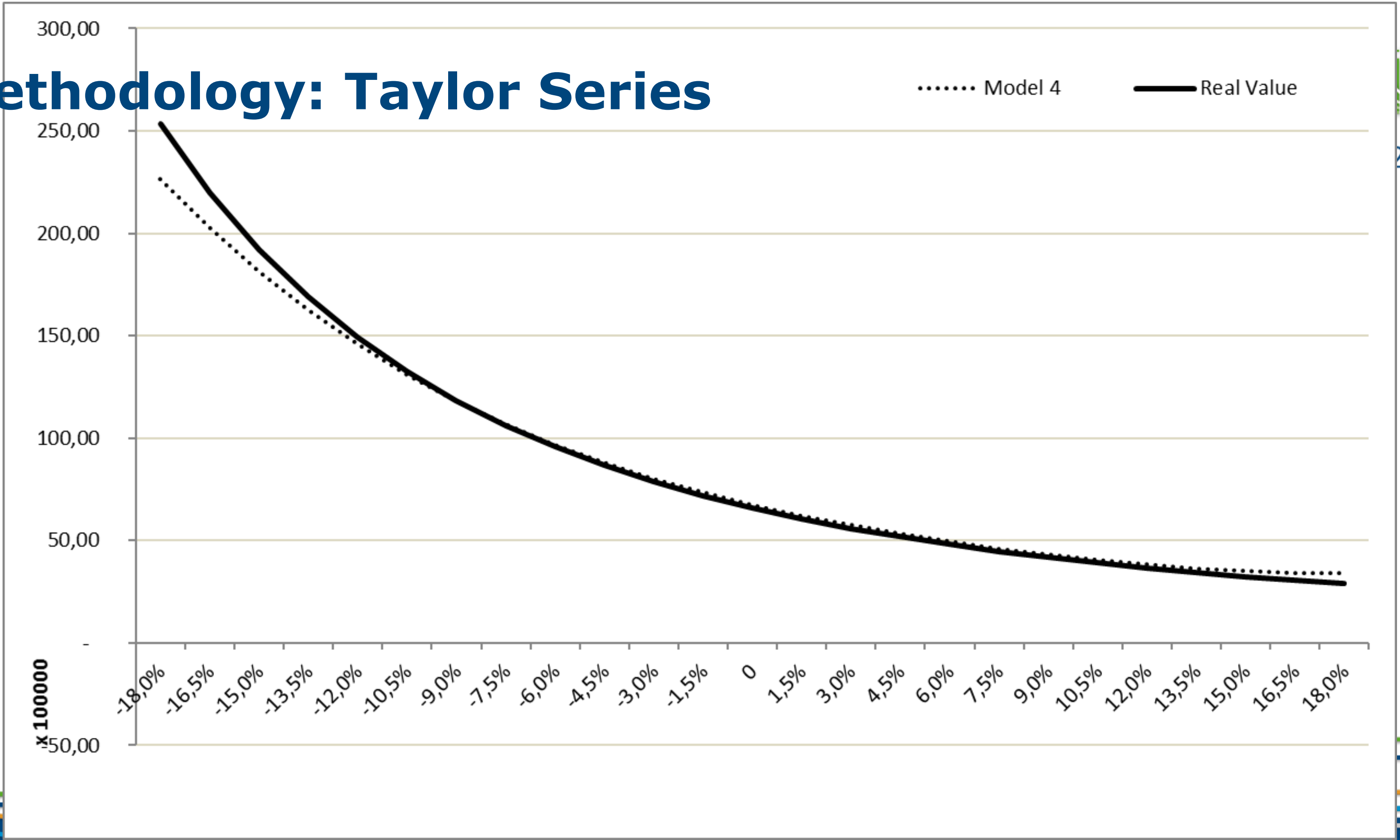
Methodology: Taylor Series



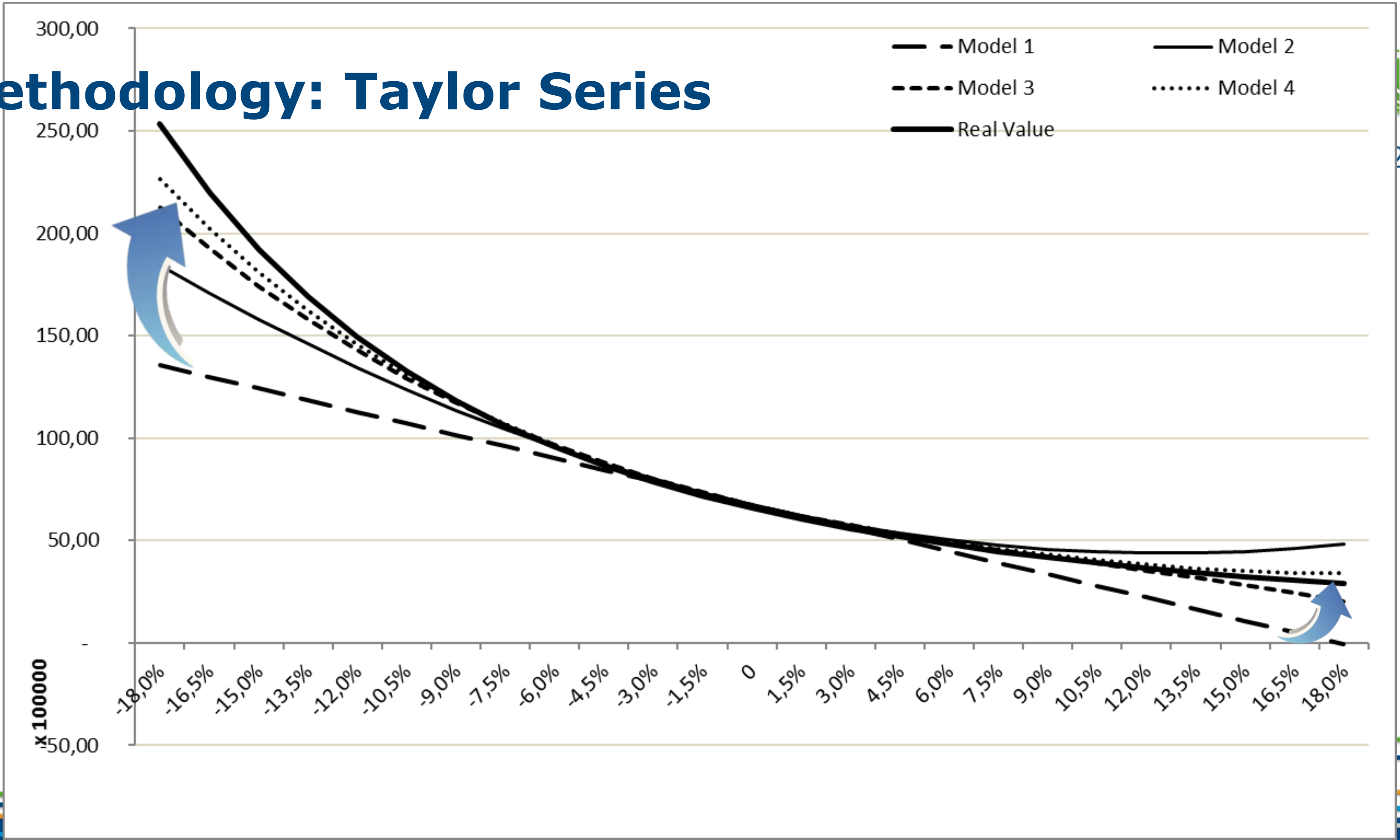
Methodology: Taylor Series



Methodology: Taylor Series



Methodology: Taylor Series



Methodology: Taylor Series

- If the fifth extension is included in the Taylor series and for a non-flat TSIR $\{\ {}_{1}i_{0}\ \neq\ {}_{2}i_{0}\ \neq \cdots \neq\ {}_{h}i_{0}\ \}$,

$$A(i)_0^* = A(i)_0 \cdot \left(1 - DMA(i)_0 \cdot \varepsilon + \frac{1}{2} \cdot CXMA(i)_0 \cdot \varepsilon^2 - \frac{1}{6} \cdot A1MA(i)_0 \cdot \varepsilon^3 + \frac{1}{24} \cdot A2MA(i)_0 \cdot \varepsilon^4\right)$$

$$L(i)_0^* = L(i)_0 \cdot \left(1 - DML(i)_0 \cdot \varepsilon + \frac{1}{2} \cdot CXML(i)_0 \cdot \varepsilon^2 - \frac{1}{6} \cdot A1ML(i)_0 \cdot \varepsilon^3 + \frac{1}{24} \cdot A2ML(i)_0 \cdot \varepsilon^4\right)$$

Methodology: Immunization procedure

- A review of known ALM techniques for pensions plans can be found in Kocherlakota, Rosenbloom and Shiu, (1988) and (1990); De Felice and Moriconi, (1991); Anderson, (1992); Bowers et al. *et al*. (1996); De la Peña, (1997); Ambachtsheer and Don Ezra, (1998); Gajek and Ostaszewski, (2002).

$$Min \sum_{\forall x_h \geq 0} x_h \cdot b_h$$

$$\forall b_h \geq 0 \quad and \quad \forall x_h \geq 0 \; for \; h \in \{1, \dots, s\}$$

$$S(i)_0^* = 0$$

Methodology: Immunization procedure

- Immunization models:

Model 1	Model 2	Model 3	Model 4
$DMA(i)_0 = DML(i)_0$	$DMA(i)_0 = DML(i)_0$	$DMA(i)_0 = DML(i)_0$	$DMA(i)_0 = DML(i)_0$
-	$CXMA(i)_0 = CXML(i)_0$	$CXMA(i)_0 = CXML(i)_0$	$CXMA(i)_0 = CXML(i)_0$
-	-	$A1MA(i)_0 = A1ML(i)_0$	$A1MA(i)_0 = A1ML(i)_0$
-	-	-	$A2MA(i)_0 = A2ML(i)_0$

- the annual balance (S_k) cannot be negative,

$$\forall k \in 0 < k < n$$

$$S_k = S_{k-1} \cdot \left(1 + {}_1f_{k-1}\right) + A_k - L_k \geq 0$$

Methodology: Immunization Testing

- Immunization Risk (GAP): *M-absolute*. Cash flow dispersion (Nawalkha and Chambers, 1996).

$$M_A = \frac{\sum^{n}_{\substack{k=1 \\ \forall h \in \{1,\ldots,s\}}} \left| x_h \cdot b_{k,h} - L_k \right| \cdot \left(1 + {}_{k-h}i_h\right)^{k-h}}{A(i)_0}$$

- That is to say, it would be that strategy in which the commitments —liabilities— would be met with the periodical revenue from the assets, with no price or investment risk.

Methodology: Immunization Testing

- And what happend if there is any surplus?

- flows prior to the investment horizon are capitalised assuming the existence of an interest rate of reinvestment at the forward interest. All future flows that occur following the investment horizon are evaluated at the corresponding spot interest rates.

$$S_t = S_t^{PAST} + S_t^{FUTURE}$$

Methodology: Immunization Testing

- final value of the assets minus the payments matured to date (t)

$$S_t^{PAST} = \sum_{\substack{k=1 \\ \forall h \in \{1,\ldots,s\}}}^{t} x_k \cdot b_{k,h} \cdot \prod_{j=1}^{t-1} (1 + {}_1f_j) - \sum_{\substack{k=1 \\ \forall h \in \{1,\ldots,s\}}}^{t} L_k \cdot \prod_{j=1}^{t-1} (1 + {}_1f_j)$$

- the present value of the future assets minus the liabilities to be paid at the end of the horizon in Year 10,

$$S_t^{FUTURE} = \sum_{\substack{k=t+1 \\ \forall h \in \{1,\ldots,s\}}}^{15} x_k \cdot b_{k,h} \cdot (1 + {}_{k-t}i_t)^{t-k} - \sum_{\substack{k=t+1 \\ \forall h \in \{1,\ldots,s\}}}^{10} L_k \cdot (1 + {}_{k-t}i_t)^{t-k}$$

Methodology: Immunization Testing

- the effectiveness of the kth strategy on the strategy of the duration - Model 1- can be obtained through the mean value of the Surplus

$$E(M)_{1,k} = \frac{Mean_1 - Mean_k}{Mean_1}$$

- And standard deviation

$$E(SD)_{1,k} = \frac{SD_1 - SD_k}{SD_1}$$

- Reduction in the Surplus equal to the Value at Risk –VaR- with a level of 99.5% (Ngai & Sherris, 2011),

$$E(VaR)_{1,k} = \frac{VaR_1 - VaR_k}{VaR_1}$$

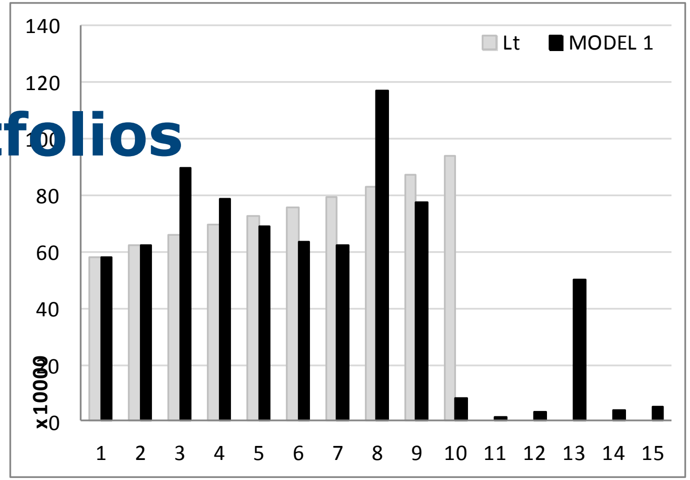
Results: Empirical aplication

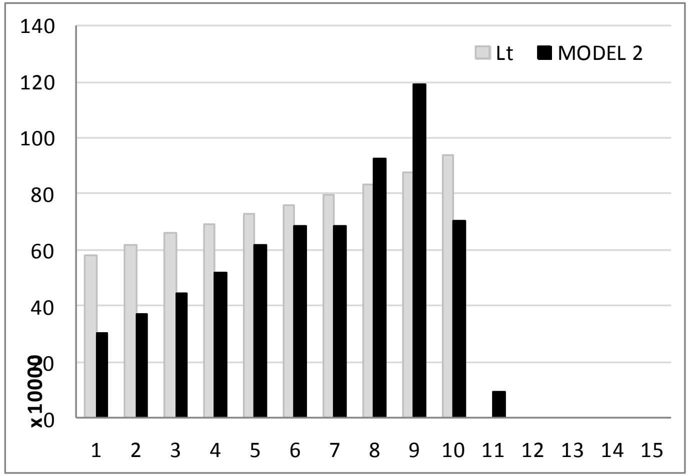
- Portfolio of 1,000 contracts of life insurance for death. Insurees aged 55 with coverage of €100,000 for death in the next 10 years. The commitments are guaranteed on the basis of the Spanish PASEM 2010 unisex mortality tables, ensuring an annual interest rate of 1.5%.
- This entire portfolio represents an actuarial liability of

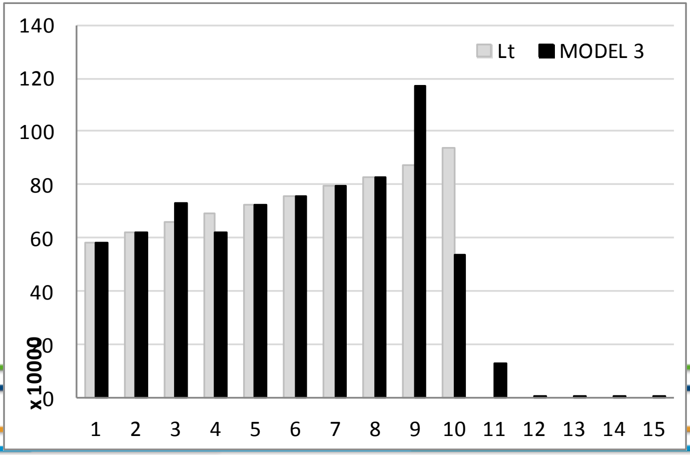
€6,807,636.19.

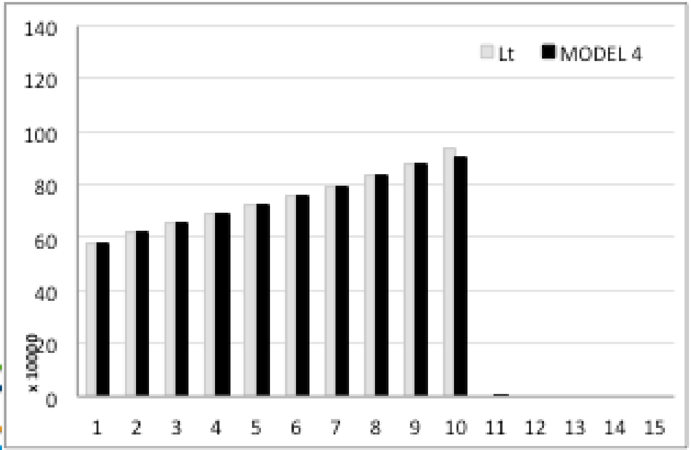
Results: Portfolios

- The paradigm is cash flow matching









Results: Testing

- 20,000 simulations are performed of the TSIR to fifteen years (*T*=15). In this case the following values are obtained as mean, standard deviation and Value at Risk –VaR- with a 99.5% confidence level (risk level of 1 in 200 simulations) as required by Solvency II in Europe

- we simulate the TSIR using the model by Cox-Ingersoll-Ross (1985), obtained using the following stochastic differential equation

$$dr = \kappa\left(\theta - r\right)dt + \sigma\sqrt{r}dz$$

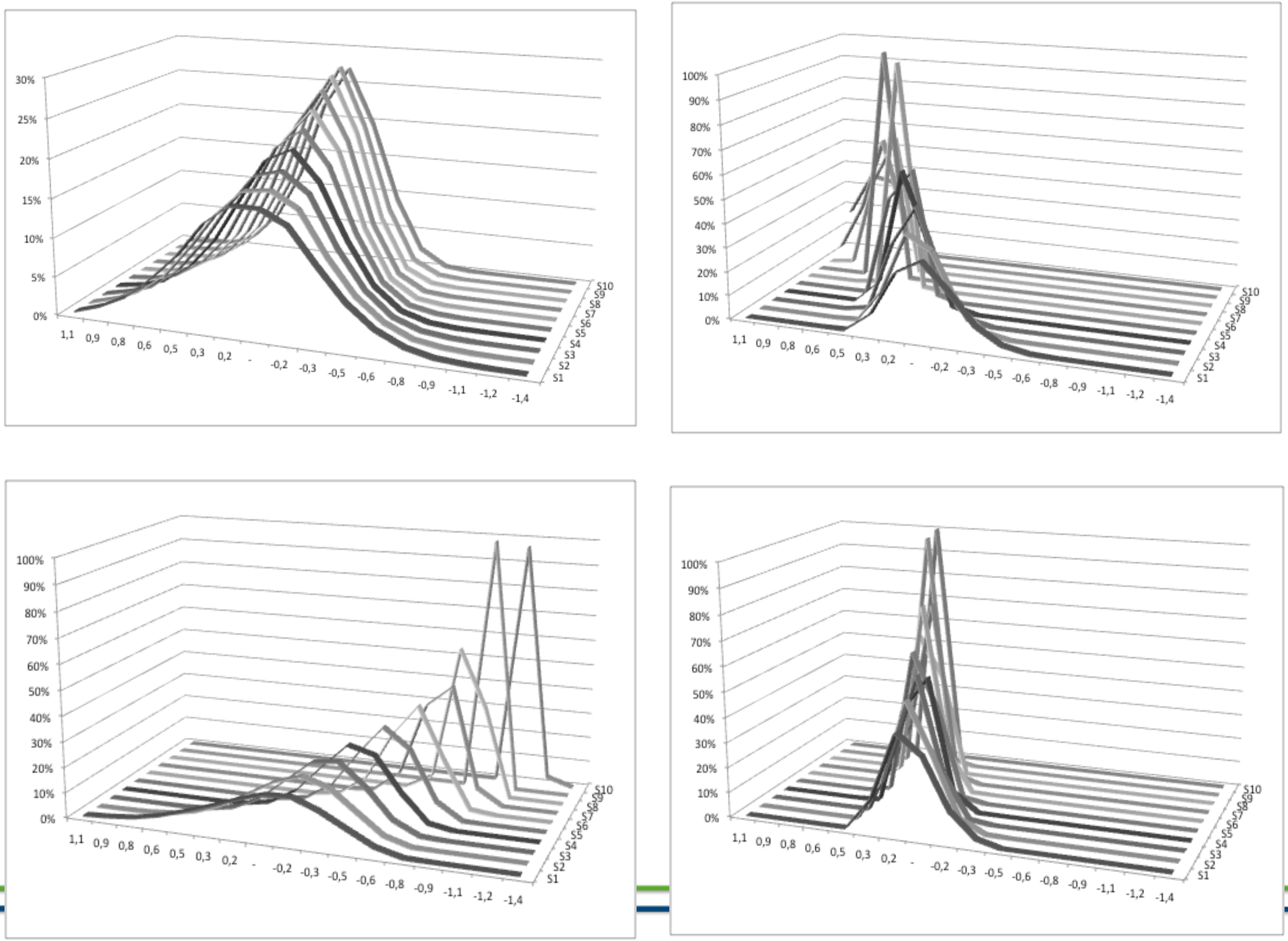
Results: Testing

- Results of the 20,000 generated TSIR

	1	2	3	4	5	6	7	8	9	10
					Mean					
$E(M)_{1,2}$	9.6294	9.6542	9.6795	9.7130	9.7515	9.7968	9.8392	9.8839	9.9269	9.9849
$E(M)_{1,3}$	-20.3517	-20.4063	-20.4701	-20.5397	-20.6525	-20.7168	-20.7868	-20.8641	-20.9469	-21.0363
$E(M)_{1,4}$	2.6397	2.6438	2.6486	2.6539	2.6595	2.6652	2.6713	2.6776	2.6840	2.6909
					Standard Deviation					
$E(SD)_{1,2}$	0.0027	0.0027	0.0025	0.0024	0.0024	0.0023	0.0022	0.0020	0.0017	0.0015
$E(SD)_{1,3}$	0.0909	0.2167	0.2978	0.3307	0.3809	0.4355	0.5413	0.6940	0.9035	0.8402
$E(SD)_{1,4}$	0.9239	0.9344	0.9408	0.9436	0.9458	0.9489	0.9557	0.9662	0.9802	0.9995
					VaR					
$E(VaR)_{1,2}$	0.7565	0.7966	0.8210	0.8468	0.8831	0.9334	0.9620	0.8779	0.7680	0.6566
$E(Var)_{1,3}$	0.0909	0.2167	0.2978	0.3307	0.3809	0.4355	0.5413	0.6940	0.9035	0.8402
$E(Var)1,4$	0.9239	0.9344	0.9408	0.9436	0.9458	0.9489	0.9557	0.9662	0.9802	0.9995

Results: Testing

- including more terms reduces the extension of possible results in different TSIRs, precisely by reducing the immunization risk.



Comments and Conclusions

- As a greater number of factors are provided as a requirement in the investment strategy, this strategy **converges** towards absolute matching.

- Use of a CFM strategy means that in the **submodule of interest risk**, a capital required for SCR solvency **is not considered**.

- Including more conditions limit immunization risk and so, the possibility that it might affect non-parallel variations in the TSIR. Nonetheless, if the immunization strategy takes into account a number of additional terms (CX; AP1; AP2), the SCR may be **reduced** or cancelled by converging towards a *de facto* **CFM strategy**.

Thank you very much for your attention!

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