

Risk analysis of annuity conversion options with a special focus on decomposing risk

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Risk analysis of annuity conversion options with a special focus on decomposing risk

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About the speaker

Introduction

Risk analysis of annuity conversion options

Risk decomposition methods from literature

MRT decomposition

Application of MRT decomposition to annuity conversion options

Contact details

About the speaker

Dr. Katja Schilling



- Master of Science (Mathematics, Illinois State University, USA, 2010)
- Diploma (Mathematics and Management, University of Ulm, 2011)
- Ph.D. (University of Ulm, 2017)
- since 2015 consultant at Allianz Pension Consult
(Allianz Pension Consult is a subsidiary of Allianz Lebensversicherungs-AG, the largest life insurance company in Germany, with main focus on structuring and arranging tailor-made solutions in the field of occupational pensions for medium-sized and large companies)
- junior member of the German Society for Actuarial and Financial Mathematics (DGVMF)
- candidate for the membership of the German Society of Actuaries (DAV)

About the speaker

Dr. Alexander Kling



Institut für Finanz- und Aktuarwissenschaften (ifa)

- ifa is an independent actuarial consulting firm.
- Our consulting services in all lines of insurance business include:
 - typical actuarial tasks and actuarial modelling
 - insurance product development
 - risk management, Solvency II, asset liability management
 - data analytics
 - market entries (cross-border business, setup of new insurance companies, Fintechs)
 - professional education
 - academic research on actuarial topics of practical relevance
- located in Ulm, Germany
- currently about 30 consultants
- academic cooperation with the University of Ulm (offering the largest actuarial program in Germany)



- joined ifa in 2003
- qualified actuary (German Association of Actuaries DAV, 2007)
- Master of Science (University of Wisconsin, Milwaukee, 2002)
- Master of Science (Mathematics and Management, University of Ulm, 2003)
- Ph.D. (University of Ulm, 2007)
- lecturer at Ludwig-Maximilians-Universität München, University of Ulm, German Actuarial Academy (DAA), European Actuarial Academy (EAA)

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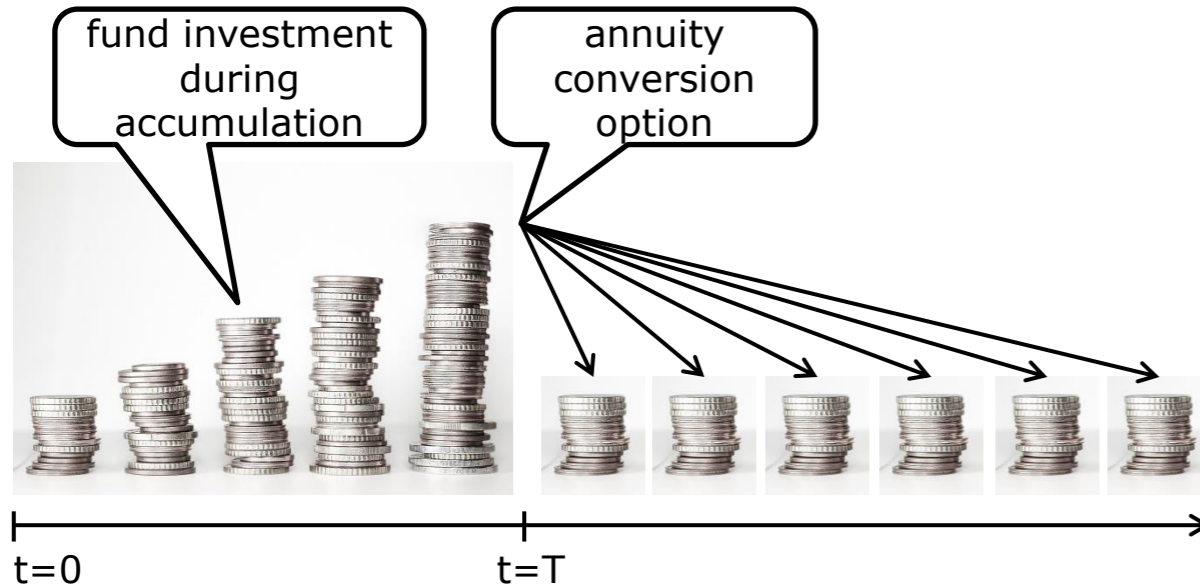
Application of MRT decomposition to annuity conversion options

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Introduction

Annuity conversion options

(Unit-linked) deferred annuities



- Money is allocated in some fund during a deferment period.
- At the end of the deferment period, the **accumulated fund value is converted into a lifelong annuity.**

Different annuity conversion options

- Guaranteed annuity option (GAO)
- GAO on a limited amount (Limit)
- Guaranteed minimum income benefit (GMIB)

Annuity conversion options are influenced by various risk sources such as

- equity, interest rate, and mortality

Introduction

Annuity conversion options



Existing literature on annuity conversion options

- measure the total risk by advanced stochastic models
- typically no decomposition of the total risk into risk factors

Our research interests

- (1) Theory:
 - How can the randomness of liabilities be allocated to different risk sources?
- (2) Application to annuity conversion options:
 - What is the dominating risk in annuity conversion options?
 - What is the relative importance of different risk sources?
- (3) Risk management of annuity conversion options:
 - How can the single risks be managed by product design or internal hedging?

Our contributions

- **Risk analysis of annuity conversion options in a stochastic mortality environment**
 - Katja Schilling, Alexander Kling, Jochen Ruß (2014)
 - ASTIN Bulletin 44 (2), 197 - 236.
- **Decomposing life insurance liabilities into risk factors**
 - Katja Schilling, Daniel Bauer, Marcus C. Christiansen, Alexander Kling (2018)
 - under review: Management Science
- **Comparing financial and biometric risks in annuity conversion options via the MRT decomposition**
 - Katja Schilling (2018)
 - under review: Insurance: Mathematics and Economics

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Different annuity conversion options



Guaranteed annuity option (GAO)

- minimum conversion rate g for converting the account value into a lifelong annuity at time T

$$L_T^{GAO,i} = \mathbb{I}_{\{\tau_x^i > T\}} \cdot g \cdot A_T \cdot \max\{a_T - \frac{1}{g}, 0\}$$

GAO with limit (Limit)

- upper bound L (limit) to which the conversion rate g at most applies

$$L_T^{Limit,i} = \mathbb{I}_{\{\tau_x^i > T\}} \cdot g \cdot \min\{A_T; L\} \cdot \max\{a_T - \frac{1}{g}, 0\}$$

Guaranteed minimum income benefit (GMIB)

- fixed minimum annuity amount $M (= g \cdot G)$

$$L_T^{GMIB,i} = \mathbb{I}_{\{\tau_x^i > T\}} \cdot \max\{g \cdot G \cdot a_T - A_T, 0\}$$

Notation

- T : deferment period/retirement date
- x : policyholder's age at inception of the contract ($t = 0$)
- τ_x : remaining lifetime
- A_T : account value at the end of the deferment period
- a_T : present value of an immediate annuity of amount 1 p.a.

Risk analysis of annuity conversion options

Insurer's strategies and stochastic model



Risk management strategies

	No hedging	Hedging
No option fee	Strategy A	-
Option fee	Strategy B	Strategy C

▪ Strategy A

- Insurer charges no option fee and does not hedge.

▪ Strategy B

- Insurer charges an option fee which is simply invested in money market instruments (no hedging).

▪ Strategy C

- Insurer charges an option fee to buy a static hedge against the financial risk during the deferment period.
- Assumption: option fee = hedging costs

Stochastic model

Fund

- Geometric Brownian motion

$$dS(t) = (r(t) + \lambda_S) \cdot S(t)dt + \sigma_S \cdot S(t)dW_S(t), \quad S(0) > 0$$

Interest rate

- Cox-Ingersoll-Ross model

$$dr(t) = \kappa \cdot (\theta - r(t))dt + \sigma_r \cdot \sqrt{r(t)}dW_r(t), \quad r(0) > 0$$

Stochastic mortality

- 6-factor forward model (cf. Bauer et al., 2008)

$$d\mu(t, T, x) = \alpha(t, T, x) dt + \sigma(t, T, x) dW_\mu(t), \quad \mu(0, T, x) > 0$$

Risk analysis of annuity conversion options

Sample results

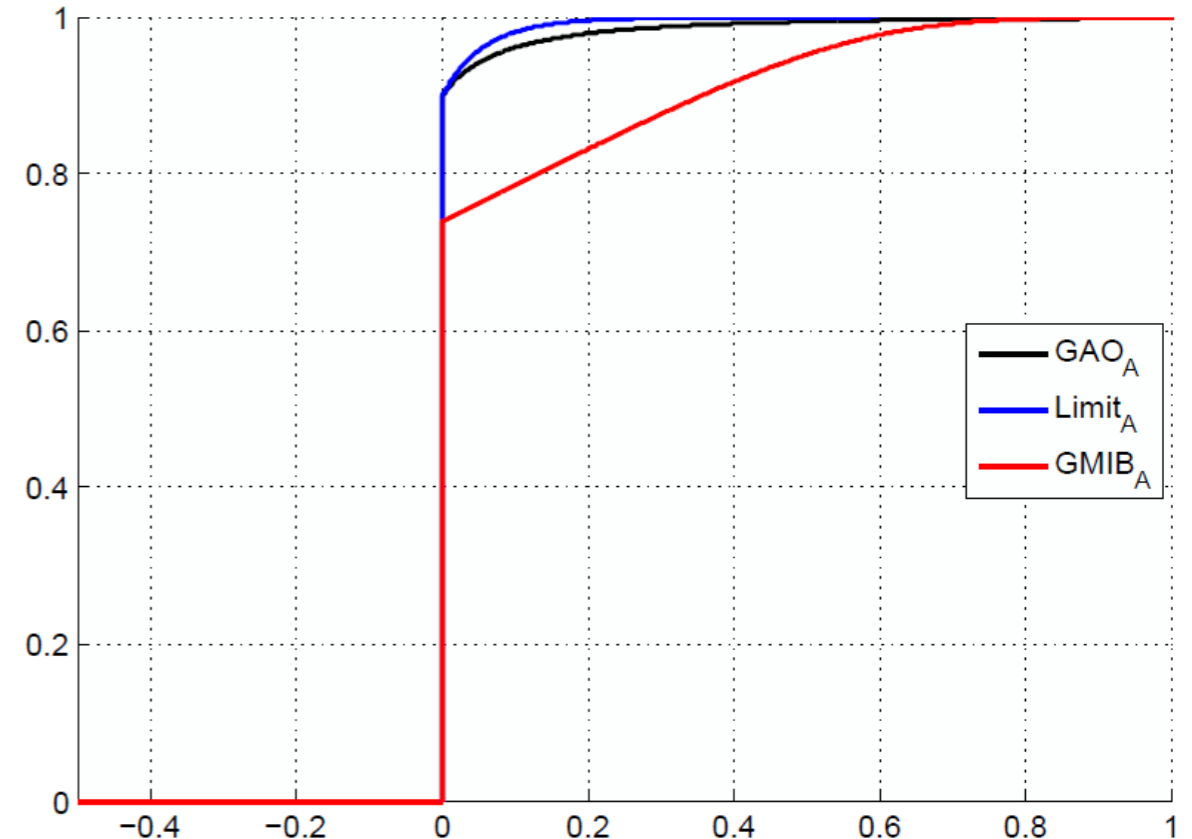


Risk of different annuity conversion options **without hedging (under strategy A)**

	GAO	Limit	GMIB
risk ($TVaR_{0,99}$)	0.7362	0.2214	0.7739
option value	0.0167	0.0096	0.1361

- Loss probability for GMIB much higher than for the other annuity conversion options
- Risk ($TVaR_{0,99}$) similar for GMIB and GAO
 - Limit has a much lower risk
- Option value does not reflect the risk of the annuity conversion option

Cumulative distribution function of insurer's loss **BERLIN 2018**



Risk analysis of annuity conversion options

Sample results

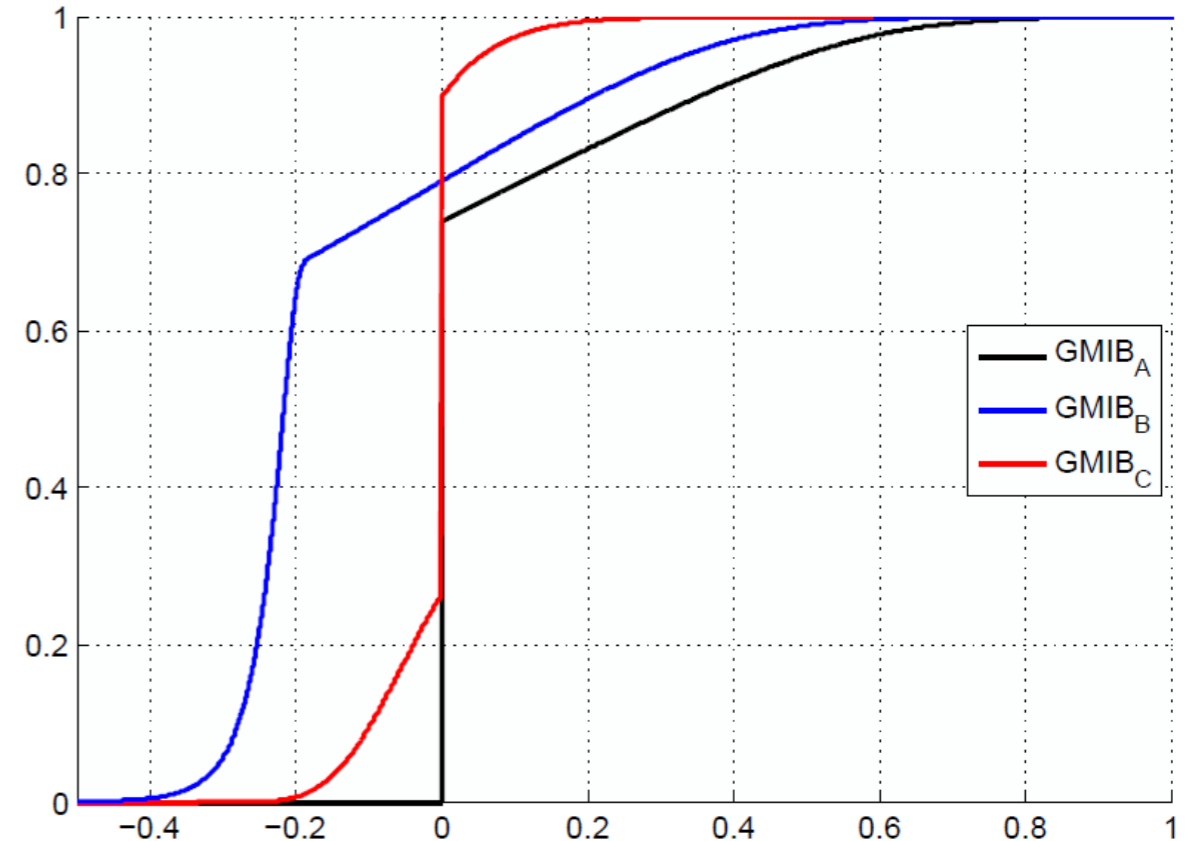
Risk of **GMIB** guarantee under **different risk management strategies**

	strategy A	strategy B	strategy C
risk ($TVaR_{0,99}$)	0.7739	0.5962	0.2385
option value	0.1361	0.1708	0.1708

- Loss probability and risk can significantly be reduced by risk management (strategies B and C)
- Risk under strategy B (option fee but no hedging) still significant
- Significant risk reduction for strategy C (option fee and hedging)
- Option value is increased if a guarantee fee is charged



Cumulative distribution function of insurer's loss BERLIN 2018



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Variance decomposition approach

Definition (given two sources of risk X_1 and X_2)

The (stochastic) variance decomposition of the risk R is defined as

$$R = \underbrace{E^P(R|X_1)}_{R_1} + \underbrace{[R - E^P(R|X_1)]}_{R_2},$$

where the risk factors R_1 and R_2 are supposed to capture the randomness caused by the sources of risk $X_1 = (X_1(t))_{0 \leq t \leq T}$ and $X_2 = (X_2(t))_{0 \leq t \leq T}$, respectively.

It follows the well-known result (not focused here):

$$\text{Var}(R) = \text{Var}(R_1) + \text{Var}(R_2)$$



Simple example

Let the insurer's risk be equal to $R = X_1(T)X_2(T)$, where X_1, X_2 are two independent (standard) Brownian motions.

Then the variance decomposition yields two different results depending on the order of X_1 and X_2 :

$$1) \quad R_1 = E^P(R|X_1) = X_1(T)E^P(X_2(T)) = 0 \\ R_2 = R - E^P(R|X_1) = X_1(T)X_2(T) - 0 = X_1(T)X_2(T)$$

$$2) \quad R_2 = E^P(R|X_2) = X_2(T)E^P(X_1(T)) = 0 \\ R_1 = R - E^P(R|X_2) = X_1(T)X_2(T) - 0 = X_1(T)X_2(T)$$

=> Variance decomposition is *not order-invariant!*

Risk decomposition methods from literature

Overview



Properties of risk decomposition methods from existing literature

	random-ness	attri-bution	unique-ness	order in-variance	scale in-variance	aggre-gation	additive aggre-gation
Variance decomposition cf. Bühlmann (1995)	✓	✓	✓	✗	✓	✓	✓
Hoeffding decomposition cf. Rosen & Saunders (2010)	✓	✓	✓	✓	✓	✗	✗
Taylor expansion cf. Christiansen (2007)	✓	✓	✗	✓	✗	✗	✗
Solvency II approach cf. Gatzert & Wesker (2014)	✓	✗	✗	✓	✓	✗	✗
MRT decomposition	✓	✓	✓	✓	✓	✓	✓

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Modeling framework



Let all **financial and demographic sources of risk** be modeled by a $(k - 1)$ -dimensional Itô process:

$$dX(t) = \theta(t)dt + \sigma(t)dW(t), \quad t \in [0, T], \quad X(0) = x_0 \in \mathbb{R}^{k-1}$$

The **number of survivors** is modeled by a doubly stochastic counting process $m - N(t)$

- $(N(t))_{t \in [0, T]}$ number of deaths with jump intensity $(\mu(t))_{t \in [0, T]}$

Notation

- $(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{P})$ probability space
- T time horizon
- W d -dimensional Brownian motion with filtration $\mathbb{G} = (\mathcal{G}(t))_{t \in [0, T]}$; \mathbb{G} is a sub-filtration of \mathbb{F}
- $m \in \mathbb{N}$ denotes the initial number of policyholders
- τ_x^i remaining life time of insured $i = 1, \dots, m$

MRT decomposition

Definition and properties



Definition

The **MRT decomposition** of $R = L - E^P(L)$ is defined as

$$R = \underbrace{\sum_{i=1}^{k-1} \int_0^T \psi_i^W(t) dM_i^W(t)}_{\text{risk factor } R_i} + \underbrace{\int_0^T \psi^N(t) dM^N(t)}_{\text{risk factor } R_k}$$

for some \mathbb{F} -predictable processes $\psi_i^W(t)$ and $\psi^N(t)$.

The processes $M_i^W(t)$ and $M^N(t)$ denote the martingale part of $X_i(t)$ and $N(t)$, respectively.

Theorem

Let L be \mathcal{F}_T -measurable and square integrable, $k - 1 = d$, and $\det(\sigma(t)) \neq 0$ for all t .

Then the MRT decomposition of $R = L - E^P(L)$ **exists** and satisfies **all properties** (i.e. randomness, attribution, uniqueness, order invariance, scale invariance, aggregation, additive aggregation).

Further properties

- **Applicability:** Explicit formulas for the integrands $\psi_1^W(t), \dots, \psi(t)_{k-1}^W(t), \psi^N(t)$ are derived (within a life insurance context)
- **Convergence:** Unsystematic mortality risk factor (R_k) is diversifiable; all other risk factors converge to a non-zero limit as the portfolio size increases

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MRT decomposition for a GAO



For the insurer's (discounted) loss $L = e^{-\int_0^T r(s)ds} (m - N(T)) g A_T \max\{a_T - \frac{1}{g}, 0\}$ from a GAO it holds (given a slightly modified stochastic model compared to slide 10):

- There exists a **measurable function** h such that $h(S(T), r(T), \mu(T)) = g A_T \max\{a_T - \frac{1}{g}, 0\}$
- The function $f(t, S(t), r(t), \mu(t)) := E^P \left(e^{-\int_t^T (r(s) + \mu(s)) ds} h(S(T), r(T), \mu(T)) | \mathcal{G}_t \right)$ is in $\mathcal{C}^{1,2}$



The **unique MRT risk factors** of the insurer's risk $R = L - E^P(L) = R_1 + R_2 + R_3 + R_4$ are given by

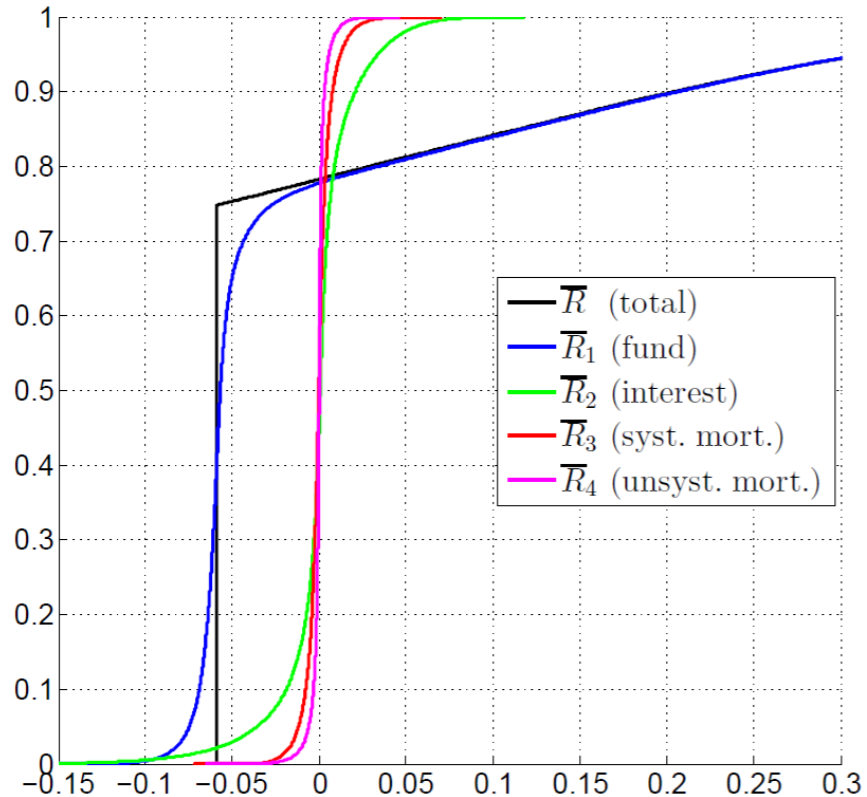
$$\begin{aligned} R_1 &= \int_0^T (m - N(t-)) e^{-\int_0^t r(s)ds} \frac{\partial f}{\partial x_1}(t, X(t)) \sigma_S S(t) dW_S(t) && \text{(fund risk)} \\ R_2 &= \int_0^T (m - N(t-)) e^{-\int_0^t r(s)ds} \frac{\partial f}{\partial x_2}(t, X(t)) \sigma_r \sqrt{r(t)} dW_r(t) && \text{(interest risk)} \\ R_3 &= \int_0^T (m - N(t-)) e^{-\int_0^t r(s)ds} \frac{\partial f}{\partial x_3}(t, X(t)) \sigma_\mu(t) \sqrt{\mu(t)} dW_\mu(t) && \text{(systematic mortality risk)} \\ R_4 &= \int_{0+}^T e^{-\int_0^t r(s)ds} f(t, X(t)) dM^N(t) && \text{(unsystematic mortality risk)} \end{aligned}$$

Application of MRT decomposition to annuity conversion options

Numerical results for a GMIB



Cumulative distribution functions of the total risk and the four risk factors



Relative risk contributions of the four sources of risk under **different risk measures**

	$TVaR_{0,99}$	Standard deviation
Fund	89.8 %	96.5 %
Interest	8.3 %	3.0 %
Syst. mortality	1.2 %	0.4 %
Unsys. mortality	0.7 %	0.1 %

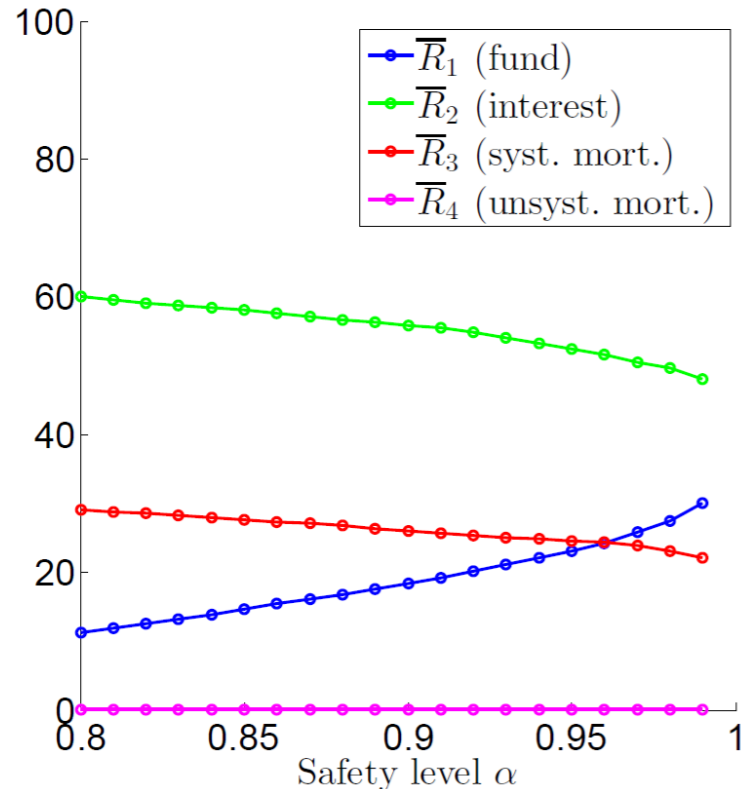
- Fund risk dominates total risk
- Interest risk is slightly relevant in the tail
- Mortality risks are negligible

Application of MRT decomposition to annuity conversion options

Numerical results for a GAO



Relative risk contributions of the four sources of risk under $TVaR_\alpha$ (in %) for different safety levels α



- Interest risk dominates total risk
- Systematic mortality is significant
- Fund risk is particularly responsible for high risk outcomes
- Unsystematic mortality risk is negligible

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Literature



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