Capital Relief Criteria Derivation For Comparing The Economic Efficiency Of Reinsurance Risk Transfer Instruments

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Motivation & Setting

- Insurers and reinsurers have a wide spectrum of risk-transfer instruments
- Each of these instruments may be characterized by a number of parameters:
  - Price or premium (P)
  - Capacity or limit (I)
  - Probability of being triggered
  - Duration of the cover (n)
  - Capital relief it provides
- Need of analytic criteria to compute the economic efficiency of different risk-transfer instruments : based on cost of capital (CoC) relief
- Focus on standard ILS (Cat & Mortality) and their economic costs :
  - Strong development in the last 10 years
  - Portfolio diversification : uncorrelated with financial markets
  - Low yield environment
  - Multi-year feature : economic efficiency hard to assess
- Hybrid vs standalone bonds

Economic Cost of a risk-transfer instrument

- Without taking into account discount and tax factors, we can write the economic cost EC as

$$\begin{aligned}\text{EC} &= \text{premium} + \text{fees} - \text{recovery} - \text{reduction in cost of capital} \\ &= \text{P} + \text{F} - \text{R} - \text{C}\end{aligned}$$

- Problem : EC can be a random variable.
- For a non-proportional cover with limit l and maturity n, the conditional mean of EC becomes

$$\begin{aligned}\mathbb{E}\left(\text{EC} \mid l, n\right) &= \mathbb{E}\left(\text{premium} \mid l, n\right) + \mathbb{E}\left(\text{fees} \mid l, n\right) \\ &\quad - \mathbb{E}\left(\text{recovery} \mid l, n\right) - \mathbb{E}\left(\text{reduction in cost of capital} \mid l, n\right) \\ \overline{\text{EC}}\left(l, n\right) &= \overline{\text{P}}\left(l, n\right) + \overline{\text{F}}\left(l, n\right) - \overline{\text{R}}\left(l, n\right) - \overline{\text{C}}\left(l, n\right)\end{aligned}$$

- the reduction in CoC also becomes stochastic if the cover runs for several years : e.g. in the ILS case. For a n-year Cat bond allowing an annual capital relief Δ, the reduction in CoC after n years can be e.g.:
  - $n \times \text{CoC} \times \Delta \times l$ if the bond is not triggered during its lifetime
  - $\text{CoC} \times \Delta \times l$ if the bond is triggered and fully exhausted during the first year

How to compare different covers from an economic point of view?

- EC comparisons make sense for covers providing a same capital relief we target
- Consider two covers with capacities $l_1,\ l_2$, and $n_1 = n_2 = 1$. We have

$$\overline{\mathrm{EC}}_1(l_1) = \overline{\mathrm{P}}_1(l_1) + \overline{\mathrm{F}}_1(l_1) - \overline{\mathrm{R}}_1(l_1) - \overline{\mathrm{C}}_1(l_1)$$
$$\overline{\mathrm{EC}}_2(l_2) = \overline{\mathrm{P}}_2(l_2) + \overline{\mathrm{F}}_2(l_2) - \overline{\mathrm{R}}_2(l_2) - \overline{\mathrm{C}}_2(l_2)$$

  - Assume $\overline{\mathrm{C}}_1(l_1) < \overline{\mathrm{C}}_2(l_2)$. Then we determine $l_1' > l_1$ such that $\overline{\mathrm{C}}_1(l_1') = \overline{\mathrm{C}}_2(l_2)$.
  - We finally compare $\overline{\mathrm{EC}}_1(l_1')$ and $\overline{\mathrm{EC}}_2(l_2)$ to select the most efficient cover from an economic point of view

Characteristics of an ILS

- limit I and maturity n
- L_i : recovery from the bond in year i (as a percentage of the total limit I)
- $L := L_1$, recovery during the first year (annual first year loss)
- $\mathbb{P}_a = \mathbb{P}(L > 0)$: probability that the bond will provide a recovery during the first year
- $\mathbb{P}_e = \mathbb{P}(L = 1)$: probability that the bond will be exhausted (recovery = limit) during the first year
- The annualized expected loss $e_n := \dfrac{\sum_{i=1}^{n} \mathbb{E}(L_i)}{n}$
- In the ILS market sphere, the 'expected loss' metric considered by sponsors and investors is $\mathbb{E}(L)$ and not e_n. In practice since the average annual recovery from the bond decreases with the number of years after issuance, we have $e_n < \mathbb{E}(L)$.
- $\mathbb{E}(L) - e_n$ increases when n increases or/and $\mathbb{E}(L)$ increases.

Rate-on-Line (RoL) & Pricing

- For small values of $\mathbb{P}_a$, one can show that

$$e_n \approx \mathbb{E}(L)\left(1 - \frac{n-1}{2}\Psi_L(l)\right)$$

where

$$\Psi_L(l) := \frac{1}{l\mathbb{E}(L)}\int_0^l \mathbb{P}(L > u)\mathbb{P}(L > l - u)du$$

- It provides the following approximation for the average total recovery the sponsor gets from the bond during the n years

$$\overline{\mathrm{R}}(l, n) \approx n \times l \times e_n \approx n \times l \times \mathbb{E}(L)\left(1 - \frac{n-1}{2}\Psi_L(l)\right)$$

- RoL = annualized price of a bond (as a percentage of the limit) = annual remuneration provided to investors on the outstanding principal $\Rightarrow$ the remuneration $\overline{\mathrm{P}}(l, n)$ until maturity is stochastic but can be approximated as follows:

$$\overline{\mathrm{P}}(l, n) \approx n \times l \times \mathrm{RoL} \times \left(1 - \frac{n-1}{2}\Psi_L(l)\right)$$

Cat & Mortality Bond : notations and assumptions

- We consider either a Mortality bond protecting against a peak in mortality in a given set of regions, or a Cat bond covering a Nat Cat region-peril (e.g. Europe Wind) on a per occurrence or an aggregate basis, with limit I_X and maturity n years
- Typically, I_X is in the 50-200M \$ range and n is equal to 3 years for a Cat bond (5 years for a Mortality bond).
- $L_{X,i}$: recovery during the i^{th} year. And $L_{X,1} := L_X$
- Δ_X : annual capital relief allowed by this standalone bond in the first year following its issuance (% of I_X).
- Assumptions:
  - The company's portfolio does not evolve during the lifetime of the bond $\Rightarrow$ the bond provides a constant annual capital relief Δ_X as long as it is not triggered. (A1)
  - Once the i^{th} triggering event has occurred, the bond provides a lower annual capital relief equal to $\frac{I_X^{(i)}}{I_X}\Delta_X$, where $I_X^{(i)}$ denotes the principal after the i^{th} triggering event. This is the principal proportionality principle of the capital relief. (A2)

Economic cost of a Cat or Mortality bond issued alone

- Under (A2), we can obtain the following approximation for the expected capital relief over n years

$$\overline{\mathsf{C}_X}\left(I_X, n\right) \approx n \times I_X \times \mathsf{CoC} \times \Delta_X \times \left(1 - \frac{n-1}{2}\mathbb{E}\left(L_X\right)\right)$$

- This provides an expected economic cost approximation

$$\begin{aligned}\overline{\mathsf{EC}_X}\left(I_X, n\right) \approx n \times I_X \times \left(\mathsf{RoL}_X - \mathbb{E}\left(L_X\right) - \mathsf{CoC} \times \Delta_X\right) \\ \times \left(1 - \frac{n-1}{2}\mathbb{E}\left(L_X\right)\right) + \overline{\mathsf{F}_X}\left(I_X, n\right),\end{aligned}$$

which can be rearranged as follows

$$\overline{\mathsf{EC}_X}\left(I_X, n\right) - \overline{\mathsf{F}_X}\left(I_X, n\right) \approx n \times \underbrace{\left(\overline{\mathsf{EC}_X}\left(I_X, 1\right) - \overline{\mathsf{F}_X}\left(I_X, 1\right)\right)}_{\text{1-year bond expected EC, no fees}}$$

$$\times \underbrace{\left(1 - \frac{n-1}{2}\mathbb{E}\left(L_X\right)\right)}_{\text{Correction Factor}}$$

Economic cost of two ILS issued simultaneously

- Here we consider two bonds X, Y (Cat or Mort.) which we refer to as ILS
- We note Δ_{X+Y} the annual capital relief allowed by both bonds together (on a **standalone** basis) in the first year following their issuance.
- We note δ_{X+Y} the difference

$$\delta_{X+Y} = \Delta_X + \Delta_Y - \Delta_{X+Y}$$

- Under (A1) and (A2), we also assume that the two bonds provide together a capital relief equal for $t \in [0, n]$ to
- Under these assumptions the expected economic cost of both bonds together can be approximated by

$$\overline{\mathrm{EC}_{X+Y}}(I, n) \approx \overline{\mathrm{EC}_X}(I, n) + \overline{\mathrm{EC}_Y}(I, n) + \underbrace{n \times I \times \mathrm{CoC} \times \delta_{X+Y}}_{\text{Correction factor}}$$

Hybrid bond : notations

- Combining two risks in a single tranche with capacity l would enable to reduce the cost of the protection : i.e. lower than the sum of the prices we would have to pay for two standalone tranches (each one with limit l).
- Less fees but also a bit less protection...and potentially a lower capital relief.
- Does the spread and fees saving allowed by the mix counterbalance the difference in protection / capital relief it induces?
- Let Q be a mixed bond, mixing X and Y, with capacity l_Q and lifetime $n_Q = n = n_X = n_Y$.
- Let us note Δ_Q the annual capital relief allowed by the hybrid bond in the first year following its issuance. When $l_Q = l_X = l_Y$, we have $\Delta_Q \leq \Delta_X + \Delta_Y$ since the hybrid bond necessarily provides less or equal protection than the two standalone bonds together.
- Again, we assume that the portfolio of the company does not evolve during the lifetime of the bond and that the capital

Hybrid bond : modelling & economic cost

- We suppose $F_{(X,Y))}(x,y) = C_\theta\left(F_X(x), F_Y(y)\right)$, where

$$C_\theta\left(u_1, u_2\right) := \left(u_1^{-\theta} + u_2^{-\theta} - 1\right)^{\frac{-1}{\theta}}, \quad \theta \in [-1, \text{inf}) \setminus \{0\}$$

  i.e. a Clayton copula modelling.
- The average economic cost of the hybrid bond can be written

$$\begin{aligned}
&\overline{\text{EC}_Q}\left(I_Q, n\right) \approx n \times I_Q \\
&\times \left(\text{RoL}_Q \times \left(1 - \frac{n-1}{2}\mathbb{E}\left(L_Q\left(X, Y, \theta\right)\right)\right) - \mathbb{E}\left(L_Q\left(X, Y, \theta\right)\right) \times \left(1 - \frac{n-1}{2}\mathbb{E}\left(L_Q\left(X, Y, \theta\right)\right)\right)\right) \\
&- n \times I_Q \times \text{CoC} \times \Delta_Q\left(X, Y, \theta\right) \times \left(1 - \frac{n-1}{2}\left(\mathbb{E}\left(L_X\right) + \mathbb{E}\left(L_Y\right)\right)\right) + \overline{\text{F}_Q}\left(I_Q, n\right)
\end{aligned}$$

Comparing the hybrid bond and the simultaneous standalone bonds

- We now determine l'_Q such that $\overline{\mathrm{C}_Q}\left(l'_Q, n\right) = \overline{\mathrm{C}_{X+Y}}\left(l, n\right)$

$$l'_Q = l \times \frac{\Delta_X \times \left(1 - \frac{n-1}{2}\mathbb{E}\left(L_X\right)\right) + \Delta_Y \times \left(1 - \frac{n-1}{2}\mathbb{E}\left(L_Y\right)\right) - \delta_{X+Y}}{\Delta_Q\left(X, Y, \theta\right) \times \left(1 - \frac{n-1}{2}\left(\mathbb{E}\left(L_X\right) + \mathbb{E}\left(L_Y\right)\right)\right)} = l \times \alpha\left(X, Y, \theta\right)$$

- Then the hybrid bond is more advantageous than the separate bonds if and only if

$$\overline{\mathrm{P}_Q}\left(l'_Q, n\right) + \overline{\mathrm{F}_Q}\left(l'_Q, n\right) - \overline{\mathrm{R}_Q}\left(l'_Q, n\right) < \overline{\mathrm{P}_{X+Y}}\left(l, n\right) + \overline{\mathrm{F}_{X+Y}}\left(l, n\right) - \overline{\mathrm{R}_{X+Y}}\left(l, n\right)$$

- When $\theta \to 0$, we obtain the independence copula and the relation

$$\Delta_Q\left(X, Y, \theta\right) \sim \Delta_{X+Y}, \ \theta \to 0$$

- When $\theta \to \infty$, we converge to the comonotonic copula and get the relation

$$\Delta_Q\left(X, Y, \theta\right) \sim \frac{1}{2}\Delta_{X+Y}, \ \theta \to \infty$$