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Granular Loss Modeling With Copulae

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About the speaker

- **Michal Pešta**
- Assistant Professor
- Research areas: errors-in-variables, change point, bootstrap, loss reserving, oncology

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- **Charles University, Prague**
- 670 years of history (founded in 1348)
- Faculty of Mathematics and Physics
- Department of Probability and Mathematical Statistics

Agenda

Aims

Methodology

Results

Conclusions

Agenda

Aims

- Loss Reserving
- Lifetime of a Claim

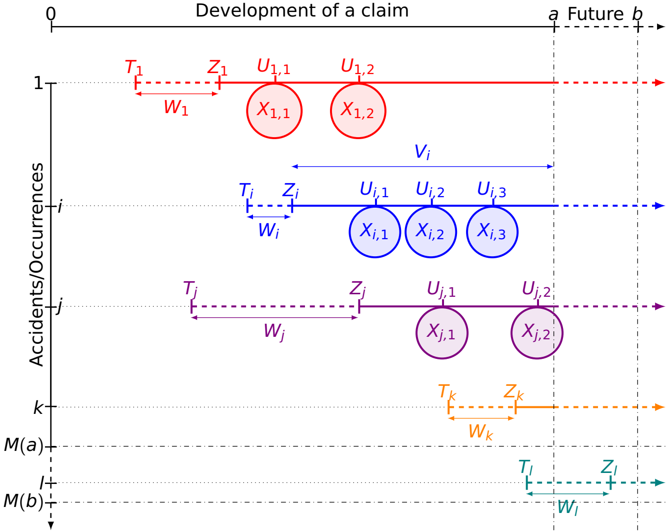
Two Main Goals

- Primary: predict future claim cash flows in non-life insurance and their uncertainty
- Secondary: back-predict incurred but not reported claims due to truncated data

Aggregated vs Granular

- Pitfalls of the conventional reserving techniques:
  - loss of information from the policy and the claim’s development due to the aggregation, cf. Norberg (1993)
  - usually small number of observations in the triangle
  - only few observations for recent accident years
  - sensitivity to the most recent paid claims
- How to possibly overcome the issues:
  - individual/claim-by-claim/micro-level/granular data, which do not represent a mainstream in the reserving field, e.g., Antonio & Plat (2014)

Illustration



Agenda

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Methodology

- Reporting Dates and Delays
- Payment Dates and Amounts
- Truncation of Accident Dates
- Dependence Between LoBs

Reporting Dates

- The time ordered reporting dates $\{Z_i\}_{i \in \mathbb{N}}$ are arrival times of a non-homogeneous Poisson process $\{M(t)\}_{t \geq 0}$ with a parametric intensity $\psi(t; \boldsymbol{\rho})$ such that

$$M(t) = \sum_{i=1}^{\infty} \mathbb{1}\{Z_i \leq t\}$$

- The cumulative intensity $\Psi(t; \boldsymbol{\rho}) := \int_0^t \psi(v; \boldsymbol{\rho}) dv$ diverges if $t \to \infty$

Reporting Delays

- The reporting delays W_i's are independent random variables
- Sequence $\{W_i\}_{i\in\mathbb{N}}$ is stochastically independent of $\{Z_i\}_{i\in\mathbb{N}}$
- Given $Z_n = z$, W_n has a parametric density $f_W(\cdot, z; \boldsymbol{\theta})$

Payment Dates

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- The time ordered payment delays $\{U_{i,1}-Z_i, U_{i,2}-Z_i, \ldots\}$ of the ith claim are arrival times of a non-homogeneous Poisson process $\{N_i(t)\}_{t\geq 0}$
- $N_i(t)|W_i, Z_i$
- Processes $\{N_i(t)\}_{t\geq 0}$, $i=1,2,\ldots$ are independent with a parametric intensity $\lambda_i(t;\boldsymbol{\nu},\boldsymbol{\beta})$ such that

$$N_i(t)=\sum_{k=1}^{\infty}\mathbb{1}\{U_{i,k}-Z_i\leq t\}$$

- The cumulative intensity $\Lambda_i(t;\boldsymbol{\nu},\boldsymbol{\beta}):=\int_0^t\lambda_i(v;\boldsymbol{\nu},\boldsymbol{\beta})\mathrm{d}v$ converges if $t\to\infty$

Payment Amounts

- Sets of the payment amounts $\{X_{ij}\}_j$'s are independent random sequences
- Sequence $\{X_{ij}\}_j$ forms an AR process
- Sequence $\{\{X_{ij}\}_j\}_{i\in\mathbb{N}}$ is stochastically independent of $\{Z_i\}_{i\in\mathbb{N}}$
- Given $Z_n = z$, the first payment X_{n1} has a parametric density $f_X(\cdot, z; \boldsymbol{\zeta})$

Accident Dates

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- Accident dates as displacement of the reporting dates
- Reporting dates are fully observed, accident dates are truncated
- The displacement theorem (Kingman, 1993) provides that accident dates T_i's are arrival times of another non-homogeneous Poisson process with a parametric intensity

$$\mu(t;\boldsymbol{\rho},\boldsymbol{\theta}) = \int_{\mathbb{R}} \psi(z;\boldsymbol{\rho}) f_W(t,z;\boldsymbol{\theta})\mathrm{d}z$$

- Back-fit the incurred but not reported claims

Copula

- Suppose that $Y_t^{(\ell)}$ is a loss amount for a time time period t (e.g., month) and for a LoB ℓ
- Assume that the dependence between lines of business (LoBs) is modeled via a parametric copula (possibly time-varying in order to capture dynamic behavior)
- E.g., $\ell \in \{1, 2\}$ ... two LoBs (material damage and bodily injury)

$$\mathbb{P}\left[Y_t^{(1)} \leq y^{(1)}, Y_t^{(2)} \leq y^{(2)}\right] = C\left(\mathbb{P}\left[Y_t^{(1)} \leq y^{(1)}\right], \mathbb{P}\left[Y_t^{(2)} \leq y^{(2)}\right]; \boldsymbol{\alpha}(t)\right)$$

By-product for Stochastic Theory

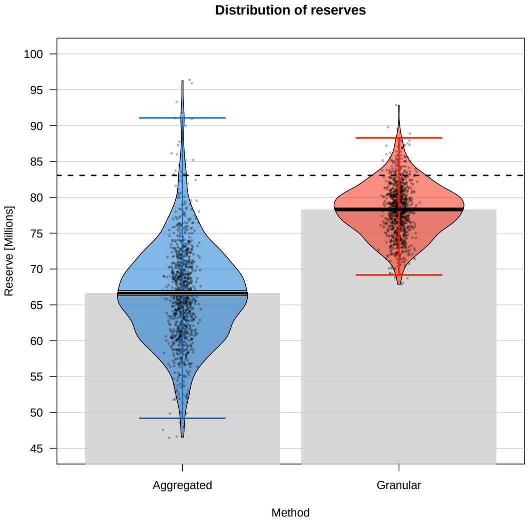
- Probabilistic framework for the n.i.n.i.d. observations
- Time-varying models in an unbalanced panel data setup
- Maximum likelihood estimators derived
- Proved consistency and asymptotic normality of the estimators
- Justification for usage of the method

Agenda

Results

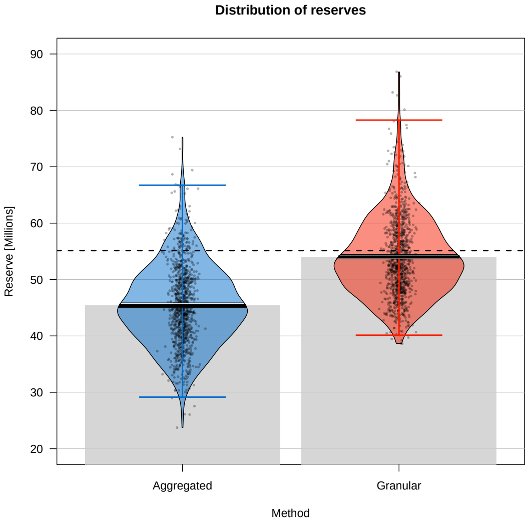
- Loss Reserves
- Incurred But Not Reported

Material



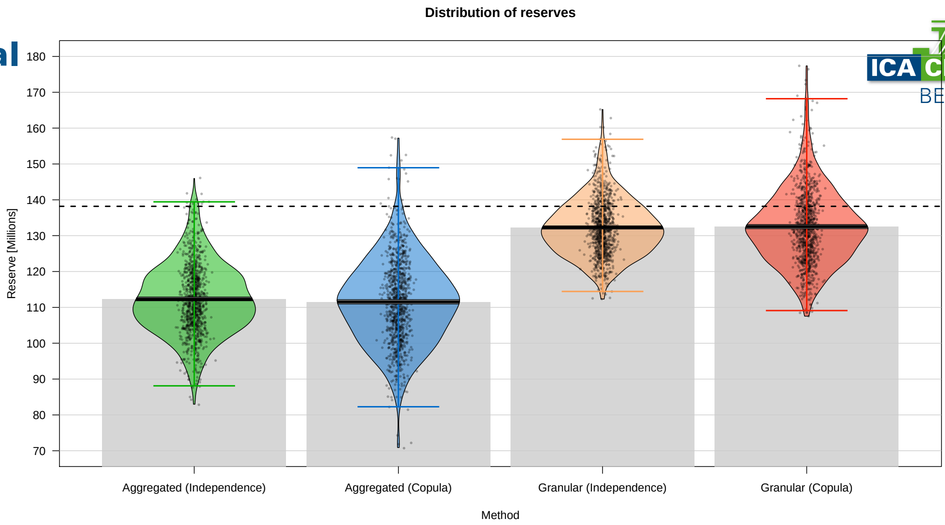
Distribution of reserves
Reserve [Millions]
100
95
90
85
80
75
70
65
60
55
50
45
Aggregated
Granular
Method

Bodily

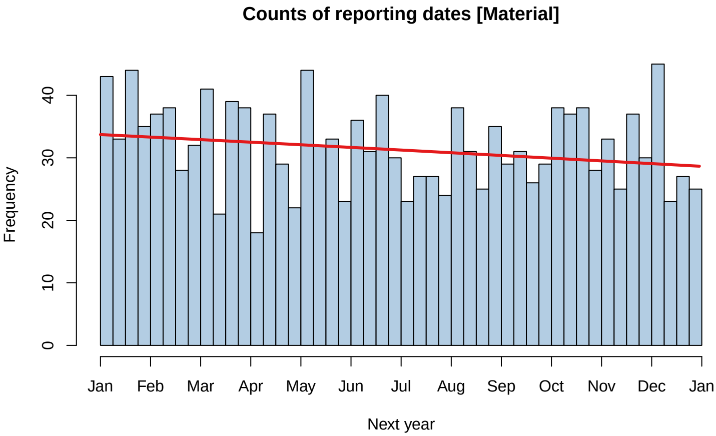


Distribution of reserves
90
80
70
60
50
40
30
20
Reserve [Millions]
Aggregated
Granular
Method

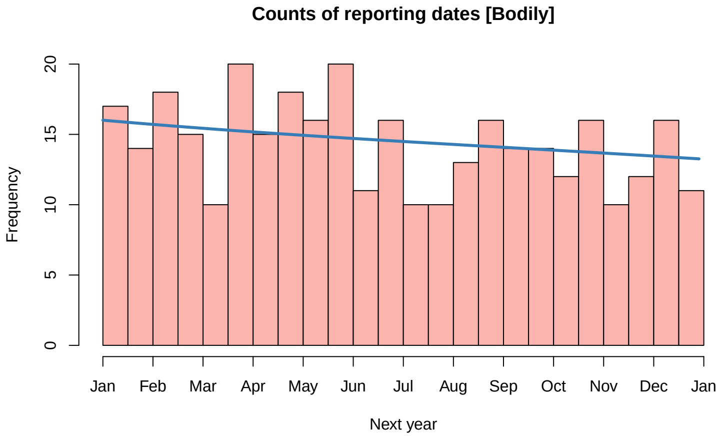
Total


Distribution of reserves
180
170
160
150
140
130
120
110
100
90
80
70
Reserve [Millions]
Aggregated (Independence)
Aggregated (Copula)
Granular (Independence)
Granular (Copula)
Method

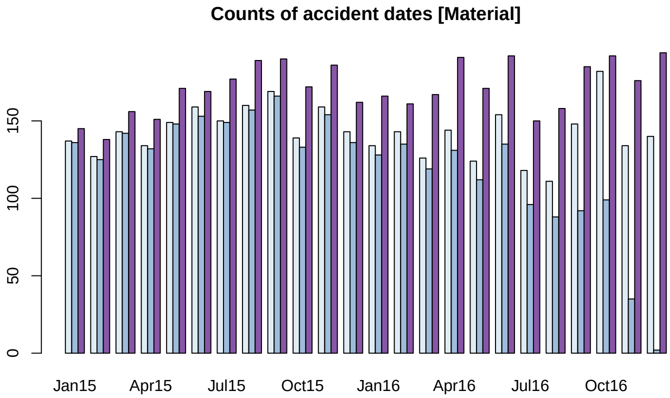
Predicted Future Reportings



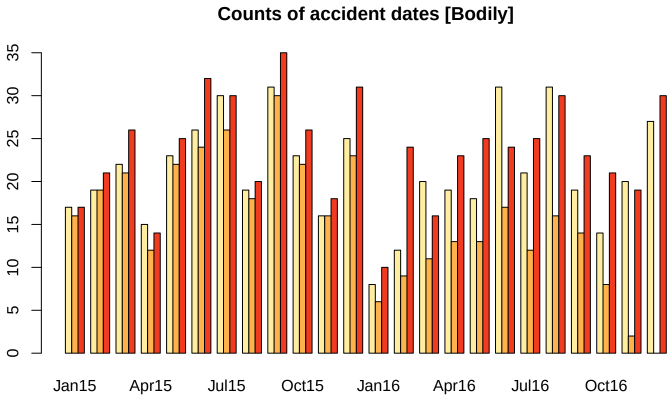
Predicted Future Reportings



Back-fitted Recent Accidents



Back-fitted Recent Accidents



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Conclusions

References

Summary

- Focus on three synergic research areas
- Inventing stochastic methods for loss reserving based on claim-by-claim data
- Using a dynamic copula framework for modeling dependencies among types of claims
- Deriving appropriate statistical inference for these approaches

Thank you very much for your attention!

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References

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