Projections and Profitability of Unit-Linked Products

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About the speaker

- **Kristian Juul Schomacker**
- Product manager within life insurance at Edlund A/S where he has worked since 2009.
- Before entering Edlund A/S and the world of life insurance, he worked in the financial sector.

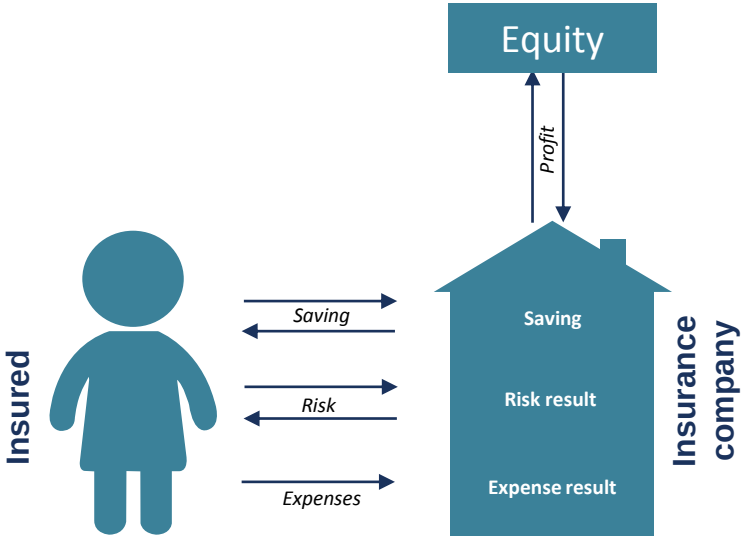
- **Edlund A/S**
- Comprehensive IT solutions for managing portfolios in life and pension.
- Innovative IT solutions are robust, reliable and at the same time flexible.
- More than 250 talented employees, most of whom work with software development.

Agenda

- Setup and motivation
- Profitability
- The forward-backward problem
- Financial results and cash flows
- Measurability
- Examples
- Conclusion

Setup

- Focus is unit-linked
- Calculating cash flows
  a) Insurance company↔ Insured
  b) Flow to equity
- Calculate risk cash flow correctly
  - Taking the disability into account



Future profits

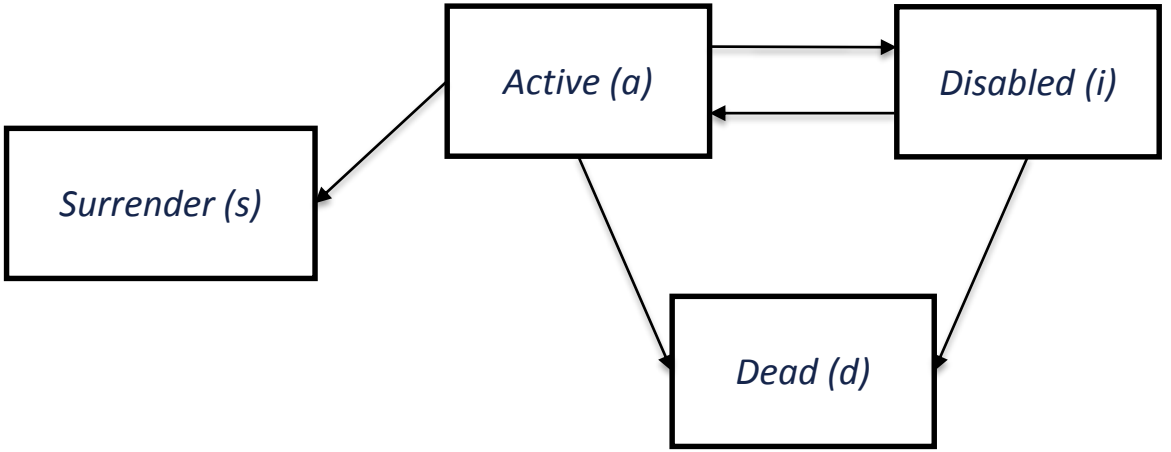
- Future profits $(0) =$

$$E_0\left[\sum_t \text{discount factor}(t)(\text{risk result}(t) + \text{expense result}(t))\right]$$

- Risk result(1)
- Expense result(1)

- Risk result(2)
- Expense result(2)

State model



The forward-backward problem

- The active reserve is given by

$$\frac{d}{dt}V^a(t) = r(t)V^a(t) + \pi^a(t) - \mu^{ai}(t)\left(V^i(t) - V^a(t)\right) - \mu^{ad}(t)\left(b^{ad}(t) - V^a(t)\right) - \mu^{as}(t)\left(b^{as}(t) - V^a(t)\right)$$

$$V^a(0) = 0$$

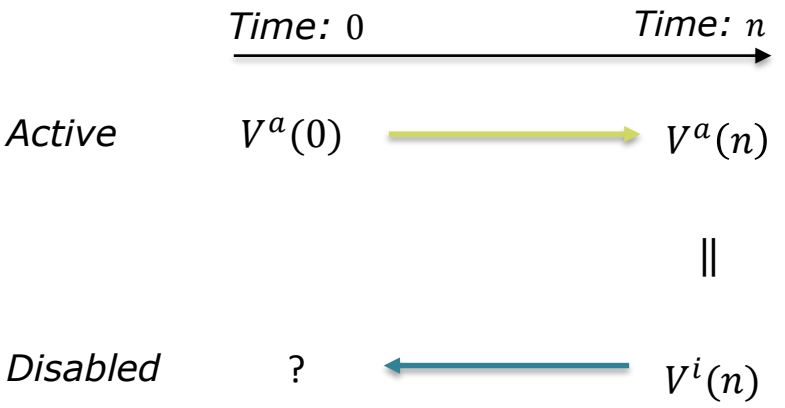
  - Solved *forward*

- The disability reserve is given by

$$\frac{d}{dt}V^i(t) = r(t)V^i(t) - b^i(t) - \mu^{ia}(t)\left(V^a(t) - V^i(t)\right) - \mu^{id}(t)\left(b^{id}(t) - V^i(t)\right)$$

$$V^i(n-) = \Delta B^i(n) \equiv \Delta B^a(n) \equiv V^a(n)$$

  - Due to the waiver of premiums during disability
  - Solves *backward*
  - n is the retirement age



Forward backward problem: Important special cases

- Forward backward problems are in general difficult to solve
- Important and relevant special cases
  - $\mu^{id}(t) = \mu^{ad}(t)$
  - $b^{id}(t) = b^{ad}(t) = c + V^{a}(t)$

    (The death sum is affine in the active reserve)

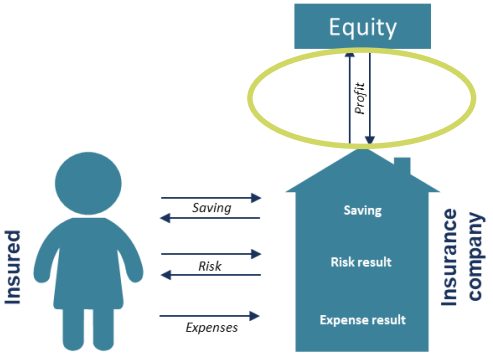
Expected risk result

- Expected risk result is the difference between
  - Risk premiums paid *to* the insurance company
  - Risk premiums paid *by* the insurance company

- Expected risk result

$$p_{ja}(0,t)[\underbrace{\pi_a(t)}_{\text{Risk premiums}} - \mu_{ai}(t)\underbrace{R_{ai}^{risk}(t)}_{\text{Sum at risk}} - \mu_{ad}(t)\underbrace{b_{ad}(t)}_{\text{Payments upon death from active}}]$$

$$+p_{ji}(0,t)[-\mu_{ia}(t)\underbrace{R_{ia}^{risk}(t)}_{\text{Sum at risk}} - \mu_{id}(t)(\underbrace{b_{id}(t)}_{\text{Payments upon death from disabled}} + \underbrace{R_{ia}^{risk}(t)}_{\text{Sum at risk}})]$$



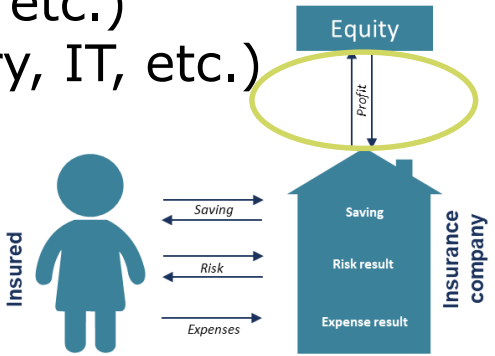
Expected expense result

- Expected expense result is the difference between
  - Expenses paid *to* the insurance company (fees, etc.)
  - Expenses paid *by* the insurance company (salary, IT, etc.)

- Expected expense result

$$p_{ja}(0,t)\Big(\underbrace{\Delta^{a}_{inv}(t)}_{\text{Difference in investment expenses}} \cdot \underbrace{X(t)}_{\text{Saving}} + \underbrace{\Delta_{a}(t)}_{\text{Difference in expenses in active state}} + \underbrace{\Delta_{as}(t)}_{\text{Surrender cost}} \mu_{as}(t)\Big) +$$

$$p_{ji}(0,t)\Big(\underbrace{\Delta^{i}_{inv}(t)}_{\text{Difference in investment expenses}} \cdot \underbrace{X(t)}_{\text{Saving}} + \underbrace{\Delta_{i}(t)}_{\text{Difference in expenses in disabled state}} \Big)$$



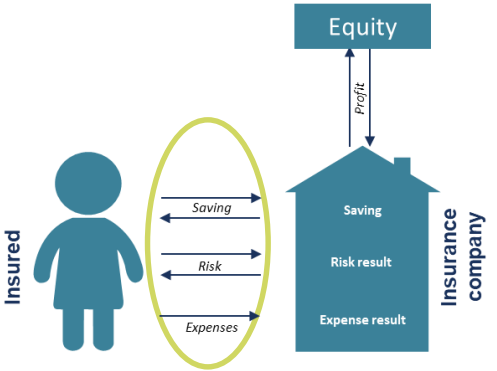
Expected cash flow to the customer

- Expected premium cash flow
  $p_{ja}(0,t)\pi(t)$

- Expected benefit cash flow

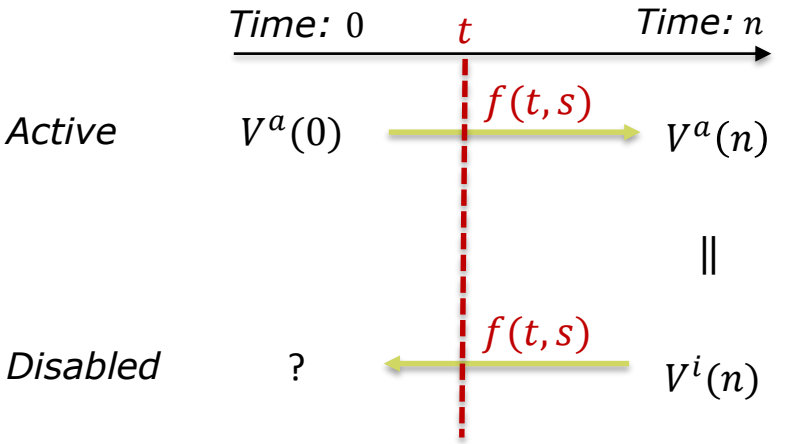
$$p_{ja}(0,t)[\mu_{ad}(t)\ \underbrace{G_a(\overbrace{X(t)}^{\text{Saving}})}_{\text{Payments upon death}} + \mu_{as}(t)\ \underbrace{H(X(t))}_{\text{Payment upon Surrender}}\ ] + p_{ja}(0,\ \underbrace{n}_{\text{Time of retirement}}\ )\ \underbrace{I_a(X(t))}_{\text{Payment at time of retirement}}$$

$$p_{ji}(0,t)[\mu_{id}(t)\ \underbrace{G_i(X(t))}_{\text{Payments upon death}} + b_i(t)] + p_{ji}(0,n)[\ \underbrace{I_i(X(t))}_{\text{Payment at time of retirement}}\ ]$$

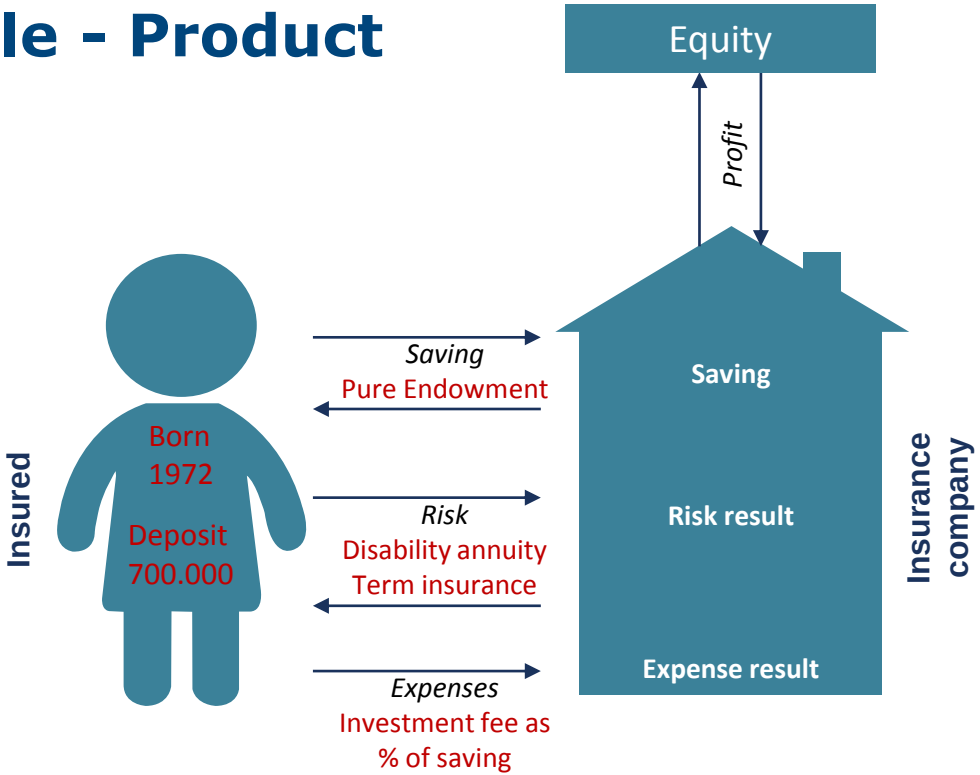


Measurability

- Profitability$(0) =$
  $E_0\left[\sum_t \text{discount factor}(t)(\text{risk result}(t) + \text{expense result}(t))\right]$
- The risk result contains
  $\underbrace{R_{ai}^{risk}(t)}_{\text{Sum at risk}} = (V_i - V_a)(t;\ \underbrace{f(t,s)_{s\geq t}}_{\text{Forward rates at time t}})$
  - Similar for $R_{ia}^{risk}(t)$
- Points
  - Use in general $f(t,s)_{s\geq t}$
  - $f(0,s)_{s\geq 0}$ correct under linearity



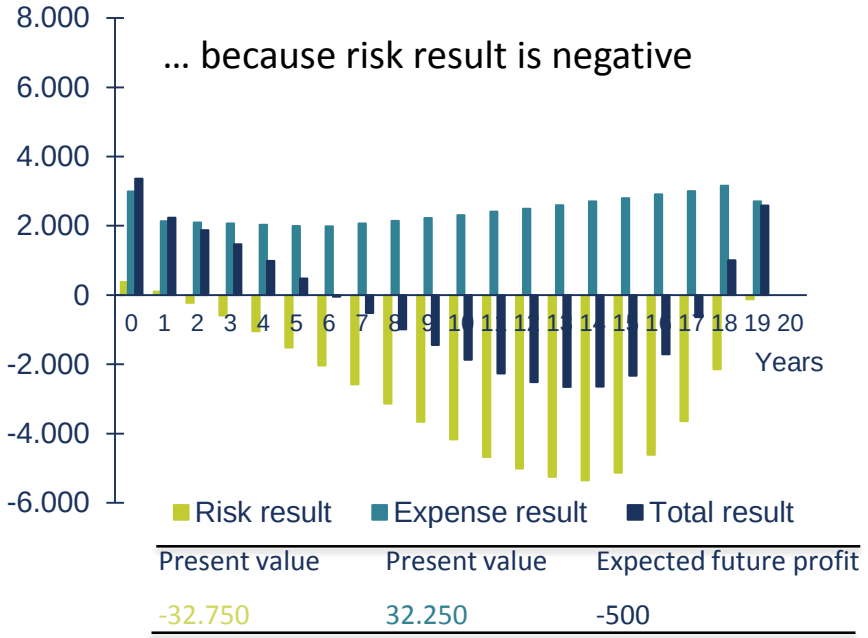
Example - Product



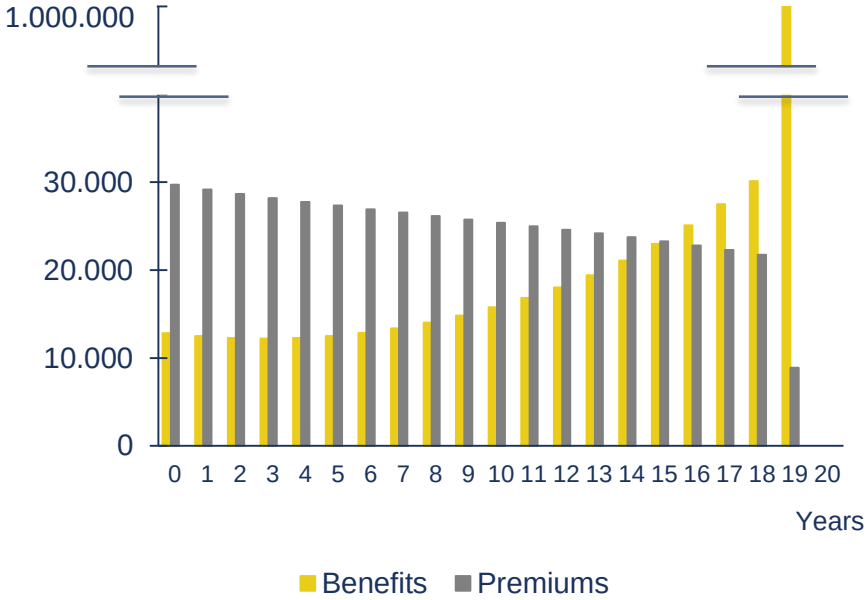
Examples calculated by

ACTULUS

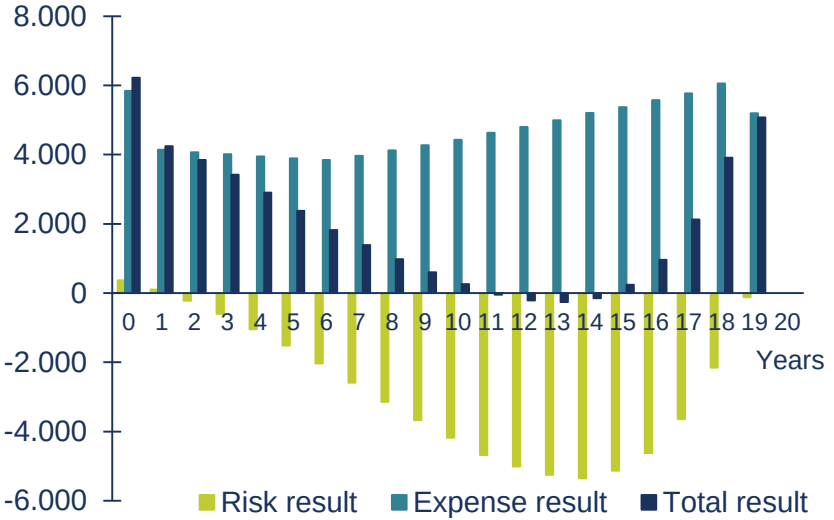
Example 1 – Total result approx zero…



Present value	Present value	Expected future profit
-32.750	32.250	-500

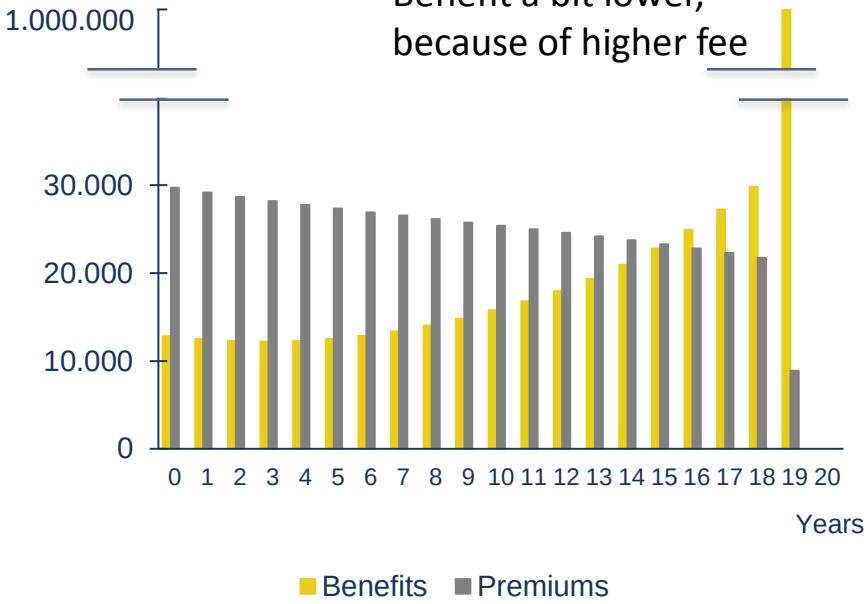


Example 2 – Total result improves by doubling investment management fee

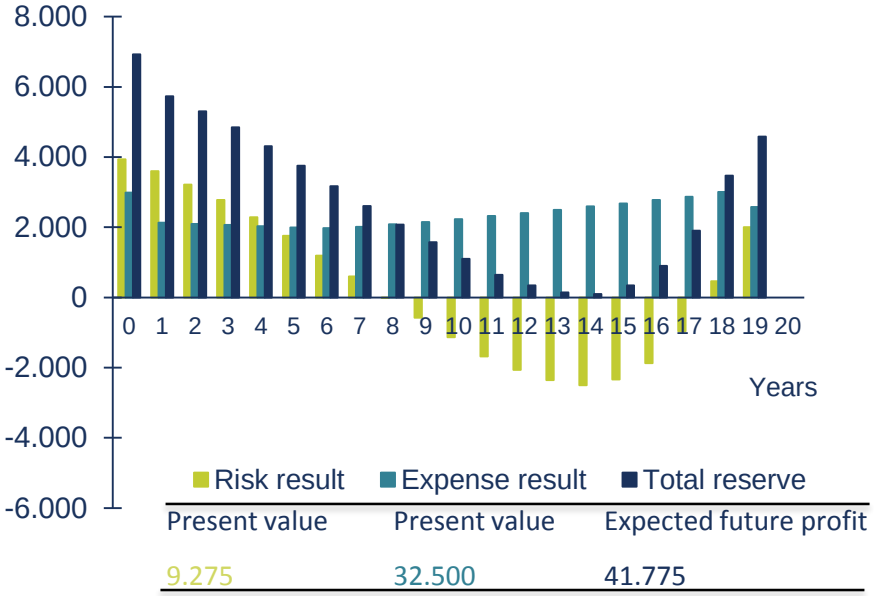


Present value	Present value	Expected future profit
-32.750	62.175	29.425

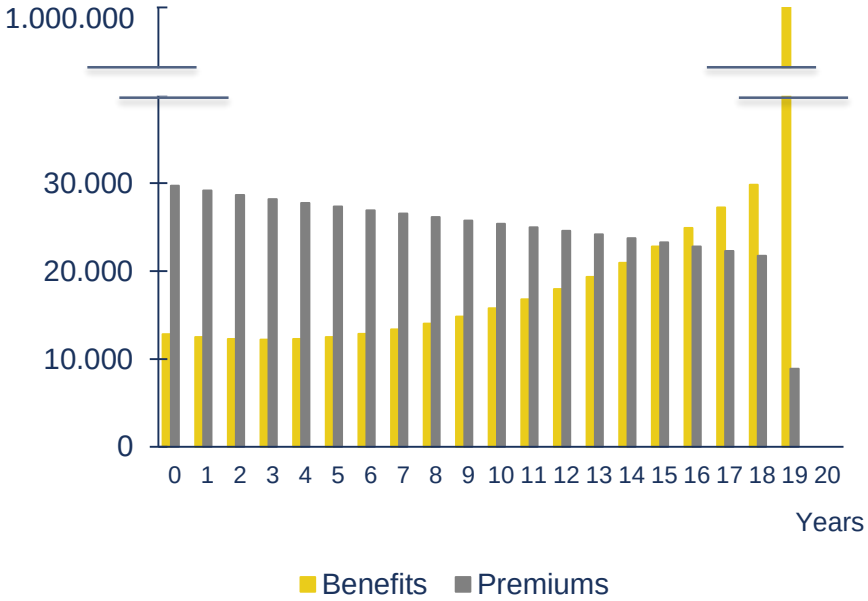
Benefit a bit lower, because of higher fee



Example 3 – Alternative:
Increase premiums for risk coverage by 100%

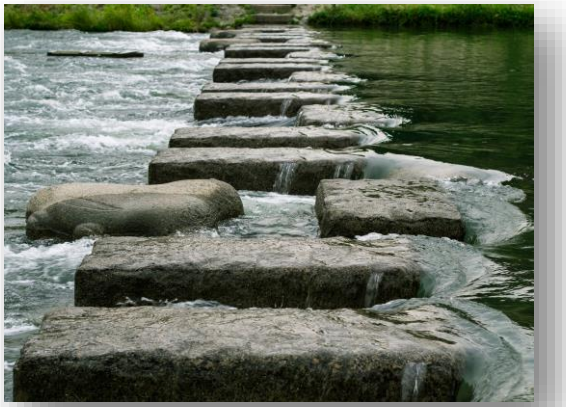


Present value	Present value	Expected future profit
9.275	32.500	41.775



Conclusion

- Profitability
  - Split in risk and expense
- The forward-backward problem
  - In general difficult to solve
  - Important special cases
- Measurability
  - Use in general $f(t,s)_{s\geq t}$
  - $f(0,s)_{s\geq 0}$ correct under linearity



Thank you very much for your attention!

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