Looking at concentration and composition in the life insurance context

Walter Olbricht, University of Bayreuth
Josée Kaulich-Bartz, Swiss Re

About the speaker

- **Walter Olbricht**
- Akademischer Direktor
- Studies: Maths & Stats in Germany and UK, Positions: Bochum, MIT (USA), Bayreuth

UNIVERSITÄT BAYREUTH

- **University of Bayreuth**
- Young research university
- 'Almost' as famous as Wagner's operas…

About the speaker

- **Josée Kaulich-Bartz**
- Manager Experience Studies & Analytics CEME, Head Statistical Working Commission of Swiss Insurance Association
- Studies: Maths at ETH Zürich, FQA SAA
  now does *real* work…

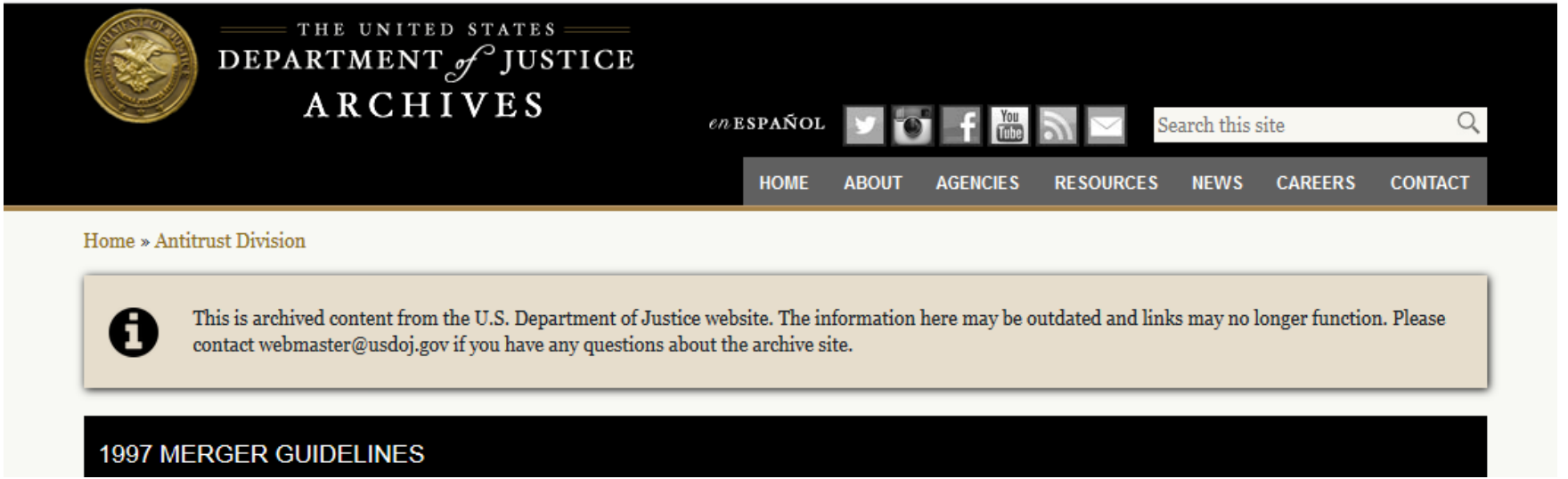
- **Swiss Re**
- One of the two biggest reinsurance companies in the world
- (At this conference) even 'more' famous than Wagner's operas…

Concentration: a first glimpse

- *Static* versus *dynamic*
- *Absolute* versus *relative*
  (*concentration* versus *disparity*)
- *Special aspects* of concentration
  (e.g. *temporal* or *spatial*)

Herfindahl-index and US-merger-guidelines

Source: https://www.justice.gov/archives/atr/1997-merger-guidelines



Herfindahl-index and US-merger-guidelines

Definition: $H = \sum_{i=1}^{n} q_i^2 = \sum_{i=1}^{n} \left(\frac{x_i}{N}\right)^2$ (where $N = \sum_{i=1}^{n} x_i$)

Example: descending shares: (0.4, 0.2, 0.2, 0.1, 0.1)

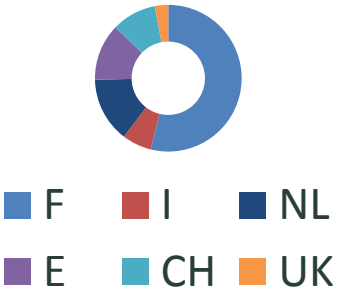
$$H = 0.4^2 + 0.2^2 + 0.2^2 + 0.1^2 + 0.1^2 = 0.26$$

(minimum value: 0.2)

Ranges:

Not concentrated	Moderately concentrated	Highly concentrated
$H < 0.1$	$0.1 < H < 0.18$	$0.18 < H$

US-merger-guidelines and AIDS-study



Country	No. of patients	Proportion	Cohorts
France (F)	22'760	54%	1) French Hospital Database on HIV (ANRS CO4 FHDH) 2) ANRS CO3 Aquitaine Cohort France
Netherlands (NL)	6'060	14%	AIDS Therapy Evaluation Project Netherlands (ATHENA)
Spain (E)	5'285	13%	1) CoRIS Spain 2) PISCIS , Catalonia and Balearic islands , Spain 3) VACH Spain
Switzerland (CH)	4'162	10%	Swiss HIV Cohort Study (SHCS)
Italy (I)	2'743	6%	Italian Cohort of Antiretroviral-Naive Patients (ICONA)
United Kingdom (UK)	1'250	3%	1) Royal Free Hospital Cohort, London 2) The EuroSIDA Study Group
Total	42'266	100%	

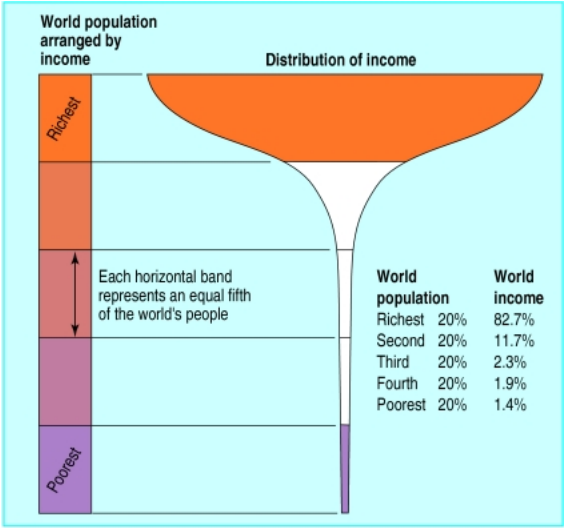
$H = 0.34$

Minimum: 0.167

Pareto, a recap

Pareto's 20-80-rule:

- 1906: 20% of the population possess 80% of the land
- United Nations Development Programme (champagne glass)



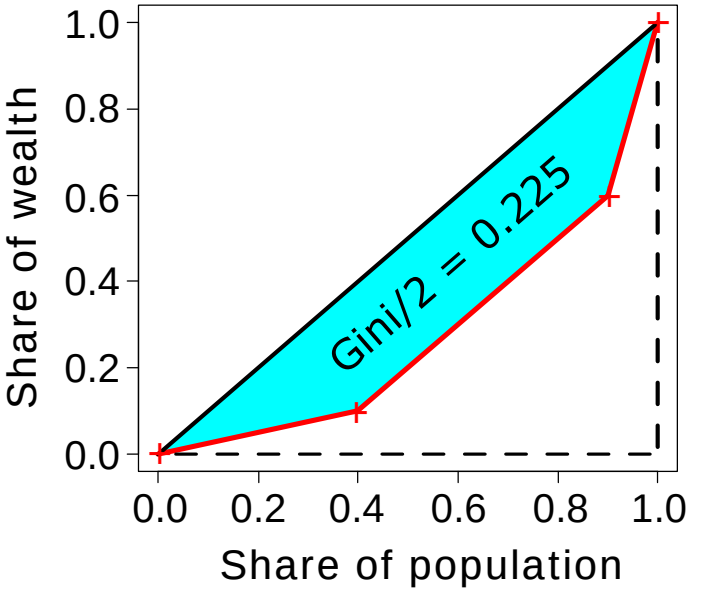
(http://www.ncbi.nlm.nih.gov/pmc/articles/PMC1115376/)

Fifth	'Mean' %
5th	55
4th	21
3rd	13
2nd	8
1st	3

(28 companies, annuity insurance)

Lorenz curve and Gini coefficient

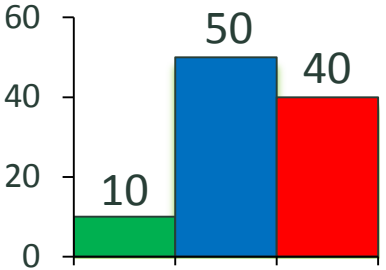
- Lorenz curve (extreme cases: total 'equality' (solid) and total 'inequality' (dashed))
- Gini coefficient (between 0 and 1 after normalization)
- Widely used for (cross-) comparisons



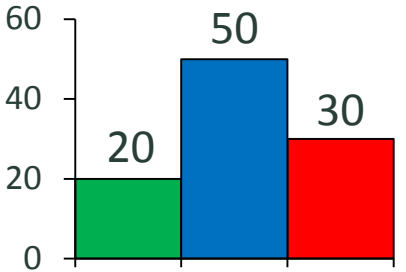
Palma index

- Observation: middle 50% possess roughly 50% of the wealth
- Variation in wealth distribution is determined by ratio of share of upper 10% to share of lower 40%

lower 40% middle 50% upper 10%
(entries are percentages)



Gini = 0.45
Palma = 4.0



Gini = 0.3
Palma = 1.5

Credit: https://www.meissner-cartoons.com/dirk-meissner/

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(My free translation:

To start, let us figure out which curve is really relevant to us.)

Palma index: applicability / interpretability

- For 27 of the 28 companies the 'middle 50%' of policies (between 40% and 90%) account for 47%-55% of the total sum insured
- Ratio of sum insured of top tenth to that of the lower four tenths of the portfolio, or alternatively -

  Ratio of average sum in top tenth to 4*(average sum in lower four tenths)
- Changes most visible in the extreme regions (early warning)

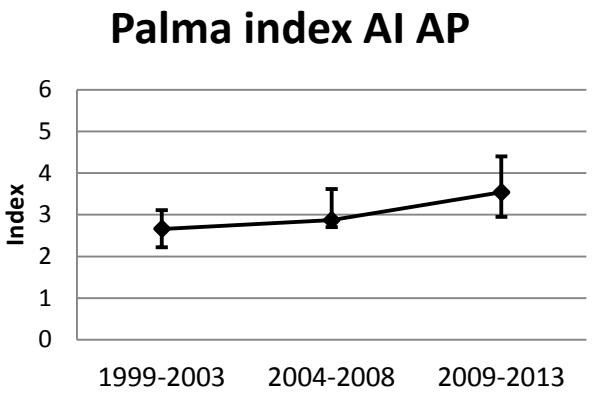
Palma index: Study

- 2 types: annuity insurance (AI) and term life insurance (TLI)
- 3 periods: 2004 (i.e. 1999 – 2003)
  2009 (i.e. 2004 – 2008)
  2014 (i.e. 2009 – 2013)
- For each type 13 companies with complete data (intersection: 12 companies)
- 2 different analyses: all policies (AP) and new policies (NP)

Results for annuity insurance / all policies

Year	Q1	Q2	Q3
2004	2.2	2.7	3.1
2009	2.7	2.9	3.6
2014	3.0	3.5	4.4

(Qi ~ quartiles)



Trend	No.
Up	9
Mixed	4
Down	0

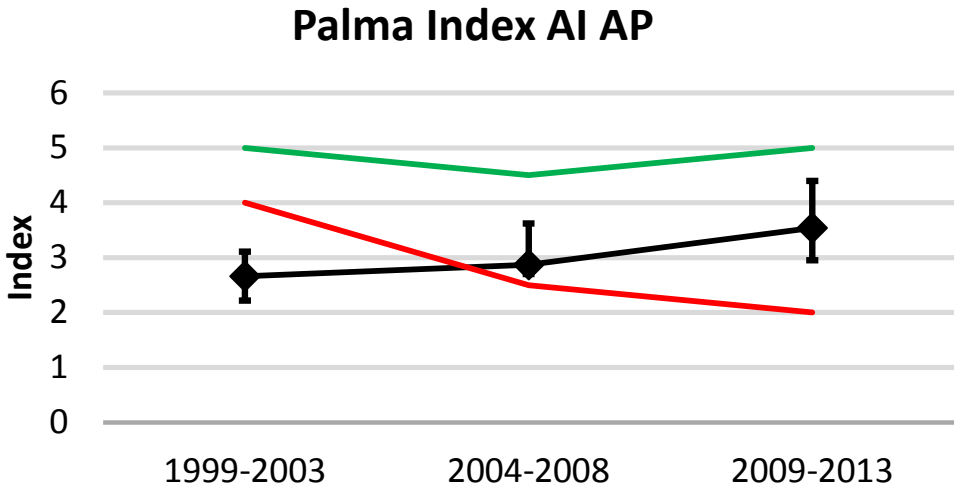
(of 13 companies)

Conclusions from the study

- Clear result: increasing values of Palma index
- More pronounced with AI than with TLI (plausible because of closer relation to capital)
- Similar results but more volatile for NP than for AP (plausible because of numbers of policies)
- Similar results but less pronounced for Gini or Herfindahl

Pool comparisons

The position of individual companies:



From concentration to composition

- The ingredients behind the Palma index (lower, middle, upper), e.g. (20%, 50%, 30%), add up to 100%.
- 'Compositional' data
- Common in chemistry and the geosciences
- Representation by bar charts, pie charts and...

Credit: https://www.meissner-cartoons.com/dirk-meissner/

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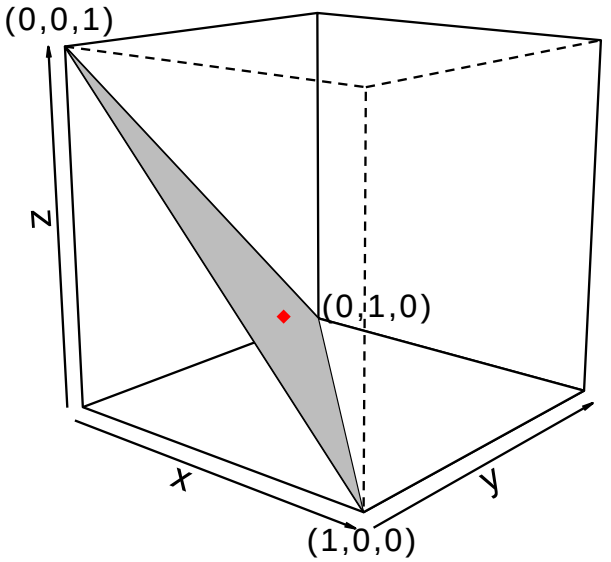
(My free translation:

Let us just start with methodology – the goals will become clear to us later!)

Ternary diagrams



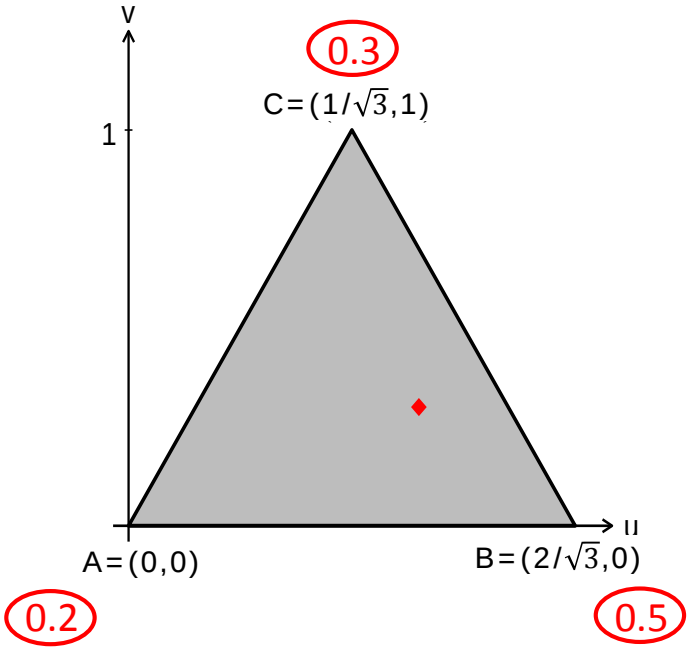
Each vector (x, y, z) with x + y + z = 1 and x, y, z > 0 can be written as

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = x \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + y \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + z \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

and hence is a convex combination of the vertices and corresponds, therefore, to a point in the simplex S^2.

Example: (0.2, 0.5, 0.3)

Ternary diagrams



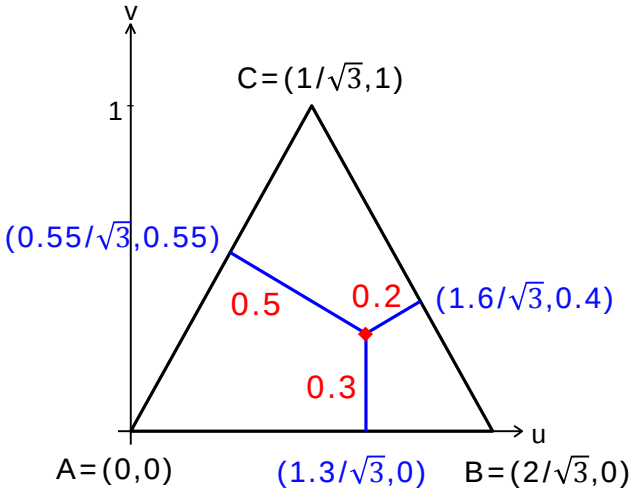
Alternatively (x, y, z) can be regarded as barycentric coordinates in an equilateral triangle ABC with height 1. Then:

$$\begin{pmatrix} u \\ v \end{pmatrix} = x \begin{pmatrix} 0 \\ 0 \end{pmatrix} + y \begin{pmatrix} 2/\sqrt{3} \\ 0 \end{pmatrix} + z \begin{pmatrix} 1/\sqrt{3} \\ 1 \end{pmatrix}$$

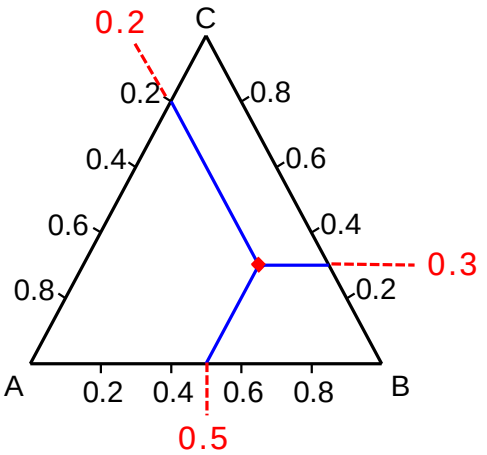
Example: $(1.3/\sqrt{3},\ 0.3)$

Ternary diagrams

Representation of data points (Example: (0.2, 0.5, 0.3))



Viviani



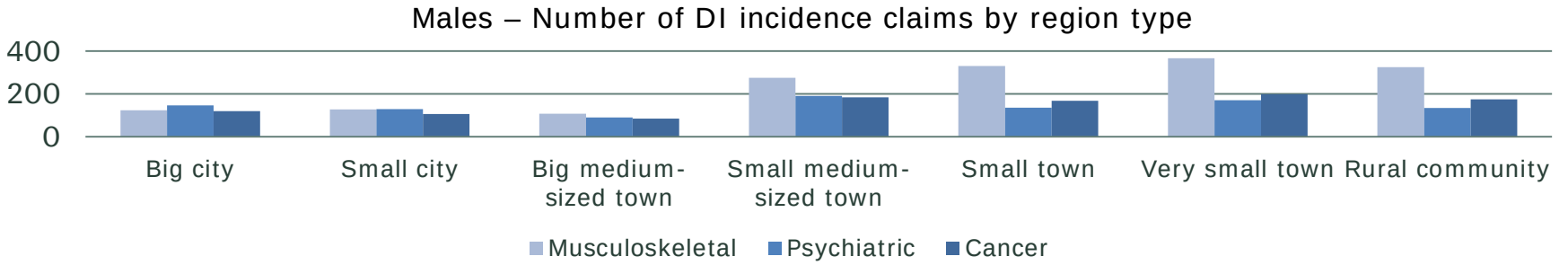
Projection

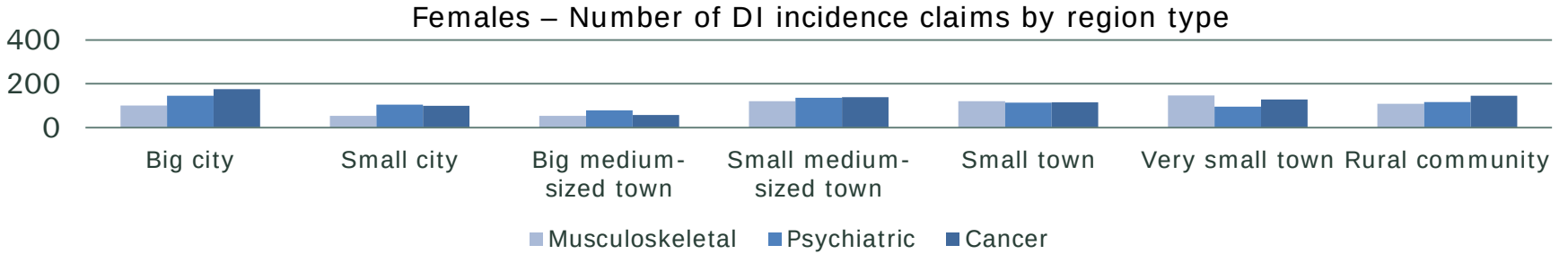
Application

City / region type	Label	Number of inhabitants (*) from	to
Big city	1	500'000	
Small city	2	100'000	499'999
Big medium-sized town	3	50'000	99'999
Small medium-sized town	4	20'000	49'999
Small town	5	10'000	19'999
Very small town	6	5'000	9'999
Rural community	7	1	4'999

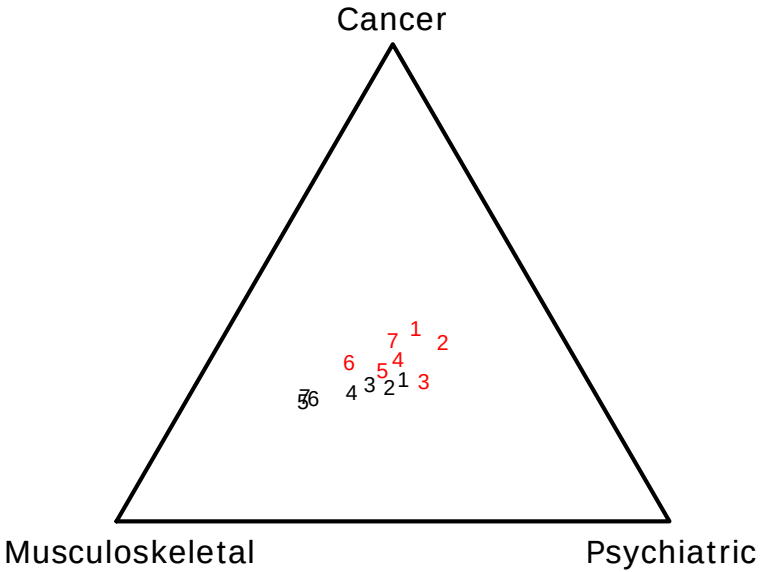
(*) Classification of the Federal Institute for Research on Building, Urban Affairs and Spatial Development (BBSR)

Application





Application



Males			
Label	Musculoskeletal	Psychiatric	Cancer
1	0.32	0.38	0.31
2	0.35	0.36	0.29
3	0.38	0.32	0.30
4	0.42	0.29	0.28
5	0.52	0.21	0.26
6	0.50	0.23	0.27
7	0.51	0.21	0.27

Application

- Male (black) vs. female (red)
- Spread (with males)
- Clustering (with males)
- Coherence (with males)

Important: (only) relative cross-comparisons

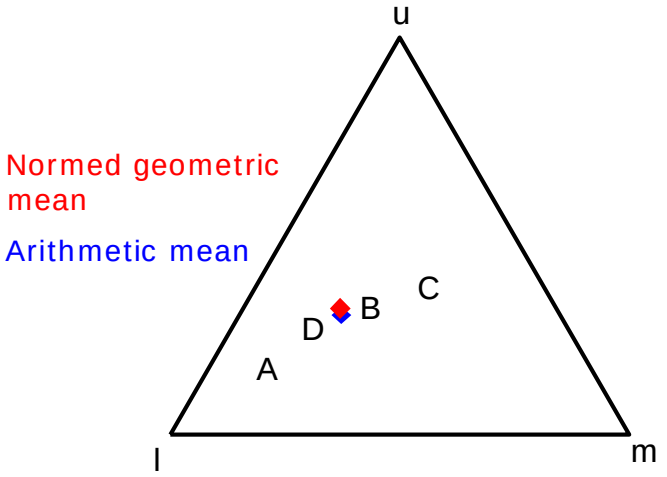
Extensions

Example: 4 Companies, 'Palma-classes', entries in %

	lower 40 %	middle 50 %	upper 10 %
Company A	70	10	20
Company B	40	25	35
Company C	25	35	40
Company D	55	15	30
Arithmetic mean	47.50	21.25	31.25
Normed geometric mean	47.32	20.33	32.34

Extensions (Aitchison approach)

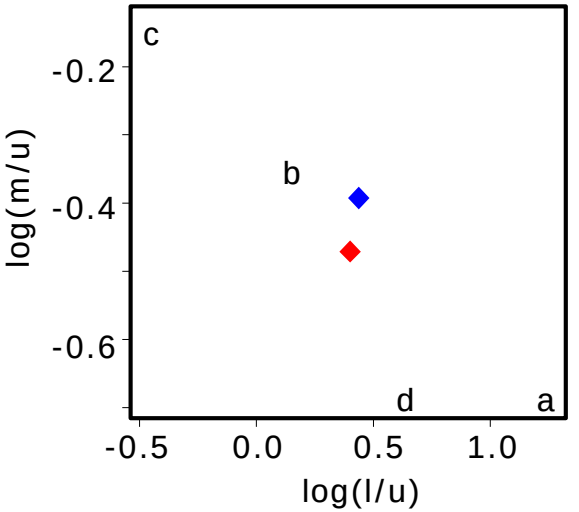
Staying-in-the-simplex



Normed geometric mean

Arithmetic mean

Additive logratio transformation



Transformed normed geometric mean

Transformed arithmetic mean

Conclusions

- Looking at concentration (Palma, Gini, Lorenz) and composition (ternary diagrams) offers great potential for the actuarial field.
- In particular exploratory considerations are recommended; inference (e.g. by means of the 'Aitchison approach') is possible but requires further considerations.
- The interplay of theory and practice is both necessary but also rewarding.

Special Thanks

M. Bauer, S. Brunner, W. Dam, T. Engler, R. Krüger and in particular the participants of the Swiss Re data monitoring workshops for many stimulating and helpful discussions

Credit: https://www.meissner-cartoons.com/dirk-meissner/

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(My free translation:

Would you please be so kind so as to use ordinary paper format for your suggestions. That will make my job so much easier.)

Thank you for your attention!

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