



Interrelations between certain regulatory Requirements, Investments, Strategies and Security in IORPs

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About the speaker



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- Chief Executive Officer
- Born in 1967, he studied mathematics (including actuarial science) and economics at University of Cologne, Germany. After he received his PhD in mathematics in 1994 he started his career in the life-insurance-industry and joined Bayer in 1998, where he worked in several finance-functions. In 2003 he became Chief Financial Officer and in January 2017 additionally Chief Executive Officer (CEO) of all German pension institutions related to Bayer, especially Bayer-Pensionskasse VVaG. He is responsible for all financial results, risks and status of Bayer's DB pension plans world-wide.



- **Bayer-Pensionskasse VVaG**
- Almost 100.000 members
- Size of balance sheet ca. 9 billion EUR
- Assets under management for Bayer pension institutions worldwide ca. 21 billion EUR

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Preconditions (1)

■ Given:

❖ Investment universe with n asset classes ($n \in \mathbb{IN}$)

❖ Investment strategies $\zeta := (z_1, \dots, z_n) \in \mathfrak{R}^n$

z_i = weight of i -th asset-class within the strategy ζ

$r(\zeta, j)$ = (random) return of strategy ζ within the time-period $[j-1;j]$, ($j \in \mathbb{IN}$)

$r_i(j)$ = (random) return of i -th asset class within the time-period $[j-1;j]$

❖ Set of admitted investment strategies (without regulatory maximum weights for asset classes)

$$\mathfrak{S} := \left\{ \zeta \in \mathfrak{R}^n : 0 \leq z_i \wedge \sum_i z_i = 1 \right\}$$

Hence:

- Portfolio always fully invested
- No short-selling

Preconditions (2)

▪ Further given:

- ❖ contributions in time-period $[j-1;j]$:
- ❖ benefits (incl. costs) in the period $[j-1;j]$:
- ❖ pension obligation (technical provision) in j :
- ❖ (random) value of investment portfolio in j (depending on all $r(\zeta, t)$ for all $t \leq j$):

$B(j)$

$L(j)$

$V(j)$

$X(j)$



deterministic

Regulation (1)

■ Regulatory requirements to be analyzed:

- ❖ maximum weights M_i ($i=1,\dots,n$) for (all) single asset classes

Then:
$$\mathfrak{S} := \left\{ \zeta \in \mathbb{R}^n : 0 \leq z_i \leq M_i \wedge \sum_i z_i = 1 \right\}$$

- ❖ Minimum funding requirements:

$$X(j) \geq \alpha(j) \cdot V(j) \quad \forall j \geq 0, \text{ whereas:}$$

$\alpha(j)$ = minimum funding degree, which is required for time j

Since pensions must be paid when due $\implies \alpha(j) \geq L(j)/V(j)$

- ❖ Example: current regulation for a „Pensionskasse“ in Germany: $\alpha(j) = 1 \quad \forall j \geq 0$
(if additional solvency requirements are not taken into account)

Regulation (2)

- ❖ „All benefits are financed when due“ means

$$X(j) \geq L(j) \quad \forall j \geq 0$$

- ❖ Remark 1: This is the weakest possible funding requirement:

$$\alpha(j) = L(j)/V(j)$$

- ❖ Remark 2: If there is no early termination risk (e.g. in case of obligatory memberships within the IORP) this is the real corporate objective of the IORP

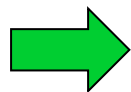
Risk Measures

- Basic Assumption: The investor determines his investment strategy using i.a. two steering parameters: risk and expected return

❖ Continuous risk measure defined on the set of admitted investment strategies $\rho : \mathcal{R}^n \rightarrow \mathcal{R}_+$

Let the risk measure used be convex, i.e. for two investment strategies ζ_1, ζ_2 we have:

$$\rho(\lambda\zeta_1 + (1-\lambda)\zeta_2) \leq \lambda\rho(\zeta_1) + (1-\lambda)\rho(\zeta_2) \quad \forall \lambda \in [0;1]$$



„Diversification doesn't harm from a risk point of view“

Usually the risk measure is short-term oriented (e.g. 1-periodic)
and is used to take short-term balance-sheet restrictions into account

Risk Measures - Examples

- Examples for classic risk measures (1-periodic with regard to return)¹⁾:

- ❖ standard deviation σ of annual returns is convex
- ❖ expected Shortfall with confidence level

$$\alpha: \text{ES}_\alpha(X) = \frac{1}{1-\alpha} \int_\alpha^1 \text{VaR}_z(X) dz \quad \text{is convex}$$

- ❖ $\text{ES}_q(X)$ coincides with $\text{CVAR}_q(X)$ in case of a continuous distribution
- ❖ $\text{VaR}_q(X)$ is generally not convex

1) see e.g. Kriele/Wolf, „Wertorientiertes Management von Versicherungsunternehmen“, Springer Verlag 2012

Definitions:

- If all random returns of the i-th asset class are distributed independantly and identically in the single periods of time, we define:

$$\mu_i := E(r_i(j)) \quad \forall j \quad (\text{expected return for i-th asset-class})$$

$$\mu(\zeta) := E(r(\zeta, j)) = \langle \zeta; (\mu_1, \dots, \mu_n) \rangle \quad \text{for } \zeta \in \mathfrak{T} \text{ and all } j$$

$$\mu(s) := \sup\{\mu(\zeta) : \zeta \in \mathfrak{T} \wedge \rho(\zeta) = s\} \quad \text{for positive real numbers } s$$

$$\rho_0 := \min\{\rho(\zeta) : \zeta \in \mathfrak{T}\}$$

Then: If for $s \geq 0$ $\mu(s)$ exists (i.e. the set on which the supremum is taken is not empty), we have:

$$\mu(s) := \max\{\mu(\zeta) : \zeta \in \mathfrak{T} \wedge \rho(\zeta) = s\}$$

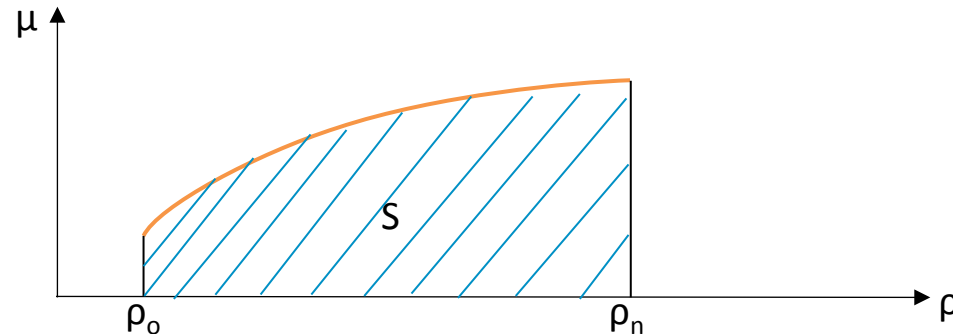
Proof: As a function μ is continous on \mathfrak{T} and the set on which the supremum is taken is a closed subset of the compact set \mathfrak{T} and hence compact itself.

Shape of the efficiente Frontier (1)

Theorem: (i) If $\mu(\rho_1)$ and $\mu(\rho_2)$ exist, then $\mu(\rho)$ exists for all $\rho \in [\rho_1; \rho_2]$

(ii) The set $S := \{(x, y) \in \mathbb{R}^2 : \mu(x) \text{ exists and } 0 \leq y \leq \mu(x)\}$ is convex

(iii) If the asset-class having the highest risk, w.l.o.g. ρ_n , has also the highest expected return, then: $\rho_1 < \rho_2 \Rightarrow \mu(\rho_1) \leq \mu(\rho_2)$, if μ for ρ_1 and ρ_2 exists



(iv) $\mu(\rho)$ is a continuous function in ρ where it is defined

Risk Measure and Regulation (1)

The random returns of the investment strategy depend on the chosen strategy ζ , and hence especially (inter alia) on $\mu(\zeta)$ and $\rho(\zeta)$.

Hence all $X(j)$ and their distributions depend on ζ . Therefore we write $X_\zeta(j)$. We now define the probability always to fulfil the regulatory minimum funding requirements („funding probability“) resp. always being able to pay the (guaranteed) benefits when due („financing probability“).

$$\left. \begin{array}{l} \Phi_\zeta := P(X_\zeta(j) \geq \alpha(j) \cdot V(j) \quad \forall j \geq 0) \\ \text{resp.} \\ \Psi_\zeta := P(X_\zeta(j) \geq L(j) \quad \forall j \geq 0) \end{array} \right\} \Rightarrow \Psi_\zeta \geq \Phi_\zeta$$

We request, that ρ and the regulatory funding requirements resp. the ability of being able to pay all due benefits are coherent to each other. That means:

$$\mu(\zeta_1) = \mu(\zeta_2) \wedge \rho(\zeta_1) < \rho(\zeta_2) \Rightarrow \Phi_{\zeta_1} \geq \Phi_{\zeta_2} \quad \forall \zeta_1, \zeta_2 \quad \text{and} \quad \mu(\zeta_1) > \mu(\zeta_2) \wedge \rho(\zeta_1) = \rho(\zeta_2) \Rightarrow \Phi_{\zeta_1} \geq \Phi_{\zeta_2} \quad \forall \zeta_1, \zeta_2$$

(analogously for Ψ)

Risk Measure and Regulation (2)

Lemma: If Φ (resp. Ψ) depends continuously on the (variable) parameters μ with $\mu(\rho_0) \leq \mu \leq \mu(\rho_n)$ and ρ with $\rho_0 < \rho < \rho_n$, then we can define analogously, that ρ is coherent with the regulatory funding requirements (resp. with the ability of being able to pay all benefits when due):

$$\mu_1 = \mu_2 \wedge \rho_1 < \rho_2 \Rightarrow \Phi_{\mu_1, \rho_1} \geq \Phi_{\mu_2, \rho_2} \quad \forall (\mu_1, \mu_2, \rho_1, \rho_2) \in [\mu(\rho_0); \mu(\rho_n)]^2 \times [\rho_0; \rho_n]^2$$

In this case Φ depends only on the variable model parameters μ and ρ (and not on other variable parameters*).

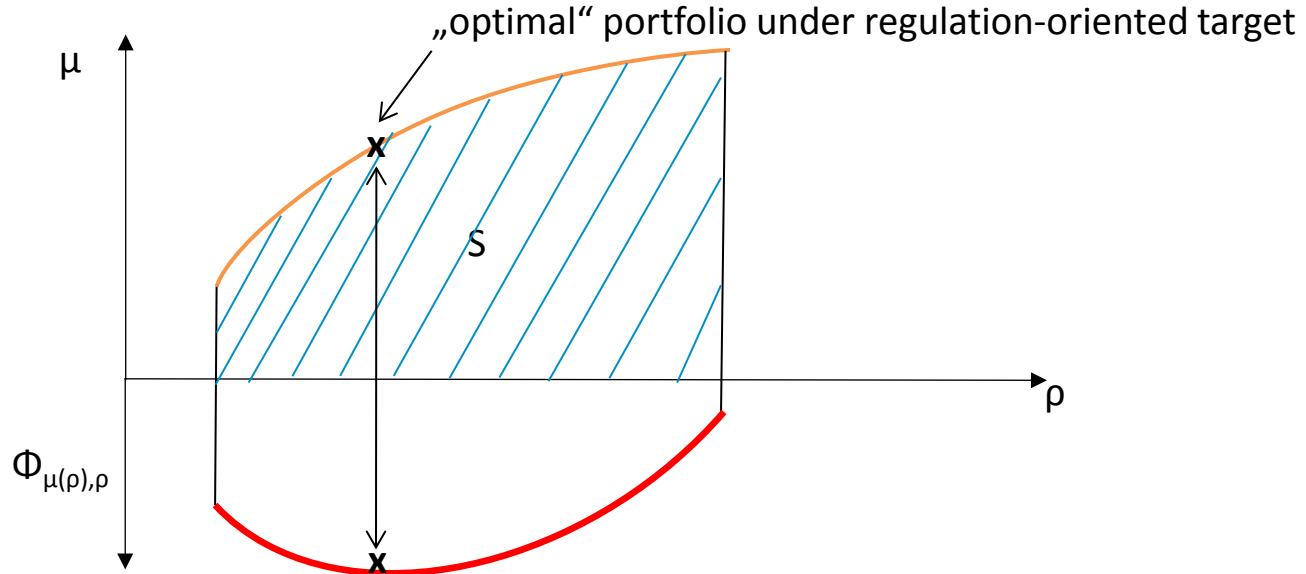
Proof: If we assume that Φ would depend on (at least) one further variable parameter β $\Phi = \Phi_{\mu, \rho, \beta}$ then there would exist $\beta_1 \neq \beta_2$, such that w.l.o.g.: $\Phi_{\mu, \rho, \beta_1} < \Phi_{\mu, \rho, \beta_2}$
Because of $\rho_0 < \rho < \rho_n$ and because of the continuity of Φ there is an $\varepsilon > 0$,
such that: $\Phi_{\mu, \rho, \beta_1} < \Phi_{\mu, \rho + \varepsilon, \beta_2}$ This contradicts the coherence condition.

*) Of course Φ can depend on further parameters, but they have then to be set constant in the assessment of Φ and the coherence in the model

Investor's Behaviour, purely oriented on Regulation

Target of the investors: Find an efficient portfolio such that the probability of fulfilling the regulatory funding requirements at any time is maximised.

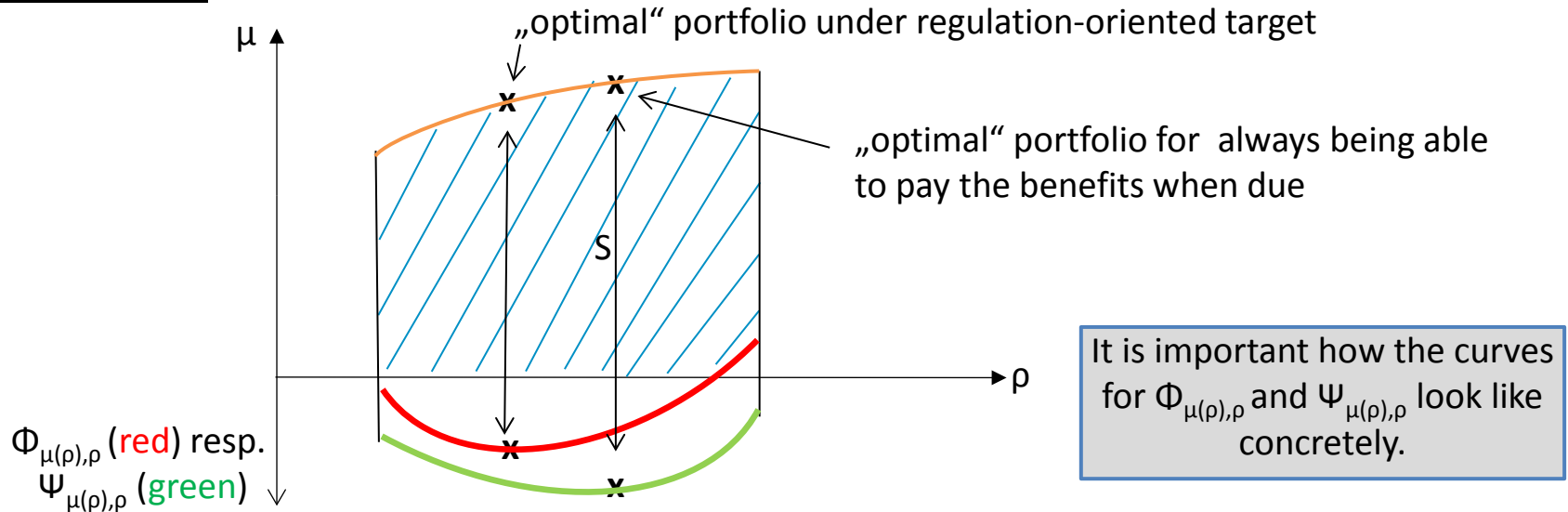
Picture:



ALM-based Investor's Behavior and Regulation

But: The optimum for $\Psi_{\mu(\rho),\rho}$ does not need to coincide with the optimum for $\Phi_{\mu(\rho),\rho}$ (based on experience this is not the case)

Picture (Example):



The Effect of maximum Weights for Asset-Classes



Given: Two regulatory regimes with maximum weights
 $M_i (i=1, \dots, n)$ resp. $\tilde{M}_i (i=1, \dots, n)$
 \implies set of admitted investment strategies \mathfrak{S} resp. $\tilde{\mathfrak{S}}$

We have: $M_i < \tilde{M}_i \forall_i$ (in an extreme case $\tilde{M}_i = 1 \forall_i$)
 $\implies \mathfrak{S} \subset \tilde{\mathfrak{S}}$

If $\tilde{\mu}$ is defined analogously to μ (for the regulatory regime with the \tilde{M}_i),
then:

- (i) the set of definition for μ is contained in the one for $\tilde{\mu}$
(or they both coincide)
- (ii) $\mu(\rho) \leq \tilde{\mu}(\rho) \forall_{\rho \in [\rho_o, \rho_n]}$
- (iii) $\Phi_{\mu(\rho), \rho} \leq \Phi_{\tilde{\mu}(\rho), \rho}$ and $\Psi_{\mu(\rho), \rho} \leq \Psi_{\tilde{\mu}(\rho), \rho} \forall_{\rho \in [\rho_o, \rho_n]}$

Conclusion: By using more generous maximum weights for asset-classes one can in general improve funding and financing probabilities. In a world of rationally acting investors this would speak in favor of a „prudent person principle“.

When do the strict inequalities and relations hold on the previous slide?

Differentiability-Condition:

Let Φ resp. Ψ be continuously differentiable in the variables z_1, \dots, z_n being the weights of the different asset-classes within the investment strategy*

Then: If Φ resp. Ψ takes its maximum on \mathfrak{S} in a point of the subset
$$\partial\mathfrak{S}_{j,k} := \left\{ \zeta \in \mathfrak{R}^n : 0 \leq z_i \leq M_i \forall_i \wedge \sum_{i=1}^n z_i = 1 \wedge z_j = M_j \wedge z_k > 0 \right\}$$
 for $j, k \in \{1, \dots, n\}, j \neq k$,
of the boundary of \mathfrak{S} , and if $\frac{\partial\Phi}{\partial z_j} > \frac{\partial\Phi}{\partial z_k}$ holds in that point,

than in (iii) of the previous slide the strict inequality holds. Analogously for Ψ .

* Remark: In practice this differentiability-condition is not unrealistic, since otherwise we could approximate Φ resp. Ψ by suitable C^∞ -functions with arbitrary exactness (on a compact area of definition)

Choosing the “optimal” Portfolio?



Differentiability-Condition:

(i) $\mu(\rho)$ is two times continuously differentiable in ρ

$$\Rightarrow \mu'(\rho) \geq 0 \quad \mu''(\rho) \leq 0 \quad \text{on } [\rho_o, \rho_n]$$

(let μ also in ρ_o and ρ_n be two times continuously differentiable from the right resp. from the left*)

(ii) Φ and Ψ are two times differentiable in the parameters μ and ρ , $(\mu, \rho) \in [\mu(\rho_o), \mu(\rho_n)] \times [\rho_o, \rho_n]$ **)

** Remark: In practice this differentiability-condition is not unrealistic, since otherwise we could approximate by suitable C^∞ -functions with arbitrary exactness (on a compact area of definition)

* here we can allow, that the absolute value of the (unilateral) differential in ρ_o gets infinite

The Shape of Φ resp. Ψ

Consider the map $F_{\Phi}: \rho \longrightarrow \Phi_{\mu(\rho), \rho}$

- ❖ It is two times continuously differentiable (because of assumptions on previous slide)
- ❖ For the first and second order differential of F_{Φ} we get:

$$F_{\Phi}'(\rho) = \frac{\partial \Phi}{\partial \mu} \cdot \mu'(\rho) + \frac{\partial \Phi}{\partial \rho}$$

$$F_{\Phi}''(\rho) = (\mu'(\rho), 1) \cdot H(\Phi) \cdot (\mu'(\rho), 1)^T + \frac{\partial \Phi}{\partial \mu} \cdot \mu''(\rho),$$

whereas $H(\Phi)$ is the Hesse-Matrix of Φ in the point $(\mu(\rho), \rho)$

- ❖ Analogously for F_{Ψ}

Convention: we will write F , if als well F_{Φ} and also F_{Ψ} can be meant.

Optima

For the maximum of F on $[\rho_0, \rho_n]$ we have:

(i) F assumes its maximum either in one of the boundary points ρ_0 or ρ_n

OR

(ii) there is otherwise a point $\rho_{\max} \in [\rho_0, \rho_n]$ with:

$$\mu' \cdot \frac{\partial \Phi}{\partial \mu} = - \frac{\partial \Phi}{\partial \rho}$$

$$\wedge (\mu', 1) \cdot H(\Phi) \cdot (\mu', 1)^T < - \frac{\partial \Phi}{\partial \mu} \cdot \mu''(\rho)$$

Remark: Because of $\frac{\partial \Phi}{\partial \mu} \geq 0$ and $\mu''(\rho) \leq 0$

the right side in the last condition is always ≥ 0 . Hence it is sufficient for the last condition, that

$$\frac{\partial^2 \Phi}{\partial \mu^2} < 0 \wedge \frac{\partial^2 \Phi}{\partial \mu^2} \cdot \frac{\partial^2 \Phi}{\partial \rho^2} - \left(\frac{\partial^2 \Phi}{\partial \mu \partial \rho} \right)^2 > 0$$

Behavior of an ALM-focussed Investor

Assumption: the ALM-focussed investor either tries to

(i) maximize F_Φ : $F_\Phi(\rho) = \Phi_{\mu(\rho),\rho} \rightarrow \max$

OR

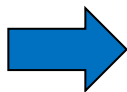
(ii) maximize F_Ψ under the boundary condition, that:

$$\Phi_{\mu(\rho),\rho} \geq 1 - \alpha$$

($1 > \alpha > 0$ is the confidence-level, e.g. $\alpha = 0,5\%$)

Remark: in (ii) we look for the maximum of F_Ψ on the set

$$\mathbf{M}_{\Phi,\alpha} := \{\rho \in [\rho_0, \rho_n] : \Phi_{\mu(\rho),\rho} \geq 1 - \alpha\}$$



To which extend a deregulation (here: an easing of the minimum funding requirements, i.e. making the $\alpha(j)$ smaller) has in total a positive effect, depends on the shape of F_Φ and F_Ψ relative to each other.

Change of Minimum Funding Requirements



Given: Two regimes with minimum funding degrees

$$\alpha(j) \quad (j \geq 0) \quad \text{resp.} \quad \tilde{\alpha}(j) \quad (j \geq 0)$$

We have: If $\tilde{\Phi}$ is defined analogously to Φ (only for the regime with minimum funding-degrees $\tilde{\alpha}(j)$), then from

$\alpha(j) \geq \tilde{\alpha}(j) \quad \forall_j$ (extreme case : $\tilde{\alpha}(j) = L(j)/V(j)$), it follows, that

$$\Phi_{\mu(\rho),\rho} \leq \tilde{\Phi}_{\mu(\rho),\rho} \wedge M_{\Phi,\alpha} \subseteq M_{\tilde{\Phi},\alpha}$$

Proof: The claims follow immediately out of the definitions.

Effect from Easing of Minimum Funding Requirements

Let Φ depend 2x continuously differentiable on the minimum funding degrees $\alpha(j)$

Let $\rho_{\max} := \text{maximum point of } F_{\Phi} \text{ in } [\rho_0, \rho_n], \tilde{\rho}_{\max}$ for $F_{\tilde{\Phi}}$ analogously

(I) Assumption, that the investor tries to maximise F_{Φ} .

Theorem I:

(a) If $\rho_{\max} < \rho_n$ and if at ρ_{\max} we have $\mu' \cdot \frac{\partial \Psi}{\partial \mu} > -\frac{\partial \Psi}{\partial \rho}$ and also $F_{\Psi}(\rho) \leq F_{\Psi}(\rho_{\max}) \forall \rho < \rho_{\max}$

then: by a sufficient easing of minimum funding requirements we can achieve that

$$\Psi_{\mu}(\tilde{\rho}_{\max}, \tilde{\rho}_{\max}) > \Psi_{\mu}(\rho_{\max}, \rho_{\max}) ;$$

in this case the easing leads to a more risky investment strategy.

(b) If $\rho_{\max} > \rho_0$ and if at ρ_{\max} we have $\mu' \cdot \frac{\partial \Psi}{\partial \mu} < -\frac{\partial \Psi}{\partial \rho}$ and also $F_{\Psi}(\rho) \leq F_{\Psi}(\rho_{\max}) \forall \rho > \rho_{\max}$

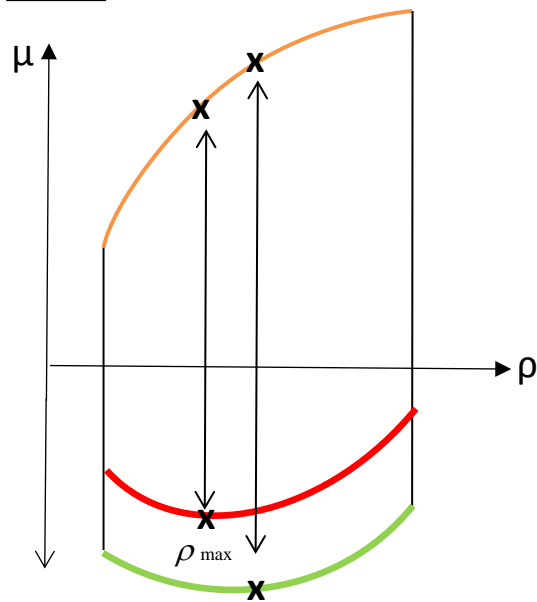
then: by a sufficient easing we can achieve $\Psi_{\mu}(\tilde{\rho}_{\max}, \tilde{\rho}_{\max}) > \Psi_{\mu}(\rho_{\max}, \rho_{\max})$

In this case the easing leads to a less risky investment strategy .

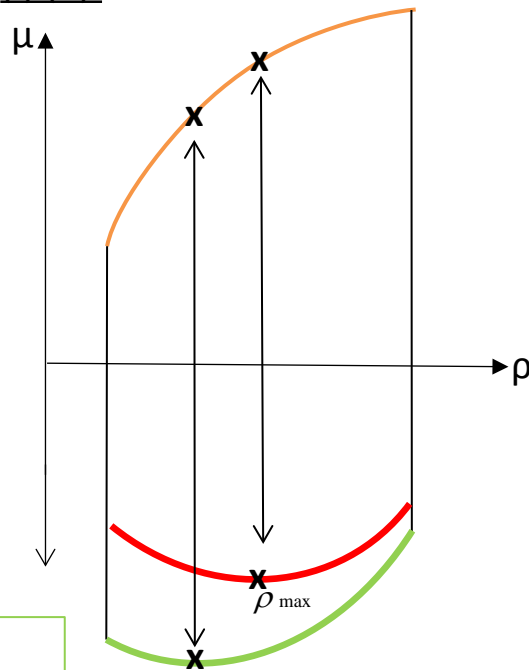
(c) If F_{Ψ} is optimal at ρ_{\max} , than an easing cannot lead to any improvement.

Consequences out of an Easing of Minimum Funding Requirements – Example-Diagrams

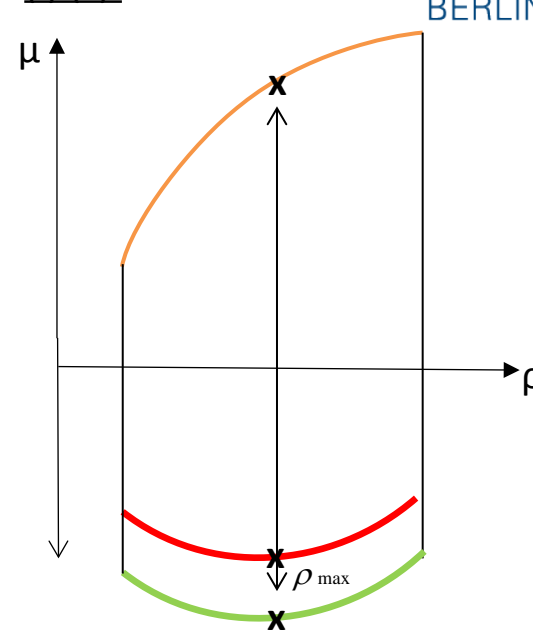
(I) (a):



(I) (b):



(I) (c):



$\Phi_{\mu(\rho),\rho}$ (red) resp. $\Psi_{\mu(\rho),\rho}$ (green)

Consequences out of an Easing of Minimum Funding Requirements

With the same assumptions and notations as on the second-last slide, we get:

(II) assumption, that the investor tries to maximize F_ψ under the boundary condition

$$\Phi_{\mu(\rho),\rho} > 1 - \alpha \quad \text{for a given („small“) } \alpha > 0$$

Theorem II:

- (a) If F_ψ assumes its maximum in ρ_n (this is e.g. the case, when $\mu' \cdot \frac{\partial \Psi}{\partial \mu} > -\frac{\partial \Psi}{\partial \rho}$ on (ρ_o, ρ_n)) and if $\rho_n \notin M_{\Phi,\alpha}$, then a sufficient easing of minimum funding requirements leads to a more risky investment strategy delivering a higher maximum value of F_ψ under the given boundary conditions.
- (b) If F_ψ assumes its maximum in ρ_o (this is e.g. the case, when $\mu' \cdot \frac{\partial \Psi}{\partial \mu} < -\frac{\partial \Psi}{\partial \rho}$ on (ρ_o, ρ_n)) and if $\rho_o \notin M_{\Phi,\alpha}$, then a sufficient easing of minimum funding requirements leads to a less risky investment strategy delivering a higher maximum value of F_ψ under the given boundary conditions.

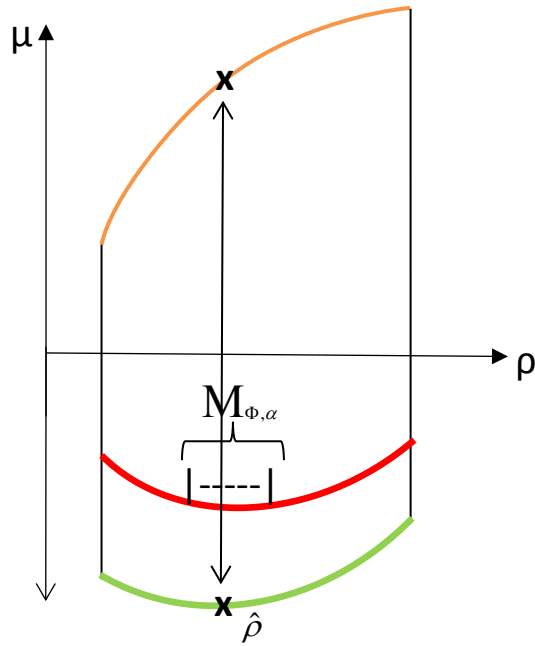
Consequences out of an Easing of Minimum Funding Requirements

(II) continued

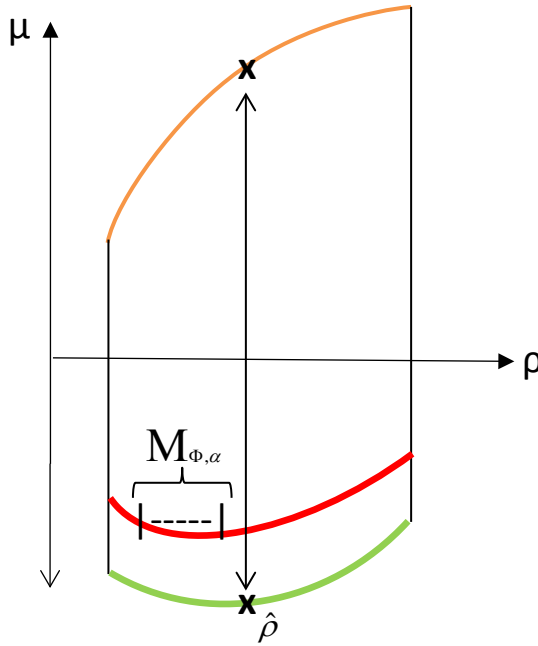
- (c) If F_ψ assumes its maximum in $\hat{\rho} \in M_{\Phi, \alpha}$, then an easing of minimum funding requirements has no impact.
- (d) If F_ψ assumes its maximum in a point $\hat{\rho} \in (\rho_o, \rho_n)$ and if $\hat{\rho} > \rho \forall \rho \in M_{\Phi, \alpha}$, then a sufficient easing of minimum funding requirements leads to a more risky investment strategy delivering a higher maximum value of F_ψ under the given boundary conditions.
- (e) If F_ψ assumes its maximum in a point $\hat{\rho} \in (\rho_o, \rho_n)$ and if $\hat{\rho} < \rho \forall \rho \in M_{\Phi, \alpha}$ then a sufficient easing of minimum funding requirements leads to a less risky investment strategy delivering a higher maximum value of F_ψ under the given boundary conditions.

Consequences out of an Easing of Minimum Funding Requirements – Example Diagrams

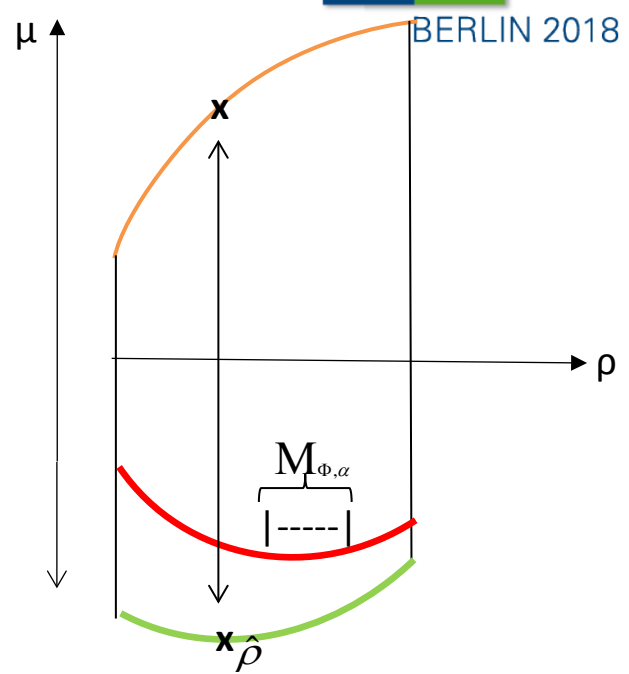
(II) (c):



(II) (d):



(II) (e):



$\Phi_{\mu(\rho),\rho}$ (red) resp. $\Psi_{\mu(\rho),\rho}$ (green)

Which situations do we typically have in which Capital Market Scenarios? (1)

- (i) (approx.) risk-free yield¹⁾ >> fixed actuarial discount rate²⁾ \oplus „over-funding“
(High-Interest-Environment)

→ $\frac{\partial \Psi}{\partial \mu}$ relatively small on (ρ_o, ρ_n) . (reason: Ψ relatively big and can hardly be further enhanced)

→ $\mu' \cdot \frac{\partial \Psi}{\partial \mu} < -\frac{\partial \Psi}{\partial \rho}$ on (ρ_o, ρ_n) . (reason: increasing risk would deliver a relatively strong negative contribution)

At the same time: analogous inequalities for Φ

→ case (I) (c) resp. (II) (c)

→ no improvement by de-regulation \oplus strategy with minimum risk

(the green arrows are not to be interpreted as mathematical consequences in a strict sense, they only make clear, what in such a scenario typically holds)

¹⁾ asset class with lowest risk, e.g. AAA-rated government bonds

²⁾ defined by tariff; in Germany: „tariflicher Rechnungszins“

Which situations do we typically have in which Capital Market Scenarios? (2)

(ii) Just like (i), only that the inequality for the risk-free yield does not hold that clearly

$$\rightarrow \mu' \cdot \frac{\partial \Phi}{\partial \mu} \begin{cases} < -\frac{\partial \Phi}{\partial \rho} \text{ near } \rho_n \\ > -\frac{\partial \Phi}{\partial \rho} \text{ near } \rho_o \end{cases}$$

(since at some point a too much increased risk diminishes the probability of always fulfilling the regulatory funding requirements too strongly)

At the same time: $\mu' \cdot \frac{\partial \Psi}{\partial \mu} < -\frac{\partial \Psi}{\partial \rho}$ either on the whole set (ρ_o, ρ_n)

or if this inequality holds for Φ (resp. Ψ) then it holds also for Ψ (resp. Φ)

\rightarrow case (I) (b) or sometimes (II) (e) or (II) (b) (resp. I (a) or sometimes II (d) or II (a))

\rightarrow improvement by deregulation \oplus less risky (resp. more risky) investment strategy

Which situations do we typically have in which Capital Market Scenarios? (3)

(iii) (approx.) risk-free yield \ll actuarial discount rate
 (“low interest environment“)

→ $\frac{\partial \Psi}{\partial \mu}$ Relatively high on $(\rho_o, \bar{\rho})$ for $\bar{\rho} \in (\rho_o, \rho_n]$
 („higher expected return desperately needed“)

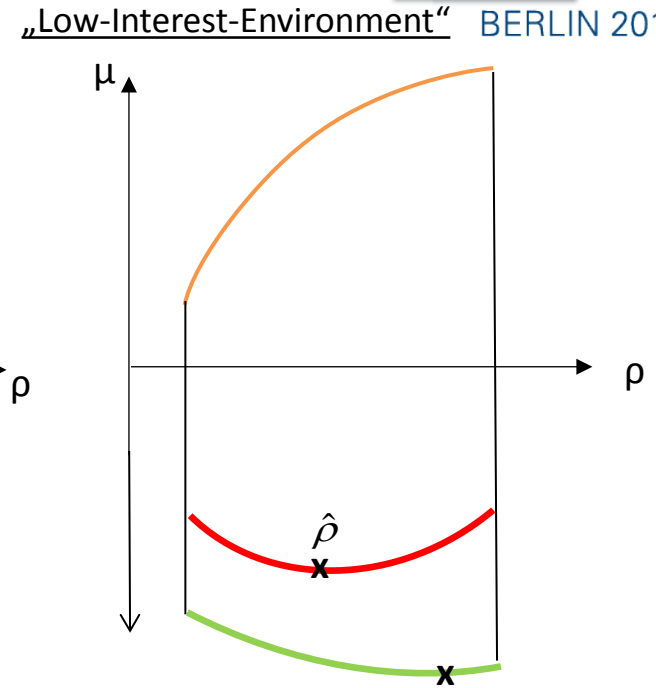
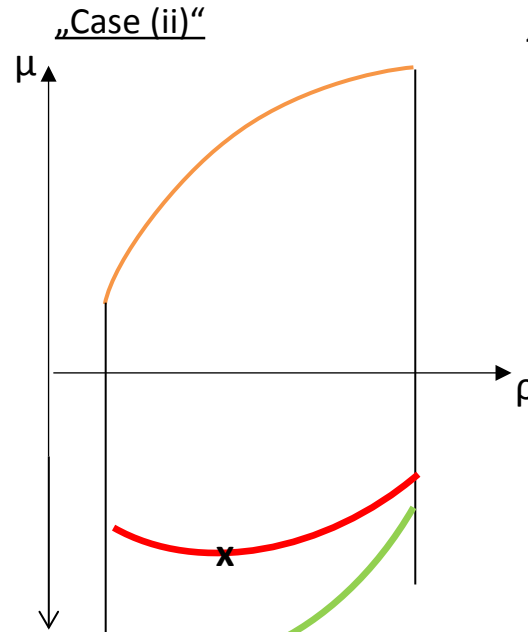
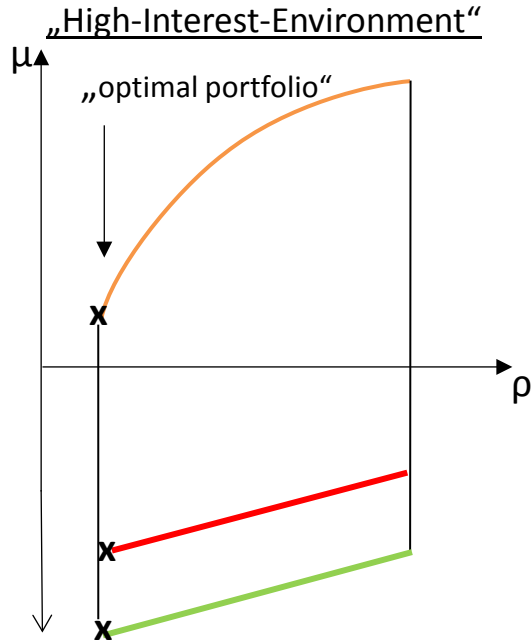
→ $\mu' \cdot \frac{\partial \Psi}{\partial \mu} > -\frac{\partial \Psi}{\partial \rho}$ („Increase in short-term risk causes only relatively little harm to the probability of being able to pay the benefits when due“)

At the same time often: $\mu' \cdot \frac{\partial \Phi}{\partial \mu} < -\frac{\partial \Phi}{\partial \rho}$ right of a point $\hat{\rho} \in [\rho_o, \bar{\rho})$

→ Case (I) (a) resp. (II) (d) or (III) (a)

→ Improvement by de-regulation \oplus more risky strategy

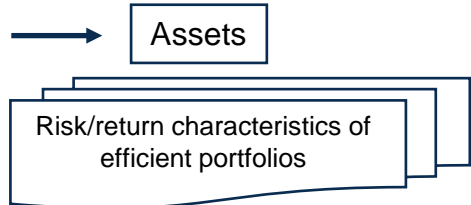
Capital Market Scenarios: Example Diagrams



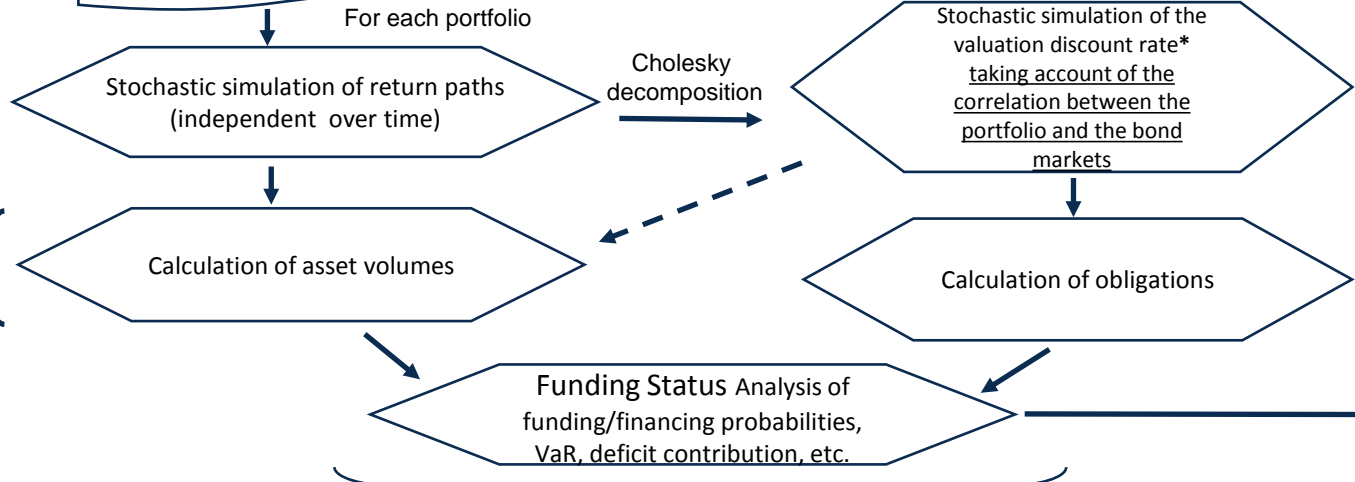
$\Phi_{\mu(\rho),\rho}$ (red) resp. $\Psi_{\mu(\rho),\rho}$ (green)

Determining F_{Φ} and F_{Ψ} by using stochastic models (Example)

Inputs:
Risk/return and correlation assumptions for various asset classes



Actuarial inputs:
Workforce structure, demographic structure of beneficiaries, etc.



New simulation, e.g. different portfolio or modified assumptions, such as deficit contributions

* in case of a **fixed** actuarial discount rate the latter has to be used

Per path (usually 3,000 paths in total) min. 15 years into the future

Application to a Solvency II Type Regime – Sketch (1)

- ❖ no fixed maximum weights for single asset classes
- ❖ no fixed minimum funding requirements
- ❖ instead: $\alpha(j) = 1 + R + z(j)$, with
 - R = fixed margin (% of technical provisions)
 - $z(j)$ = solvency capital requirement at time j
- ❖ z depends on all asset- and liability related risks (incl. operative risk)
- ❖ hence: z depends on ρ and $\mu(\rho)$
- ❖ hence: funding requirements are functions of ρ and $\mu(\rho)$:

$$\alpha(j, \rho) = : \alpha_j(\rho)$$

Application to a Solvency II Type Regime – Sketch (2)

- ❖ compare two regimes $\alpha_j(\rho)$ and $\tilde{\alpha}_j(\rho)$ (second regime could e.g. lead to lower funding requirements, if a less conservative calibration for stress parameters is used)
- ❖ define $\Phi, \tilde{\Phi}, F_\Phi, F_{\tilde{\Phi}}, \Psi, F_\Psi$ analogously as before
- ❖ they will depend on $\alpha_j(\rho)$ and $\tilde{\alpha}_j(\rho)$
- ❖ for the first and second order differential of F_Φ we get

$$F_\Phi'(\rho) = \frac{\partial \Phi}{\partial \mu} \cdot \mu'(\rho) + \frac{\partial \Phi}{\partial \rho} + \sum_{j=1}^{\omega} \frac{\partial \Phi}{\partial \alpha_j} \cdot \alpha_j'(\rho) \qquad F_\Phi''(\rho) = \left(\mu'(\rho), 1, \alpha_1'(\rho), \dots, \alpha_\omega'(\rho) \right) \cdot H(\Phi) \cdot \left(\mu'(\rho), 1, \alpha_1'(\rho), \dots, \alpha_\omega'(\rho) \right)^T + \frac{\partial \Phi}{\partial \mu} \cdot \mu''(\rho) + \sum_{j=1}^{\omega} \frac{\partial \Phi}{\partial \alpha_j} \cdot \alpha_j''(\rho),$$

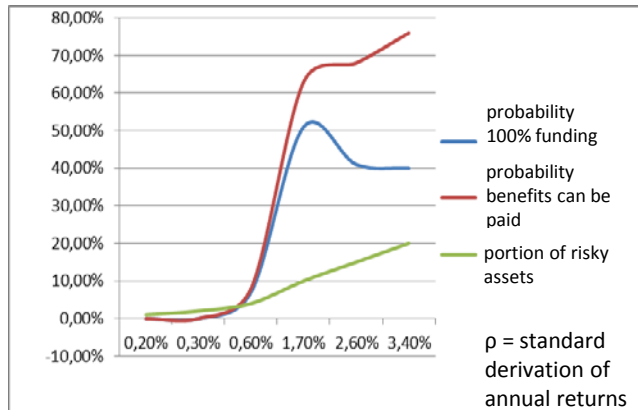
- ❖ the rest of the theory can be done analogously as before, especially analoga for Theorem I and Theorem II
- ❖ assessment for capital market scenarios also analogously yielding similar statements if one assumes $\frac{\partial \alpha_j}{\partial \rho} \geq 0$

Concrete Examples for F_{Φ} and F_{Ψ} (1)

Simulation of a (fictive) IORP, without new members entering the scheme

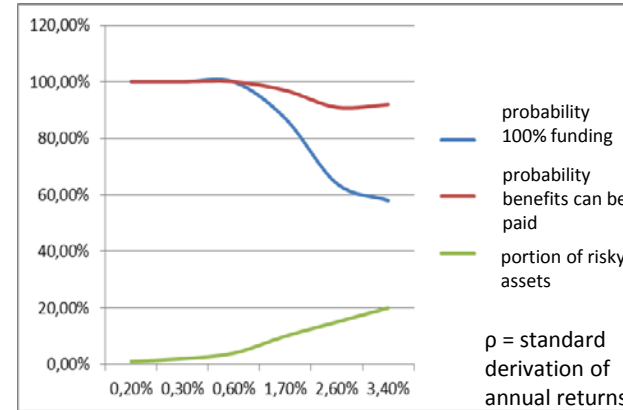
Example 1:

Asset-class with lowest risk has a return expectation being below the actuarial discount rate



Example 2:

Asset-class with lowest risk has a return-expectation of above the actuarial discount rate

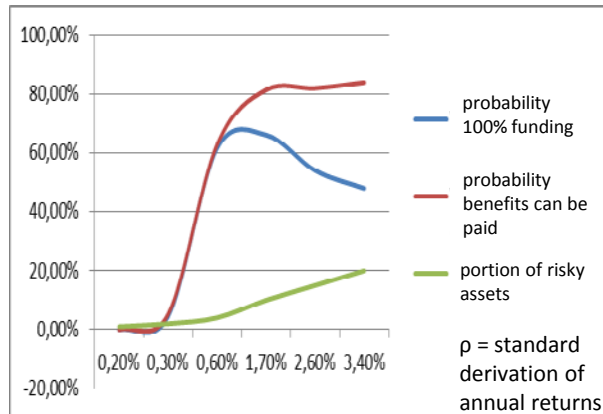


Concrete Examples for F_Φ and F_Ψ (2)

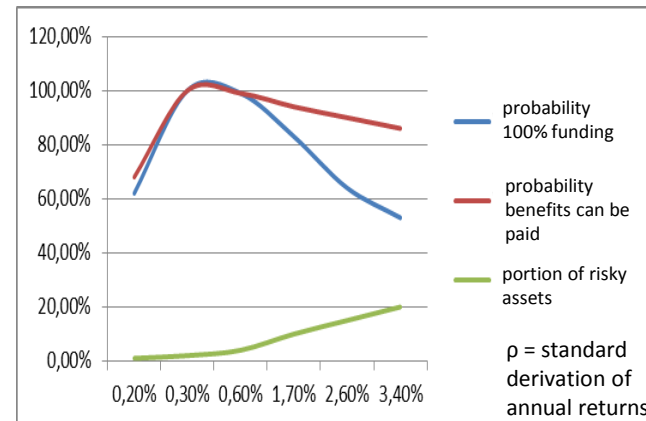
Simulation of a (fictive) IORP, without new members entering the scheme

Example 3:

Asset-class with the lowest risk has a return expectation of 7 bp above the actuarial discount rate



Example 4: Asset-class with the lowest risk has a return-expectation of 27 bp above the actuarial discount rate



Conclusion and closing Remarks



- ❖ This is no argumentation against regulation!
- ❖ Regulation is necessary to limit potential negative consequences out of investor's irrational behavior
- ❖ Maximum weights for asset-classes should be set that way, that they cannot be completely exhausted by ALM-focussed investors anyhow, in case a „prudent person principle“ is not possible or not wanted.
- ❖ An easing of minimum funding requirements does not automatically lead to more risky investment strategies.
- ❖ Especially for IORP's with obligatory memberships (no early termination risks) any regulation using strict minimum funding requirements should be seen critical. It only has to be assured, that the guaranteed benefits can be paid when due.
- ❖ A regulation being flexible on time should be considered (flexible reaction to prevailing capital market scenarios). Such a regulation should have a more anti-cyclical character.

Thank you very much for your attention!



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Backup: Sketches of Proof



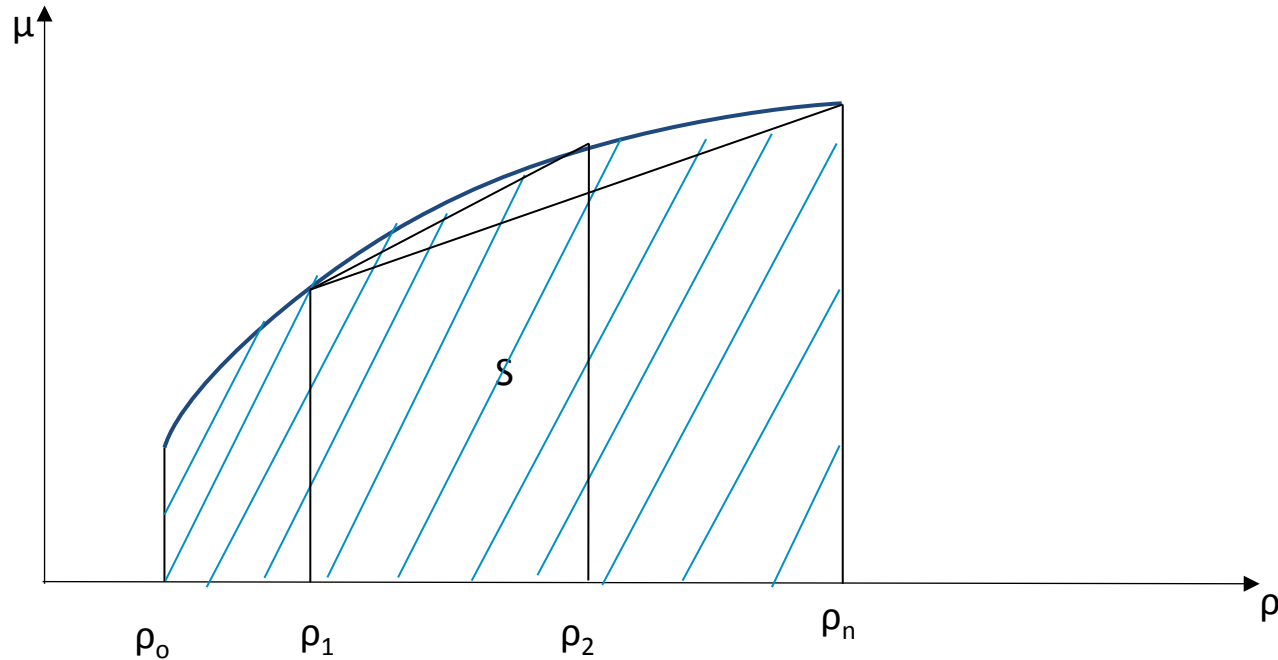
Sketch of Proof regarding the Theorem on the Shape of the Efficient Frontier

- (i) Choose ζ_1 and ζ_2 so that $\rho_i = \rho(\zeta_i)$ for $i=1$ resp. 2. Since ρ is continuous in ζ , the term $\rho(\lambda\zeta_1 + (1-\lambda)\zeta_2)$ assumes any value between ρ_1 and ρ_2 for $\lambda \in [0;1]$.
- (ii) Making use of (i) this follows immediately from $\rho(\lambda\zeta_1 + (1-\lambda)\zeta_2) \leq \lambda\rho(\zeta_1) + (1-\lambda)\rho(\zeta_2)$ and $\mu(\lambda\zeta_1 + (1-\lambda)\zeta_2) = \lambda\mu(\zeta_1) + (1-\lambda)\mu(\zeta_2)$.
- (iii) Let w.l.o.g. the n -th asset-class being the one having the highest risk. If ρ_1 is arbitrary, but fixed, then for an arbitrary $\rho_2 > \rho_1$ the curve

$$\gamma(t) := \begin{pmatrix} \rho_1 + t(\rho_2 - \rho_1) \\ \mu(\rho_1 + t(\rho_2 - \rho_1)) \end{pmatrix}, \quad t \in [0;1]$$

lies above or on the straight line through the points $(\rho_1; \mu(\rho_1))$ and $(\rho_n; \mu(\rho_n))$ because of (ii). But this straight line is increasing by definition.

Illustrative Picture to (iii) of the last Proof



Sketch of Proof (continued)

- (iv) We prove this by contradiction: if we assume, that $\mu(\rho)$ is not continuous at some $r \in (\rho_0, \rho_n)$, then there would exist an $\varepsilon > 0$, so that for all $\delta > 0$ there would be an $s \in (\rho_0, \rho_n)$ so that $|s - r| < \delta$ and $|\mu(s) - \mu(r)| > \varepsilon$.
With this the absolute value of the slope m of the line segment between the points $(r, \mu(r))$ and $(s, \mu(s))$ can get arbitrarily large, only if δ is chosen small enough, because:

$$|m| = \frac{|\mu(s) - \mu(r)|}{|s - r|} \geq \frac{\varepsilon}{\delta} \rightarrow \infty \text{ for } \delta \rightarrow 0$$

If we now choose δ sufficiently small, we can achieve the following:

If $s < r$, then the line segment between $(s, \mu(s))$ and $(r, \mu(r))$ lies below the line segment between $(\rho_0, \mu(\rho_0))$ and $(r, \mu(r))$ in the (ρ, μ) -coordinate-system. If $s > r$, then the line segment between $(\rho_0, \mu(\rho_0))$ and $(s, \mu(s))$ lies above the line segment between $(s, \mu(s))$ and $(r, \mu(r))$. Because of the convexity of the set S from (ii) this contradicts the definition of $\mu(s)$ resp. $\mu(r)$ (as being the maximum achievable expected return for the given risk level s resp. r).

Sketch of Proof: Implications of regulatory Maximum Quotas



Proof:

- (i) follows from $\mathfrak{S} \subset \tilde{\mathfrak{S}}$
- (ii)
$$\begin{aligned} \mu(\rho) &= \sup\{\mu(\zeta) : \zeta \in \mathfrak{S} \wedge \rho(\zeta) = \rho\} \\ &\leq \sup\{\mu(\zeta) : \zeta \in \tilde{\mathfrak{S}} \wedge \rho(\zeta) = \rho\} = \tilde{\mu}(\rho) \end{aligned}$$
- (iii) follows directly from (ii) because of the definition of Φ resp. Ψ and because of the coherence assumption stipulated in chapter 2.

Sketch of Proof for Theorem I and Theorem II

Proof of Theorem I

- (a) $\tilde{\Phi}$ depends continuously differentiable from all $\alpha(j)$, and $\tilde{\Phi}$ becomes Ψ if all $\tilde{\alpha}(j)$ become $L(j)/V(j)$, and hence $\tilde{\rho}_{\max}$ then becomes the point where F_{Ψ} takes its optimum. Since $F_{\Psi}(\rho) \leq F_{\Psi}(\rho_{\max})$ for all $\rho < \rho_{\max}$ and since $F_{\Psi}'(\rho_{\max}) > 0$ this point is located on the right hand side of ρ_{\max} . Since $F_{\Psi}'(\rho_{\max}) > 0$ the maximum value of F_{Ψ} is greater than $F_{\Psi}(\rho_{\max})$. From this our claim follows.
- (b) can be proven analogously
- (c) is trivial. Q.E.D

The proof of Theorem II is more or less obvious.