Interrelations between certain regulatory requirements, investment strategies and security of benefits in occupational pension institutions (IORPs)

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1. Introduction

This paper analyses the interrelations between regulatory requirements for IORPs and the probability Ψ, that all guaranteed benefits can be paid over a long-term time horizon, when they are due. We call Ψ the financing probability. In this context we assess further, which impact regulatory frameworks have on the structure of investment strategies applied by the IORPs. In our analysis we point out, how these results can look like in certain different capital market environments. We always assume, that an IORP shows a rational investment behavior, which is strictly asset-liability-management-(ALM)-based. Introductions to ALM can e.g. be found in [5] or [1].

There is a given investment universe available for the IORP consisting out of n ϵ IN asset classes (European equities, European bonds, …, US equities, …, real estate …). An investment strategy is a vector $\zeta := (z_1,...,z_n) \in \Re^n$, where z_i is the (relative) weight of the i-th asset class within the investment strategy. Leaving (for the time being) regulatory maximum quotas for certain different asset classes aside, we define the set of all permitted investment strategies as: $\Im := \left\{ \zeta \in \Re^n : 0 \le z_i \wedge \sum_i z_i = 1 \right\}$

Since $z_i \geq 0$ for all i, we assume, that short-selling is excluded, and since the sum of all z_i is 1, we assume, that the portfolio is always completely invested. For all time-periods [j-1;j] (j ϵ IN_+) let the contribution payments B(j), and the benefit payments L(j) as well as the actuarial (net) benefit obligations V(j) be given. Due to the nature of the problem, which this paper is treating, we assume them to be deterministic. In contrast, the value of the investment portfolio X(j) at the end of the respective time-period is a random variable, which depends from all (random) investment yields $r(\zeta,t)$ for t ϵ IN_+, t ≤ j, which could be achieved by applying the investment strategy ζ in the time-periods [t-1;t].

2. Portfolio Efficiency

We assume, that the IORP, when choosing the investment strategy, considers inter alia the expected annual investment yield for the strategy and also the level of risk of that strategy. We assume further, that the IORP, in order to measure this risk, uses a continuous risk measure $\rho : \Re^n \to \Re_+$, which is convex, i.e. for each two investment strategies ζ_1, ζ_2 the following inequality holds:

$$\rho(\lambda\zeta_1 + (1-\lambda)\zeta_2) \le \lambda\rho(\zeta_1) + (1-\lambda)\rho(\zeta_2) \quad \forall \lambda \in [0;1]$$

In common language this means, that diversification is not disadvantageous under risk aspects. Typically this risk measure is often short-term-oriented (e.g. towards one accounting period) and can serve to take short-term balance sheet requirements of the IORP into account. Examples for convex risk measures are the standard deviation σ of returns or the Expected Shortfall[1] for a given confidence level

[1] In case of a continuous probability distribution this coincides with the Conditional Value at Risk (CVAR).

$$\mathrm{ES}_\alpha(X) = \frac{1}{1-\alpha}\int_\alpha^1 \mathrm{VaR}_z(X)\,dz$$

The Value at Risk $VaR_\lambda(x)$ for a given confidence level λ however is in general not convex (see [2]).

We assume, that the random returns of each asset class for all different single time-periods [j-1;j] follow the same probability distribution (of course, for two different asset classes the probability distributions may be different). Additionally, let them be stochastic independent for different such time-periods. If $r_i(j)$ is the return for the i-th asset class in the period [j-1;j] then for the expected return $\mu(\zeta)$ of an investment strategy ζ (independent of j) we have: $\mu(\zeta) = \langle \zeta; (\mu_1,...,\mu_n)\rangle$ whereas $\mu_i := E(r_i(j)) \;\; \forall j$. We now assign to each level of risk s (s is a positive real number) the maximum achievable expected return by setting: $\mu(s) := \sup\{\mu(\zeta) : \zeta \in \Im \wedge \rho(\zeta) = s\}$.

Lastly we define: $\rho_0 := \inf\{\rho(\zeta) : \zeta \in \Im\}$. With this we obtain the following

Lemma: If for a real positive s μ(s) exists (i.e. the set, on which the supremum is taken, is not empty), then $\mu(s) = \max\{\mu(\zeta) : \zeta \in \Im \wedge \rho(\zeta) = s\}$

Proof: μ as a function on $\Im$ is continuous and the set, on which the supremum is taken, is a closed subset of the compact set $\Im$, and hence itself compact. Q.E.D.

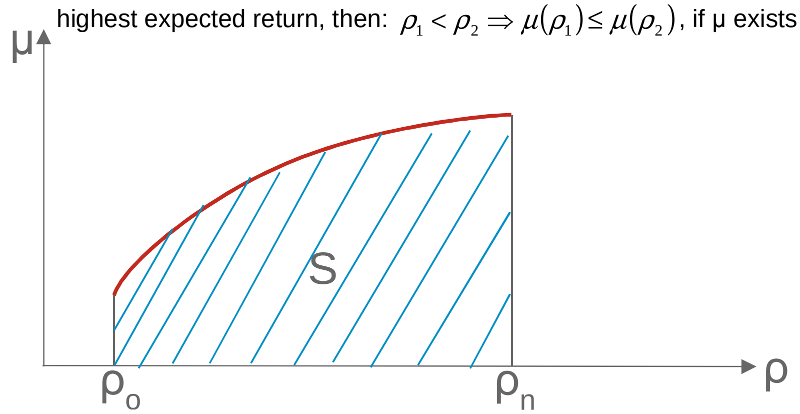
For the same reason we can substitute in the definition for ρ_o the infimum by a minimum. We can now show, that under these conditions the curve (s, μ (s)), i.e. the „efficient frontier“ , has a comparable shape, as we know it from classical capital market theory[2] (for classical capital market theory see [3] or e.g. chapter B in [6]).

Theorem:

(i) If $\mu(\rho_1)$ and $\mu(\rho_2)$ exist, then $\mu(\rho)$ exists for all $\rho \in [\rho_1;\rho_2]$

(ii) The set $S := \{(x,y) \in \Re^2 : \mu(x) \text{ exists and } 0 \le y \le \mu(x)\}$ is convex

(iii) If the asset class with the highest level of risk, without loss of generality ρ_n, also delivers the highest expected return, then: $\rho_1 < \rho_2 \Rightarrow \mu(\rho_1) \le \mu(\rho_2)$, if μ exists for ρ_1 and ρ_2



(iv) μ(ρ) is continuous as a function of ρ on its domain of definition

Proof:

[2] There much more special assumptions and preconditions are used, e.g. using the standard deviation of returns as risk measure and normally it is also implicitly assumed, that returns are normally distributed.

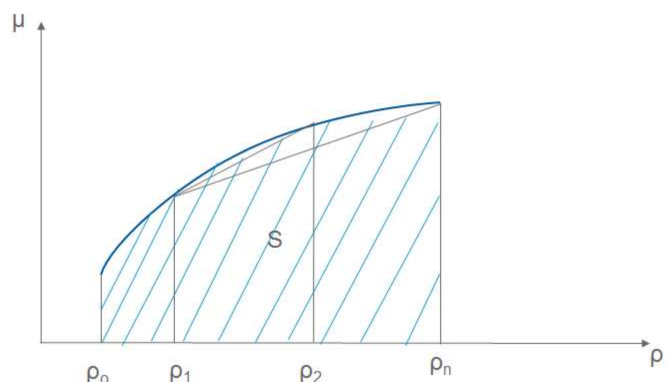
(i) Choose ζ_1 and ζ_2 so that $\rho_i=\rho(\zeta_i)$ for i=1 resp. 2. Since ρ is continuous in ζ, the term $\rho(\lambda\zeta_1+(1-\lambda)\zeta_2)$ assumes any value between ρ_1 and ρ_2 for λϵ[0;1].

(ii) Making use of (i) this follows immediately from $\rho(\lambda\zeta_1+(1-\lambda)\zeta_2) \le \lambda\rho(\zeta_1)+(1-\lambda)\rho(\zeta_2)$ and $\mu(\lambda\zeta_1+(1-\lambda)\zeta_2) = \lambda\mu(\zeta_1)+(1-\lambda)\mu(\zeta_2)$.

(iii) Let w.l.o.g. the n-th asset-class being the one having the highest risk. If ρ_1 is arbitrary, but fixed, then for an arbitrary $\rho_2>\rho_1$ the curve

$$\gamma(t):=\begin{pmatrix}\rho_1+t(\rho_2-\rho_1)\\ \mu(\rho_1+t(\rho_2-\rho_1))\end{pmatrix},\quad t\in[0;1]$$

lies above or on the straight line through the points $(\rho_1;\mu(\rho_1))$ and $(\rho_n;\mu(\rho_n))$ because of (ii). But this straight line is increasing by definition.



(iv) We prove this by contradiction: if we assume, that μ(ρ) is not continuous at some r ϵ (ρ_o, ρ_n), then there would exist an ε>0, so that for all $\delta>0$ there would be an $s\in(\rho_o;\rho_n)$ so that $|s-r|<\delta$ and $|\mu(s)-\mu(r)|>\varepsilon$
With this the absolute value of the slope m of the line segment between the points $(r,\mu(r))$ and $(s,\mu(s))$ can get arbitrarily large, only if δ is chosen small enough, because:

$$|m|=\frac{|\mu(s)-\mu(r)|}{|s-r|}\ge\frac{\varepsilon}{\delta}\to\infty \text{ for } \delta\to 0$$

If we now choose δ sufficiently small, we can achieve the following:
If s < r, then the line segment between $(s,\mu(s))$ and $(r,\mu(r))$ lies below the line segment between $(\rho_o,\mu(\rho_o))$ and $(r,\mu(r))$ in the (ρ,μ)-coordinate-system. If s > r, then the line segment between $(\rho_o,\mu(\rho_o))$ and $(s,\mu(s))$ lies above the line segment between $(s,\mu(s))$ and $(r,\mu(r))$. Because of the convexity of the set S from (ii) this contradicts the definition of μ(s) resp. μ(r) (as being the maximum achievable expected return for the given risk level s resp. r).

Q.E.D.

The random investment returns depend on the chosen investment strategy ζ, and hence especially on μ(ζ) and ρ(ζ).

It follows that all X(j) and their probability distribution depend on ζ. We therefore write $X_\zeta(j)$.

We now define the probabilities to always fulfil the regulatory funding requirements at all times (in short: the “funding probability“) resp. to be always able to pay all pension benefits when they become due (in short: the “financing probability”); in this context we assume, that all required regulatory minimum funding ratios $\alpha(j)\ge 0$, j ϵ {1,…,ω} for ω ϵ IN (last point in time to be considered), are given:

$$\left.\begin{array}{l}\Phi_\zeta := P\big(X_\zeta(j) \ge \alpha(j)\cdot V(j)\ \ \forall j \ge 0\big) \\ \text{resp.} \\ \Psi_\zeta := P\big(X_\zeta(j) \ge L(j)\ \ \forall j \ge 0\big)\end{array}\right\} \Rightarrow \Psi_\zeta \ge \Phi_\zeta \text{ in practice (see Remark 2)}$$

We now require, that the risk measure ρ, which is used by the IORP and the regulatory requirements resp. the target, that all liabilities are sufficiently funded, are coherent. For this we have to request:

$$\mu(\zeta_1) = \mu(\zeta_2) \wedge \rho(\zeta_1) < \rho(\zeta_2) \ \Rightarrow\ \Phi_{\zeta_1} \ge \Phi_{\zeta_2} \forall \zeta_1, \zeta_2 \text{ and } \mu(\zeta_1) > \mu(\zeta_2) \wedge \rho(\zeta_1) = \rho(\zeta_2) \ \Rightarrow\ \Phi_{\zeta_1} \ge \Phi_{\zeta_2} \forall \zeta_1, \zeta_2$$

We make of course an analog request for Ψ.

Remark 1: These coherence conditions mean, that, if two investment strategies deliver the same return expectation, then the one with the lower risk level will have a higher (or equal) funding- and financing probability. It means also, that, if two investment strategies have the same risk level, then the one with the higher return expectation will have a higher (or equal) funding- and financing probability. Both are conditions, which one would normally expect in practice.

Remark 2: The target of being able to pay all benefits when they become due, which the financing probability is referring to, can be seen as a very special case of the requirement to fulfil certain funding ratios $\alpha(j)$ at all times j, namely as the "weakest" special case, which is practically possible. In this case we would have $\alpha(j)$ = L(j)/V(j) for all j. If however the IORP now is not exposed to any early termination risks, its real corporate objective is to ensure, that all benefits can be paid when they are due, and NOT, that already at earlier points in time certain defined minimum funding ratios $\alpha(j)$ are met.

If now Φ (resp. Ψ) is depending continuously on the variable parameters μ with $\mu(\rho_0) \le \mu \le \mu(\rho_n)$ and ρ with $\rho_0 < \rho < \rho_n$, which themselves depend on ζ we write for simplicity reasons $\Phi_{\mu,\rho}$. In this notation our coherence conditions from above read:

$$\mu_1 = \mu_2 \wedge \rho_1 < \rho_2 \ \Rightarrow\ \Phi_{\mu_1,\rho_1} \ge \Phi_{\mu_2,\rho_2} \forall (\mu_1, \mu_2, \rho_1, \rho_2) \in [\mu(\rho_0); \mu(\rho_n)]^2 \times [\rho_0; \rho_n]^2$$

$$\mu_1 > \mu_2 \wedge \rho_1 = \rho_2 \ \Rightarrow\ \Phi_{\mu_1,\rho_1} \ge \Phi_{\mu_2,\rho_2} \forall (\mu_1, \mu_2, \rho_1, \rho_2) \in [\mu(\rho_0); \mu(\rho_n)]^2 \times [\rho_0; \rho_n]^2$$

Lemma:

In this case Φ is depending *only* on the variable parameters μ and ρ, which themselves depend on ζ (and not on other variable parameters[3]).

Proof:

We conduct an indirect proof: we assume, that Φ is depending on (at least) one further variable parameter β: $\Phi = \Phi_{\mu,\rho,\beta}$. Then there would exist $\beta_1 \ne \beta_2$, such that w.l.o.g. $\Phi_{\mu,\rho,\beta_1} < \Phi_{\mu,\rho,\beta_2}$. Because of $\rho_0 < \rho < \rho_n$ and because Φ is continuous in ρ there is an ε>0 with $\Phi_{\mu,\rho,\beta_1} < \Phi_{\mu,\rho+\varepsilon,\beta_2}$. But this is a contradiction to our coherence condition.

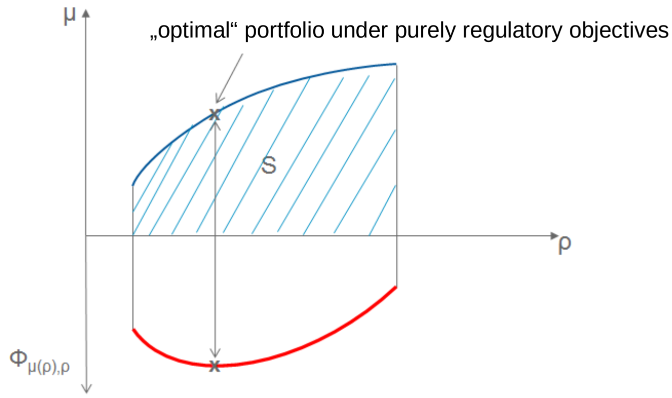
Q.E.D.

[3] Φ can of course depend on further parameters, but in the context of an analysis of Φ and the coherence conditions these have then to be seen as constants. Especially we do NOT assume a two-parametric probability distribution.

3. General investment behavior of the IORP

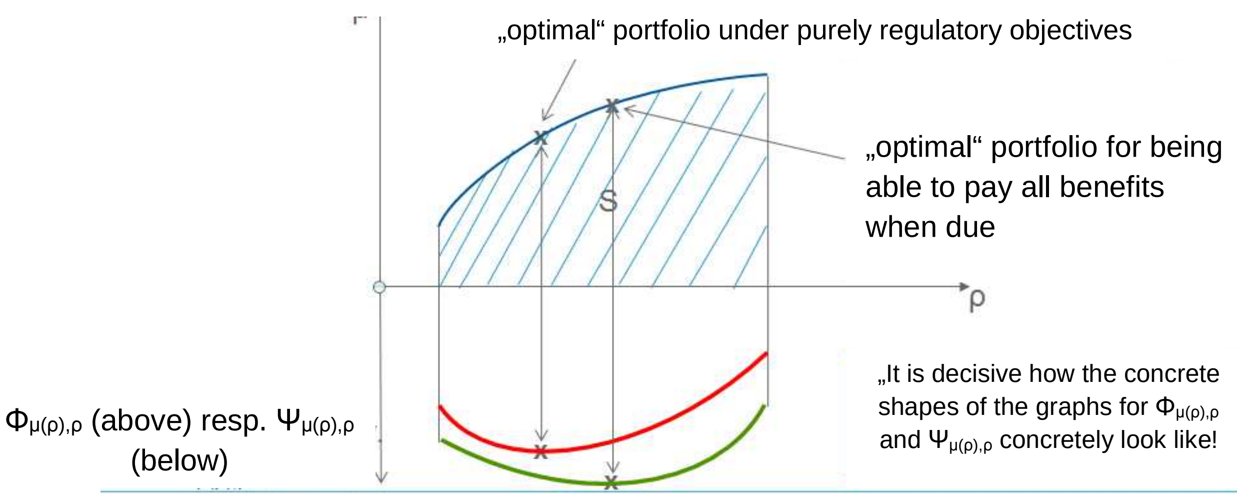
If the IORP aligns its business behavior exclusively with the objective of being compliant with the regulatory minimum funding ratios at any time, then it will try to find an efficient investment strategy (portfolio), i.e. a strategy , which delivers for a given level of risk ρ the maximum possible expected return $\mu(\rho)$, whereas ρ is determined, so that $\Phi_{\mu(\rho),\rho}$ is maximal.

Figure:



The problem now is, that the level of ρ, where $\Psi_{\mu(\rho),\rho}$ reaches its optimal value (please remember, that the real business objective of the IORP must be to maximize $\Psi_{\mu(\rho),\rho}$) often does not coincide with the optimal level of ρ for $\Phi_{\mu(\rho),\rho}$ and hence, that an IORP, which is only focused on its real business objective, would choose a different optimal investment strategy than if it were focused on being compliant with the regulatory minimum funding ratios (at any time).

Figure (example):



4. Implications of regulatory maximum quotas for certain asset-classes on the IORP

We consider two different regulatory regimes: one with maximum quotas M_i, i.e. it is required, that $z_i \leq M_i$ for all i, and one with maximum quotas $\tilde{M}_i$ i.e. $z_i \leq \tilde{M}_i$ for all i. Then we get the respective sets of all permitted investment strategies as

$$\Im := \left\{ \zeta \in \Re^n : 0 \leq z_i \leq M_i \wedge \sum_i z_i = 1 \right\} \text{ resp. } \quad \tilde{\Im} = \left\{ \zeta \in \Re^n : 0 \leq z_i \leq \tilde{M}_i \wedge \sum_i z_i = 1 \right\}$$

Obviously: $M_i < \tilde{M}_i \forall_i$ (in an extreme case $\tilde{M}_i = 1 \forall_i$) $\Rightarrow \Im \subset \tilde{\Im}$

If we define $\tilde{\mu}$ analogously as μ (as a function in ρ, like in the theorem in paragraph 2, only for the regulatory regime with $\tilde{M}_i$), then

(i) the domain of definition of μ is contained in the one for $\tilde{\mu}$ (or they both coincide)

(ii) $\mu(\rho) \leq \tilde{\mu}(\rho) \forall_{\rho \in [\rho_o, \rho_n]}$

(iii) $\Phi_{\mu(\rho),\rho} \leq \Phi_{\tilde{\mu}(p),\rho} \wedge \Psi_{\mu(\rho),\rho} \leq \Psi_{\tilde{\mu}(p),\rho} \forall_{\rho \in [\rho_o, \rho_n]}$

Proof:

(i) follows from $\Im \subset \tilde{\Im}$

(ii)
$$\begin{aligned} \mu(\rho) &= \sup\{\mu(\zeta) : \zeta \in \Im \wedge \rho(\zeta) = \rho\} \\ &\leq \sup\{\mu(\zeta) : \zeta \in \tilde{\Im} \wedge \rho(\zeta) = \rho\} = \tilde{\mu}(\rho) \end{aligned}$$

(iii) follows directly from (ii) because of the definition of Φ resp. Ψ and because of the coherence assumption stipulated in chapter 2.

Q.E.D

Consequence: By introducing more generous maximum quotas for the single asset classes one can in general improve the funding probability and the financing probability. In the case of rationally acting investors (IORPs) this would in general speak in favor of some kind of "prudent person principle" to be used as a regulatory regime.

In this case it is interesting to analyze, when the strict inequality in (iii) holds.

If Φ resp. Ψ depends continuously differentiable on the coordinates ζ_i of ζ, then the following holds:

If the maximum of Φ resp. Ψ in $\Im$ is assumed in in a point of the following subset

$$\partial\Im_{j,k} := \left\{ \zeta \in \Re^n : 0 \leq \zeta_i \leq M_i \forall_i \wedge \sum_{i=1}^{n} \zeta_i = 1 \wedge \zeta_j = M_j \wedge \zeta_k > 0 \right\} \text{for j, k} \in \{1,\dots n\},\ j \neq k,$$

of the boundary of $\Im$, and if there the inequality $\frac{\partial \Phi}{\partial \zeta_j} > \frac{\partial \Phi}{\partial \zeta_k}$ holds, then obviously the strict inequality holds in (iii). Analogously for Ψ.

Remark: In practice this differentiability condition is not an unrealistic requirement, since otherwise Φ resp. Ψ can be approximated arbitrarily exact by C^∞-functions (on a compact domain of definition).

From this it follows, that, if a supervisor wants to use maximum quotas for asset-classes at all, they should be chosen so generously, that rationally acting and asset-liability-management-focussed investors anyhow do not completely exhaust them in practice. In that case they could be interpreted as a pure protection against irrational behavior.

5. Implications of Funding Requirements for the IORP

We shall assume, that the following conditions of differentiability hold:

(i) μ(ρ) is two times continuously differentiable in ρ

$$\Rightarrow \mu'(\rho) \geq 0 \text{ and } \mu''(\rho) \leq 0 \text{ on } [\rho_o, \rho_n]$$

(also in ρ_o and in ρ_n let μ be two times continuously differentiable from the right resp. from the left, whereas we can permit, that the absolute value of the (unilateral) differential in ρ_o gets infinite)

(ii) Φ and Ψ depend two times continuously differentiable on the parameters μ and ρ, $(\mu,\rho) \in [\mu(\rho_o),\mu(\rho_n)] \times [\rho_o,\rho_n]$

Remark: Again, in practice this differentiability conditions are no unrealistic requirements, since otherwise Φ resp. Ψ can be approximated arbitrarily exact by C^∞-functions (on a compact domain of definition).

We consider the map $F_\Phi: \rho \longrightarrow \Phi_{\mu(\rho),\rho}$

This map is two times continuously differentiable.

For the first and second order differential of F_Φ we get:

$$F_\Phi'(\rho) = \frac{\partial \Phi}{\partial \mu} \cdot \mu'(\rho) + \frac{\partial \Phi}{\partial \rho}$$

$$F_\Phi''(\rho) = (\mu'(\rho),1) \cdot H(\Phi) \cdot (\mu'(\rho),1)^T + \frac{\partial \Phi}{\partial \mu} \cdot \mu''(\rho),$$

whereas H(Φ) is the Hesse-matrix of Φ at the point (μ(ρ), ρ). Analogously for F_Ψ.

Convention: We will write just F, if F_Φ as well as F_Ψ can be meant.

Our task is now to maximize F_Φ resp. F_Ψ .

For the wanted maximum of F on $[\rho_0,\rho_n]$ the following holds:

(i) F assumes its maximum either in one of the boundary points ρ_o or ρ_n

OR

(ii) there otherwise exists a $\rho_{\max} \in \left]\rho_0;\rho_n\right[$ where the following holds:

$$\mu' \cdot \frac{\partial \Phi}{\partial \mu} = -\frac{\partial \Phi}{\partial \rho} \quad \wedge \; (\mu',1) \cdot \mathrm{H}(\Phi) \cdot (\mu',1)^{\mathrm{T}} < -\frac{\partial \Phi}{\partial \mu} \cdot \mu''(\rho)$$

Remark: Because of $\frac{\partial \Phi}{\partial \mu} \geq 0$ and $\mu''(\rho) \leq 0$ the right hand side of the last inequality is always ≥ 0.

Hence it is sufficient for condition (ii), if $\frac{\partial^2 \Phi}{\partial \mu^2} < 0 \wedge \frac{\partial^2 \Phi}{\partial \mu^2} \cdot \frac{\partial^2 \Phi}{\partial \rho^2} - \left(\frac{\partial^2 \Phi}{\partial \mu \partial \rho}\right)^2 > 0$, which means, that H(Φ) is negative definite.

Analogously for F_{Ψ}.

We now assume two different possible behaviors of an asset-liability-management-focused investor.

This investor either tries

(i) to maximize F_{Φ} : $F_{\Phi}(\rho) = \Phi_{\mu(\rho),\rho} \rightarrow \max$

OR

(ii) to maximize F_{Ψ} under the side condition, that: $\Phi_{\mu(\rho),\rho} \geq 1-\lambda$

(whereas 1>λ>0 is a pre-defined confidence level)

Remark: In (ii) we will search the maximum of F_{Ψ} within the set

$$M_{\Phi,\lambda} := \{\rho \in [\rho_0, \rho_n] : \Phi_{\mu(\rho),\rho} \geq 1-\lambda\} \quad .$$

To what extent a deregulation (which in this context shall be understood as an easing of the prevailing funding requirements, which is a reduction of the α(j)) has in total a positive impact, is depending on, how the graphs of F_{Φ} and F_{Ψ} are located and shaped relative to each other.

In order to assess the impact of a potential easing of minimum funding requirements, we analyze two different regulatory regimes, one having minimum funding ratios $\alpha(j)$ and a different one having minimum funding ratios $\tilde{\alpha}(j)$ $(j \geq 0)$.

If now $\tilde{\Phi}$ is defined analogously as Φ, only for the regulatory regime having the minimum funding ratios $\tilde{\alpha}(j)$ then $\alpha(j) \geq \tilde{\alpha}(j) \forall_j$ - in an extreme case $\tilde{\alpha}(j) = L(j)/V(j)$ - implies: $\Phi_{\mu(\rho),\rho} \leq \tilde{\Phi}_{\mu(\rho),\rho} \wedge M_{\Phi,\lambda} \subseteq M_{\tilde{\Phi},\lambda}$

Remark: This follows immediately out of the definitions.

We first assume, that the considered IORP is with regard to ist behavior solely focused on regulatory requirements. If Φ and $\tilde{\Phi}$ depend additionally two times continuously differentiable on all required regulatory minimum funding ratios $\alpha(j)$ and if we define $\rho_{\max}$:= place, where F_{Φ} takes its maximum in $[\rho_0, \rho_n]$, and if $\tilde{\rho}_{\max}$ is defined analogously for $F_{\tilde{\Phi}}$, and if additionally the maximum of F_{Ψ} is strict, we get the following

Theorem I:

(a) If $\rho_{max}<\rho_n$ and if at ρ_{max} we have : $\mu' \cdot \frac{\partial \psi}{\partial \mu} > -\frac{\partial \psi}{\partial \rho}$ and if $F_\Psi(\rho) \le F_\Psi(\rho_{max}) \forall_{\rho<\rho_{max}}$

Then the following holds: by a sufficient easing of the required minimum funding ratios we can achieve

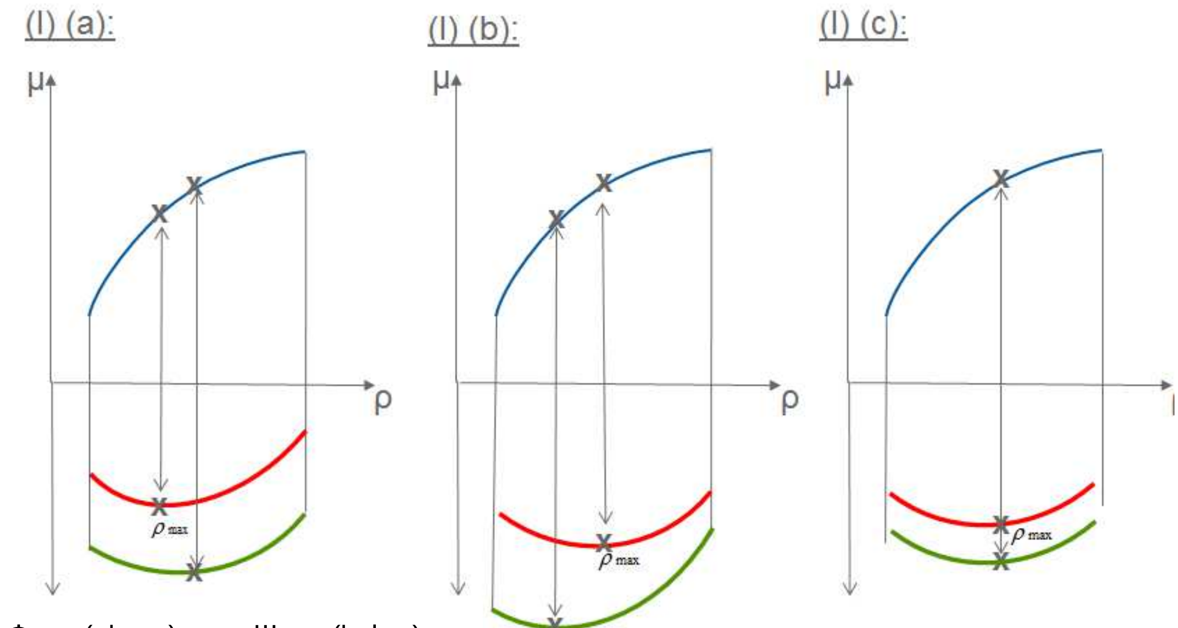
$\Psi_{\mu(\tilde{\rho}_{max}),\tilde{\rho}_{max}} > \Psi_{\mu(\rho_{max}),\rho_{max}}$. In this case easing of funding requirements leads to a riskier investment strategy.

(b) If $\rho_{max} > \rho_o$ and if at the point ρ_{max} we have $\mu' \cdot \frac{\partial \Psi}{\partial \mu} < -\frac{\partial \Psi}{\partial \rho}$ and also $F_\Psi(\rho) \le F_\Psi(\rho_{max}) \forall_{\rho>\rho_{max}}$,

then the following holds: by a sufficient easing of minimum funding requirements it can be achieved, that $\Psi_{\mu(\tilde{\rho}_{max}),\tilde{\rho}_{max}} > \Psi_{\mu(\rho_{max}),\rho_{max}}$. In this case this kind of easing would lead to a less risky investment strategy.

(c) If F_Ψ is optimal at ρ_{max}, an improvement by easing of funding requirements cannot be achieved.

We can visualize this – in a strongly simplified manner- within the following drawings:



$\Phi_{\mu(\rho),\rho}$ (above) resp. $\Psi_{\mu(\rho),\rho}$ (below)

Proof:

(a) $\tilde{\Phi}$ depends continuously differentiable from all α(j), and $\tilde{\Phi}$ becomes Ψ if all $\tilde{\alpha}(j)$ become L(j)/V(j), and hence $\tilde{\rho}_{max}$ then becomes the point where F_Ψ takes its optimum. Since $F_\Psi(\rho) \le F_\Psi(\rho_{max})$ for all $\rho < \rho_{max}$ and since $F_\Psi'(\rho_{max}) > 0$ this point is located on the right hand side of ρ_{max}[4]. Since $F_\Psi'(\rho_{max})>0$ the maximum value of F_Ψ is greater than $F_\Psi(\rho_{max})$. From this our claim follows.

[4] Therefore we need $\rho_{max} < \rho_n$!

(b) can be proven analogously

(c) is trivial. Q.E.D

If we now assume, that the IORP tries to maximize F_Ψ under the side condition $\Phi_{\mu(\rho),\rho} > 1-\lambda$ for a given ("small") λ > 0, then the following holds:

Theorem II:

(a) If F_Ψ assumes its maximum value in ρ_n (this is e.g. the case, if $\mu' \cdot \frac{\partial\Psi}{\partial\mu} > -\frac{\partial\Psi}{\partial\rho}$ on (ρ_o, ρ_n)) and if $\rho_n \notin M_{\Phi,\lambda}$, then a sufficiently strong easing of minimum funding ratios will lead to a riskier investment strategy delivering a higher maximum value of F_Ψ under the side condition $\Phi_{\mu(\rho),\rho} \geq 1-\lambda$.

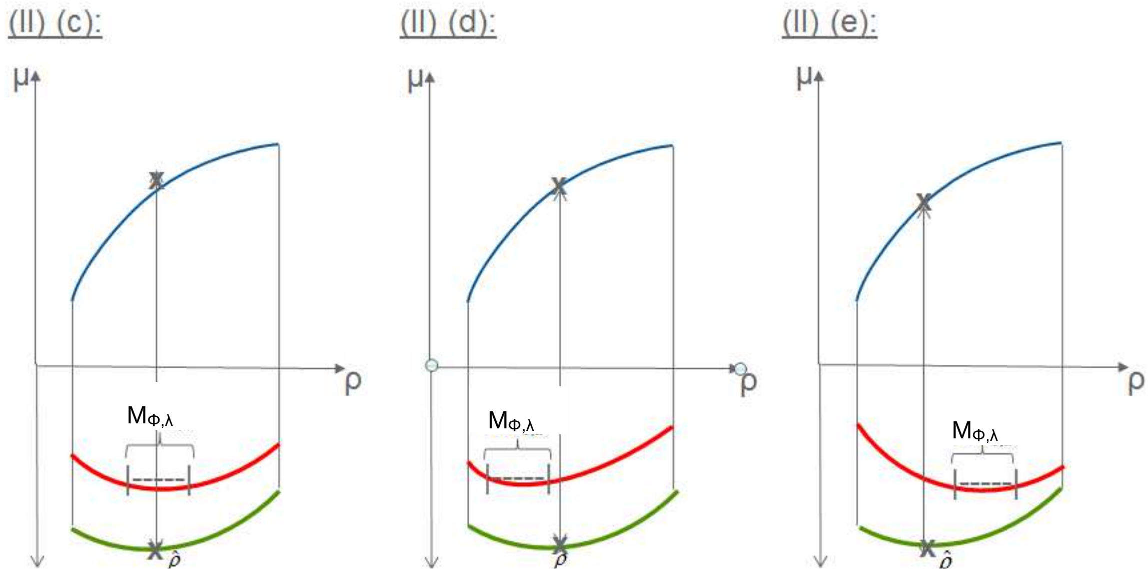
(b) If F_Ψ assumes its maximum in ρ_o (this is e.g. the case, if $\mu' \cdot \frac{\partial\Psi}{\partial\mu} < -\frac{\partial\Psi}{\partial\rho}$ on (ρ_o, ρ_n)) and if $\rho_o \notin M_{\Phi,\lambda}$ then a sufficiently strong easing of minimum funding ratios will lead to a less risky investment strategy delivering a higher maximum value of F_Ψ under the aforementioned side condition.

(c) If F_Ψ assumes its maximum at some $\hat{\rho} \in M_{\Phi,\lambda}$, then an easing of minimum funding ratios will have no impact.

(d) If F_Ψ assumes its maximum at some $\hat{\rho} \in (\rho_o, \rho_n)$ and if $\hat{\rho} > \rho \forall \rho \in M_{\Phi,\lambda}$, then a sufficiently strong easing of minimum funding ratios will lead to a riskier investment strategy delivering a higher maximum value of F_Ψ under the aforementioned side condition.

(e) If F_Ψ assumes its maximum value at some $\hat{\rho} \in (\rho_o, \rho_n)$ and if $\hat{\rho} < \rho \forall \rho \in M_{\Phi,\lambda}$, then a sufficiently strong easing of minimum funding ratios will lead to a less risky investment strategy delivering a higher maximum value of F_Ψ under the aforementioned side condition.

Again, we visualize II(c), II(d), and II(e) – in a strongly simplified manner- using the following drawings:



$\Phi_{\mu(\rho),\rho}$ (above) resp. $\Psi_{\mu(\rho),\rho}$ (below)

The proof of these claims is more or less obvious.

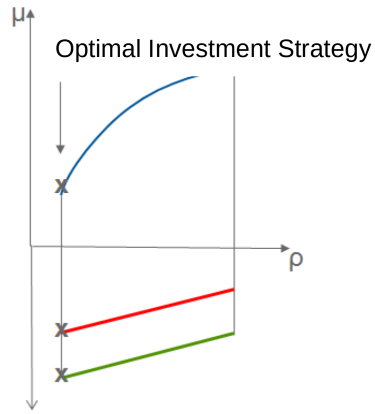
5.a Appearance of these Constellations in Practice

Now we want to show in form of some practical examples, which of the aforementioned constellations often occur in which concrete capital market environments. We will treat two extreme situations and all shades between them are imaginable.

5.a (I) High-Interest Environment

If the (approximately) risk free interest rate (which should be the expected yield of the asset-class bearing the lowest level of risk) is far above the fixed discount rate (defined by tariff; in German: "tariflicher Rechnungszins"), which the IORP uses in its valuation of pension liabilities, and if the current funding ratio of the IORP is above 100 %, then by experience $\frac{\partial \Psi}{\partial \mu}$ is relatively small on (ρ_o,ρ_n), since Ψ is already relatively high – potentially close to 100 % - and hence can only hardly be increased any more. In this situation we often have $\mu' \cdot \frac{\partial \Psi}{\partial \mu} < -\frac{\partial \Psi}{\partial \rho}$ on (ρ_o,ρ_n) since the integration of additional risk in the portfolio can almost only have a negative impact. At the same time in this situation often the corresponding inequalities hold for Φ. Because of the general explanations above, in this case a further easing of minimum funding requirements would not help to improve the financing probability Ψ further and the IORP would invest anyhow into a portfolio, whose structure is quite close to the one of the investment strategy bearing the lowest possible risk.

High-Interest Environment



5.a(II) Low-Interest Environment

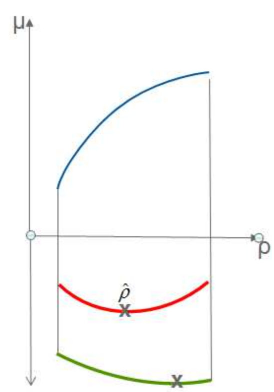
If in contrast the (approximately) risk-free interest rate is clearly below the fixed discount rate (defined by tariff; in German: “tariflicher Rechnungszins”), which the IORP uses in its valuation of pension liabilities, then often $\frac{\partial \Psi}{\partial \mu}$ is relatively high on a domain $(\rho_o, \bar{\rho})$ for some $\bar{\rho} \in (\rho_o, \rho_n)$ since in this situation higher return expectations are desperately needed. At the same time an increase of shorter term risk parameters will cause only relatively little harm with regard to the probability of being able to pay the guaranteed benefits when due. This means: $\mu' \cdot \frac{\partial \Psi}{\partial \mu} > -\frac{\partial \Psi}{\partial \rho}$

At the same time we often have

$\mu' \cdot \frac{\partial \Phi}{\partial \mu} < -\frac{\partial \Phi}{\partial \rho}$ on the right hand side of some $\hat{\rho} \in [\rho_0, \bar{\rho})$ (since Φ is more risk sensitive).

If this constellation is given, an easing of minimum funding requirements would contribute to a clear increase of the probability of being able to pay the guaranteed benefits when due. But this would mean at the same time, that the IORP would have to use a riskier investment strategy.

Low-Interest Environment



5.b Sketch of an Application of the Theory to a Solvency II Type Regime

The theory described so far can be also easily applied to a Solvency II type regime. In this case we would have no fixed regulatory maximum quotas for certain asset classes, since every asset allocation is allowed, which would lead to a solvency capital requirement, which is sufficiently low. In this context “sufficiently low” means, that all technical provisions (incl. risk margin) plus the total solvency capital requirement can be covered by the market value of the corresponding assets. Furthermore we would also have no fixed required regulatory minimum funding ratios α(j). Instead, the minimum funding ratios can be interpreted as functions, that depend not only on future points in time j, but also on the required solvency capital requirements in j and a risk margin R (in % of the technical provisions). Hence:

$\alpha(j) = 1 + R + z(j) > 1$,

where z(j) represents the solvency capital requirement expressed in % of the technical provisions. This z will depend on the different risks, which are resulting out of the liabilities and also from other risks like operative risks, early termination risks etc. (more details and an overview of risks to be taken into account in hierarchical form can be found in technical specifications to Solvency II as published by EIOPA or in [7]). But especially z will also depend on the risks resulting out of the chosen investment strategy. So, for this consideration we can assume, that z depends on ρ and μ(ρ). Hence our minimum funding requirement at the time j is also a function depending (especially) on ρ and μ(ρ) and we can interpret the α´s as functions $\alpha(j,\rho)$. We can now compare a solvency regime using the $\alpha_j(\rho) := \alpha(j,\rho)$ against a different regime using $\tilde{\alpha}_j(\rho)$. The second regime could e.g. lead ceteris paribus to lower funding requirements at all times j by using a less conservative calibration of stress parameters for different asset classes or by requesting a lower risk margin etc. Then we can define $\Phi, \tilde{\Phi}, F_\Phi, F_{\tilde{\Phi}}, \Psi, F_\Psi$ analogously as we did before, taking into account, that the funding probabilities depend on the new $\alpha_j(\rho)$ and $\tilde{\alpha}_j(\rho)$.

Remark: Since the capability of the IORP of being able to pay all guaranteed benefits when due does (ceteris paribus) not depend on regulatory requirements, if an investment strategy is fixed and given, these new $\alpha_j(\rho)$ have not to be taken into account when evaluating the financing probability Ψ and the function F_Ψ.

For the first and second order differentials for F_Φ we similarly get (assuming sufficient differentiability for the $\alpha_j(\rho)$):

$$F_\Phi'(\rho) = \frac{\partial \Phi}{\partial \mu} \cdot \mu'(\rho) + \frac{\partial \Phi}{\partial \rho} + \sum_{j=1}^{\omega} \frac{\partial \Phi}{\partial \alpha_j} \cdot \alpha_j'(\rho)$$

$$F_\Phi''(\rho) = \left(\mu'(\rho), 1, \alpha_1'(\rho), \ldots, \alpha_\omega'(\rho) \right) \cdot H(\Phi) \cdot \left(\mu'(\rho), 1, \alpha_1'(\rho), \ldots, \alpha_\omega'(\rho) \right)^T$$
$$+ \frac{\partial \Phi}{\partial \mu} \cdot \mu''(\rho) + \sum_{j=1}^{\omega} \frac{\partial \Phi}{\partial \alpha_j} \cdot \alpha_j''(\rho),$$

And again, of course F assumes its maximum on $[\rho_0,\rho_n]$ either in one of the boundary points ρ_0 or ρ_n or there is an inner point $\rho_{\max} \in \left]\rho_0; \rho_n\right[$ at which the following holds:

$F_\Phi'(\rho) = 0$ and $F_\Phi''(\rho) < 0$

For F_Ψ the formulas for the differentials are the same as at the beginning of chapter 5.

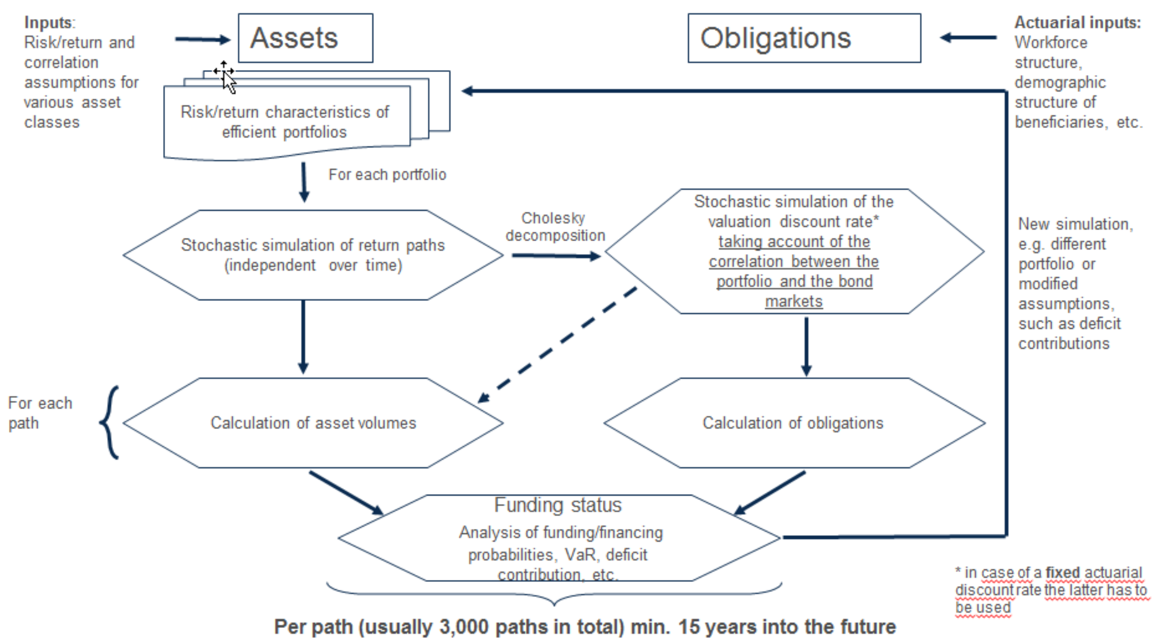
We consider the regime using the $\tilde{\alpha}_j(\rho)$ as an easing of the regime using $\alpha_j(\rho)$, if:
$\alpha_j(\rho) > \tilde{\alpha}_j(\rho) \forall j, \rho$

With this the whole rest of the theoretical analysis in chapter 5 can be done quite analogously. This is especially true for an analogon of Theorem I and for an analogon of Theorem II.

The assessments for certain capital market environments may be done analogously as in 5.a (I) and 5.a (II), however in this case also the sensitivities of the funding probability against changes in the (risk-depending) funding requirements $\alpha_j(\rho)$ must be taken into account. In this context one reasonably would assume, that $\frac{\partial \alpha_j}{\partial \rho} \geq 0$ and one could also come to similar statements as in 5.a(I) and 5.a(II) using the assumptions mentioned above.

6. Application in Practice

In practice one can determine F_Φ and F_Ψ by using methods of stochastic modelling in a suitable way. A flow chart for a possible methodology is given below:



Practical experience shows, that stochastic simulations being done with this methodology confirm the results, which are described above in 5.a(I) and 5.a(II). A more detailed description of this technique can e.g. be found in [4].

7. Conclusion and final Remarks

From the author´s point of view the following conclusions can be drawn from the analyses done in the course of this paper:

1. This paper is not a plea against regulation and regulatory requirements!
2. Regulation and regulatory requirements are inevitable only for the reason to avoid or limit possible negative consequences resulting out of irrational behavior of investors.
3. Regulatory maximum quotas for certain asset classes should be set that way, that in practice they will not be exhausted by rationally acting and asset-liability-management-focused investors anyhow, if the regulator/lawmaker is not capable or not willing to change the regime towards a "prudent person principle".
4. An easing of regulatory minimum funding requirements does not automatically lead to riskier investment strategies.
5. Especially for IORP´s, which are not exposed to significant early termination risks, a requirement like „100 % funding at any time" should be critically challenged. In the end it must (only) be sufficiently sure, that the guaranteed benefits can be paid when due.
6. One could also think about a regulatory regime, which is flexible over time, so that the regulatory requirements could be adapted to existing capital market environments. In general, such a regime would be more anti-cyclical from its character.

Related Literature:

[1] Hoevenaars/Molenaar/Steenkamp, 2003, Simulation for the Long Run in: Asset and Liability Management Tools, Risk Books London;

[2] Kriele/Wolf, 2012, Wertorientiertes Management von Versicherungsunternehmen, Springer-Verlag Berlin Heidelberg;

[3] Markowitz, 1952, Portfolio Selection, Journal of Finance, Volume 7;

[4] Nellshen, 2011, Das Risikomanagement institutioneller Kapitalanleger insbesondere im Bereich der Kapitalanlage in: Handbuch Investmentfonds für institutionelle Anleger, Uhlenbruch Verlag Bad Soden/Ts.;

[5] Thurnes, 2012, Kapitalanlageprozesse in der betrieblichen Altersversorgung in: Kapitalanlage in der betrieblichen Altersversorgung (H-betrAV), C.F.Müller Verlagsgruppe Hüthig Jehle Rehm Heidelberg München Landsberg Berlin;

[6] Betsch/Groh/Lohmann, 1998, Corporate Finance, Verlag Vahlen, München

[7] Peek/Reuss/Scheuenstuhl, 2008, Evaluating the Impact of Risk Based Funding Requirements on Pension Funds, Financial Market Trends, OECD 2008 (Risklab)