

*Hedging mortality risk in order to decrease the regulatory capital requirement under the new Australian prudential standards in effect since 1 January 2013.*

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**Abstract**

*Mortality derivatives have been developed in recent years to mitigate mortality and longevity risks of life insurers. As risk management tools, these derivatives have potential impacts on the regulatory capital requirement of an insurer under its local prudential standards. On the 1st January 2013, the Australian regulator adopted new prudential standards for insurance companies, on the model of Solvency II. These new standards are more sensitive to risks and will lead to significant changes in the capital requirement of insurance companies. In the first section of this paper, we describe the new Australian prudential standards applicable to life insurers. In the second section, we describe how mortality derivatives can be used to hedge mortality risk in this new environment. We propose a new way of pricing mortality swaps by considering their impact on the regulatory capital requirements of two counterparties under the prudential standards. We present a simple example where a Lump Sum insurer and an Annuity insurer swap mortality risks. We show that if the Lump Sum insurer places 30% of policy-holder benefits in a mortality swap with an Annuity insurer, the two counterparties can get an average reduction of the prudential capital requirement coming from mortality risks of close to 20%; this reduction of 20% gives them the option of releasing capital or strengthening their solvency position.*

**Key words:** Longevity, Mortality, Derivatives, Mortality swap, Capital Requirement, LAGIC, Solvency, Risk Management

*Couverture du risque de mortalité afin de réduire le capital réglementaire requis dans le contexte des nouvelles normes prudentielles australiennes effectives depuis le 1 janvier 2013.*

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### **Abstract**

*Les dérivées de mortalité ont été développées ces quelques dernières années pour atténuer les risques de mortalité et de longévité des assureurs vie. En tant qu'outil de gestion du risque, ces dérivées ont un possible impact sur le capital réglementaire requis d'un assureur soumis à des normes prudentielles locales. Au 1 janvier 2013, le régulateur australien a adopté de nouvelles normes prudentielles pour les compagnies d'assurance, s'inspirant du modèle de Solvabilité II. Les nouvelles normes sont plus sensibles aux risques et vont conduire à des changements significatifs du capital requis des compagnies d'assurance. Dans la première partie de cet article, nous décrivons les nouvelles normes prudentielles australiennes applicables aux assureurs vie. Dans la deuxième partie, nous décrivons comment les dérivées de mortalité peuvent être utilisées pour couvrir le risque de mortalité dans ce nouvel environnement. Nous proposons une nouvelle manière de tarifer des swaps de mortalité en considérant leur impact sur le capital réglementaire requis de deux contreparties soumises aux normes prudentielles. Nous présentons un exemple simple où un assureur en cas de décès place 30% de ses prestations aux assurés dans un swap de mortalité contracté avec un assureur de rentes. Les deux contreparties peuvent obtenir une réduction moyenne du capital réglementaire requis pour couvrir les risques sur la mortalité de près de 20% ; cette réduction de 20% leur donne le choix entre libérer du capital ou renforcer leur solvabilité.*

*Mots-clés : Longévité, mortalité, dérivées, swap de mortalité, capital requis, LAGIC, solvabilité, gestion du risque*

## INTRODUCTION AND PRELIMINARY REMARKS

As of 1 January 2013, the Australian insurance industry has come under a new supervisory regime that is in many aspects more sensitive to risks than the previous one. The Australian Prudential Regulation Authority (APRA) – the body that regulates the Australian insurance industry - has introduced a three pillar supervisory framework for general insurers and life insurers which is similar to the supervisory framework of Solvency II, but with distinctive features relevant to the local market. These 3 pillars are the following:

Pillar 1: Framework for the calculation of the Prescribed Capital Requirement and the Capital Base. These requirements may be subject to a transitional period for companies that have to increase their capital under the new framework.

Pillar 2: The risk management process or ICAAP (Internal Capital Adequacy Assessment Process) is the equivalent of the ORSA in Solvency II. Pillar 2 includes the review by APRA who may require a supervisory adjustment to capital if the ICAAP is not deemed satisfactory.

Pillar 3: Reporting and disclosure to the market.

The first part of this paper will look at the transition of the Australian prudential framework that is taking place for Life insurance companies and we will describe Pillar 1 components of the Prescribed Capital Requirement. These considerations will set the context for the mortality derivatives and how they can be relevant in this new prudential framework.

The second part of this paper will describe mortality derivatives and a methodology for pricing the derivatives under the new prudential framework. In order to do that, we will take into account the impact of the mortality derivatives on the capital requirement of the life insurer.

This paper will show it is possible to determine a price for a mortality arrangement between two counterparties by quantifying the impact of the arrangement on the capital requirement of the counterparties. Numerical examples using a simple scenario will be provided.

Preliminary remarks:

- As the author is more familiar with the Australian prudential framework than with the Solvency II framework, this paper will not discuss how mortality derivatives may or may not be used for the capital requirement of a life insurer in a Solvency II context. This will be an area of future improvement.
- Aspects the Australian prudential framework that are not relevant to the pricing of mortality derivatives (Such as pillar 2 or pillar 3) will be briefly mentioned but will not be developed here. In particular, we look at the framework applicable to life insurers only. For a comprehensive description of the Australian prudential framework, refer to the APRA website (see bibliography).

## 1. THE AUSTRALIAN PRUDENTIAL FRAMEWORK

The Australian prudential framework for the life insurance industry is set by the Australian Prudential Regulation Authority as per *Life Insurance Act 1995*. As previously mentioned, the Australian prudential framework transitioned on the 1 January 2013. Here is a brief summary of what has changed for life insurers.

### *a. Previous prudential framework*

Until the 31/12/2012, Life insurers in Australia had to meet two prudential requirements:

- The Solvency requirement, which covered asset and liability risks for a statutory fund closed to new business for the purpose of the exercise (Run-off scenario). The Solvency requirement was calculated using stressed assumptions that were mostly prescribed by the regulator, so that insurers' capital position would be easily comparable to each other.
- The Capital Adequacy requirement, which covered risks for a statutory fund that is open to new business. The Capital Adequacy requirement was calculated using stressed assumptions that reflected the business, although within a prescribed range. The Capital Adequacy requirement gave a more realistic representation of the insurers' capital position.

A statutory fund had to be solvent under both requirements. The shareholder fund of the insurer was to meet only one requirement, the Management Capital Requirement.

Refer to Life Prudential Standard 2.04 published by APRA for a full description of the former prudential framework.

### *b. New prudential framework*

The new framework – called “LAGIC” for Life and General Insurance Capital – came into effect on the 1 January 2013 and the first quarterly Capital report will be due to APRA in April 2013. Under the new regime, there is now only one capital position to comply with under Pillar 1, the Prescribed Capital Amount (or PCA). The PCA of a fund is calculated such that if the statutory fund was to start the year with a Capital Base equal to the PCA, and losses occurred at a 99.5% confidence level then the assets remaining would be at least sufficient to provide for the adjusted policy liabilities of the fund at the end of the year. (LPS 110.30)

The PCA can be calculated using the regulator's standard method set out in the Prudential Standard or by the insurer's internal model (Internal Model-Based method, which must be approved by APRA).

*Note: The Internal Model-Based method is a new feature of the prudential framework, although so far APRA hasn't approved any such method. For the purpose of this paper, we will only consider the standard method*

For insurers using the standard method, the Prescribed Capital Amount is determined as follows:

$$\begin{aligned} &\text{Prescribed Capital Amount (PCA)} \\ &= \\ &\quad \text{Insurance Risk Charge} \\ &\quad + \\ &\quad \quad \text{Asset Risk Charge} \\ &\quad + \\ &\quad \quad \text{Asset Concentration Risk Charge} \\ &\quad + \\ &\quad \quad \text{Operational Risk Charge} \\ &\quad - \\ &\quad \quad \text{Aggregation Benefit} \\ &\quad + \\ &\quad \quad \text{Combined Stress Scenario Adjustment} \end{aligned}$$

The PCA, together with any supervisory adjustment determined by the regulator on a discretionary basis, constitute the Prudential Capital Requirement (PCR) – which is the minimum level of capital to be held by the fund before active intervention of the regulator. LPS 110.23 states that “A life insurer must ensure that the life insurer and each of its funds have a Capital Base, at all times, in excess of its the Prudential Capital Requirement.”

APRA has advised that “To the extent that an insurer has a higher operational risk profile or an inadequate approach to operational risk management, APRA would increase the insurer’s PCR by making a supervisory adjustment. This is intended to provide incentives for insurers to improve their operational risk management.”

$$\begin{aligned} &\text{Prudential Capital Requirement (PCR)} \\ &= \\ &\quad \text{Prescribed Capital Amount (PCA)} \\ &\quad + \\ &\quad \quad \text{Supervisory Adjustment} \end{aligned}$$

### *c. Components of the Prescribed Capital Amount*

Here is a description of each component of the Prescribed Capital Amount and how these components may be affected by mortality derivatives that we will describe in the second part of this paper.

#### (1) Insurance Risk Charge

The Insurance Risk Charge (IRC) is the component of the PCA relating to mortality, morbidity, longevity, lapses and other insurance risks. Refer to Prudential Standard LPS 115.

The IRC is the reduction in the Capital Base that would occur if the adjusted policy liabilities were changed to an amount equal to stressed policy liabilities. Stresses are described in the aforementioned Prudential Standard and aim at a 99.5% level of sufficiency within a year.

The Insurance Risk Charge is the component of the PCA that is directly impacted by mortality derivatives described in this paper. More details about the IRC will be provided when we quantify the impact of mortality derivatives on the IRC.

## (2) Asset Risk Charge

The Asset Risk Charge (ARC) is the component of the PCA relating to asset risks (Refer to Prudential Standard LPS 114). The ARC is calculated using stress tests on various metrics impacting the valuation of assets – stresses apply to real interest rates, expected inflation, currency, equity, propriety, credit spreads and default.

The ARC is not sensitive to the mortality derivatives described in the second part of this paper because it depends only on assets while the mortality derivatives we describe involve future cashflows but no assets. However, the ARC has an indirect effect on the mortality derivatives' pricing through the Aggregation Benefit that depends on the ARC and the IRC.

## (3) Asset Concentration Risk Charge

The Asset Concentration Risk Charge (ACRC) is the component of the PCA relating to concentrations in individual assets or counterparties (Refer to Prudential Standard LPS 117). Limits are set according to the type of assets and the rating of counterparties, any excess over these limits incurs a charge in the Capital Base.

The ACRC is not sensitive to the mortality derivatives described in the second part of this paper as long as the exposure to the counterparty of the mortality derivatives doesn't breach the prescribed limit.

The standard doesn't explicitly provide such limits for mortality derivatives, however we can assume it would fall under exposure category (e) or (f) of LPS 117 Attachment A, that concern reinsurance arrangements. The exposure limit would be the greater of AUD 20m and 25% of the market value of the fund.

The mortality derivatives will need to be included in the calculation of the Asset Concentration Risk Charge, and stay under the admissible limit so that the whole purpose of the derivatives is not compromised.

For the exercise of pricing the mortality derivatives, we will remain within 'reasonable' exposure to counterparties and assume the exposure doesn't breach the limit so that the ACRC is not impacted by mortality derivatives.

## (4) Operational Risk Charge

The Operational Risk Charge (ORC) is the component of the PCA relating to the "risk of loss resulting from inadequate or failed internal processes" and is calculated using the movements in gross premiums and the policy liabilities.

The Operational Risk Charge is a feature added to the new prudential framework.

It is unclear at this point what effect mortality derivative would have on the ORC, however it seems the effect would only be marginal – the standard doesn't allow for an ORC calculated on reinsurance arrangements, so it seems that mortality derivatives would receive the same treatment and be overlooked in the calculation of the ORC under Pillar 1.

For the exercise of pricing the mortality derivatives, we will assume the ORC is not impacted by mortality derivatives.

#### (5) Aggregation Benefit

The Aggregation Benefit (AB) makes an explicit allowance for diversification between asset and insurance risks in the calculation of the Prescribed Capital Amount. It is calculated as follows:

$$AB = ARC + IRC - \sqrt{ARC^2 + IRC^2 + 2 \times Correlation \times ARC \times IRC}$$

With Correlation = 20%

The Aggregation Benefit is impacted by the mortality derivatives through the Insurance Risk Charge. As mentioned as per the Asset Risk Charge, the Aggregation Benefit will need to be taken into consideration when calculating the impact of the mortality derivative on the Prescribed Capital Amount.

#### (6) Combined Stress Scenario Adjustment

The Combined Stress Scenario Adjustment takes into account the tax impact of the different charges as stated by APRA:

“Future shareholder tax benefits arising in the stressed scenarios are recognised in determining the Insurance Risk Charge and Asset Risk Charge. A life insurer must increase the Prescribed Capital Amount by the aggregate amount of any tax benefits that cannot be netted against deferred tax liabilities as specified in LPS 112.”

In this paper, we will consider the price of the mortality derivatives ignoring the tax impact on the insurer. Therefore the Combined Stress Scenario Adjustment will be overlooked in the calculations.

### *d. Other characteristics of the new prudential framework*

The characteristics of the new prudential framework that are most relevant for the calculation of mortality derivatives are described above. However, here are some general comments on the new prudential framework, which do not require much development for the purpose of this paper, but may nonetheless be noteworthy:

#### (1) Alignment to international standards

The new prudential standards aims at facilitating alignment to the Solvency II standard, whereby multinational companies operating in Australia are able to provide a consistent approach to their capital requirement.

## (2) Alignment across industries

Alignment across industries is all the more important since many groups operate across industries such as banking, general insurance and life insurance. The new prudential framework wants to avoid “regulatory arbitrage” whereby a group can reduce its capital requirement by simply moving assets from one entity to the next.

## (3) Moving from prescriptive stresses to prescriptive confidence level

The philosophy of the new prudential framework is to require capital that corresponds to a 99.5 per cent probability of sufficiency rather than a prescribed set of stresses.

## (4) Taking into consideration the quality of capital

The new prudential framework has 2 types of capital:

- Tier 1 as the highest quality of capital, freely available to absorb losses.
- Tier 2 as any other component of the capital that may not be freely available but still contributes to the strength of the capital position.

The standard imposes a minimum for Tier 1.

## (5) Allowing for management actions

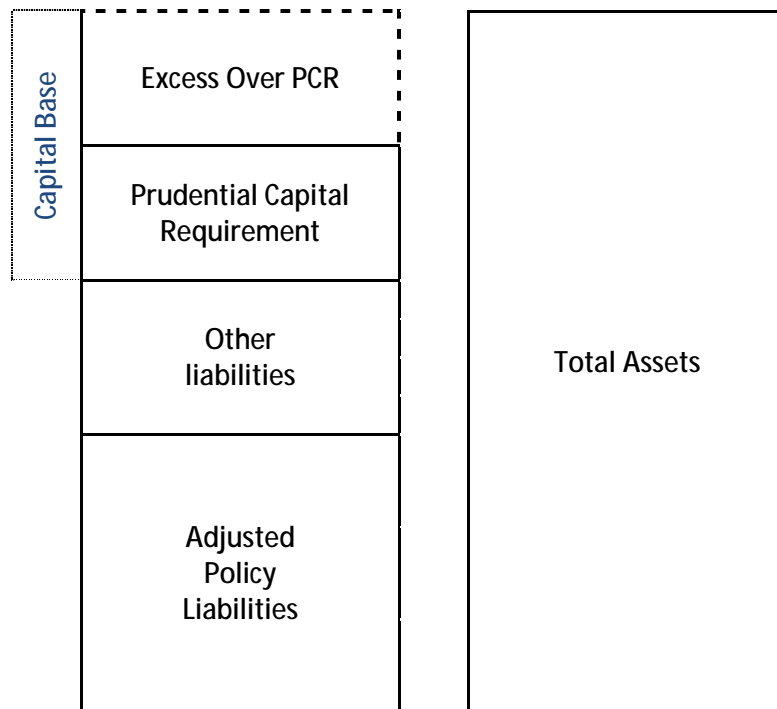
The new prudential framework allows for management actions that would be taken in adverse scenarios in order to limit the losses incurring in these scenarios.

## *e. Summary*

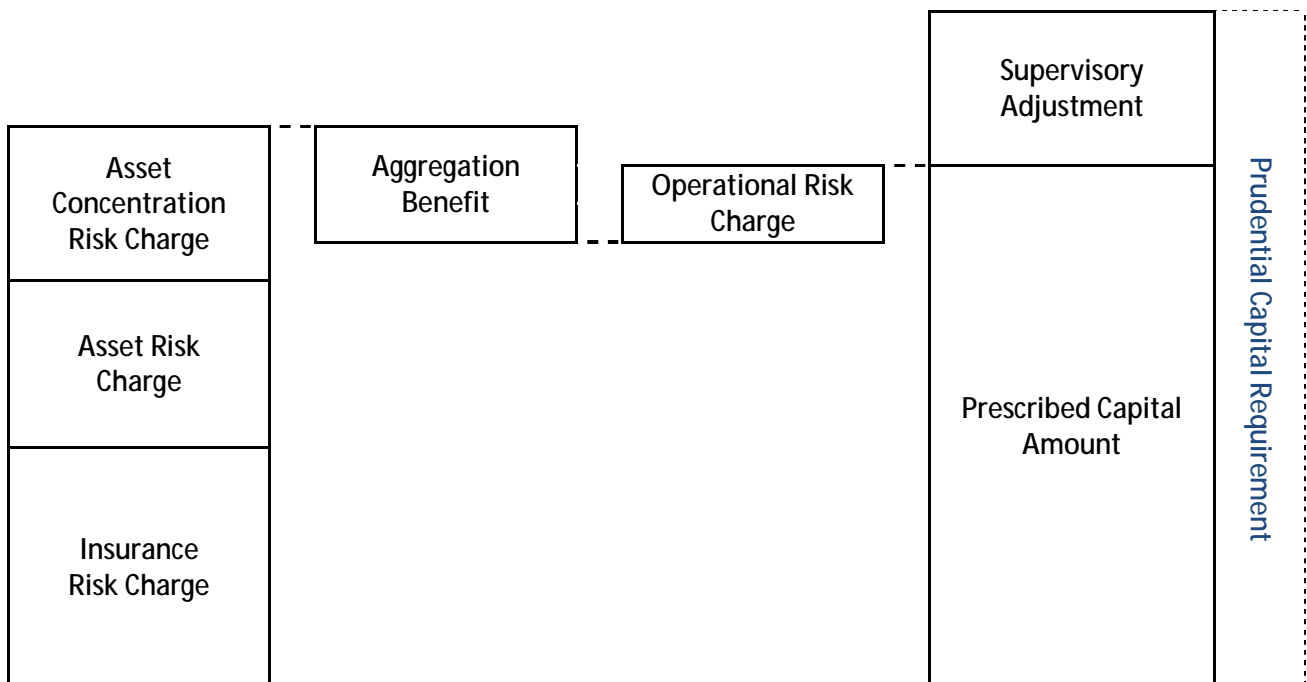
We have given an overview of the Australian prudential framework applicable to a life insurance as at the 1 January 2013. Complete description can be found on APRA’s website.

In the second section, we will look at the effect of mortality derivatives on the Prudential Capital Requirement and how we can infer a price for those derivatives. But before doing that, here is a summary of the prudential framework we are working in:

(1) Representation of the Prudential Capital Requirement in relation to the balance sheet



(2) Representation of the build-up of the Prudential Capital Requirement



## 2. MORTALITY DERIVATIVES UNDER THE AUSTRALIAN PRUDENTIAL FRAMEWORK

Let us consider the case of a life insurer that provides annuities to its policyholders. If future mortality rates are lower than the best estimate assumptions, the insurer will have to pay annuities longer than expected and this may incur a loss. If we consider on the other hand a lump sum insurer, benefits will not be as important as expected under best estimate assumptions and this may incur a profit. In the case of an insurer that provides both annuity and lump sum policies, there is a natural immunization against the changes in future mortality rates. However, a lot of insurers tend to be more exposed to one risk or the other and they are not able to hedge their risks within themselves. In order to manage the risk of mortality, different solutions have been proposed in recent years. These solutions, mortality derivatives, involve swapping a cashflow of policyholders' benefits for another cashflow, whether it is floating for floating or floating for fixed.

With the new capital requirement set by APRA, insurers find that the mortality risks they have taken create a charge that is more sensitive to stresses than it was previously. The immediate effect for them is that the Prudential Capital Requirement they need to hold is higher than the amount they previously held under the old prudential framework. This extra amount of capital that some insurers have to find incurs a cost. This cost is an incentive to develop risk management solutions and decrease the sensitivity of the insurer to adverse future mortality rates.

### *a. Description of Mortality Derivatives*

We use the description of a mortality swap proposed by Cox and Lin<sup>1</sup>. The mortality swap is a contract between an insurer A that is subject to adverse consequences of a mortality increase (Lump sum insurer / Term Life insurer) and an insurer B that is subject to adverse consequences of a mortality decrease (Annuity insurer). We assume that insurer A and insurer B are providing only one type of policies and there is no other insurance risk hedging except for the mortality swap as described here.

There is no payment at initiation of the contract, then each year, insurer A pays to insurer B an amount that is based on the total benefits paid by insurer B to its policyholders, while insurer B pays to insurer A an amount that is based on the total benefits paid by insurer A. There is no need for both counterparties to transfer the cash, the difference between the 2 amounts is to be calculated and transferred to the appropriate counterparty.

Let  $R_A$  (resp.  $R_B$ ), the benefits paid by insurer A (resp. B) to its policyholders during a given year.

The mortality swap consists of:

- Insurer A paying  $\alpha \cdot R_B$  to the insurer B
- Insurer B paying  $\beta \cdot R_A$  to the insurer A

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<sup>1</sup> Samuel Cox and Yijia Lin, 2004, *Natural Hedging of life and annuity mortality risks*

$\alpha$  and  $\beta$  are to be determined at the initiation of the contract for a duration the counterparties agree on. The ratio  $\beta/\alpha$  determines the price of the arrangement. This article will propose a methodology to calculate this ratio.

### *b. Problems of the approach of Cox and Lin*

The pricing method of mortality swaps described by Cox and Lin has an underlying assumption that the price depends on the swap being a zero-sum game. The present value of cashflow between counterparties under market conditions is nil, that is with our notations:

$$\beta.PV(R_A) = \alpha.PV(R_B)$$

One issue is that the method explicitly excludes the impact of credit rating of the counterparties. In a scenario where counterparties have the same credit rating, one can expect that the price of the mortality swap will not depend on the credit rating of the companies. However, one can expect a risk premium in the price of the mortality derivative if there is a difference of credit rating between counterparties. (Cf Biffis<sup>2</sup> for counterparty risk).

The method implicitly excludes the impact of the mortality swap on the capital requirement of counterparties. Counterparties may not have the same interest in swapping mortality risks. An annuity provider typically has assets and liabilities with long durations. As a consequence, the impact on its capital requirement of a mortality swap will look very different to that of a Lump Sum insurer that has shorter assets and liabilities.

### *c. New approach for pricing a Mortality Swap*

The method that is proposed in this article is to compare not only the cashflows directly induced by the mortality swap, but also the profit induced by the reduction in capital requirement caused by the arrangement. The zero-sum game principle will apply on profit induced by the reduction in capital requirement instead of future cashflows.

In a given year, the present value of net cashflow from insurer B to insurer A plus profit due to reduction in capital requirement of insurer A is to be equal to the present value of net cashflow from insurer A to insurer B plus profit due to reduction in capital requirement of insurer B; Or

$$\beta.PV(R_A) - \alpha.PV(R_B) + i_A \cdot \delta PCA(A) = \alpha.PV(R_B) - \beta.PV(R_A) + i_B \cdot \delta PCA(B)$$

(Eq.A)

*Notations:*

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<sup>2</sup> Biffis, Blake, Pittoti & Sun, 2011, *The cost of counterparty risk and collateralization in longevity swaps*

$PV(R_A)$ :	Present value of future benefits of the year paid by insurer A to policyholders. It is calculated with an interested rate $i_A$ + Risk-Free rate.
$i_A$ :	Cost of capital for insurer A (Dividend yield or Bond yield above risk-free rate). It is risk premium that correspond to the credit risk of insurer A determined by its rating.
$\delta PCA(A)$ :	Differential in Prescribed Capital Amount due to future cashflows created by the mortality swap.
$i_A \cdot \delta PCA(A)$ :	Profit due to the reduction in the Prescribed Capital Amount of insurer A.

*Same notations apply for insurer B.*

This formula takes into account:

- (a) The fact that the capital requirements of insurer A and insurer B may not react symmetrically to the mortality swap arrangement. They may vary according to asset allocation, credit rating, risk diversification within the insurer and many other parameters.
- (b) The fact that the cost of capital for insurer A and insurer B may differ (for example because of different credit ratings), so they don't have the same appetite for a reduction in capital requirement.

As described in Section 1, the Prescribed Capital Amount is expressed as follows:

$$PCA = IRC + ARC + ACRC + ORC - AB + CSSA$$

We consider the mortality swap has no impact on the Asset Risk Charge (ARC), Asset Concentration Risk Charge (ACRC), Operational Risk Charge (ORC) and Combined Stress Scenario Adjustment (CSSA). So the PCA is constant in relation to these items.

Only the Insurance Risk Charge is a variable in relation to the mortality swap.

The movement in PCA can be expressed as a function of the remaining items:

- Insurance Risk Charge (I)
- Asset Risk Charge (A)
- Aggregation Benefit (AB)

$$\frac{\partial PCA}{\partial I} = \frac{\partial [I + A - AB]}{\partial I}$$

$$\frac{\partial PCA}{\partial I} = \frac{\partial [I + A - (I + A - \sqrt{A^2 + I^2 + 2c \cdot A \cdot I})]}{\partial I}$$

$$\frac{\partial PCA}{\partial I} = \frac{I + c \cdot A}{\sqrt{A^2 + I^2 + 2c \cdot A \cdot I}}$$

Note that:

$$\text{If Correlation} = 1 \text{ or } A = 0 \text{ then } \frac{\partial PCA}{\partial I} = 1$$

For insurer A:

$$\partial PCA_A = 2 \cdot \varphi_A \cdot \partial I$$

With:

$$\varphi_A = \frac{I_A + c \cdot A_A}{2 \times \sqrt{A_A^2 + I_A^2 + 2c \cdot A_A \cdot I_A}}$$

For insurer B:

$$\partial PCA_B = 2 \cdot \varphi_B \cdot \partial I$$

With:

$$\varphi_B = \frac{I_B + c \cdot A_B}{2 \times \sqrt{A_B^2 + I_B^2 + 2c \cdot A_B \cdot I_B}}$$

#### *d. Calculation of the Insurance Risk Charge Post-Swap*

LPS 115.10 states: "The Insurance Risk Charge for a statutory fund is the reduction, if any, in the Capital Base that would occur if the adjusted policy liabilities were changed to an amount equal to the stressed policy liabilities determined under this Prudential Standard."

$$I = \text{Max}\{\text{SPL} - \text{APL}, 0\}$$

According to the mortality swap, a proportion  $\alpha$  of the policyholders' benefits of insurer A is covered by insurer B since insurer B has agreed to pay the same proportion of the future policyholders' benefits. So the IRC of insurer A decreases by the proportion  $\alpha$ .

However, insurer A has agreed to cover a proportion  $\beta$  of the future policyholders' benefits paid by insurer B. So the IRC of insurer A should increase by a proportion  $\beta$  of the IRC of insurer B calculated with stresses adverse to insurer A.

We note  $I_A^+$  the Insurance Risk Charge of insurer A post-swap:

$$I_A^+ = I_A \cdot (1 - \alpha) + \beta \cdot \tilde{I}_B.$$

$\tilde{I}_B$  is not the actual regulatory Insurance Risk Charge of insurer B but a theoretical Insurance Risk Charge calculated using stressed assumptions of insurer A. Since insurer A and insurer B are adversely affected by opposite stresses,  $\tilde{I}_B$  is in fact a negative charge.

The differential of Insurance Risk Charge for insurer A is as follows:

$$\partial I_A = I_A^+ - I_A = \beta \cdot \tilde{I}_B - \alpha \cdot I_A$$

The differential of IRC before has two components:

- $\beta \cdot \tilde{I}_B$ , which is a negative 'Insurance Risk Charge' of insurer B using insurer A's stresses.
- $-\alpha \cdot I_A$ , the offload of risk from insurer A to insurer B.

We deduce from the previous equations:

$$\partial PCA_A = 2 \cdot \varphi_A \times \partial I_A = 2 \cdot \varphi_A \times (\beta \cdot \tilde{I}_B - \alpha \cdot I_A)$$

(Eq.B)

If we inject this into the original equation (Eq.A), then:

$$\beta \cdot PV(R_A) - \alpha \cdot PV(R_B) + i_A \cdot 2\varphi_A \times (\beta \cdot \tilde{I}_B - \alpha \cdot I_A) = \alpha \cdot PV(R_B) - \beta \cdot PV(R_A) + i_B \cdot 2\varphi_B \times (\alpha \cdot \tilde{I}_A - \beta \cdot I_B)$$

Which leads to:

$$\frac{\beta}{\alpha} = \frac{PV(R_B) + i_B \cdot \varphi_B \cdot \check{I}_A + i_A \cdot \varphi_A \cdot I_A}{PV(R_A) + i_A \cdot \varphi_A \cdot \check{I}_B + i_B \cdot \varphi_B \cdot I_B}$$

(Eq.C)

### e. Comments

∅ We now have a pricing formula that gives the ratio  $\beta/\alpha$  - the amount insurer B will pay to insurer A for every dollar insurer A pays to insurer B. This formula requires for each insurer:

1. The present value of future cashflow for the year we consider
2. The Insurance Risk Charge pre-swap
3. The theoretical Insurance Risk Charge using the counterparty's stresses
4. The Asset Risk Charge (unchanged before and after swap)
5. The cost of capital above risk-free rate

Of all the items above, only item 3 (theoretical Insurance Risk Charge using the counterparty's stresses) is an additional item that the insurer needs to calculate especially for the swap. All other items should already be available.

∅ The formula contains the impact of the ratings of insurer A and insurer B on the price through the present values of future policyholders' benefits discounted with different rates. It can be assumed that the discount rate used on insurer A (resp. B) should be the cost of capital of insurer A (resp. B), that is  $i_A$  + Risk-free rate.

∅ If there is no Insurance Risk Charge for insurer A and insurer B, the formula is equivalent to the original formula derived from Cox and Lin:  $\beta \cdot PV(R_A) = \alpha \cdot PV(R_B)$

### f. Efficiency on the swap using Profit on Exposure

We measure the efficiency of the mortality swap by using the ratio of Profit / Exposure:

$$P_A = \frac{\beta \cdot PV(R_A) - \alpha \cdot PV(R_B) + i_A \cdot \delta PCA(A)}{\alpha \cdot PV(R_B)}$$

This represents the ratio of Profit on Exposure for insurer A, where:

- ü  $\beta \cdot PV(R_A) - \alpha \cdot PV(R_B)$  is the present value of net cashflow from insurer B to insurer A
- ü  $i_A \cdot \delta PCA(A)$  is the profit coming from the reduction in capital requirement.

ü  $\alpha \cdot PV(R_B)$  is the exposure of insurer A to insurer B.

$P_A$  represents the profit (before tax) of insurer A for each dollar of exposure to the policyholders' benefits of insurer B.

If we use equation (Eq.B) for  $\delta PCA(A)$ , we can express Profit on Exposure as:

$$P_A = \frac{\beta}{\alpha} \times \left( \frac{PV(R_A) + 2 \cdot \varphi_A \cdot i_A \times \tilde{I}_B}{PV(R_B)} \right) - \frac{2 \cdot \varphi_A \cdot i_A \times I_A}{PV(R_B)} - 1$$

With equation (Eq.C), we have already calculated the ratio  $\frac{\beta}{\alpha}$ . Which means  $P_A$  is a constant in regards to the exposure. So we can establish that the profit coming from the mortality swap is proportional to the exposure of insurer A to insurer B and vice-versa.

At this point, before we start extrapolating and try to generate profits by simply increasing the exposure, we should bear in mind there are a number of conditions behind this formula. In particular, we have assumed the Asset Concentration Risk Charge does not impact the mortality swap. If we start increasing the exposure inconsiderately, we can't keep this assumption anymore. Furthermore, we have assumed one particular stress on the mortality of insurer A for the calculation of the Insurance Risk Charge. If we increase the exposure to insurer B so that insurer A is more exposed to insurer B's risk than to its own, there is a tipping point where the stress we applied in the Insurance Risk Charge is no longer appropriate and we need to apply the very opposite stress. In summary:

- Ø The profit is proportional to the exposure as long as the exposure doesn't trigger changes in assumptions.
- Ø There is an optimal exposure and we can calculate the profit generated by the mortality swap up until this optimal exposure.

### *g. Numerical example for a swap between an Annuity insurer and a Term life insurer*

We consider an annuity insurer (Insurer A) with policyholders aged 65 who receive \$10,000 annually. We also consider a Term life insurer (Insurer B) with policyholders aged 65 and a benefit of \$250,000 payable at their death within 10 years.

The Asset Risk Charge is set at 10% of best estimate policy liabilities for the annuity provider and 5% for the Term life insurer. The difference is to account for the different durations of the assets.

The Insurance Risk Charge is calculated using a stress of:

- ü -20% on mortality rates for longevity risk as prescribed by LPS 115-38 of the APRA Standards.
- ü +20% on mortality rates for mortality risk, determined by the Actuary as per LPS 115-33.

The risk-free rate is set at 3% and the base cost of capital above risk-free rate is set at 4% for both insurers. Other assumptions are described in Appendix B.

Under these assumptions, we find a ratio  $\frac{\beta}{\alpha} = 37\%$  which determines the price of the mortality swap between the two counterparties.

- *Impact of mortality swap on the Prescribed Capital Amounts and cashflows*

We calculate the Prescribed Capital Amounts after mortality swap and the profits from net cashflows for each counterparty. Results are shown relative to the PCA before mortality swap:

Alpha	Beta	Insurer A		Insurer B	
		PCA Post-Swap / PCA Pre-Swap	Expected profit on Cashflow / PCA Pre-Swap	PCA Post-Swap / PCA Pre-Swap	Expected profit on Cashflow / PCA Pre-Swap
0%	0.0%	100.0%	0.0%	100.0%	0.0%
5%	1.8%	99.8%	0.1%	95.6%	-0.1%
10%	3.7%	99.5%	0.1%	91.2%	-0.2%
15%	5.5%	99.3%	0.2%	86.8%	-0.2%
20%	7.4%	99.1%	0.3%	82.5%	-0.3%
25%	9.2%	98.9%	0.3%	78.2%	-0.4%
30%	11.1%	98.6%	0.4%	73.9%	-0.5%
35%	12.9%	98.4%	0.5%	69.6%	-0.6%

For the annuity insurer (insurer A), the mortality swap has little impact on the PCR compared to the impact on the PCR of the Term life insurer. The profit of the annuity insurer comes mostly from incoming net cashflows.

For the Term life insurer (insurer B), the mortality swap has a significant impact on the PCR and generates profit from the reduction in PCR. This profit is shared with the annuity insurer through outgoing cashflows.

- *Efficiency of the mortality swap*

We then calculate the efficiency of the mortality swap with the ratio profit / exposure.

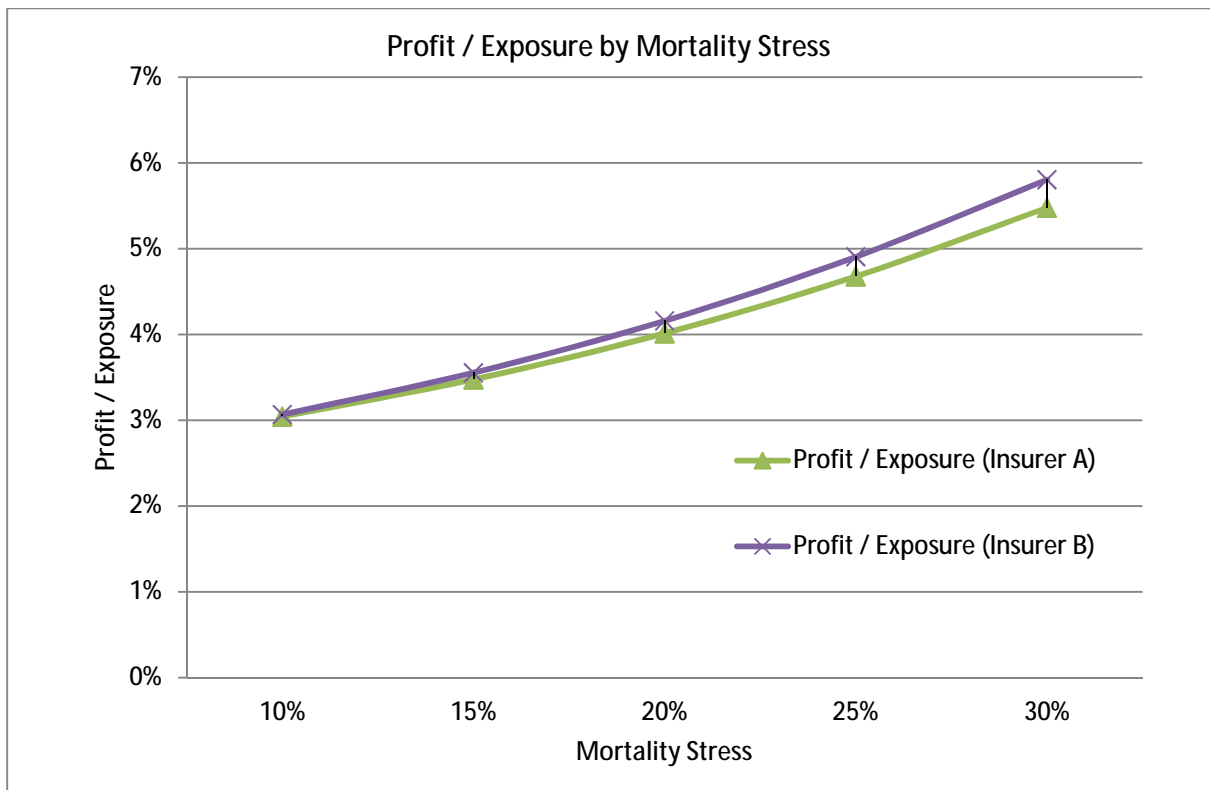
	Insurer A (Annuity)	Insurer B (Term life 10 years)
Profit coming from Net Cashflow / Exposure	3.5%	-3.6%
Profit from Reduction in PCR / Exposure	0.5%	7.8%
Total Profit / Exposure	4.0%	4.2%

In this simplified scenario, the mortality swap generates a profit of around 4% of the exposure to the counterparty's policyholders' benefits.

- *Sensitivity to the mortality stress*

We now look at the Profit on Exposure ratio when the stress on mortality risks for the Term life insurer increases (The stress on longevity for the annuity insurer is prescribed by the standard).

Here is the trend we observe if we increase the mortality stress from 10% to 30%:



As we can expect intuitively, the greater the stress applied for the calculation of the Insurance Risk Charge, the greater the efficiency of the mortality swap.

## CONCLUSION

In this paper, we have described the Australian prudential framework applicable to life insurer as of 1 January 2013 and presented the main differences in comparison to the previous framework. We have pointed out that under the new framework, life insurers have a significant incentive to hedge mortality and longevity risks, but many insurers can't hedge these risks internally.

We have then proposed mortality swaps as a risk management tool to hedge mortality and longevity risks in the context of this prudential framework, allowing life insurers to access other insurers' risk and generate mutual gain. We have provided a methodology for pricing mortality swaps between two counterparties that are exposed to opposite mortality risks.

Using a simple example of a mortality swap between an annuity insurer and a Term life insurer, we have calculated a ratio of efficiency of around 4% subject to reasonable assumptions, whereby for an exposure of \$100 to the expected benefits to counterparty's policyholders, the mortality swap generates a profit of \$4 through cashflows between counterparties or through reduction in the Prudential Capital Requirement, depending on the risk sensitivity of the insurer's Capital requirement.

Looking forward, we promote mortality swaps as instruments to mitigate capital requirements for life insurers that are over-exposed to mortality or longevity risks.

## APPENDIX A – BIBLIOGRAPHY

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## APPENDIX B - Other assumptions used in the calculation of the mortality swap

- Mortality assumptions for both insurers:

"S1" series base mortality tables: Based on 2000-2006 experience collected by 30 June 2007 from UK self-administered pension schemes, published in CMI Working papers 34 and 35, 2008.

## APPENDIX C - LEXICON

<i>APRA</i>	<i>Australian Prudential Regulation Authority</i>
<i>Asset Risk Charge</i>	<i>Component of the Prudential Capital Requirement related to asset risk, determined by applying different stresses on the asset</i>
<i>Capital base</i>	<i>Excess of assets over Adjusted policy liabilities and other liabilities</i>
<i>Insurance Risk Charge</i>	<i>Component of the Prudential Capital Requirement related to insurance risk, determined by applying different stresses on the liability</i>
<i>Asset Concentration Risk Charge</i>	<i>Component of the Prudential Capital Requirement related to over-exposure to a counterparty, the calculation depends on the type of counterparty</i>
<i>LAGIC</i>	<i>Life And General Insurance Capital, the prudential standards set by APRA</i>
<i>Prescribed Capital Amount</i>	<i>Pillar 1 capital requirement in excess of adjusted policy liabilities and other liabilities</i>
<i>Prudential Capital Requirement</i>	<i>Prescribed Capital Amount with Pillar 2 supervisory adjustment</i>