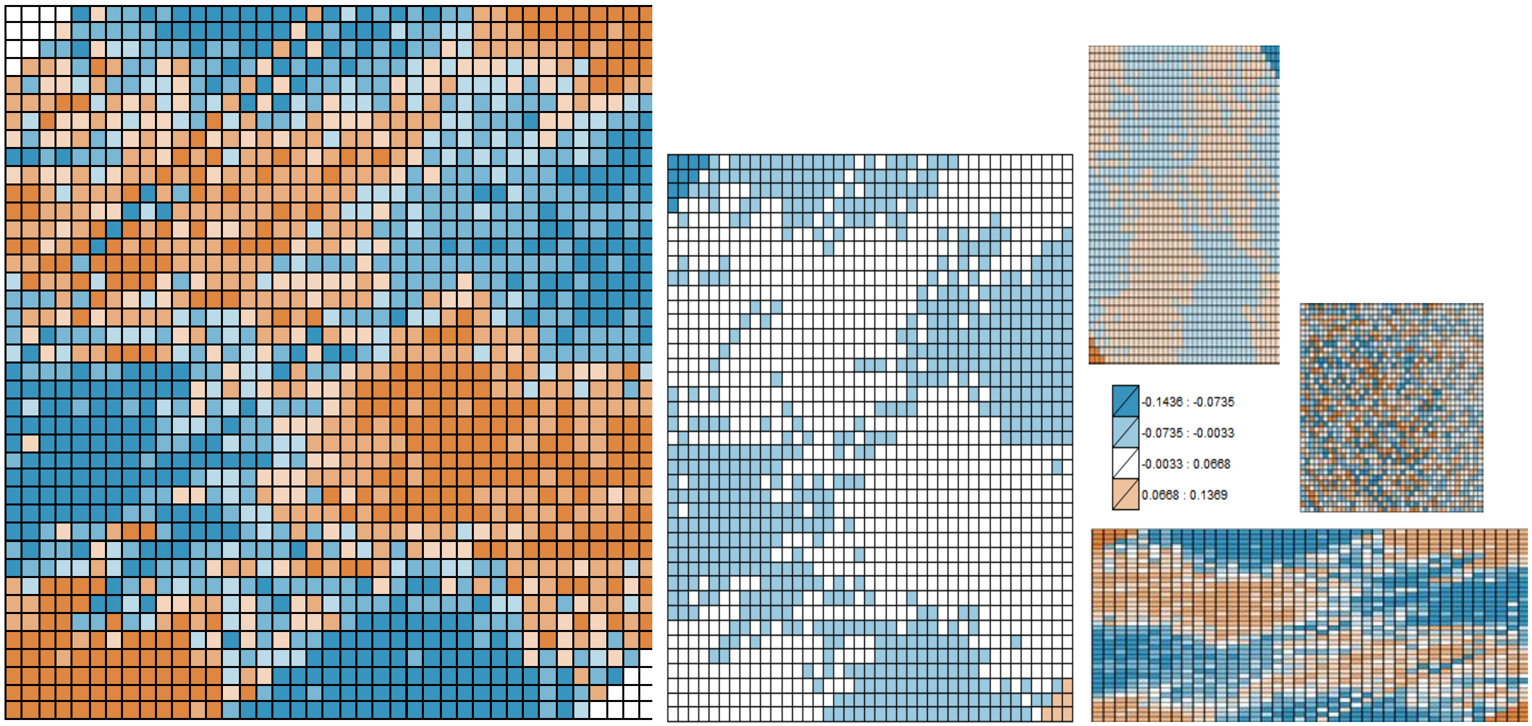

Coherent Stochastic Mortality Modelling for a small population

An application to Swiss mortality data

A presentation to 2013 AFIR Colloquium

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Outline

- **Background**
- **Model review**
- **Model proposed**
  - Case Switzerland
- **Summary and Discussion**
- **Questions**

Background

- **Growing longevity risk**
  - Europe: strong growth, small populations
    - Switzerland: generous legislation, only one table for reserve evaluation for private pension funds and one for public pension funds

- **Available data and models**
  - Could be employed to forecast the future

- **Which models are needed?**
  - Practical for pension funds, accepted by the trustees
    - e.g., Li-Lee (2005) (mentioned in “Longevity future in Switzerland”, Federal Office for Statistics, 2009)

Background

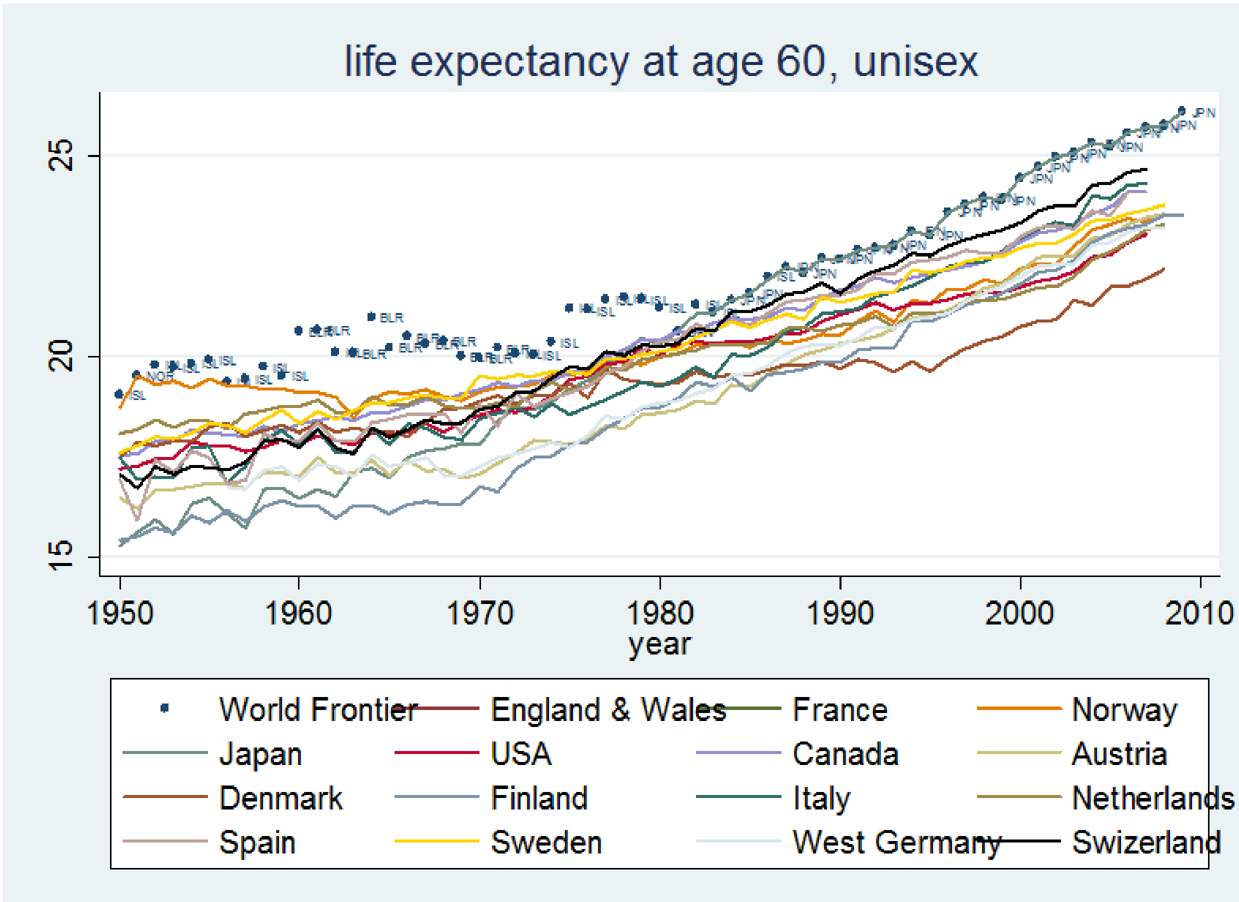

Figure 1: Life expectances at age 60 for 15 low mortality countries and a "world frontier" with lowest mortality.

Model Review

- **Models for single population**
  - Lee-Carter type
    - $\log(m(x,t)) = \beta_x^{(0)} + \beta_x^{(1)}\kappa_t^{(1)} + \text{other terms}$
  - Cairns-Blake-David (CBD) type
    - $\text{logit}(q(x,t)) = \kappa_t^{(1)} + \kappa_t^{(2)}(x - \bar{x}) + \text{other terms}$
  - Age-Period-Cohort (APC) type
  - Plat (2009)
    - $\log(m(x,t)) = \beta_x^{(1)} + \kappa_t^1 + \kappa_t^2(x - \bar{x}) + \gamma_{t-x} + \text{other terms}$

Model Review

- **Models for multiple population**
  - Li-Lee (2005)
    - $\log\big(m(x,t,i)\big) = a(x,i) + B(x)K(t) + b(x,i)k(t,i) + \varepsilon(x,t,i)$

Augmented common factor model

Extension of Lee-Carter

  - SAINT (2011)
    - $\log\big(\mu(x,t,i)\big) = \log\big(\mu(x,t)\big) + \sum_{j=1}^{3} r_j(x)k_j(t,i)$

$\mu(x,t)$, frailty model

$r_j(x)$, predefined parametric age factors

Model Proposed

- **Ideas from Li-Lee (2005)**
  - $\log\big(m(x,t,i)\big) = a(x,i) + B(x)K(t) + b(x,i)k(t,i) + \varepsilon(x,t,i),$

    decompose $a(x,i) = A(x) + \tilde{a}(x,i)$

    $$\log\big(m(x,t,i)\big) = \underbrace{A(x) + B(x)K(t)}_{\text{Common factor: Lee-Carter}} + \underbrace{\tilde{a}(x,i) + b(x,i)k(t,i)}_{\text{“Spread” factor: Lee-Carter}} + \varepsilon(x,t,i),$$

- **Ideas from the SAINT (2011)**
  - $\log\big(\mu(x,t,i)\big) = \log\big(\mu(x,t)\big) + \sum_{j=1}^{3} \underbrace{r_j(x)}_{\text{Predefined parametric age factors}} k_j(t,i)$

Model Proposed

- **Our idea:**
  - Include nice features from other models since 2005
  - More flexible

$$\log\big(m(x,t,i)\big) = \beta_x + \kappa_t^1 + \kappa_t^2(x-\bar{x}) + \gamma_{t-x} + \beta(x,i) + \sum_{j=1}^{m} b_j(x,i)k_j(t,i) + \varepsilon(x,t,i),$$

Plat (2009)

Lee-Carter typed nonparametric age factors

"Spread" factor: Lee-Carter

Common factor: Lee-Carter

Predefined parametric age factors

Model Proposed

- **Model estimation**

Stage 1:

- Plat model, reference population
  - Maximum likelihood
- Augmented Lee-Carter model, spread population
  - Singular Value Decomposition (SVD):
  - Based on the Plat model's estimation result

Stage 2:

- Dynamic processes estimation
  - Based on result from Stage 1
  - Reference
  - Spread

- **Test and Diagnostics**
  - BIC, standardized residuals, etc.

Simulation

Model Proposed: case Switzerland

Data

- **Reference population:**
  - Pooled international mortality data
    - Austria, Denmark, Finland, France, Germany, Netherland, Norway, Spain, Sweden, Switzerland, UK, USA, Japan, except Canada and Italy compared to Li-Lee (2005) due to partial lack of data
- **Spread population:**
  - Switzerland

Simulation setting: With uncertainty

- **Reference population:**
  - Random walk with drift
  - AR(1)
- **Spread population:**
  - VAR(1)

Model Proposed: case Switzerland

- **Simulation result: death rate projections**

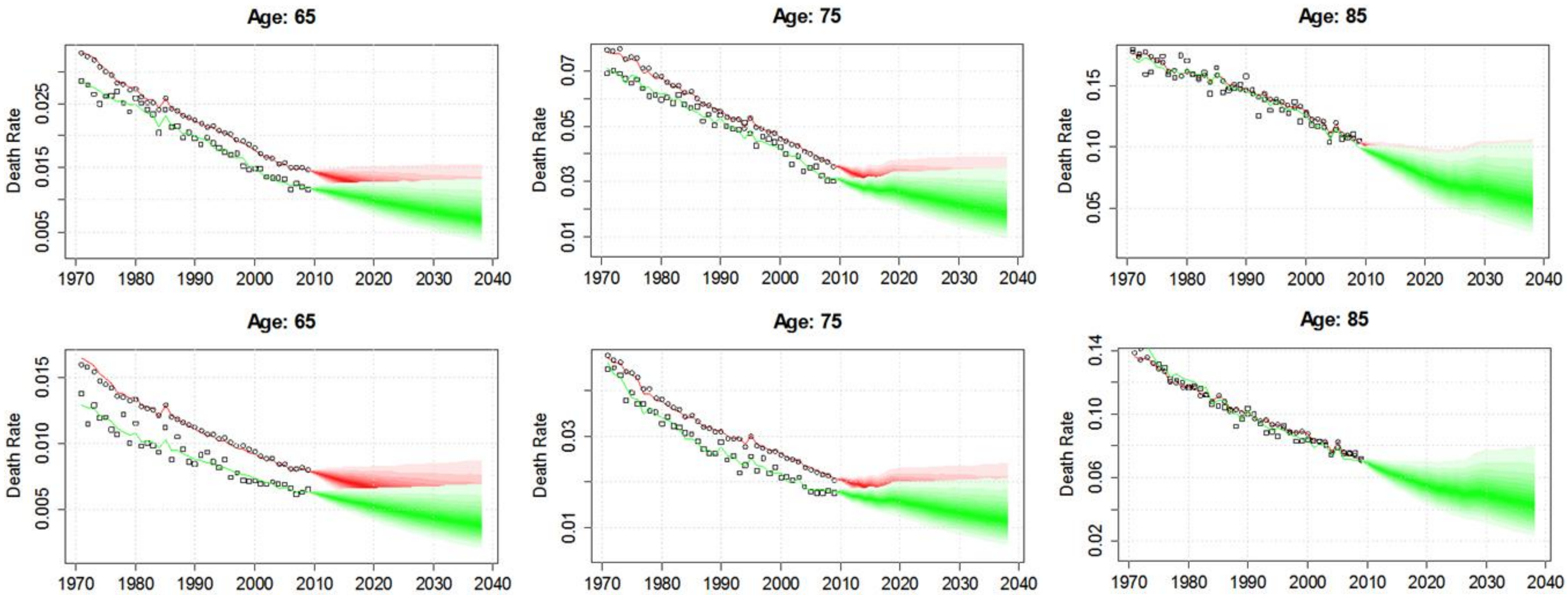

Figure 2: 5000 simulated death rate projections and raw data for Switzerland (green, square) and the international trend (red, circle) at age 65, 75 and 85. Upper: Male population; lower: Female population.

Model Proposed: case Switzerland

Life expectancy

- **Closing the life table (age 50-89 is used here)**
  - Extending mortality rate qxt
    - Log scale, logit scale
    - Bell et al. (1992)
    - Denuit and Goderniaux (2005)
  - Extending improvement
    - Linearly taper off to zero

Model Proposed: case Switzerland

■ **Simulation result: life expectancy**

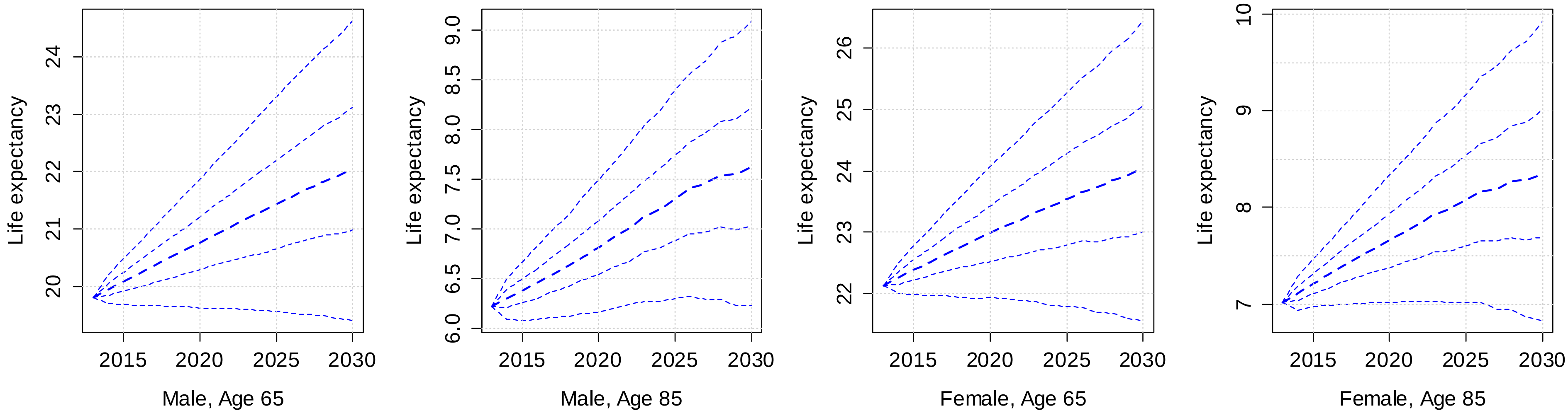

Figure 3: Period life expectancy at age 65 and 85. Blue dashed curves: Percentiles (5%, 25%, 50%, 75%, 95%) for simulated life expectancies.

Summary and Description

- **Model**

  Specification

  - Plat (reference) + augmented Lee-Carter (spread)

  Simulation

  - With uncertainty

- **Features**
  - Combine nice features from past single population modelling techniques
  - Allow for flexible age effect for spread population
  - Coherent projections
  - Simple

Summary and Discussion

Further development?

- **Model**
  - Estimation:
    - SVD ➔ maximum likelihood
  - Simulation:
    - bootstrap, could be time consuming
- **Data**
  - Target population
    - **Not** limited to Switzerland!
  - Reference population:
    - Group different countries as international data
      - Generally stable; Remove US population increases Japan's effect, population size
    - Use world frontier as reference
- **Practical issues**
  - Closing the life table
  - Sensitivity tests for choice of reference population group

Questions?

Thank you!

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