A Correlation Sensitivity Analysis for non-life underwriting risk module SCR

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Motivation

Some extracts from the Directive 2009/138/EC ...

- The main objective of (re)insurance regulation and supervision is **adequate policyholder protection**. Other objectives such as financial stability and fair and stable markets should also be taken into account but should not undermine that main objective.
- Supervision shall be based on a **prospective and risk-oriented approach**. Solvency II therefore adopts an economic risk-based approach which allows for a system that reflects the true risk profile of (re)insurance undertakings.
- The protection of policyholders presupposes that insurance and reinsurance are subject to effective solvency requirements what derives into an efficient capital allocation in the European Union. In the light of developments market, the current system is no longer adequate. It is therefore necessary to establish a **new regulatory framework**.

Motivation

Section 4 - Subsection 1 of the Directive: General provisions for the Solvency Capital Requirement using the Standard formula or an Internal Model.

- Two ways to meet the Solvency Capital Requirements.
  - Standard Model
  - Internal Model (under regulator previous authorization)

Historically risks have been considered separately (and still are) but under new regulation framework interrelations between them must be considered, so **correlation estimation between risk will be crucial** in both Standard and Internal approaches and becomes a key point.

Motivation

Some papers on SCR topics...

- Sandström, A. (2007); *Calibration for skewness. Scandinavian Actuarial Journal, 2, 126-134.*
- *Ronkainen et al. (2007); Topical modeling issues in Solvency II. Scandinavian Actuarial Journal, 2, 135-146.*
- *Pfeifer, D. and Staussburger, D. (2008); Stability problems with the SCR aggregation formula. Scandinavian Actuarial Journal, 1, 61-67.*
- *Ferri et al. (2011); Sensitivity analysis to changes on correlation between lines of business of non life underwriting module SCR based on standard formula. (submitted)*

Motivation

Our aim is analyze the influence that correlation matrix between lines of business and dependence structure has on the estimation of non life underwriting SCR estimation under both the Standard and Internal approach.

- We focus on SCR estimation of non-life underwriting risk sudmodule.
- We built and Internal Model.
- We compute the 2010-SCR with both Standard (based on QIS-5) and Internal approaches with QIS-5 parameters.
- We performed a correlation sensitivity analysis on both Standard and Internal approaches.

Standard formula

Under QIS-5 the non life underwriting SCR is obtained as the product of a volume measure and an approximation of the 99.5% Mean-VaR at a one year horizon of a lognormally distributed random variable:

$$SCR = \rho(\sigma) \cdot V$$

$$V = \sum_{i=LoB_1}^{LoB_n} V_i = \sum_{i=LoB_1}^{LoB_n} \left(max\{ \sum_{j=1}^{m} P_{t,i,j}^{written}; \sum_{j=1}^{m} P_{t-1,i,j}^{written}; \sum_{j=1}^{m} P_{t,i,j}^{earned} \} + \sum_{j=1}^{m} BE_{t,i,j} \right) \left(\frac{3}{4} + \frac{1}{4} W_i \right)$$

where W_i is a geographical diversification coefficient and

$$\rho(\sigma) = \frac{exp\left(z_{0.995} \cdot \sqrt{log\left(\sigma^2+1\right)} \right)}{\sqrt{\left(\sigma^2+1\right)}} - 1$$

where $z_{0.995}$ is the 99.5-*th* percentile of a standard normal distribution and σ is obtained in two steps as a mixture of premium standard deviation and reserve standard deviation by lines of business with weights premium and reserve volume measures by line of business in the first step and aggregating standard deviations by line of business **using correlation** between them in a second step.

The premium and reserve standard deviation by line of business are parameters to be estimated.

Internal Model

- Under the Internal Model the SCR is obtained as the 99.5% Mean-VaR at a one year horizon of a random sample from a multivariate distribution where the random variables represented by the margins are the prediction of net underwriting result of each line of business.
- The predictions are obtained through simple linear regression model based on a set of historical data of each component involved in the net result calculation.
- We use two multivariate distributions, Gaussian and t-Student, and two sets of univariate distributions for margins, Gaussian and t-Student.

Data and inputs for models

- We use deflated aggregate time serial data from 2000-2009 corresponding to the Spanish non-life underwriting market.
- Reclassification of data. From Spanish's legal branches to QIS-5's Lines of Business following recommendation of the Spanish supervisor, UNESPA, for QIS-5 spanish participants.
- For the Internal Model we consider 4 independent components:
  - Net premiums (inc. variation on reserves for non-earned premium and current risks account)
  - Net claims (inc. variation on reserves for claims)
  - Net expenses
  - Other expenses

Table: Standard model inputs

LoB	$P_i^{2009,written}$	$P_i^{2010,written}$	BE_i^{2010}(*)	(%) σ_{pr}^i	(%) σ_{res}^i	(%) σ_i
Motor vehicle liability	5.78	5.15	5.22	10	9.5	8.5
Other motor	4.81	4.54	1.00	7	10	6.8
Marine, Aviation, Transport	0.42	0.30	0.59	17	14	13.2
Fire	6.87	5.86	2.65	10	11	9.1
3rd. party liability	1.21	1.05	4.33	15	11	10.6
Credit, Suretyship	0.49	0.41	0.90	21.5	19	17.3
Legal expenses	0.16	0.16	0.12	6.5	9	6.3
Assistance	0.67	0.61	0.06	5	11	5
Miscellaneous	1.89	1.90	0.21	13	15	12.51
N.P. Property	1.85	0.41	0.00	17.5	20	16
N.P. Casualty	0.07	0.03	0.00	17	20	15.9
N.P. MAT	0.23	0.10	0.00	16	20	16.2

Source: DGSFP / (*)*Best Estimate* / Units: thousand of millions

Table: Internal model inputs

LoB	$\hat{Y}_{2010}^{i,pr}$	$\hat{Y}_{2010}^{i,cl}$	$\hat{Y}_{2010}^{i,exp}$	$\hat{Y}_{2010}^{i,o.exp}$	CV_{pr}^i(*)	CV_{cl}^i	CV_{exp}^i	$CV_{o.exp}^i$
Motor vehicle liability	6.82	5.13	1.26	-0.01	10	6	9	53
Other motor	5.41	3.86	0.92	0.03	6	2	4	21
Marine, Aviation, Transport	0.48	0.32	0.12	0.008	6	11	8	18
Fire	7.63	4.71	2.10	0.12	3	3	4	10
3rd. party liability	1.63	0.90	0.33	0.02	14	21	9	18
Credit, Suretyship	0.55	1.03	0.37	0.02	3	71	39	55
Legal expenses	0.19	0.09	0.04	0.0009	9	9	7	42
Assistance	0.74	0.53	0.13	0.01	6	8	6	8
Miscellaneous	1.96	0.77	0.69	0.04	1	2	4	11
N.P. Property	1.84	0.69	0.34	-	5	41	32	-
N.P. Casualty	0.07	0.03	0.02	-	8	12	6	-
N.P. MAT	0.23	0.46	0.39	-	8	1.38	1.75	-

Source: DGSFP / (*) (%) Variation coefficient / Units: thousand of millions

SCR. Standard versus Internal approach

Table: SCR. Standard Model *versus* Internal approach

Independence correlation matrix between LoB					
	Standard model	Gaussian Margins		t-Student Margins	
d.f.		Gaussian copula	t-Student copula	Gaussian copula	t-Student copula
4	4.15	4.15	4.62	4.72	5.28
10	-	-	4.39	4.31	4.56
35	-	-	4.22	4.17	4.25
QIS-5 correlation matrix between LoB					
	Standard model	Gaussian Margins		t-Student Margins	
d.f.		Gaussian copula	t-Student copula	Gaussian copula	t-Student coupla
4	7.18	6.74	7.30	7.65	8.53
10	-	-	7.03	7.06	7.38
35	-	-	6.83	6.84	6.94
Comonotonicity correlation matrix between LoB					
	Standard model	Gaussian Margins		t-Student Margins	
d.f.		Gaussian copula	t-Student copula	Gaussian copula	t-Student copula
4	11.63	10.22	10.25	12.93	12.92
10	-	-	10.25	11.22	11.28
35	-	-	10.22	10.47	10.49

Source: Own source / Units: thousand of millions

SCR. Standard versus Internal approach

Table: SCR. Confidence level in Internal Model to asses Standard Model SCR

Independence correlation matrix between LoB					
	Standard model	Gaussian Margins		t-Student Margins	
d.f.		Gaussian copula	t-Student copula	Gaussian copula	t-Student copula
4	99.5	99.5	99.13	99.16	99.11
10	-	-	99.30	99.37	99.19
35	-	-	99.43	99.46	99.41
QIS-5 correlation matrix between LoB					
	Standard model	Gaussian Margins		t-Student Margins	
d.f.		Gaussian copula	t-Student copula	Gaussian copula	t-Student copula
4	99.5	99.70	99.44	99.32	99.10
10	-	-	99.55	99.54	99.40
35	-	-	99.66	99.66	99.61
Comonotonicity correlation matrix between LoB					
	Standard model	Gaussian Margins		t-Student Margins	
d.f.		Gaussian copula	t-Student copula	Gaussian copula	t-Student copula
4	99.5	99.83	99.83	99.28	99.27
10	-	-	99.83	99.58	99.57
35	-	-	99.83	99.77	99.77

Source: Own source

Discussion

Some advantages and disadvantages using Standard versus Internal approaches...

- Standard Model is an easy implementing approach but not a clear understandable model.
- Despite is an easy implementing model it seems to be a too rigid model when using QIS-5 parameters depending too much on volume measure.
- SCR calculated with Standard Model is based on an approach of Mean-VaR but it's not a Mean-VaR based capital.
- Standard Model does not take into account technical premiums but underwriting premiums what could lead to problems when comparing two companies with same volume measures and different risk premium margin.

Discussion

Some advantages and disadvantages using Standard versus Internal approaches...

- We believe that our random variable reflects better premium and reserve risk since it takes into account resources and claims and translates them to the profit and losses account.
- Using copulas it's a good way to estimate Mean-VaR and helps to improve understanding in SCR estimation.

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