An Importance Sampling Algorithm for Copula Models in Insurance

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Outline

Motivations

Importance Sampling
- Motivation
- Importance Sampling

Case Study

Conclusion

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Consider a portfolio of d losses $\mathbf{X} = (X_1, \ldots, X_d)$ with joint distribution

$$\mathbb{P}(X_1 \leq x_1, \ldots, X_d \leq x_d) = C(F_{X_1}(x_1), \ldots, F_{X_d}(x_d)),$$

with

- $C : [0,1]^d \to [0,1]$ the associated copula function,
  - ✔ C captures the **dependence** between the X_i.
- F_{X_i}'s the marginal distributions,
  - ✔ Here: X_j are **loss distributions** of different Lines of Business

Motivations

Objective

Given a function Ψ_0, how to estimate $\mu = \mathbb{E}(\Psi_0(\mathbf{X}))$?

Examples of applications in Insurance

- Expected Shortfall of an aggregate loss $S = \sum_{j=1}^{d} X_j$
- Fair premium of a stop-loss cover on the aggregate loss S.

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Standard Monte Carlo

Project Ψ_0 onto the copula space:

$$\Psi(u_1, \ldots, u_d) := \Psi_0\left(F_{X_1}^{-1}(u_1), \ldots, F_{X_d}^{-1}(u_d)\right).$$

- $\mu = \mathbb{E}(\Psi_0(\mathbf{X})) = \mathbb{E}(\Psi(\mathbf{U}))$ with $\mathbf{U} \sim C$.
- Estimate μ with

$$\mu_n = \frac{1}{n}\sum_{i=1}^{n} \Psi(\mathbf{U}_i), \quad \mathbf{U}_i \overset{i.i.d}{\sim} C.$$

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$\mathrm{var}(\mu_n)$ might be large even for large n (high estimation error)

Variance Reduction Methods

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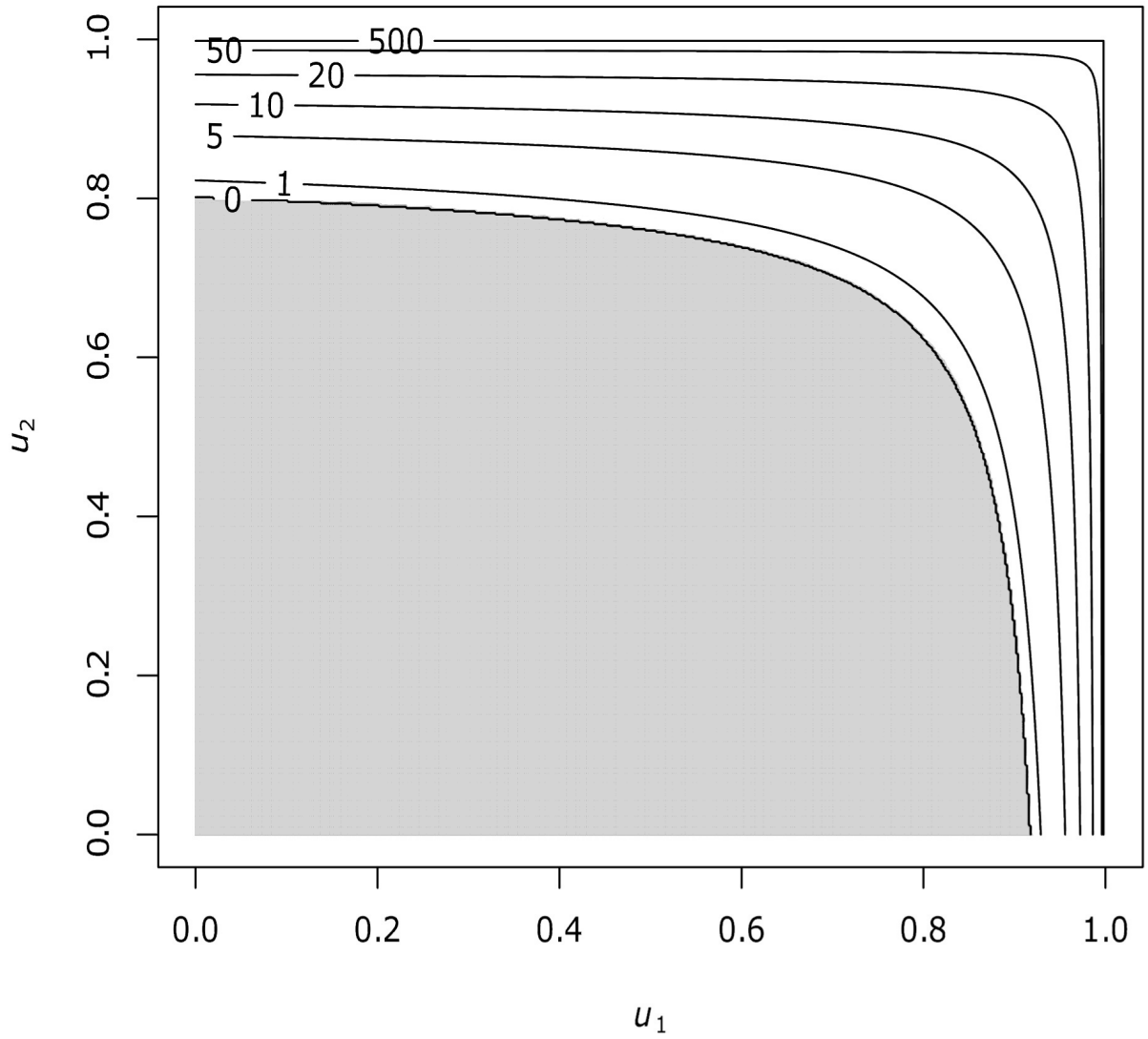
Level plot of Excess of Loss

$$\Psi(u_1, u_2) = \max\{F_{X_1}^{-1}(u_1) + F_{X_2}^{-1}(u_2) - T, 0\}$$



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Level plot of Excess of Loss with Gumbel copula sample ($\theta = 1.5$)

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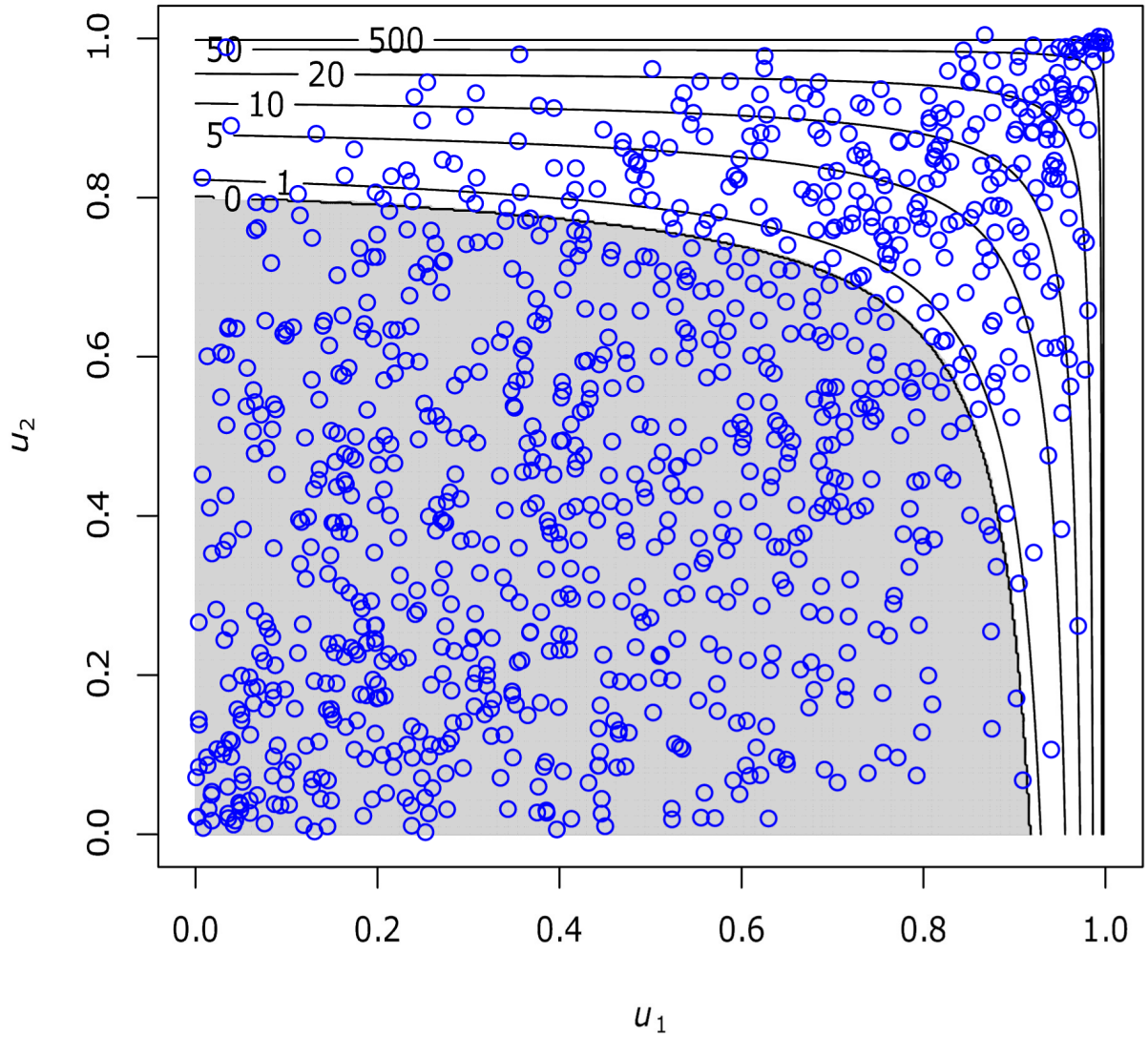

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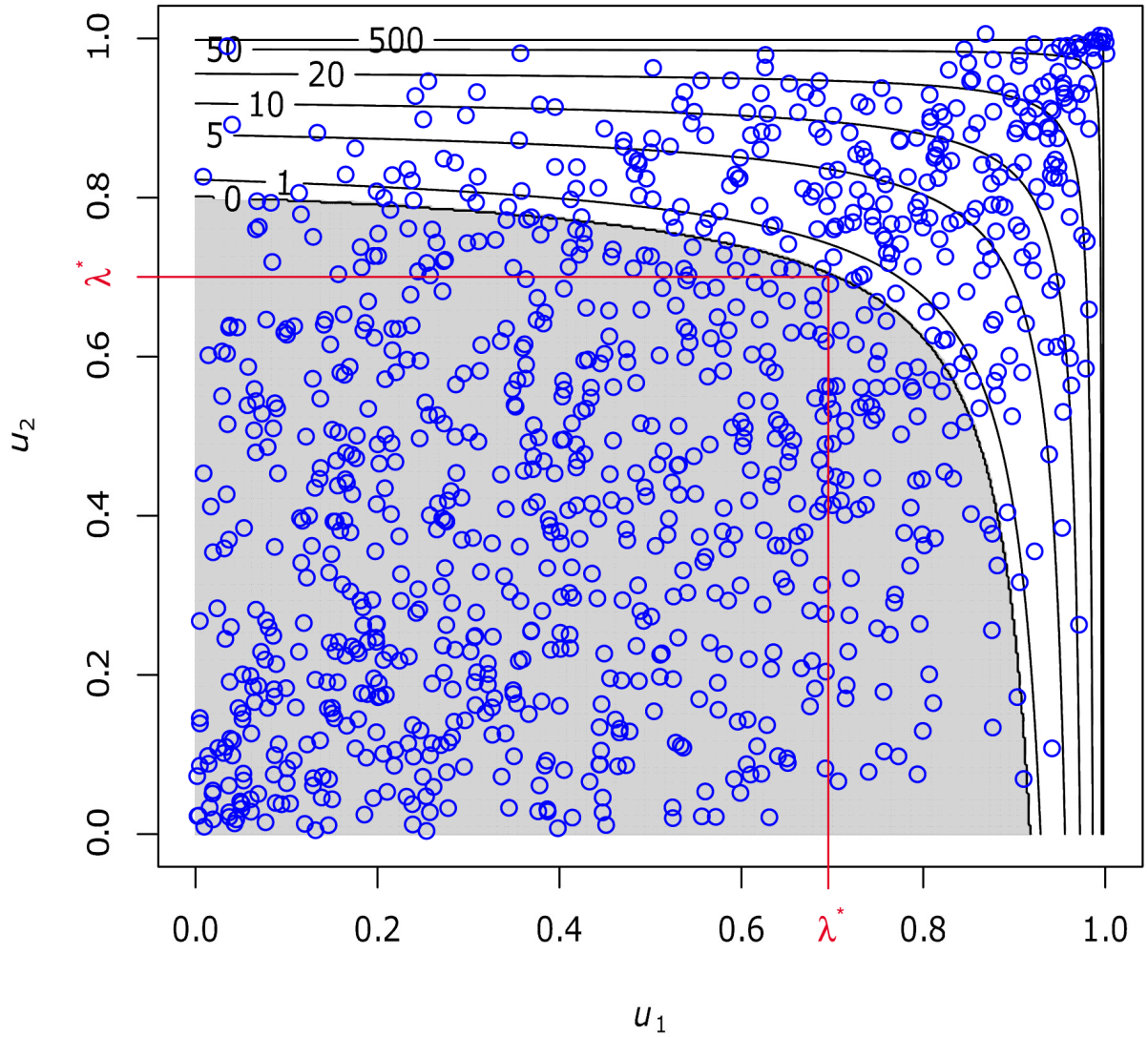

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$$C(u_1, u_2) := \mathbb{P}(U_1 \leq u_1, U_2 \leq u_2)$$

- $C^{[\lambda]}$ the conditional distribution

$$C^{[\lambda]}(u_1, u_2) := \mathbb{P}(U_1 \leq u_1, U_2 \leq u_2 | \max\{U_1, U_2\} > \lambda),$$

$F_{\mathbf{V}}$ is a mixture of $C^{[0]} = C$ and $C^{[\lambda^*]}$

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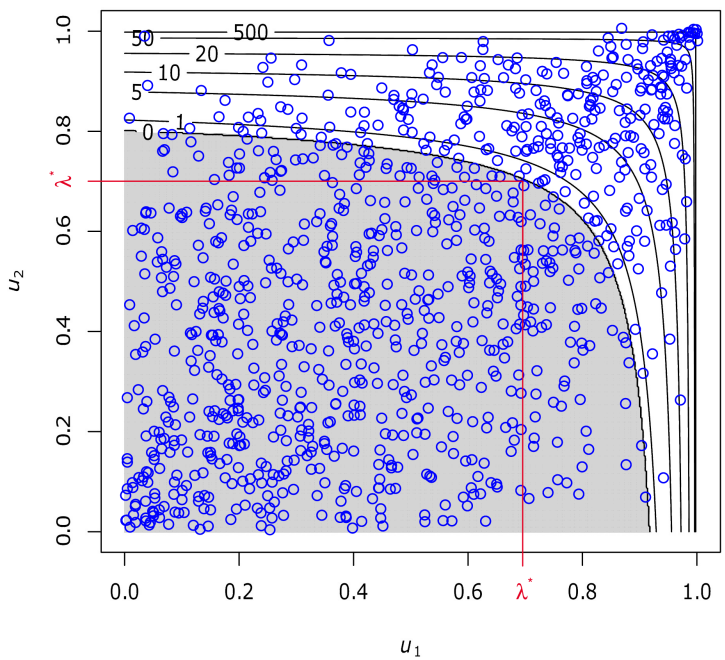

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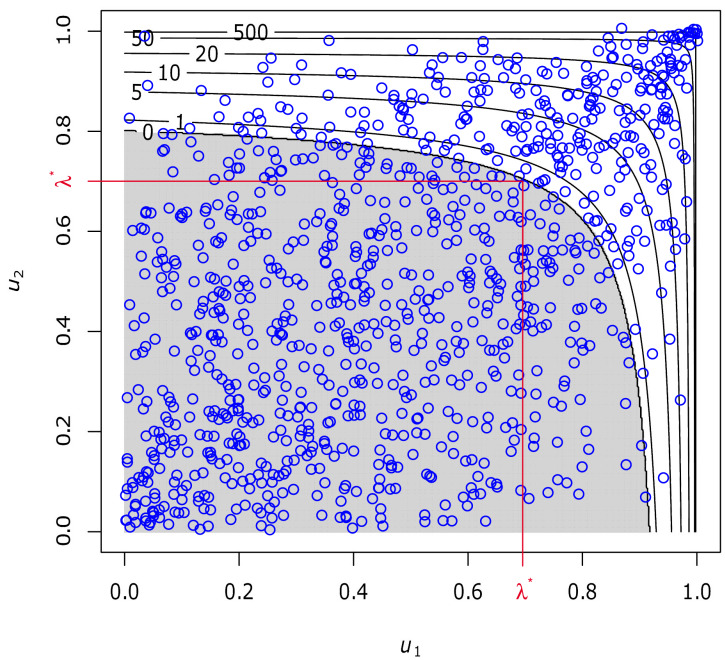

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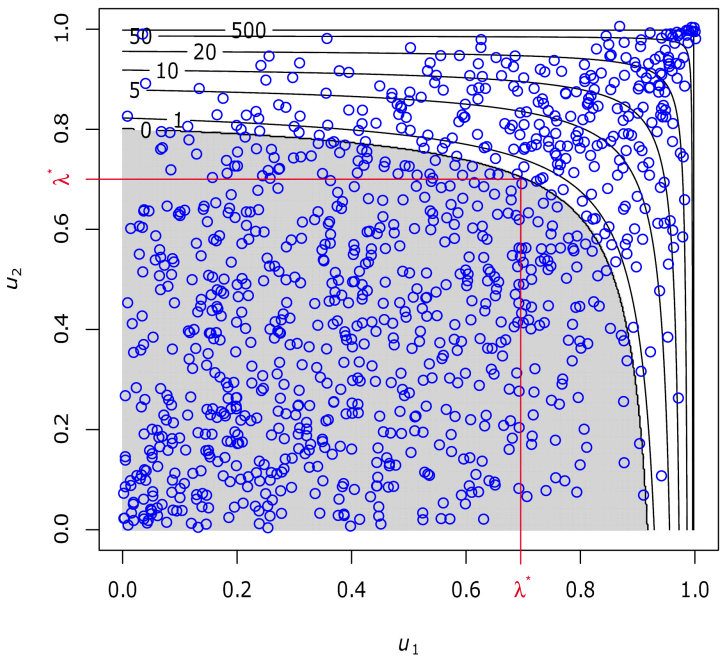

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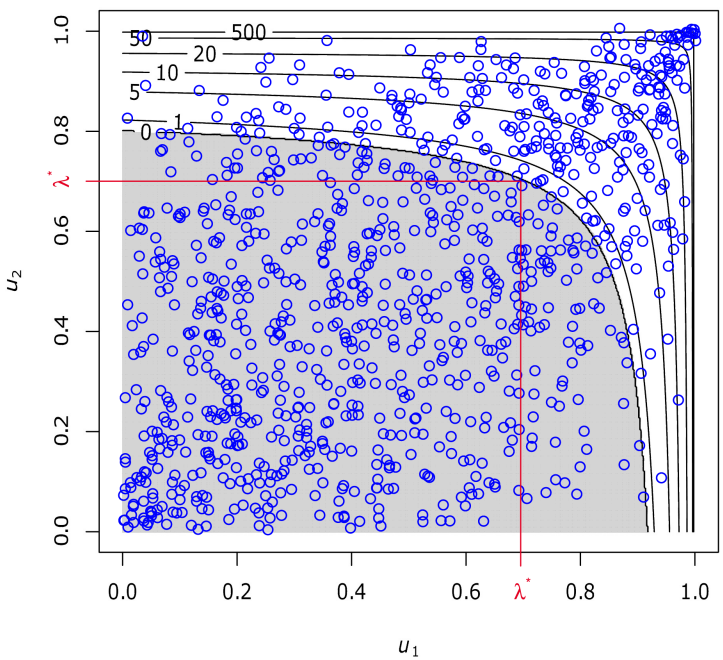
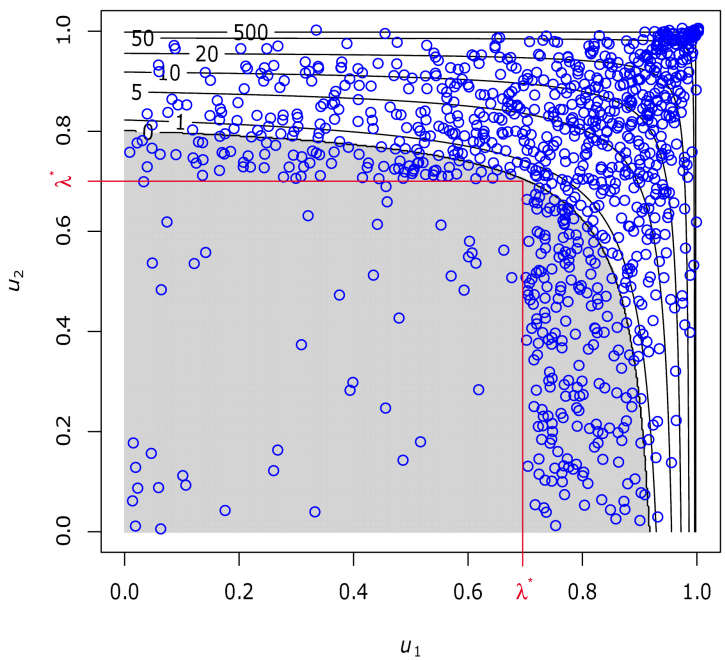


1'000 samples from $F_{\mathbf{V}}$, $p_0 = 10\%$



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1'000 samples from $F_{\mathbf{V}}(u_1, u_2) = p_0\, C^{[0]}(u_1, u_2) + (1 - p_0)\, C^{[\lambda^*]}(u_1, u_2)$

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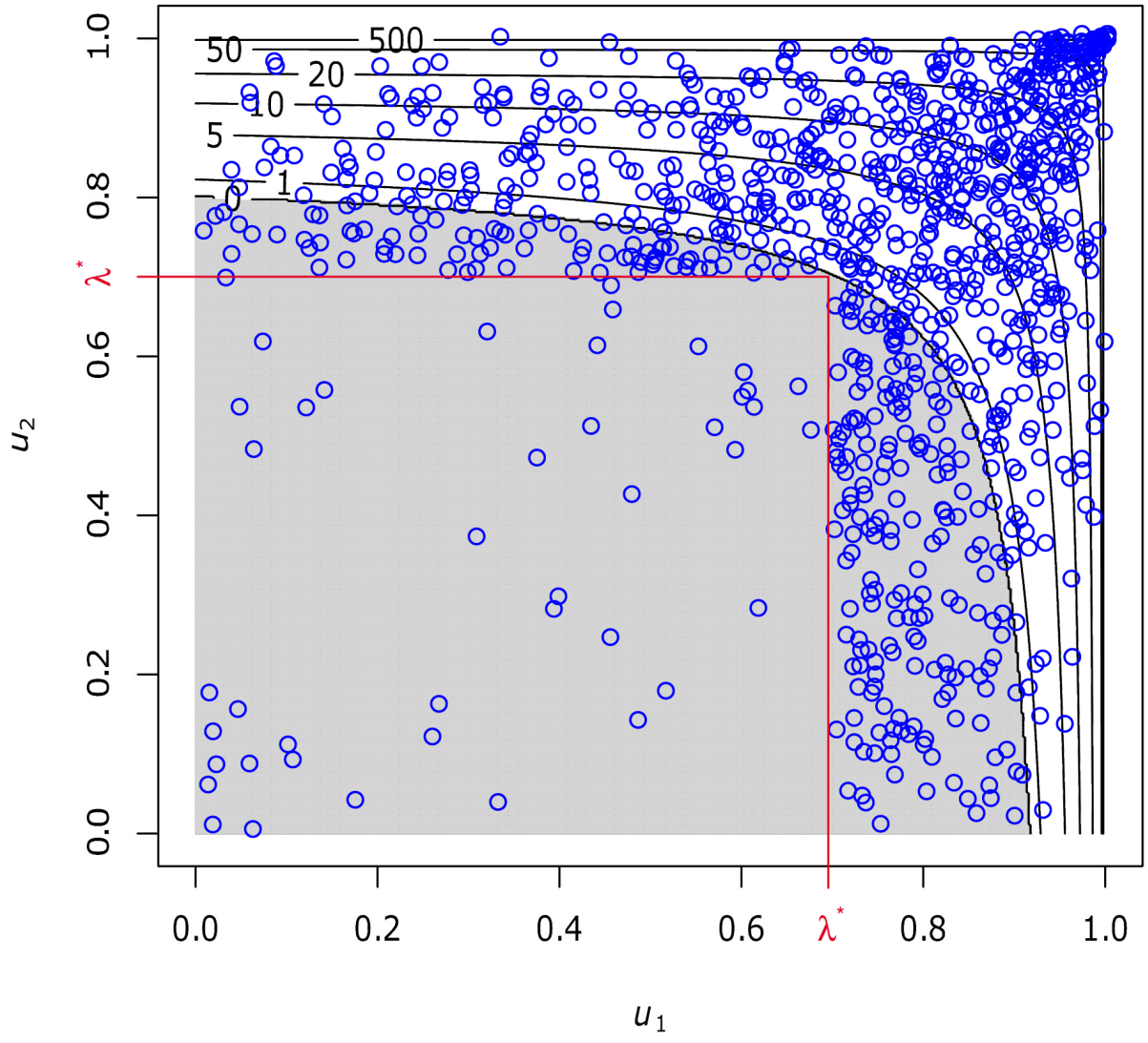

Importance Sampling

Algorithm

Fix $n \in \mathbb{N}$. For $i = 1, \ldots, n$:

1. Draw $\Lambda_i \in \{0, \lambda^*\}$ with probability $\{p_0, 1 - p_0\}$,
2. Draw $\mathbf{V}_i \sim C^{[\Lambda_i]}$ (rejection algorithm!),
3. Compute the weight $w(\mathbf{V}_i) = \frac{\mathrm{d}C(\mathbf{V}_i)}{\mathrm{d}F_{\mathbf{V}}(\mathbf{V}_i)}$ (is easy!).

Return

$$\widetilde{\mu}_n = \frac{1}{n} \sum_{i=1}^{n} w(\mathbf{V}_i) \Psi(\mathbf{V}_i)$$

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$$\widetilde{\mu}_n = \frac{1}{n}\sum_{i=1}^{n} w(\mathbf{V}_i)\Psi(\mathbf{V}_i), \quad \mathbf{V}_i \overset{i.i.d}{\sim} F_{\mathbf{V}}$$

Remarks

✔ *Unbiased estimator?* Yes

$$\mathbb{E}\left[\widetilde{\mu}_n\right] = \int_{[0,1]^2} \frac{\mathrm{d}C(\mathbf{v})}{\mathrm{d}F_{\mathbf{V}}(\mathbf{v})}\Psi(\mathbf{v})\mathrm{d}F_{\mathbf{V}}(\mathbf{v}) = \int_{[0,1]^2} \Psi(\mathbf{u})\mathrm{d}C(\mathbf{u}) = \mu.$$

✘ *Well defined weights in general?* No

$$\text{If } p_0 = 0,\ F_{\mathbf{V}}(u_1,u_2) = \begin{cases} 0 & \text{on} \quad [0,\lambda^*]^2, \\ C^{[\lambda^*]}(u_1,u_2) & \text{on} \quad [0,1]^2 \setminus [0,\lambda^*]^2. \end{cases}$$

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$\Lambda : \Omega \to [0, 1)$ with distribution F_{Λ}

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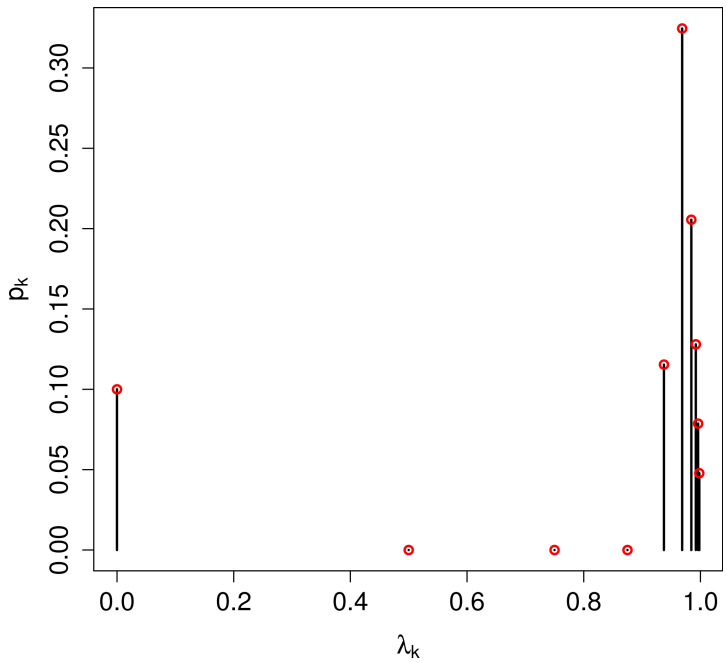

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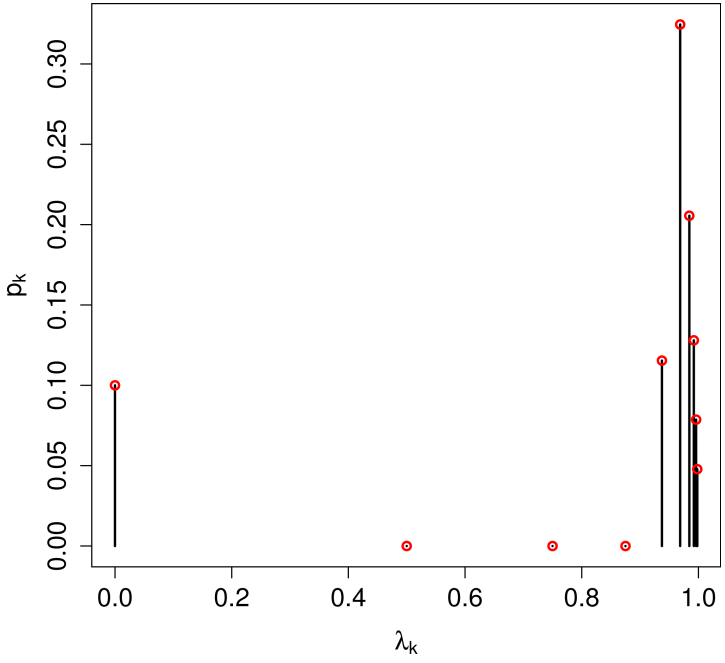

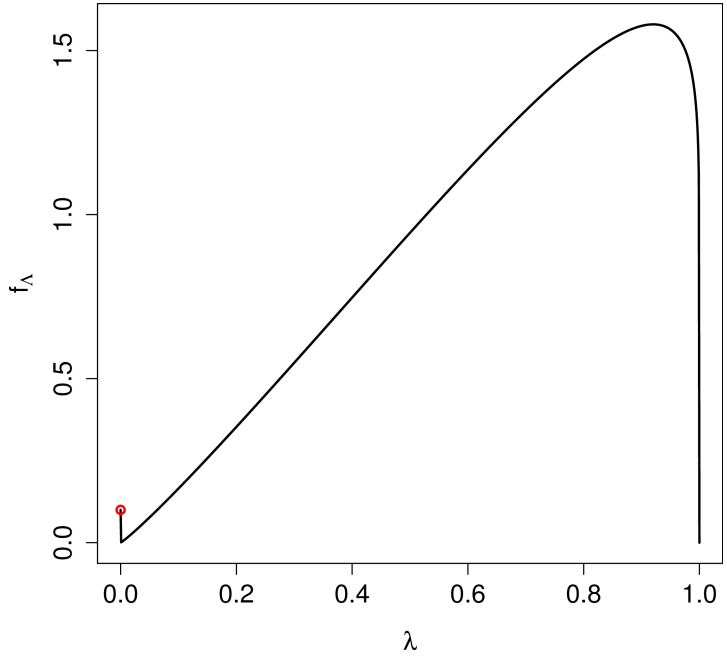

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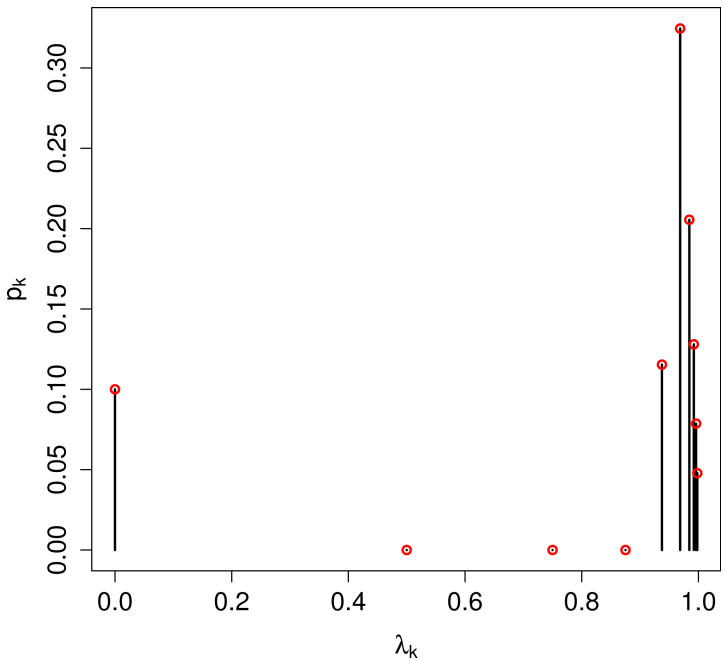
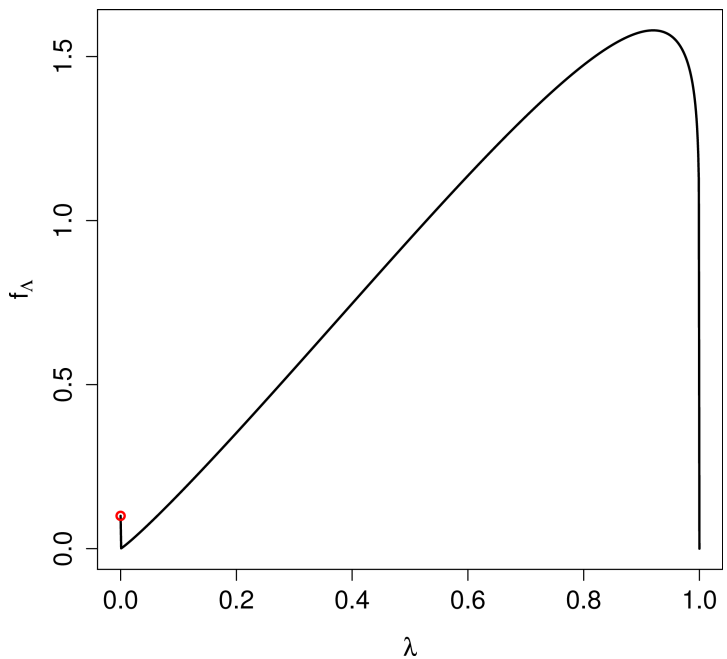
$\Lambda : \Omega \to [0, 1)$ with distribution F_Λ discrete or continuous.





Only requirement: $\mathbb{P}(\Lambda = 0) > 0$.

Outline

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- Importance Sampling

Case Study

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- $d = 2, 5$ and 25 dimensions
- $n = 100'000$ simulations, $N = 500$ repetitions
- $X_j \sim LN(10 - 0.1j, 1 + 0.2j), j = 1, \ldots, d$
- $S = \sum_{j=1}^{d} X_j$ are aggregate losses
- C is Gumbel copula ($\theta = 1.5$)

Reduction factor in estimator's variance

$$\frac{\text{var}(\mu_n)}{\text{var}(\widetilde{\mu}_n)}$$

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Objective	$d = 2$	$d = 5$	$d = 25$
$\mathbb{E}[(S - T)^+]$	81	39	17
$\text{VaR}_{0.995}(S)$	12	12	10
$\text{ES}_{0.99}(S)$	19	18	14
$\mathbb{E}[X_1 \vert S > F_S^{-1}(0.99)]$	21	19	11
$\mathbb{E}[X_d \vert S > F_S^{-1}(0.99)]$	22	18	18

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- ✔ Very weak assumptions on the copula and easy to implement
  - Only a sampling algorithm needed
  - No density required
  - No further restrictions
- ✘ Calibration of mixing distribution F_Λ is critical
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