Half-Mack Stochastic Reserving

ASTIN

Den Haag, May 2013

Agenda

- Mack Chain Ladder Procedure
- Half-Mack Procedure

The Chain Ladder Procedure

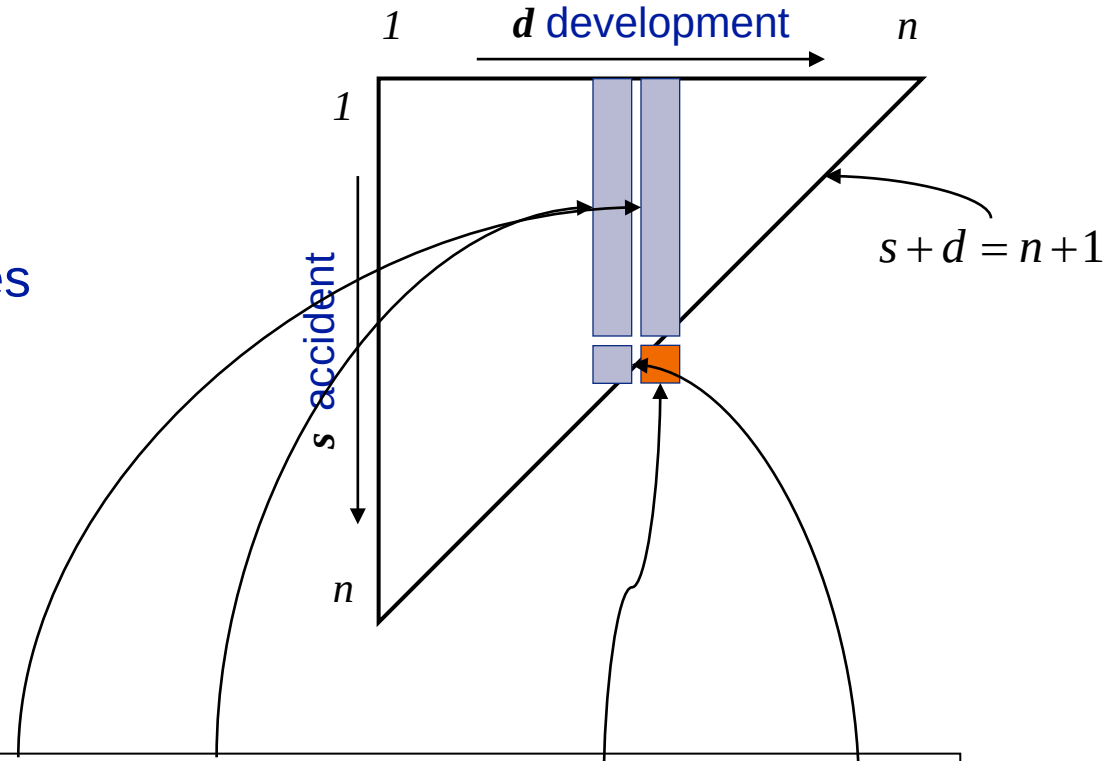
PRS EXPERTS FOR EXPERTS

Notations:

s = accident year

d = development year

$C_{s,d}$ = cumulative reserves



Algorithm:

$$\lambda_{d-1} = \sum_{s=1}^{n-d+1} C_{s,d} \Big/ \sum_{s=1}^{n-d+1} C_{s,d-1} \qquad C_{s,d} = \lambda_{d-1} \cdot C_{s,d-1}$$

The Mack Chain Ladder Procedure

Hypothesis:

$C_{s,d}$ independent for each accident year s

$$E\left[C_{s,d}\middle|C_{s,1}\cdots C_{s,d-1}\right] = \lambda_{d-1}\cdot C_{s,d-1}$$

$$V\left[C_{s,d}\middle|C_{s,1}\cdots C_{s,d-1}\right] = \sigma_{d-1}\cdot C_{s,d-1}$$

Variance proportional to the mean

Same development pattern for all accident years

Future depends only on the last diagonal

Independent accident years

The Mack Chain Ladder Procedure

PRS EXPERTS FOR EXPERTS

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$$V\left[C_{s,d}\middle|C_{s,1}\cdots C_{s,d-1}\right] = \sigma_{d-1}\cdot C_{s,d-1}$$

Consequences:

$$\begin{cases} \lambda_{d-1} = \sum_{s=1}^{n-d+1} C_{s,d} \Big/ \sum_{s=1}^{n-d+1} C_{s,d-1} & \rightarrow \quad E[R] \\ \sigma_{d-1} = \frac{1}{n-d}\sum_{s=1}^{n-d+1} C_{s,d-1}\left(\frac{C_{s,d}}{C_{s,d-1}} - \lambda_{d-1}\right)^2 & \rightarrow \quad V[R] \end{cases}$$

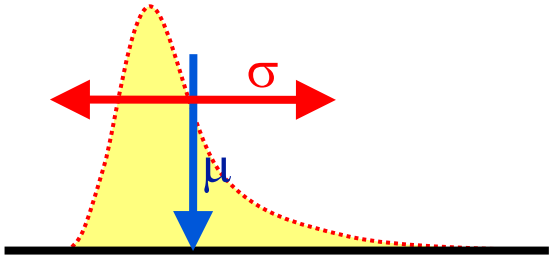
Application:

- Assume an underlying distribution of the reserves
- Fit with the first 2 moments

The Mack Chain Ladder Procedure

- Systematic & statistical errors
- Non-parametric

- No smoothing/adjustment of the development factors
- Only 2 moments
- No tail factors



The Mack Chain Ladder Procedure

PRS EXPERTS FOR EXPERTS

Hypothesis:

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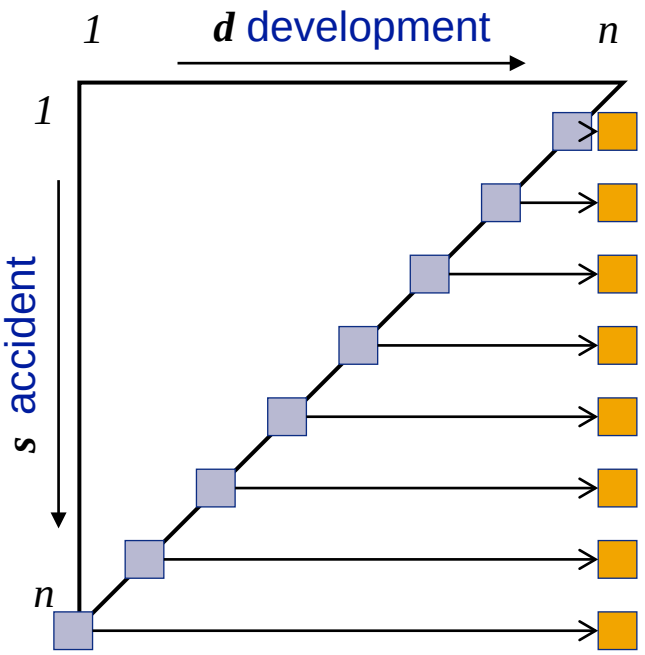
Application:

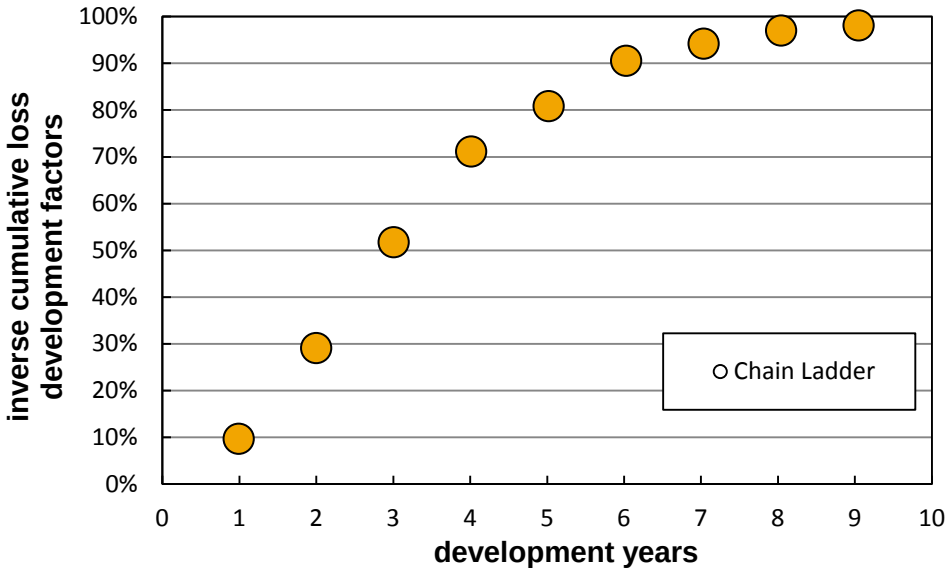
- Assume an underlying distribution of the reserves
- Fit with the first 2 moments
- Take over only the first 2 moments of the development factors
- Fit a model to the development factors

Agenda

- Mack Chain Ladder Procedure
- Half-Mack Procedure

Chain Ladder

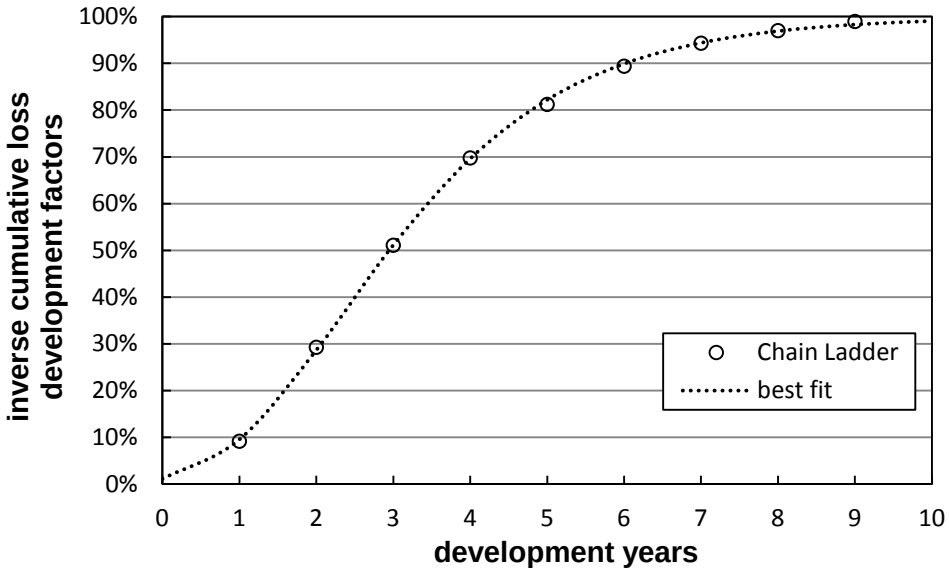




Chain Ladder Loss Saturation

Tricks of the reserving actuaries:

- Smoothing
- Adjustments to experience
- Fit to a model



loss saturation function

loss development factor

$$a2u_d \rightarrow F(t) = \left[1 - \exp\left(-\frac{t-\tau}{\lambda}\right)\right]^{\alpha}$$

location

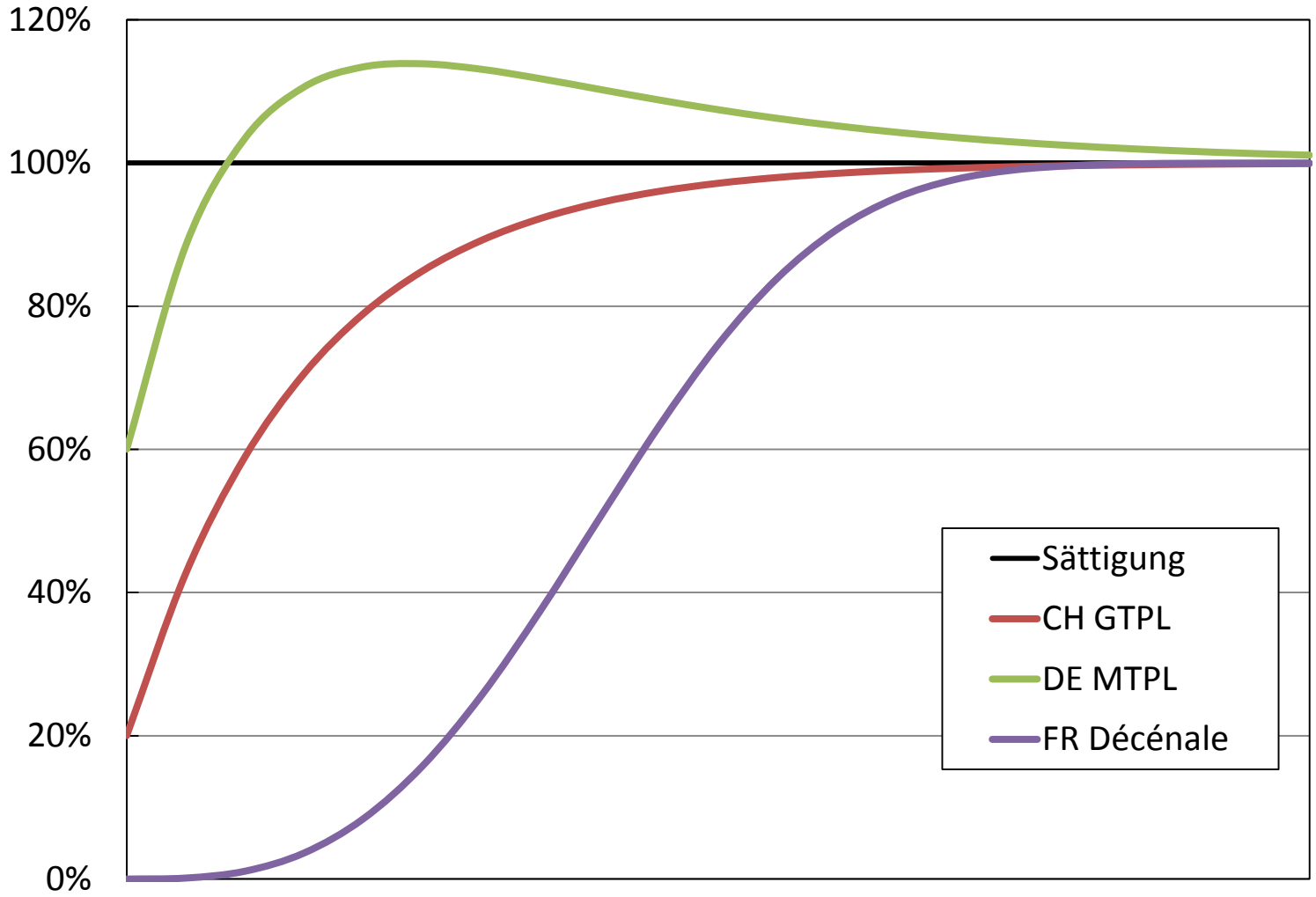
shape

development year

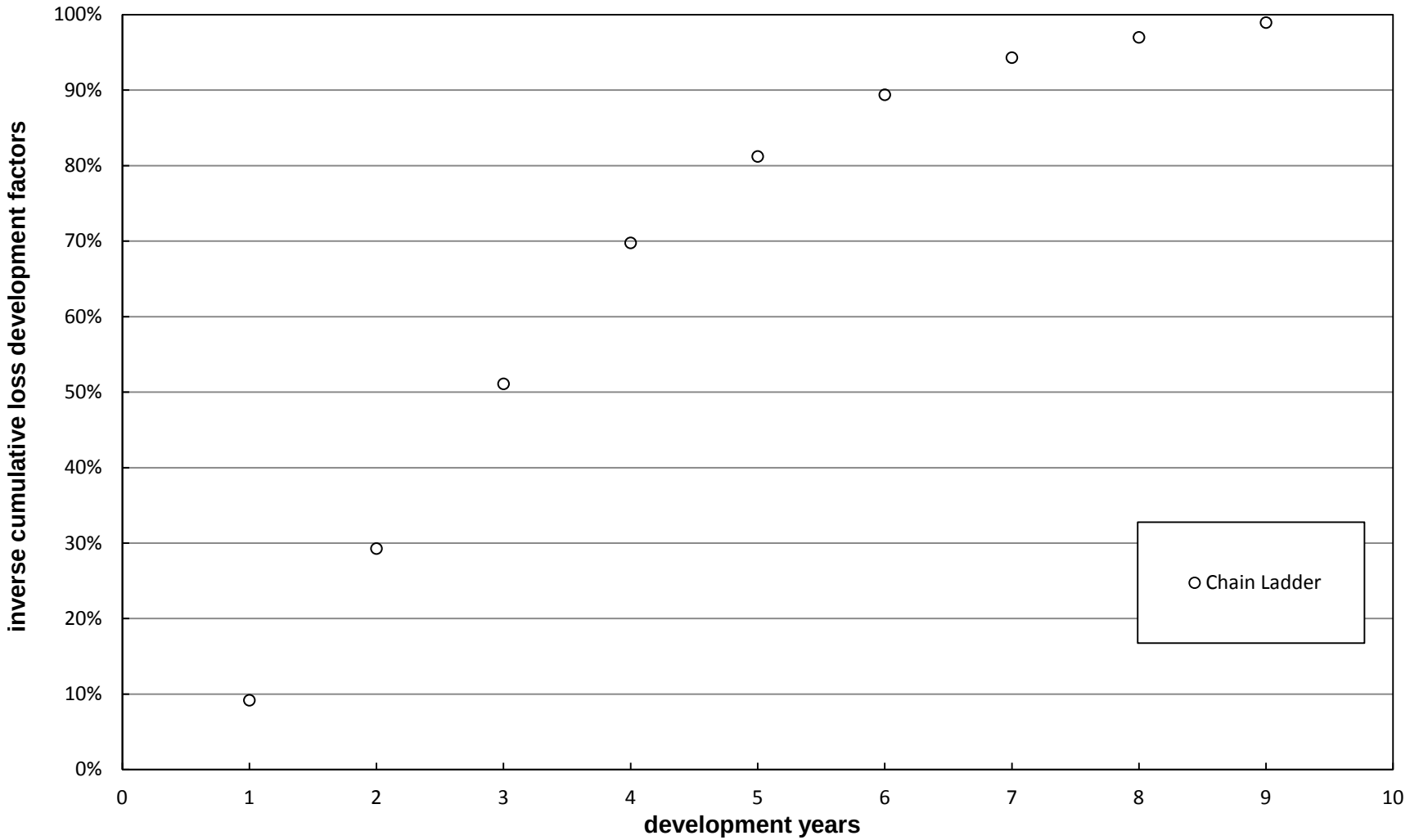
continuous time

scale

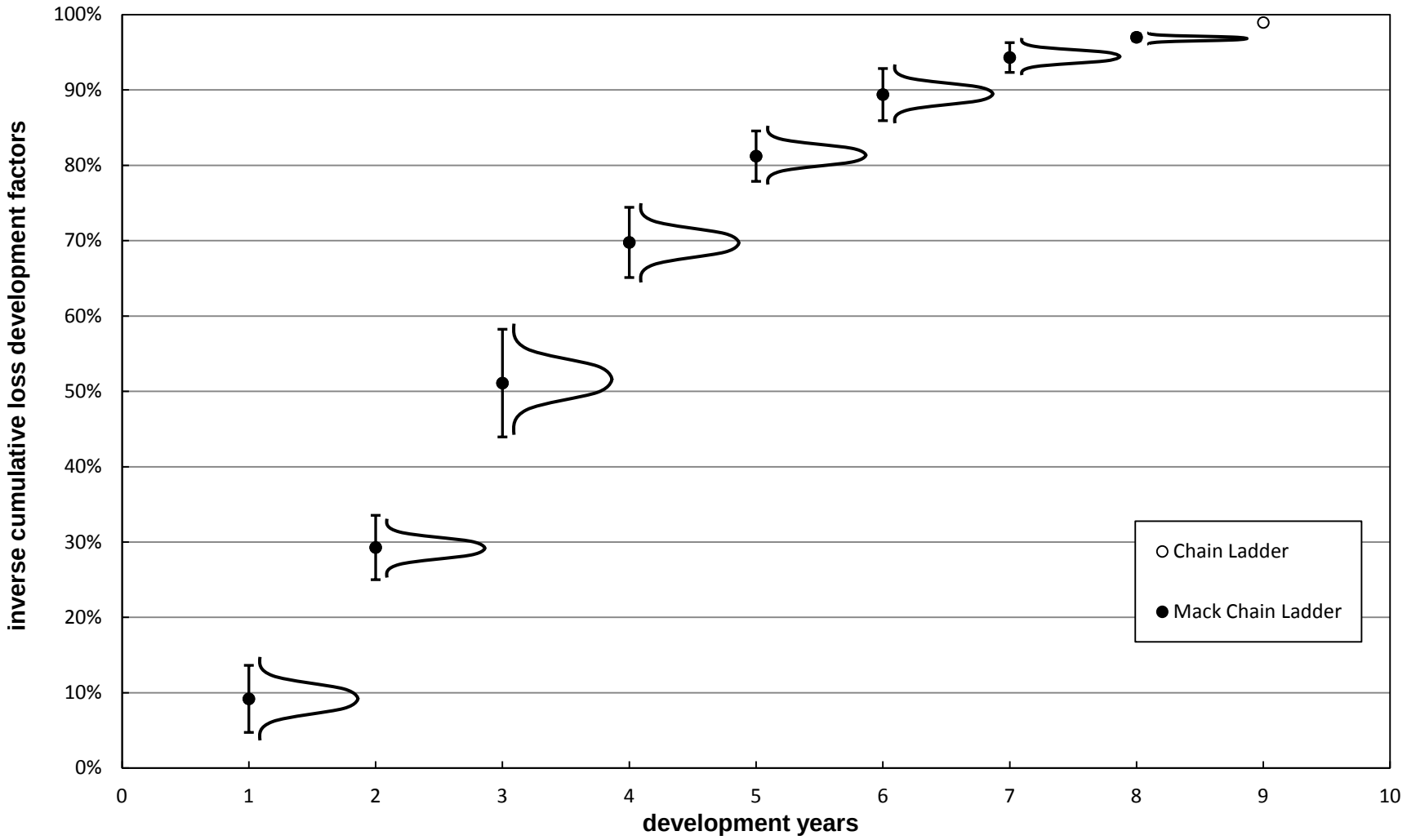
Typical Loss Saturation Patterns



Chain Ladder Loss Saturation



Mack Chain Ladder Loss Saturation



Half-Mack Chain Ladder Loss Saturation

Fit with χ^2 statistic

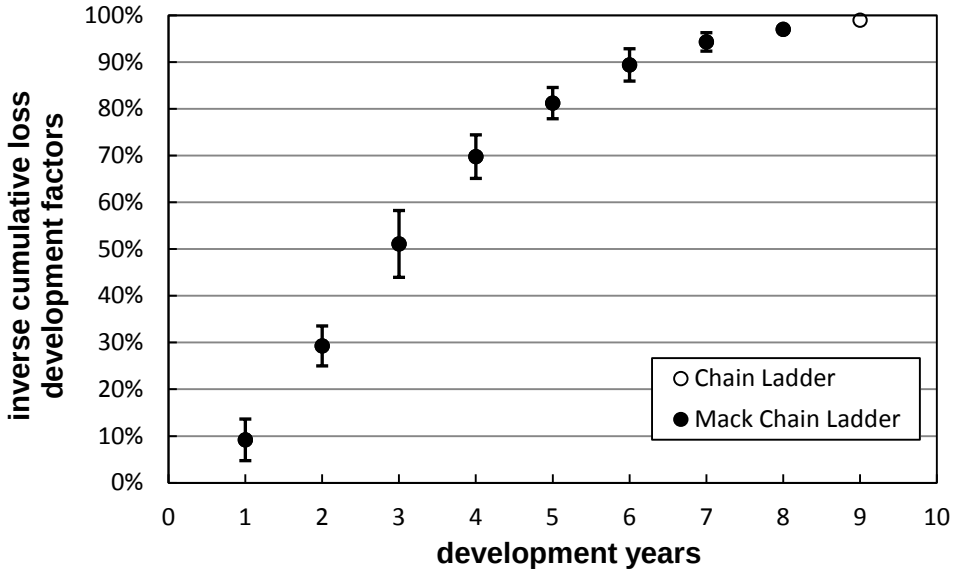
- Best estimate minimizes χ^2 function

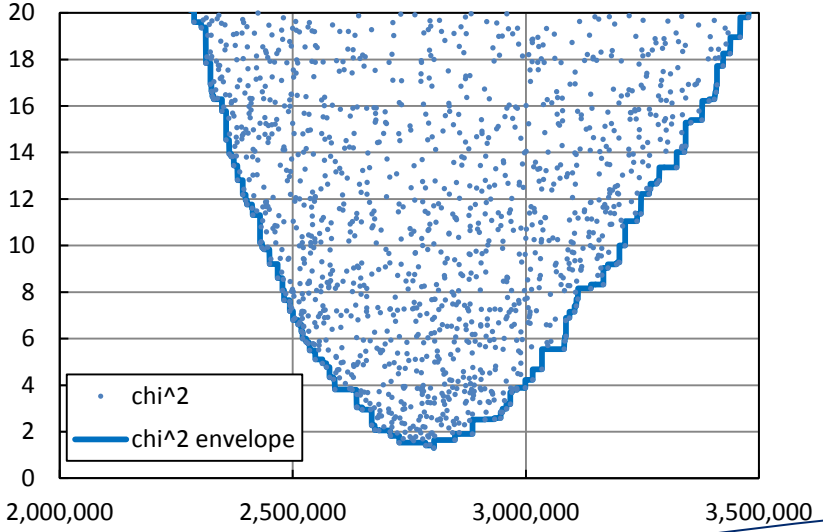
CL observation — model

$$\chi^2 = \sum_{t=1}^{T} \left(\frac{f_t - f(t)}{\Delta f_t} \right)^2$$

MCL error

- Goodness of fit $\frac{\chi^2_{\min}}{\text{dof}}$



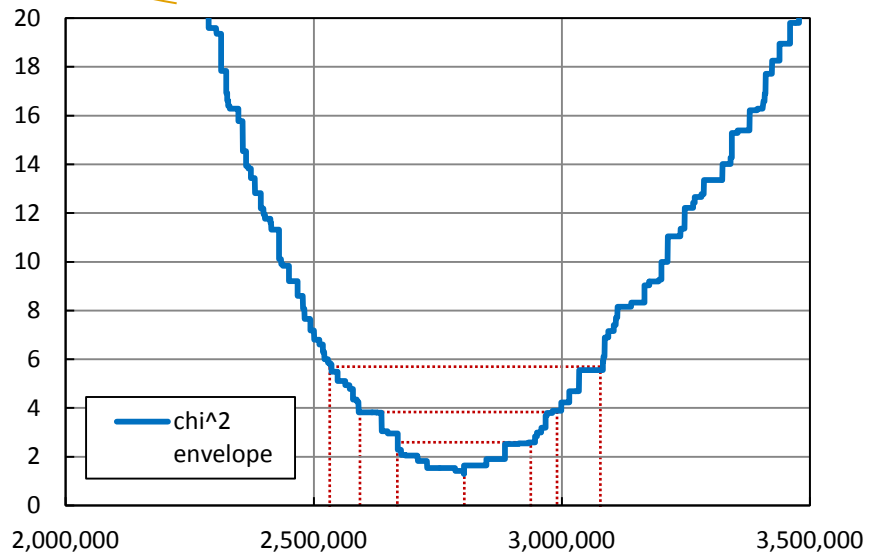


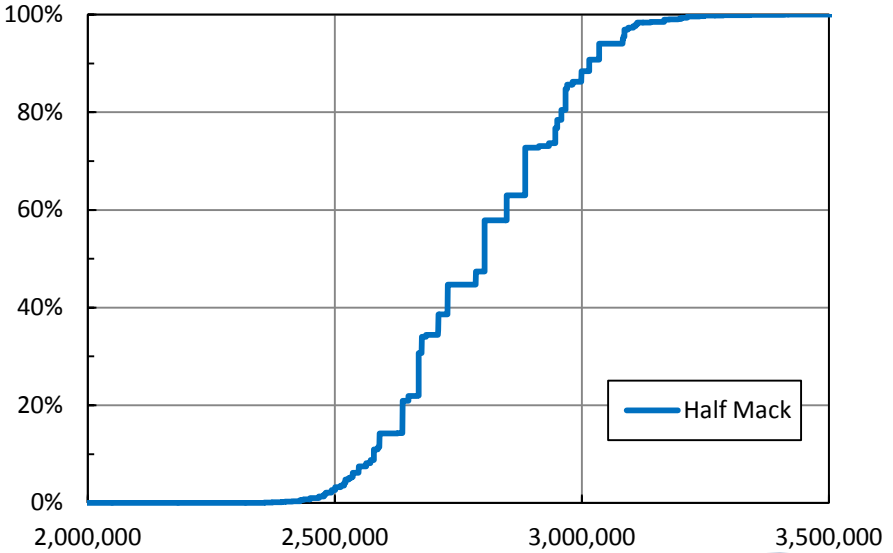
Half-Mack Chain Ladder Loss Saturation

χ^2 confidence intervals

- best fit: $\chi^2 = \chi^2_{\min}$
- 1σ interval: $\chi^2 \le \chi^2_{\min} + 1$
- p intervals: $\chi^2 \le \chi^2_{\min} + \chi^2(p)$

$\rightarrow$ reserves distribution

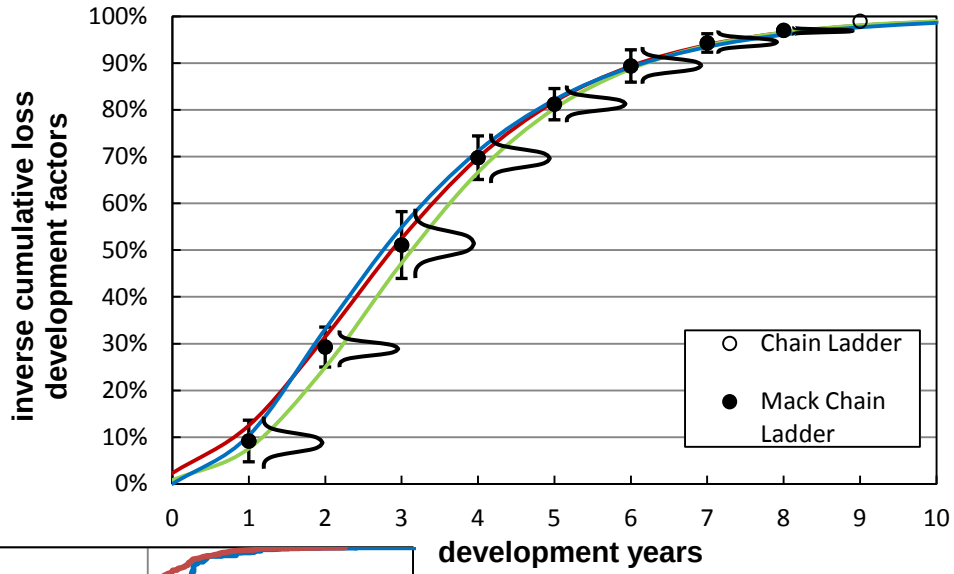


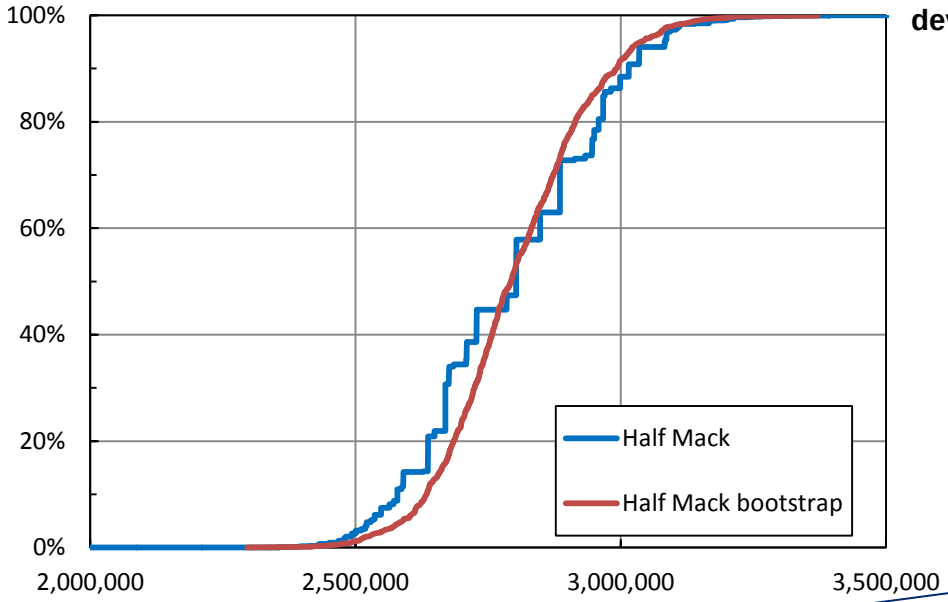


Half-Mack Bootstrap

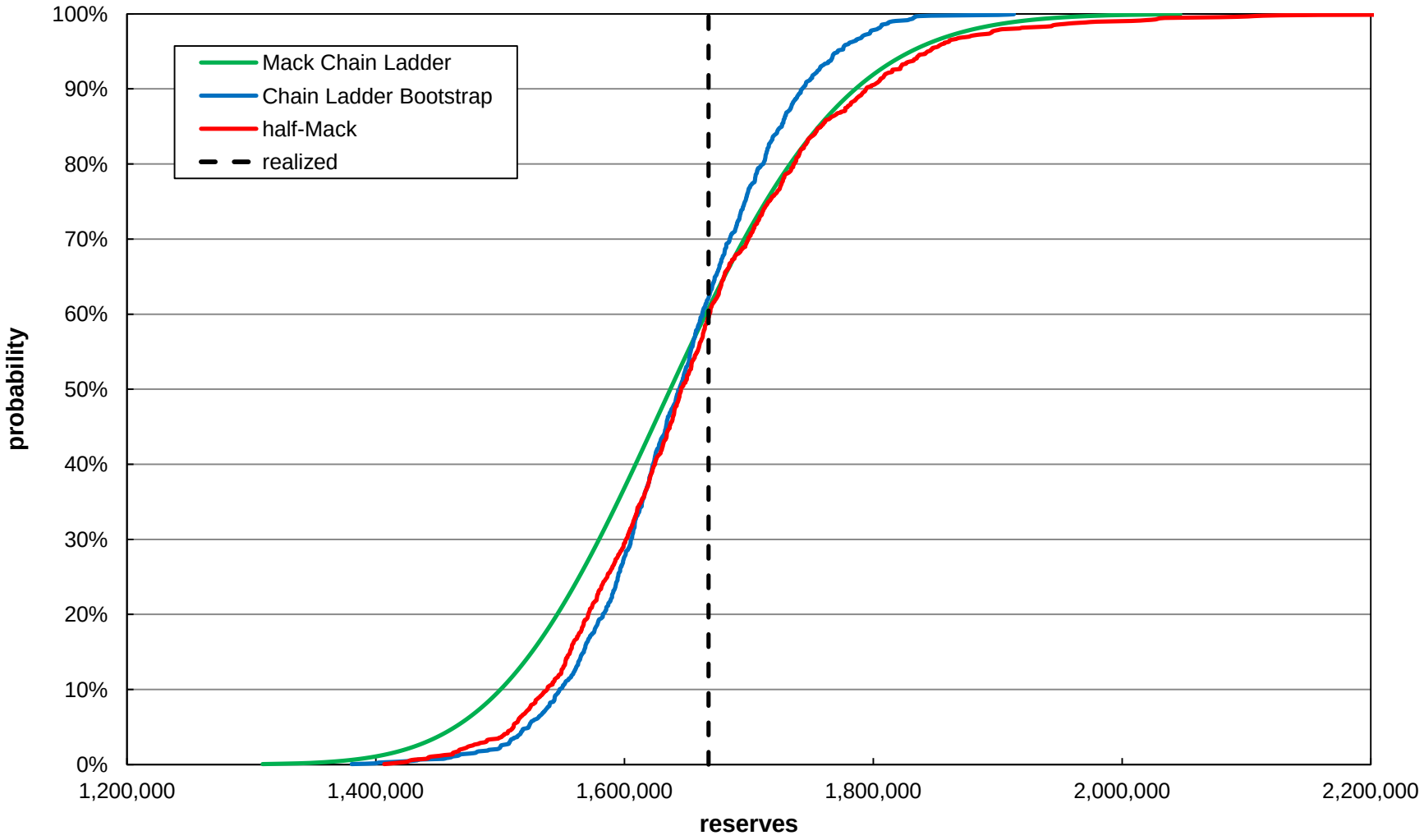
χ^2 statistic inefficient $\Rightarrow$ bootstrap

- Sample the
  loss development factors
- Fit the best
  loss saturation function





Half-Mack Reserves in Comparison



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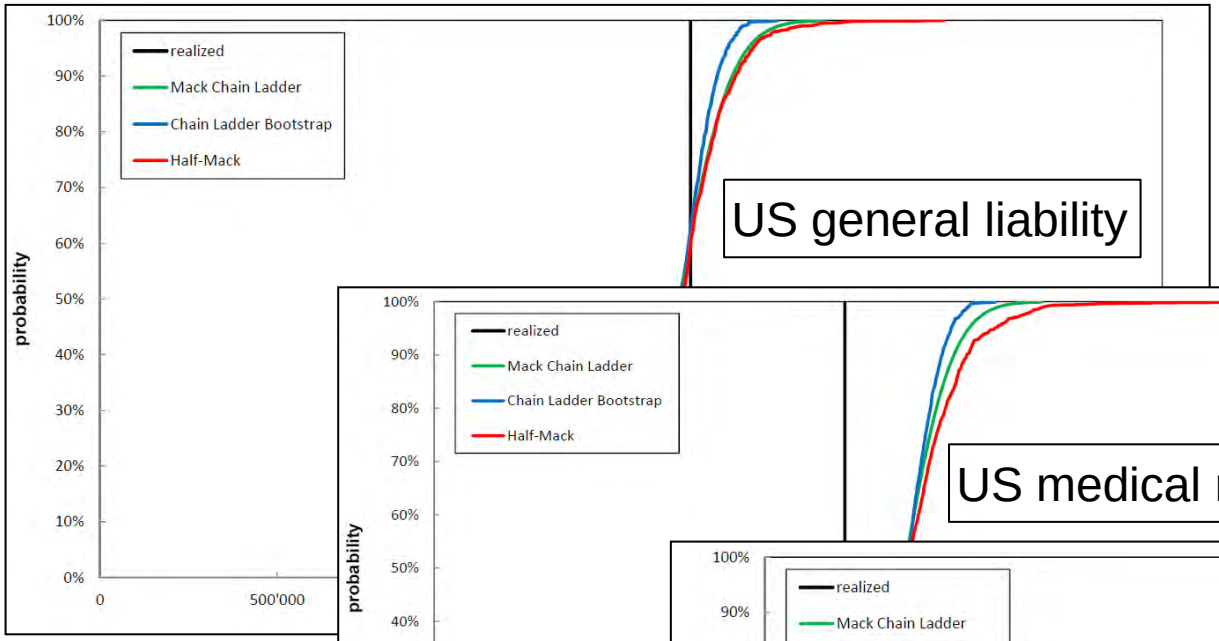
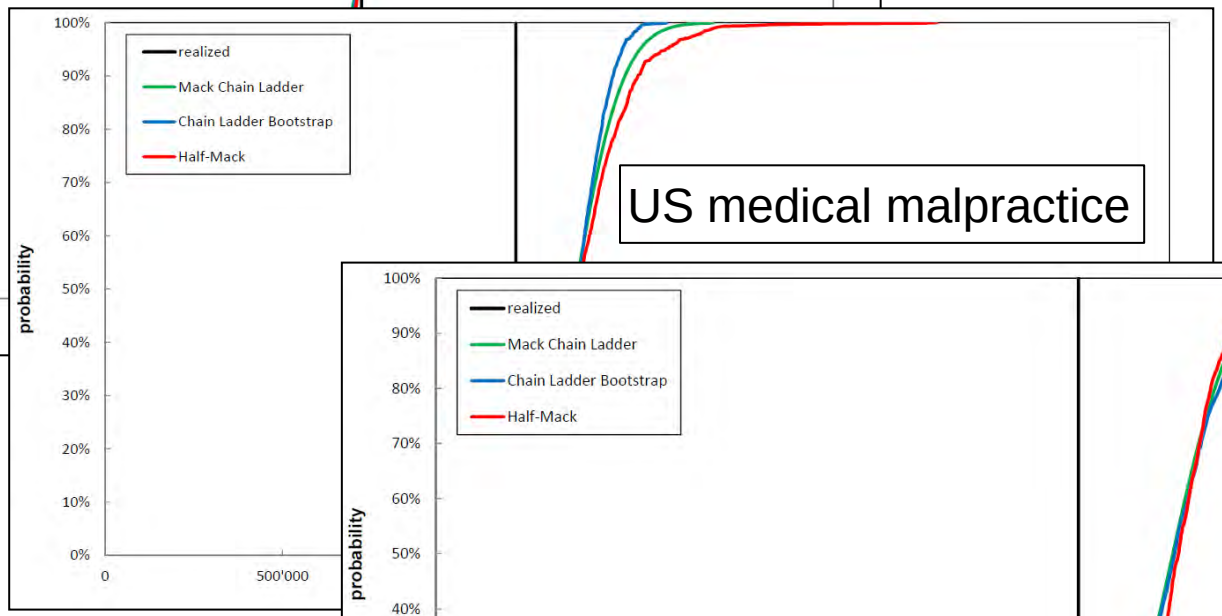
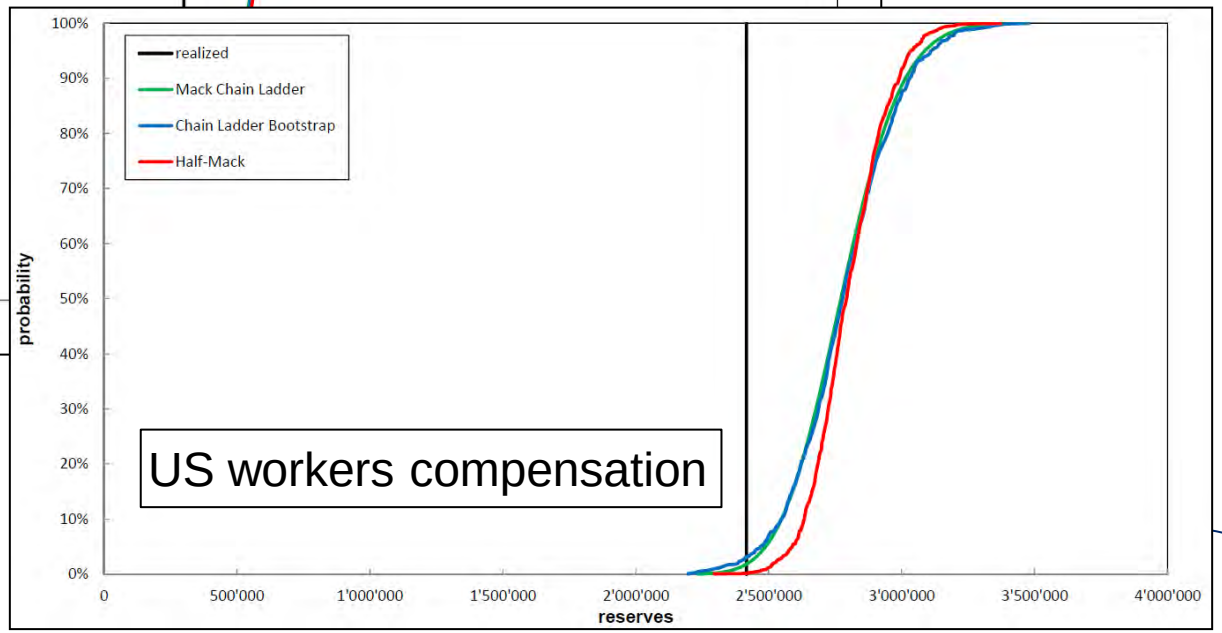






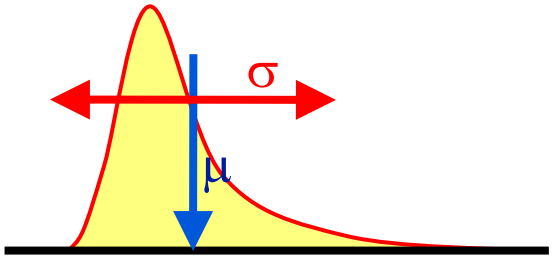
Agenda

- Mack Chain Ladder Procedure
- Half-Mack Procedure

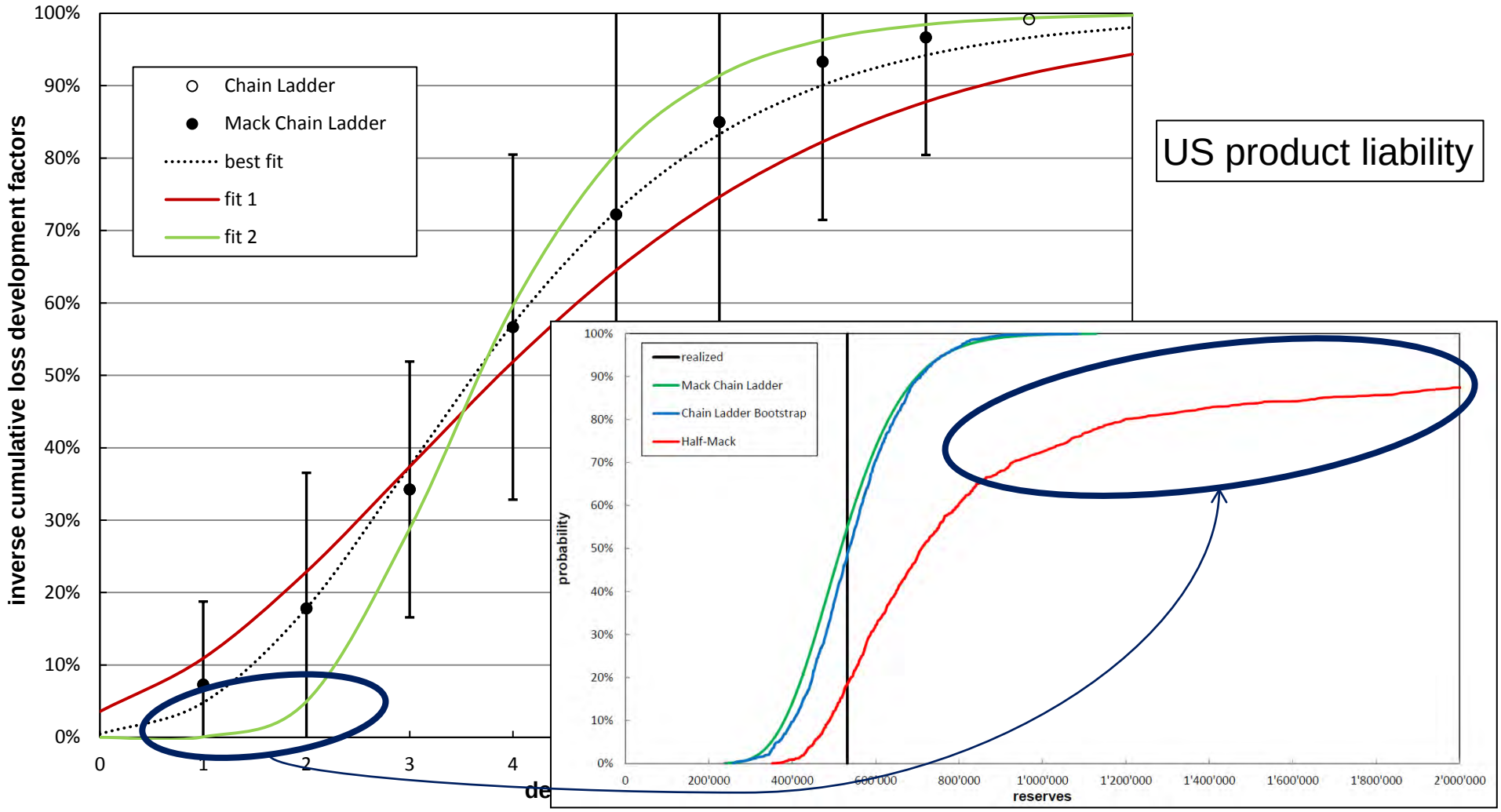
The Mack Chain Ladder Procedure

- Systematic & statistical errors
- Non-parametric
- Natural smoothing of the development factors
- Full distribution of reserves
- Automatic tail factors
- Accounts for market experience

- No smoothing/adjustment of the development factors
- Only 2 moments
- No tail factors



Half-Mack & Actuarial Judgment



US product liability

Conclusion

The half-Mack procedure is
yet another stochastic reserving tool.

There is no silver bullet:
actuarial engineering remains an art ☺

Contact

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