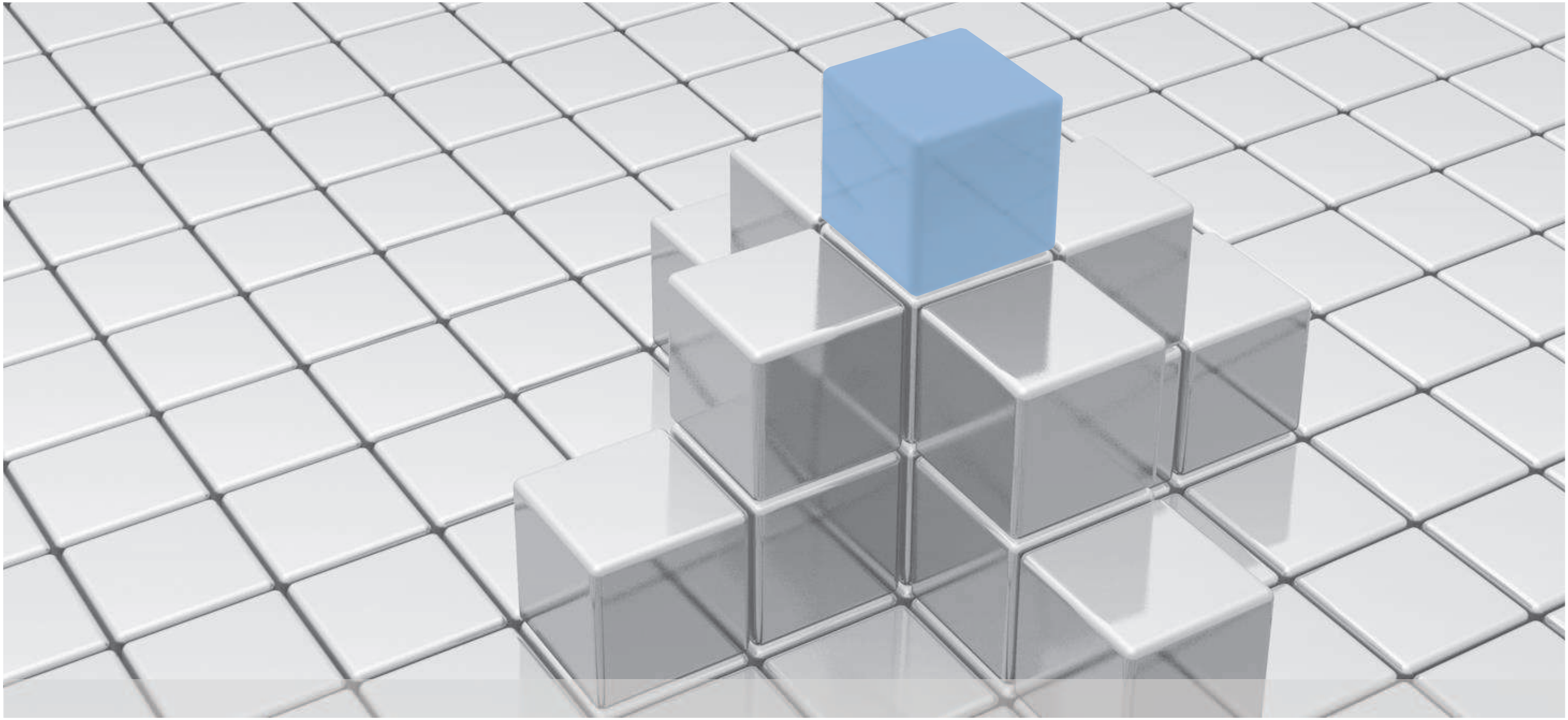

IFRS CONFIDENCE LEVEL OF RESERVE RISK MARGINS: PRACTICAL APPROXIMATIONS

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Why IFRS? Why Confidence Level? ...

The upcoming IFRS 4 Phase II brings **one specific requirement to disclose confidence level of reserve risk margin** calculated under the Solvency II Cost of Capital approach. This is an important additional requirement to the upcoming Solvency II that brings significant changes to current reporting of insurance entities in relation to valuation of insurance liabilities.

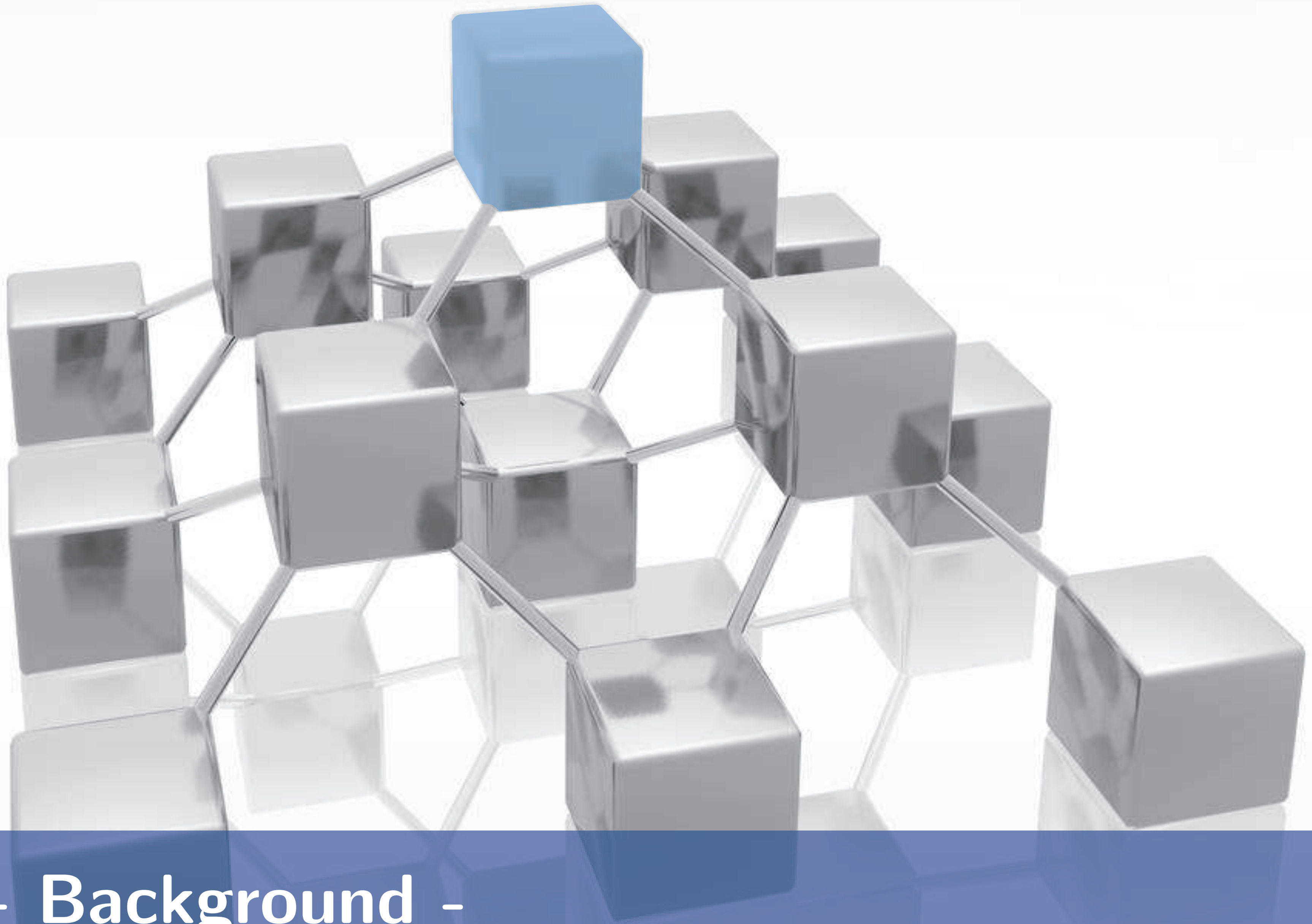
The time lag between Solvency II and IFRS 4 Phase II: majority of insurers have not started making the necessary preparations to implement and accommodate this new regulatory requirement, as they are currently busy making their final preparations for Solvency II before it comes into play in early 2016, and also the IFRS 4 Phase II will commence only in early 2017.

No specific guidance from IFRS on the calculation of confidence level of reserve risk margins. This research work

- looks for practical ways of implementing this new requirement; and
- proposes a distribution-free approach to estimating IFRS confidence level of reserve risk margins in a ‘standard formula’ style.

Outline

1. **Background**
   - *Reserve risk margin:* reserve risk and its profile characterisation
   - *Probability of Sufficiency (PoS)* as a measure of IFRS Confidence Level
   - *'Log-normality bubble':* analytical PoS solution vs. limited practical insight
   - *Distribution-free approximation of PoS:* shape of reserve risk profile and its approximation by Cornish-Fisher (C-F) expansion
2. **Distribution-free approximations of PoS**
   - *Second order C-F approximation:* utilising CoV and skewness only
   - *Third and fourth order C-F approximations:* utilising CoV, skewness and kurtosis
   - Analysis of quality of PoS approximations
   - PoS approximations for portfolio of multiple reserving classes
   - Practical implementation



- Background -

Reserve risk and its profile

- Under Solvency II the **risk** is generally defined as a possibility of having adverse performance result (e.g. insurance, investment, reserving) that results in 'low capital performance' and/or capital consumption.

- In particular, the **reserve risk** is the risk that provisions for past exposures will be inadequate to meet the ultimate costs when the business is run off to extinction. The risk of reserves developing other than expected (booked provisions) is significant for non-life insurers, especially for long tail lines of business.

- The reserve risk is materialised via its carrier - reserve value. It is the distribution of reserve risk carrier that characterises the reserve risk, and it is often referred to as the **reserve risk profile**.

- The reserve risk profile is differentiated by the type of business or reserving class:
  - personal vs. commercial insurance;
  - short vs. long tail;
  - duration and convexity of reserve claim payments, etc.

Reserve risk margin

- The reserve provisions are generally booked at the *Central Estimate* plus *Risk Margin*. **The risk margin plays the role of safety load reflecting the uncertainty in reserve central estimate, whereas the actual volatility of reserving process (reserve risk) is fully absorbed by capital requirements:**

$$
\begin{aligned}
\mathbb{E}\left[\left(CE-\widehat{CE}\right)^2\right] &= \mathbb{E}\left[\left(\left(CE-\mathbb{E}\left[\widehat{CE}\right]\right)+\left(\mathbb{E}\left[\widehat{CE}\right]-\widehat{CE}\right)\right)^2\right] \\
&= \text{bias}^2+\text{Var}\left[\widehat{CE}\right] \\
&= \text{bias}^2+\mathbb{E}\left[\text{Var}\left[\widehat{CE}\,|\,\mathcal{F}\right]\right]+\text{Var}\left[\mathbb{E}\left[\widehat{CE}\,|\,\mathcal{F}\right]\right],
\end{aligned}
$$

where $\mathcal{F}=\sigma\left\{\mathbf{M},\mathbf{CE}\right\}$ is the information filtration on a set of models $\mathbf{M}$ and a set of true parameters $\mathbf{CE}$.

- the first term is dealt within Pillar II;
- the second term is fully captured by the Solvency II capital requirements; and
- the third term is fully captured by the risk margin.

PoS as a measure of IFRS confidence level (1)

- Let X be the reserve value with distribution F_X, central estimate CE_X and variability measured via coefficient of variation CoV_X.

- The confidence level of reserve risk margin η_X per unit of CE_X (assumed to be pre-calculated and provided by reserving/capital actuaries) is defined by the probability level of provision set at $(1 + \eta_X) \times CE_X$ on the reserve risk profile F_X. It is called *Probability of Sufficiency (PoS)* and equal to:

$$PoS = F_X\left((1 + \eta_X) \cdot CE_X\right) = \mathbb{P}\left[X \leq (1 + \eta_X) \cdot CE_X\right]$$

- Examples of PoS use: in Australia the non-life technical provisions are required by APRA to be set with the minimum of 75% of PoS

PoS as a measure of IFRS confidence level (2)

PoS itself is a measure of prudence in liability valuation:

- **PoS below 50%** – the technical provisions are set below the central estimate **(under-reserved position)**;
- **PoS of 50% to 60%** – the technical provisions are set approximately at the level of central estimate **(weak prudence)**
- **PoS of around 75%** – that technical provisions are set so that likely (i.e. up to 1-in-4 years) reserve deteriorations above the central estimate are fully absorbed by the technical provisions **(adequate prudence)**; and
- **PoS above 75%** – that the technical provisions could also absorb some of unlikely reserve deteriorations above the central estimate **(strong prudence)**.

PoS in the ‘log-normality bubble’ (1)

Let $X \sim \mathcal{LN}(\mu, \sigma^2)$. Then $CE_X = e^{\mu + \frac{1}{2}\sigma^2}$, $\sigma^2 = \ln\left(1 + CoV_X^2\right)$ and $\frac{1}{CE_X}X \sim \mathcal{LN}(-\frac{1}{2}\sigma^2, \sigma^2)$.

$$\begin{aligned} PoS &= \mathbb{P}\left[X \leq (1+\eta_X) \cdot CE_X\right] \\ &= \mathbb{P}\left[\frac{X}{CE_X} \leq 1 + \eta_X\right] \\ &= \Phi\left(\frac{\ln(1+\eta_X) + \frac{1}{2}\sigma^2}{\sigma}\right) \\ &= \Phi\left(\frac{\ln\left((1+\eta_X) \cdot \sqrt{1 + CoV_X^2}\right)}{\sqrt{\ln\left(1 + CoV_X^2\right)}}\right) \end{aligned}$$

... and the problem is solved!, but ...

PoS in the ‘log-normality bubble’ (2)

- **Log-normality does not cover the whole range of practically feasible reserve risk profiles,** i.e. for $CoV_X \leq 50\%$ and disproportionally higher(lower) skewness
  - e.g. how about $CoV_X = 20\%$ and skewness γ_X of 1.0 (or 0.4) or equivalently 5 (or 2), when expressed per unit of CoV_X **(i.e. Skewness-to-CoV (SC) ratio)**?
  - **with log-normality we could only achieve the SC ratio in the range of 3 to 3.25 for $CoV_X \leq 50\%$**
- Is there a way of **estimating PoS using only** the reserve risk profile’s characteristics like **CoV and skewness** without knowing the parametric structure of reserve distribution?

Distribution-free approximation of PoS (1)

... rather inspired by the idea of being able to solve any world's problem on the back of a cocktail napkin

The art of guesstimation ...

... **Problem 1:** *how far does a football player travel during the course of a 90-minute game?*

Answer: $\approx 20\text{km}$

... **Problem 2:** *if all French baguettes sold in Paris last year were placed end-to-end, what distance would they cover?*

Answer: ???

Single Shape Parameter distributions (1)

- Most two-parameter distributions commonly used in insurance for reserving and loss modelling are of a special type:
  - their *scale* and *shape* parameters are separated
  - the shape of the distribution is fully explained by its shape parameter;
  - or equivalently, any higher-order statistic like skewness, kurtosis, etc. is fully explained by CoV
- This class of distributions is called **Single Shape Parameter (SSP) distributions**.
- Examples of SSP distributions include: Gamma, Inverse-Gaussian (Wild), Log-Normal, Dagum, Suzuki, Exponentiated-Exponential (Verhulst), Inverse-Gamma (Vinci), Birnbaum-Saunders, Exponentiated-Fréchet and Log-Logistic.
- Not all two-parameter distributions are of SSP type - e.g. Log-Gamma distribution (i.e. $\mathrm{Exp}\left[\mathrm{Gamma}(\alpha, \beta)\right]$).

Single Shape Parameter distributions (2)

The SSP distributions can be split into three main categories:

- **Moderately skewed distributions ($1.5 < SC \leq 3$)**
  - *Gamma*;
  - *Inverse-Gaussian (Wald)*;
- **Significantly skewed distributions ($3 < SC < 4$)**
  - *Log-Normal*;
  - *Suzuki*;
  - *Exponentiated-Exponential (Verhulst)*;
  - *Dagum*;
- **Extremely skewed distributions ($4 < SC < 5.5$)**
  - *Inverse-Gamma (Vinci)*;
  - *Birnbaum-Saunders*;
  - *Log-Logistic*;
  - *Exponentiated-Fréchet*.

SSP characterisation of reserves (1)

SSP characterisation of reserves:

Table 1: *Differentiation of reserve risk profile by type of reserve class.*

Type of reserving class				
Duration	**CoV range**	**Skewness (SC ratio)**	**Parametric distribution(s)**	**Example of reserving class**
Short tail	10%-12%	1.9 to 2.1	Gamma	Motor (ex Bodily Injury)
Short tail	12%-16%	2.0 to 3.0	Gamma, Inverse-Gaussian (Wald)	Home
Short tail	10%-16%	2.9 to 3.1	Inverse-Gaussian (Wald), Log-Normal	Comm Property/Fire, Comm Accident
Long tail	12%-25%	3.0 to 3.5	Log-Normal	Motor Bodily Injury, Marine
Long tail	18%-50%	3.0 to 4.0	Log-Normal, Inverse-Gamma (Vinci)	Workers Comp, Prof Liab, Comm Liab
Long tail	25%-70%	> 4	Inverse-Gamma (Vinci)	Asbestos and other long tail books

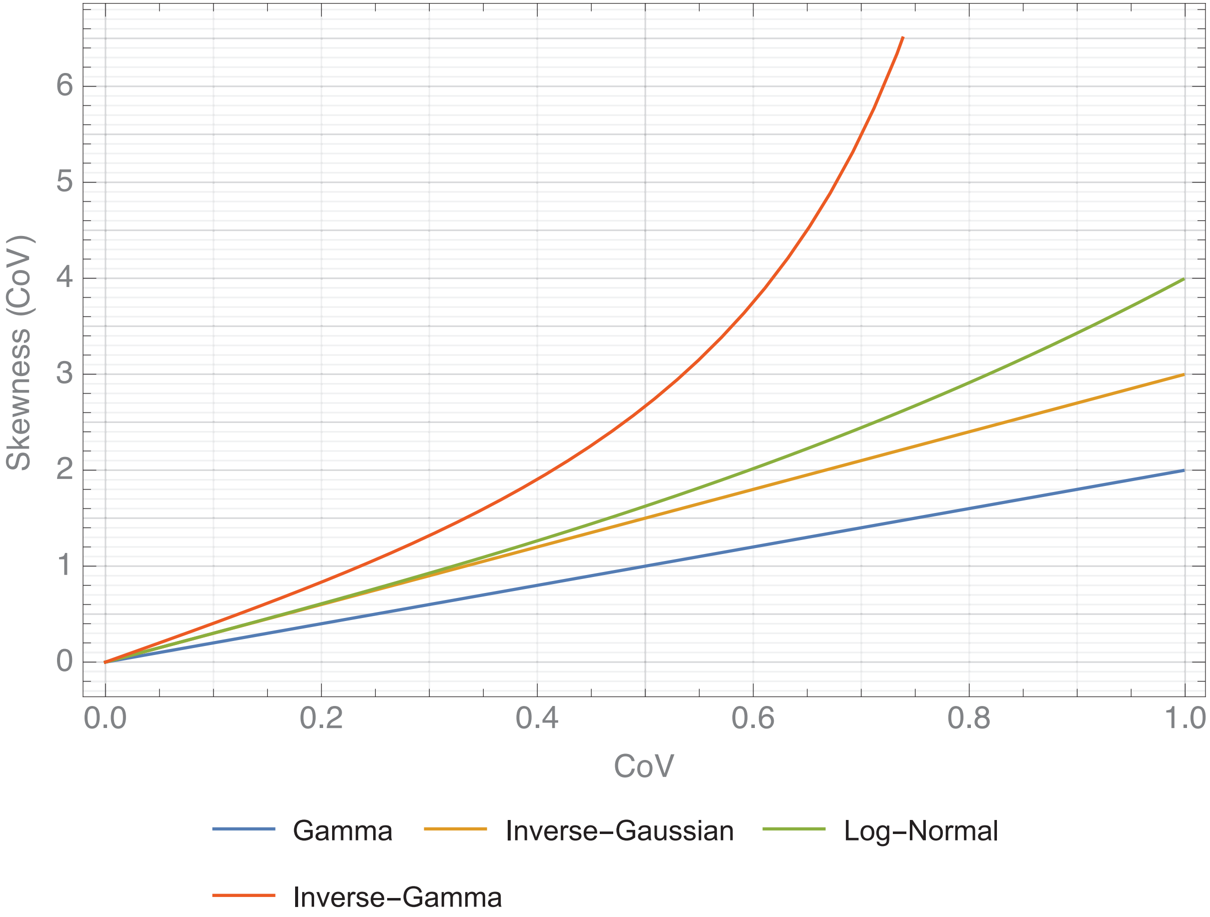
Table 2: *SC and KCsq ratios for the four parametric distributions.*

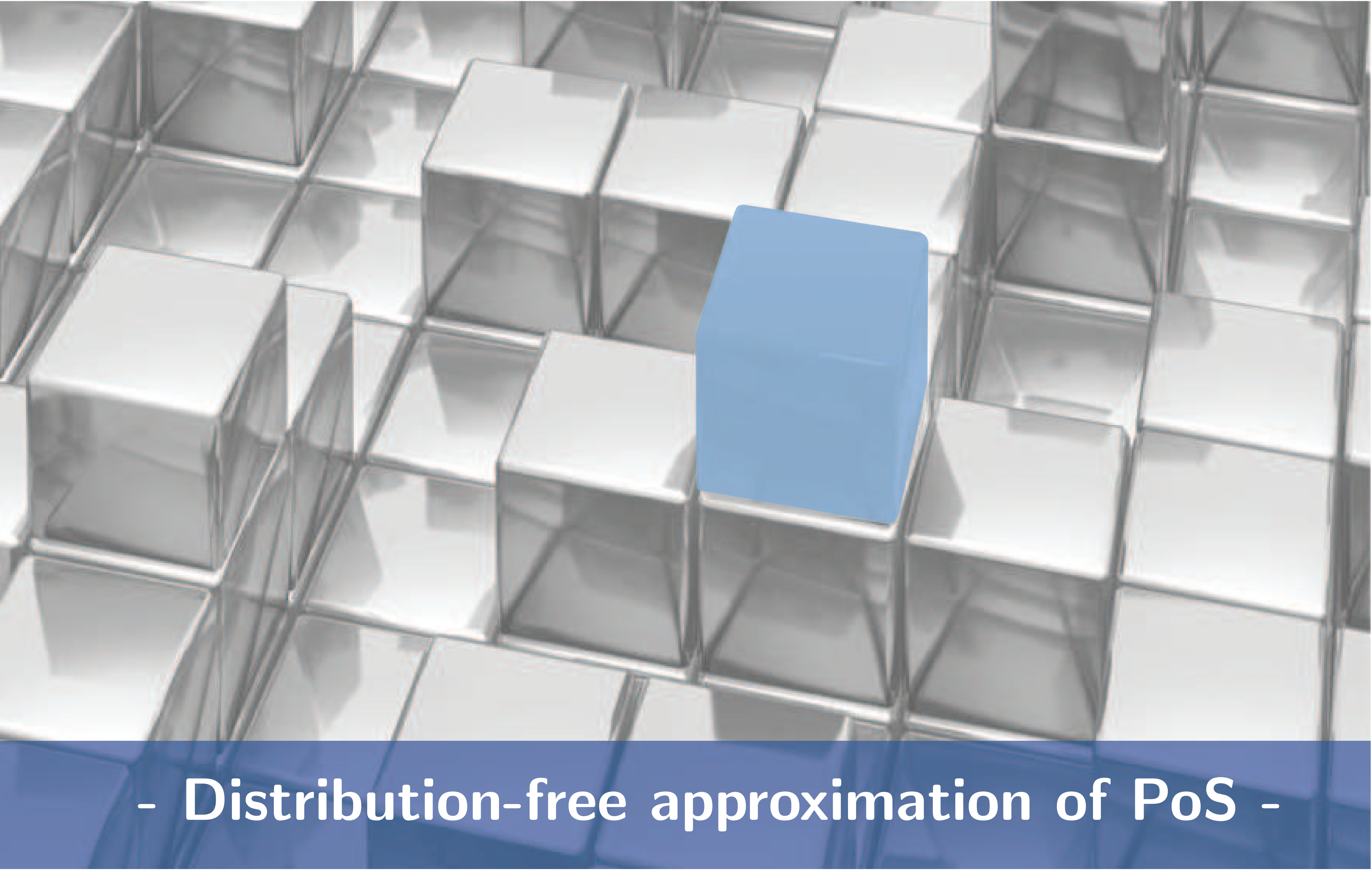
Parametric distribution	**SC ratio as a function of CoV**	**KCsq ratio as a function of CoV^2**
Gamma	2	6
Inverse-Gaussian (Wald)	3	15
Log-Normal	$3 + CoV^2 \in (3, 4),\ CoV < 100\%$	$16 + 15CoV^2 + 6CoV^4 + CoV^6 > 16$
Inverse-Gamma (Vinci)	$\frac{4}{1-CoV^2} > 4,\ CoV < 100\%$	$\frac{30\left(1-\frac{1}{5}CoV^2\right)}{\left(1-CoV^2\right)\left(1-2CoV^2\right)} > 30,\ CoV < 70\%$

SSP characterisation of reserves (2)

Skewness as a function of CoV for the main SSP distributions:

Figure 1: *Skewness as a function of CoV for the four parametric distributions.*





- Distribution-free approximation of PoS -

Setting the problem ...

$$PoS = \mathbb{P}\left[X \leq CE_X \cdot (1+\eta_X)\right] = \alpha. \tag{1}$$

To solve for unknown level of PoS, α, one would need first to invert PoS by taking a VaR of X at α

$$\mathrm{VaR}_{\alpha}(X) = CE_X \cdot (1+\eta_X)\,, \tag{2}$$

and then express the $\mathrm{VaR}_{\alpha}(X)$ through the VaR of centralised and normalised copy of X, $\widetilde{X} = \frac{X - CE_X}{CE_X \cdot CoV_X}$, and solve the following equation for α

$$CE_X \cdot \left(1 + CoV_X \cdot \mathrm{VaR}_{\alpha}\left(\widetilde{X}\right)\right) = CE_X \cdot (1+\eta_X)\,, \tag{3}$$

or equivalently the equation

$$\mathrm{VaR}_{\alpha}\left(\widetilde{X}\right) = \frac{\eta_X}{CoV_X}. \tag{4}$$

Cornish-Fisher approximation

The shape of any analytical distribution function can be approximated by Cornish-Fisher (C-F) expansion of the distribution by its cumulants.

In particular, for $\widetilde{X} = \frac{X - m_X}{m_X \cdot CoV_X}$ the fourth-order Cornish-Fisher approximation is used to estimate $\mathrm{VaR}_\alpha(\widetilde{X})$:

$$\begin{aligned} \mathrm{VaR}_\alpha(\widetilde{X}) \approx\ & z_\alpha + \gamma_X \frac{z_\alpha^2 - 1}{6} + \iota_X \frac{z_\alpha^3 - 3z_\alpha}{24} \\ & -\gamma_X^2 \frac{2z_\alpha^3 - 5z_\alpha}{36} - \gamma_X \iota_X \frac{z_\alpha^4 - 5z_\alpha^2 + 2}{24} \\ & +\gamma_X^3 \frac{12z_\alpha^4 - 53z_\alpha^2 + 17}{324}, \end{aligned} \tag{5}$$

where γ_X and ι_X are skewness and kurtosis of X respectively, and also are functions of CoV_X for SSP type of reserve risk profiles.

PoS solution (1)

C-F approximation (5) brings a polynomial (quartic) into (4). **Solving the quartic equation for z_α and taking $\Phi\left(z_\alpha\right)$ provides a PoS approximation.**

Reduced forms of C-F approximation:

- *The second-order Cornish-Fisher (Normal Power)*:

$$\mathrm{VaR}_\alpha(\widetilde{X}) \approx z_\alpha + \gamma_X \frac{z_\alpha^2 - 1}{6}. \tag{6}$$

  This requires solving a quadratic equation.

- *The third-order Cornish-Fisher approximation*:

$$\begin{aligned}\mathrm{VaR}_\alpha(\widetilde{X}) \approx\ & z_\alpha + \gamma_X \frac{z_\alpha^2 - 1}{6} + \iota_X \frac{z_\alpha^3 - 3z_\alpha}{24} \\ & -\gamma_X^2 \frac{2z_\alpha^3 - 5z_\alpha}{36}.\end{aligned} \tag{7}$$

  This requires solving a cubic equation.

PoS solution (2)

- *The second-order Cornish-Fisher:*

$$\widehat{z}_\alpha = -\frac{3}{\gamma_X} + \sqrt{\frac{9}{\gamma_X^2} + \frac{6}{\gamma_X} q + 1}. \tag{8}$$

where $q = \frac{\eta_X}{CoV_X}$.

- *The third-order Cornish-Fisher approximation:* The Cardano's method was used to solve the cubic equation. The discriminant is positive for most practically feasible values of $CoV_X \leq 50\%$ and $5\% \leq \eta_X \leq 30\%$), and thus provides one real root (see Dal Moro & Krvavych 2015).

- *The fourth-order Cornish-Fisher approximation:* The quartic equation is solved using the Ferrari's method.

Quality of PoS approximation (1)

Table 4: *PoS of Gamma distribution with* $\eta = 10\%$*: actual vs. approximated.*

CoV =	**Parametric distr**	**Quadr approx**	**Cubic approx**	**Quartic approx**	**Quadr approx** Δ %	**Cubic approx** Δ %	**Quartic approx** Δ %
5%	0.97462	0.97460	0.97462	0.97461	-0.00251	-0.00005	-0.00076
10%	0.84172	0.84134	0.84172	0.84163	-0.04474	0.00019	-0.01084
15%	0.75656	0.75574	0.75657	0.75663	-0.10792	0.00184	0.00860
20%	0.70899	0.70772	0.70903	0.70964	-0.17897	0.00581	0.09127
25%	0.68089	0.67914	0.68098	0.68265	-0.25672	0.01300	0.25963
30%	0.66357	0.66130	0.66373	0.66709	-0.34193	0.02423	0.53011
35%	0.65276	0.64991	0.65302	0.65871	-0.43571	0.04033	0.91290
40%	0.64612	0.64264	0.64652	0.65524	-0.53909	0.06207	1.41080
45%	0.64233	0.63814	0.64291	0.65529	-0.65295	0.09024	2.01815
50%	0.64055	0.63557	0.64136	0.65798	-0.77794	0.12557	2.72020

Quality of PoS approximation (2)

Table 10: *PoS of Log-Normal distribution with η = 10%: actual vs. approximated.*

	Parametric distr	**Quadr approx**	**Cubic approx**	**Quartic approx**	**Quadr approx** Δ %	**Cubic approx** Δ %	**Quartic approx** Δ %
CoV =							
5%	0.97334	0.97330	0.97333	0.97330	-0.00455	-0.00099	-0.00412
10%	0.84264	0.84134	0.84264	0.84219	-0.15331	0.00079	-0.05275
15%	0.76224	0.75950	0.76259	0.76258	-0.35945	0.04542	0.04405
20%	0.71914	0.71518	0.72035	0.72232	-0.55074	0.16833	0.44209
25%	0.69505	0.69012	0.69784	0.70360	-0.70877	0.40223	1.23119
30%	0.68134	0.67571	0.68667	0.69797	-0.82595	0.78161	2.44139
35%	0.67377	0.66772	0.68283	0.70085	-0.89846	1.34492	4.01961
40%	0.67004	0.66384	0.68435	0.70903	-0.92471	2.13642	5.81980
45%	0.66882	0.66277	0.69027	0.71988	-0.90484	3.20757	7.63451
50%	0.66929	0.66367	0.70019	0.73128	-0.84037	4.61660	9.26212

Quality of PoS approximation (3)

Table 13: *PoS of I-Gamma distribution with* $\eta = 10\%$*: actual vs. approximated.*

	Parametric distr	**Quadr approx**	**Cubic approx**	**Quartic approx**	**Quadr approx** Δ %	**Cubic approx** Δ %	**Quartic approx** Δ %
CoV =							
5%	0.97211	0.97202	0.97208	0.97199	-0.00894	-0.00316	-0.01150
10%	0.84387	0.84134	0.84392	0.84273	-0.29948	0.00596	-0.13519
15%	0.76828	0.76315	0.76958	0.76910	-0.66883	0.16866	0.10589
20%	0.72955	0.72256	0.73412	0.73736	-0.95826	0.62619	1.07058
25%	0.70925	0.70131	0.72024	0.72957	-1.11964	1.54823	2.86444
30%	0.69882	0.69093	0.72107	0.73559	-1.12789	3.18495	5.26202
35%	0.69402	0.68731	0.73520	0.74760	-0.96663	5.93428	7.72048
40%	0.69259	0.68827	0.76448	0.75964	-0.62481	10.37882	9.67967
45%	0.69324	0.69257	0.81052	0.76848	-0.09563	16.91873	10.85441
50%	0.69515	0.69948	0.86485	0.77331	0.62387	24.41323	11.24424

PoS approximation for reserve portfolio (1)

$$X_{\Sigma} = \sum_{i=1}^{m} X_i, \tag{9}$$

where each i-th class reserve value is approximated by the *Fleishman polynomial* of a standard normal random variable

$$X_i \approx CE_i \cdot (1 + CoV_i \cdot P_3(Z_i)), \tag{10}$$

where $P_3(Z_i) = a_i z_i + b_i \left(z_i^2 - 1\right) + c_i z_i^3$.

Reserving classes have a Gaussian dependence structure (calibrated to rank correlations) with Gaussian linear correlation coefficients ρ_{ij}.

The idea is to

- calculate CoV, skewness and kurtosis at the portfolio level; and
- use them in the PoS approximations derived earlier for a standalone class.

PoS approximation for reserve portfolio (2)

Portfolio Variance

$$\begin{aligned}\mathrm{Var}[X_\Sigma] &= \mathbb{E}\left[\left(\sum_{i=1}^{m}\sigma_i\cdot P_3(Z_i)\right)^2\right]\\ &= \sum_{i=1}^{m}\sigma_i^2+2\sum_{ij}\sigma_i\sigma_j\cdot\mathbb{E}\left[P_3(Z_i)P_3(Z_j)\right],\end{aligned}$$

where $\sigma_i = CE_i \cdot CoV_i$, and $\mathbb{E}\left[P_3^2(Z_i)\right] = 1$.

$$\begin{aligned}\mathbb{E}\left[P_3(Z_i)P_3(Z_j)\right] &= \rho_{ij}\left(a_ia_j+2b_ib_j\rho_{ij}+3\left(a_ic_j+a_jc_i\right)\right.\\ &\quad\left.+3c_ic_j\left(3+2\rho_{ij}^2\right)\right).\end{aligned}$$

or when $c_i = 0$ this is reduced to

$$\mathbb{E}\left[P_3(Z_i)P_3(Z_j)\right] = \rho_{ij}\left(a_ia_j+2b_ib_j\rho_{ij}\right).$$

PoS approximation for reserve portfolio (3)

Portfolio third central moment

$$\begin{aligned}
\mathbb{E}\left[(X_\Sigma - CE_\Sigma)^3\right] &= \mathbb{E}\left[\left(\sum_{i=1}^{m} \sigma_i \cdot P_3(Z_i)\right)^3\right] \\
&= \sum_{i=1}^{m} \sigma_i^3 \cdot \gamma_i \\
&\quad +3 \sum_{ij} \sigma_i^2 \sigma_j \cdot \mathbb{E}\left[P_3(Z_i)^2 P_3(Z_j)\right] \\
&\quad +6 \sum_{ijk} \sigma_i \sigma_j \sigma_k \cdot \mathbb{E}\left[P_3(Z_i) P_3(Z_j) P_3(Z_k)\right],
\end{aligned}$$

and for $c_i = 0$:

$$\begin{aligned}
\mathbb{E}\left[P_3(Z_i)^2 P_3(Z_j)\right] &= 2\rho_{ij}\left(2a_i a_j b_i + \left(a_i^2 + 4b_i^2\right) b_j \rho_{ij}\right) \\
\mathbb{E}\left[P_3(Z_i) P_3(Z_j) P_3(Z_k)\right] &= 2\left(a_j a_k b_i \rho_{ij} \rho_{ik} + a_j a_i b_k \rho_{jk} \rho_{ik}\right. \\
&\quad \left. + a_i a_k b_j \rho_{ij} \rho_{ik}\right) + 8 b_i b_j b_k \rho_{ij} \rho_{ik} \rho_{jk}
\end{aligned}$$

- Conclusions -

Conclusions

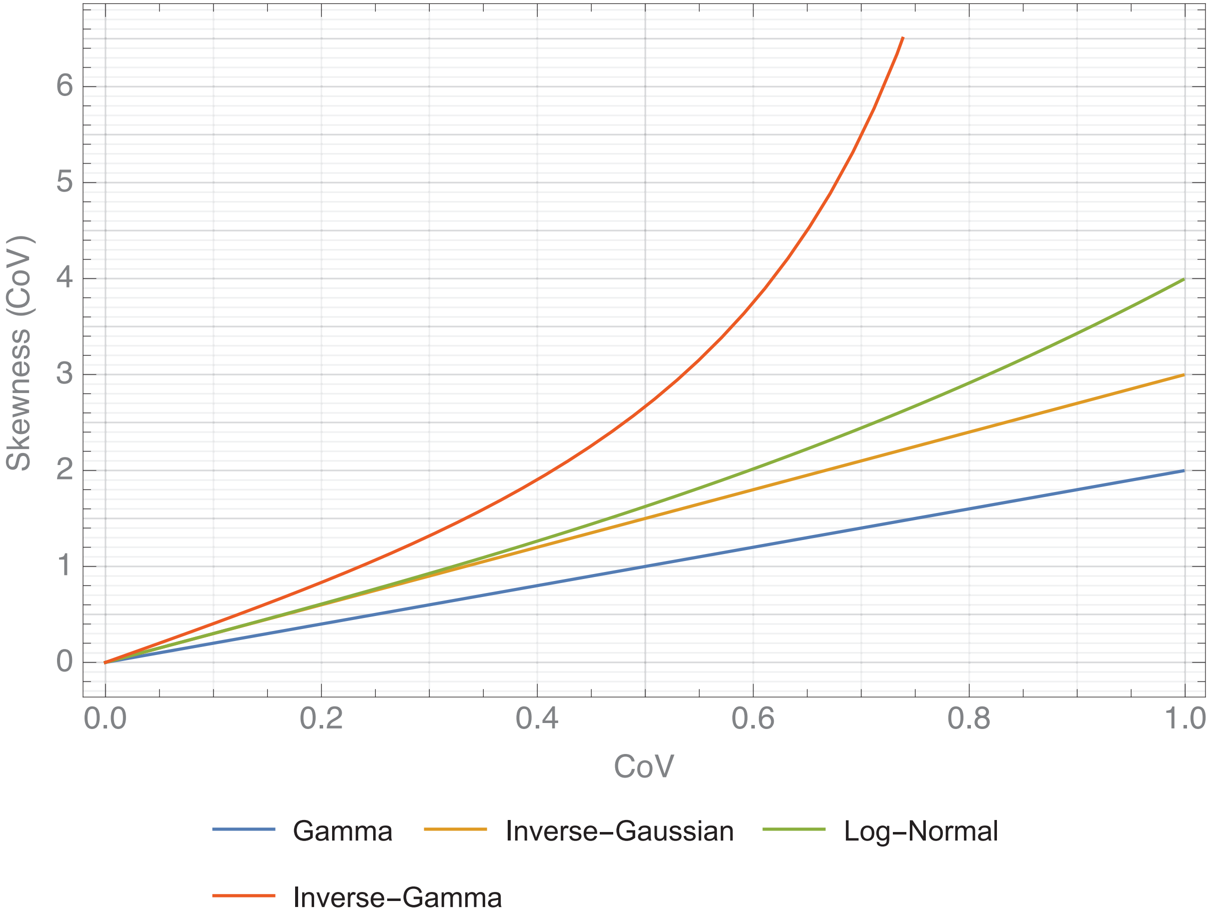
Key takeaway points

- **Looks like we have got a plan**
    - we have done our preparations;
    - and it is now time to 'finish the assignment on the back of a cocktail napkin'
- **Practical implementation**
    - *distribution-free approximation of PoS* – use the best suitable variant of the proposed aproach;
    - *locate the reserve risk profile with respect to known SSP distributions* – by comparing its SC (KCsq) ratio to those of the known parametric SSP distributions;
    - interpolate the correction factor between the parametric distributions adjacent to the reserve risk profile; and
    - use it to adjust the initial distribution-free approximation of PoS.

Location of reserve risk profile ...

... with respect to four known SSP parametric distributions:

Figure 1: *Skewness as a function of CoV for the four parametric distributions.*



References

Dal Moro, E. and Krvavych, Y. Modelling Probability of Sufficiency of Reserve Risk Margins under Solvency II Cost of Capital Approach: practical approximations, *ASTIN Colloquium*, Sydney, 2015. [submitted]

Adam, J. et al. *Guestimation: solving the world's problems on the back of a cocktail naprkin*, Princeton University Press, 2008.

Thank You

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Q & A