Closing-out the Algerian life tables : For more accuracy and adequacy at the old ages

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Outline of the Talk

- Motivation
- Data
- Old age mortality models
- Old age mortality extrapolation results

  Some methological orientations
  Models evaluation / selection
  Model calibration under age limit assumption
  Coherence constraints

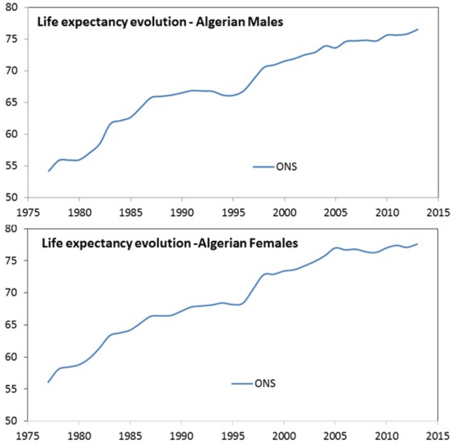
- Final Results
- Concluding remarks

Motivation

- Mortality data in Algeria
- National / International Estimates
- Data quality
- Closing out methodology

Mortality Data in Algeria

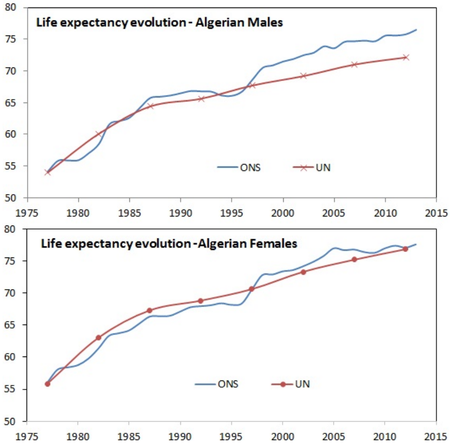
- Annually published by the Office of National Statistics (ONS) starting from 1977;



- No technical report

National / International estimates

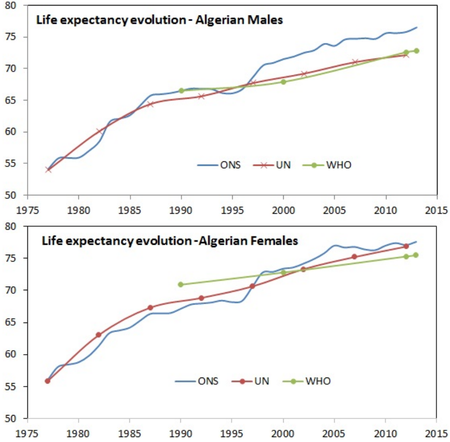
- Some differences with NGO estimates;



- **United Nations Organization (UNO)**

National / International estimates (2)

- Some differences with NGO estimates;



- **United Nations Organization (UNO) / World Health Organization (WHO)**

Data Quality

- Investigate the quality and the reliability of ONS estimates

Flici and Hammouda (2016). 'Review of half-Century of mortality changes in Algeria : 1992-2012'. Under Review

''.... Life expectancy evolution in Algeria (ONS) has been affected by a set of methodological changs ..."
'' .... Closing out method could be inappropriate .... ''

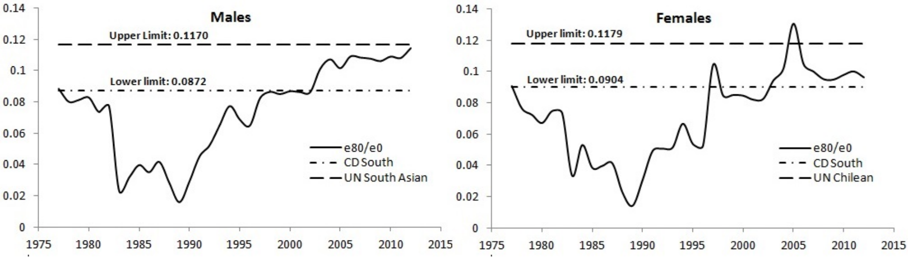
Closing-out methodology : the Algerian case

- Age Specific Mortality rates until the age of 75 or 80 (by five-age groups);
- Mortality rates between 10 and 75 years to Select an adequate model life table (UN model life tables or Coale Demeny life tables);
- Residual life expectancy at the closing age is estimated;
- The life expectancy at birth can be deduced by recurence;

Lets observe the report between the life expectancy at birth and the life expectancy at the age 80 . . . .. And compare to the model life tables

Closing-out methodology inadequacy

- Evolution of $\frac{e_{80}}{e_0}$ in Official publications;



- Comparison to $\frac{e_{80}}{e_0}$ $\{\min, \max\}$ issued from all the 9 families of the Model life tables (UN and CD);
- Each MLT family has a fixed $\frac{e_{80}}{e_0}$ $\{\min, \max\}$.

Under estimation of e_{80}compared to e_0 in the Algerian official life tables

Closing-out methodology unadequacy (2)

- A such imperfection is common for African Countries;

Ita and Banjan (1984) :

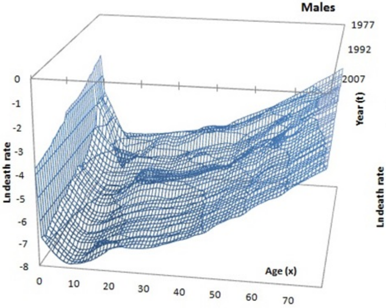
' ... we need to select an adequate model life tables among CDMLT or the UNMLT. Between the two, African countries prefer use the first model life tables to complete their mortality data. Mortality of the northern African countries is generally represented by the south type of CD life tables while the North type is used to describe the mortality pattern of sub-Saharan countries.... ''

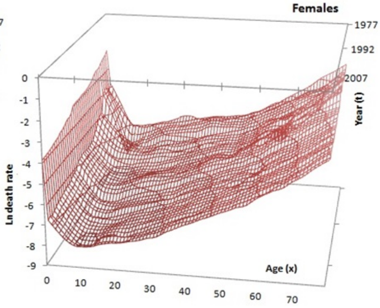
- No one African countries has been considered to construct MLT (Inadequacy);
- Risk of model selection : The use of the stable population method (Coale and al, 1966 and 1983) doesn't guarantee a good model selection;
- LSE can also be used (Coale and al. 1966), but African use the first method;

We propose an alternative way to close out the Algerian life table : Old age mortality extrapolation models

Data : Algerian Mortality Surface

- ONS annual publications
- Some years are missing : 1979, 1984, 1986, 1988, 1990, 1992 and 1997
- Some life tables were closed out earlier than the age 80 : 1983, 1985, 1987, and [1993 - 1996]

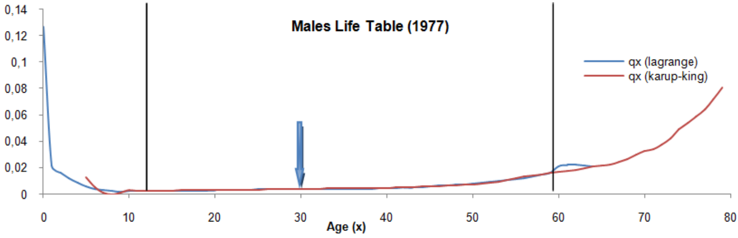




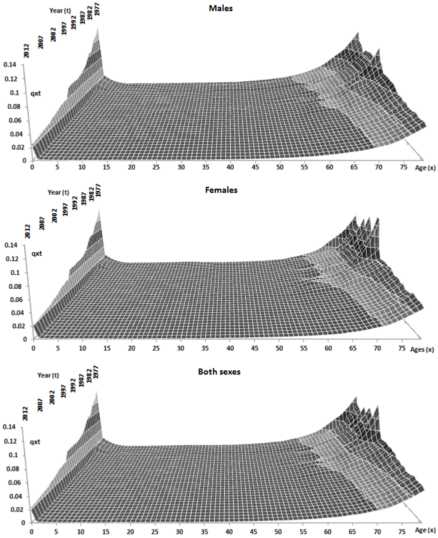
- Missing data estimation : FLICI, F (2014). "*Estimation of the missing data in the Algerian mortality surface*". Conference paper. SMTDA2014.

Single ages mortality interpolation

- Almost mortality models are based on a single ages mortality discription
- Combine Lower ages with Higher age interpolation proprieties;
- Lagrange for lower ages / Karup-King for Higher ages
- Jonction point : $MinA = \frac{1}{5} \sum_{x=k-2}^{k+2} \left\| q_k^* - q_k^+ \right\|, \ldots..k = 20 - 50,$



Single ages mortality Surfaces



Old ages mortality extrapolation methods

- Gompertz-Makeham model (GPZ, MKM)
- Weibull's model (WBL)
- Helligman and Pollard model (HP)
- Coale-Guo and Coale-Kisker methods (CK)
- Himes, Preston and Condran (HPC)
- Perks, Logistic, Kannisto, Thatcher models (PRK, LOG, KST, THT)
- Denuit et Goderniaux method (DG)

Gompertz-Makeham model (GPZ, MKM)

- Gompetrz (1825): $\mu_x = \alpha.e^{\beta x}$
- Makeham (1867) :$\mu_x = c + \alpha.e^{\beta x}$
- A Makeham-type function was used to extend the United Nation's model life tables beyond the age of 80 (UN, 1981):

$$\frac{{}_nq_x}{1-{}_nq_x} = A + B^x$$

Age Range Calibration : [50-75], closing-out $=>$ $l_x = 0$

Gavrilov and Gavrilova (2011)

"... Old people are generally exempt from any risky activity...." so, at older ages MKM model doesnt have any added value compared to GPZ model.

- Gompertz model was used in 1983 to extend the Coale-Demeny model life table until the age of 100 (Coale and al, 1983).

Old age mortality Data Improvement

- When we extend mortality just until the age of 100, the choice of the extrapolation method is of a little importance.
- Since the 80's, life expectancy has improved as well as the number of centenarians (Robine and Vaupel, 2001; Buettner, 2002).
- Models comparison on real Data;
- It turned out that the old age mortality does not follow a Gompertzian function, but slows down slightly (old age mortality deceleration : Kannisto and al., 1994; Kannisto, 1992; Coale and Guo, 1989).
- Gavrilov and Gavrilova (2011) : no deceleration = Old age mortality data quality effect;
- In the absence of data allowing to verify these two hypothesis on the Algerian population, we keep supposing that the first assumption is more near to be verifiable.

Weibull's model (WBL)

- Weibull (1951):

$$\mu_x = \alpha . x^{\beta}$$

- Linear transformation

$$ln(\mu_x) = ln(\alpha) + \beta . ln(x)$$

Helligman and Pollard model (HP)

- Helligman-Pollard model (1980):

$$\frac{q_x}{1-q_x} = A^{(x+B)^c} + D.exp(-E(ln(x) - ln(F))^2 + G.H^x$$

- At the age of 40 or 50, the two first terms become useless :

$$Logit[q_x] = \pi + \varphi.x \;\; with\ \pi = ln(G)\ and\ \varphi = ln[H]$$

- This last formula is similar to the Gompertz mortality law used on q_x.
- This method was examined by the UN Population Division for old age mortality extrapolation issue (Buettner, 2002).

Coale-Guo and Coale-Kisker methods (CK)

- Coale-Guo model (1989) : Linear deceleration trend beyond age 80
- Death rates growth rates (x, x+5, x+10 ...):

$$k_x = ln[\frac{{}_5m_x}{{}_5m_{x-5}}]$$

- $k_x$is increasing by a constant R:

$$k_{x+5} = k_x - R$$

- Extend 5 age death rates from 75 to 110 with known k_{80} :

$${}_5m_{x+5} = {}_5m_x.exp(k_{80} - \frac{(x-80)}{5}.R)$$

- General formula:

$${}_5m_{80+5.i} = {}_5m_{75}exp(ik_{80} - \frac{i(i+1)}{2}R) \ \ i = 1, 2, 3, ...$$

- Arbitrary closure constraint :${}_5m_{105} - {}_5m_{75} = 0.66$

Coale-Guo and Coale-Kisker methods (CK)

- Coale and Kisker (1990): Coale-Guo model for single ages mortality rates.

$$\hat{u}_x = \hat{u}_{x-1} exp(k_{80} + s.(x-80)) \; x = 80, 81, ..., 109.$$

- k_{80}- the growth rate of mortality at 80 :

$$k_{80} = \frac{ln(\frac{\hat{u}_{80}}{\hat{u}_{65}})}{15}$$

- Closure age constraint:

$$\hat{u}_{110} = \begin{cases} 1 \; for \; males \\ 0.8 \; for \; females \end{cases}$$

- This leads to:

$$s = -\frac{ln(\frac{\hat{u}_{79}}{\hat{u}_{110}}) + 31 k_{80}}{465}$$

Coale-Guo and Coale-Kisker methods (CK)

- Coale-Kisker = Quadratic model (Roli, 2008. Thatcher, 1999).
- Let :

$$ln(\mu_x) = a + bx + cx^2$$

- Mortality growth rate :

$$k_x = ln[\frac{\mu_x}{\mu_{x-1}}] = a + bx + cx^2 - a - b(x-1) - c(x-1)^2 = c(2x-1)$$

- For a starting age of 80:

$$k_x - k_{80} = 2.c.(x - 80)$$

- and that leads to the same formula seen earlier :

$$\hat{u}_x = \hat{u}_{x-1} exp(k_{80} + 2.c.(x - 80)) \; x = 80, 81, ..., 109.$$

Himes, Preston and Condran (HPC)

- Himes, Preston and Cardan (1994) :
- Smooth the observed mortality curve by an 2-steps 3rd order moving average:

$$g_x = \frac{\mu_{x-1}+\mu_x+\mu_{x+1}}{3}$$

$$h_x = \frac{g_{x-1}+g_x+g_{x+1}}{3}$$

- Similar to a GPZ model or to the third terms of HP model.
- Extend mortality until the age of 115 years:

$$Logit(\mu_x) = ln(\frac{\mu_x}{1-\mu_x}) = \theta + \Pi.x$$

- θ and Π are estimated on the age ranges [45-79].

Perks, Logistic, Kannisto, Thatcher models (PRK, LOG, KST, THT)

Perks (1932):

$$\mu_x = \frac{\alpha.e^{\beta x}}{1+\gamma.e^{\beta x}}$$

General form of logistic model :

$$\mu_x = \theta + \frac{\lambda.\alpha.e^{\beta x}}{1 + \alpha.e^{\beta x}}$$

Thatcher et al. (1998) (data of 13 developed countries) found that λ is very close to 1:

$$\mu_x = \theta + \frac{\alpha.e^{\beta x}}{1 + \alpha.e^{\beta x}}$$

Perks, Logistic, Kannisto, Thatcher models (PRK, LOG, KST, THT)

- at older age, θ become negligible as the constant accident risk in Makeham's model (Kannisto, 1992):

$$\mu_x \approx \frac{\alpha.e^{\beta x}}{1+\alpha.e^{\beta x}}$$

in the Logit form:

$$Logit(\mu_x) = ln(\alpha) + \beta.x$$

- That is the same formula used independently in 1994 by Himes, Preston and Cardan.

Denuit et Goderniaux method (DG)

Denuit et Goderniaux (2005):

- A 2-polynomial for log mortality rate

$$ln(q_x) = a + bx + cx^2$$

- Age limit constraint : $q_{130} = 1$and $P_{130} = 0$.

Old age mortality models - Summary

- **Mortality Measure :** μ_x**or** q_x**:**

Gavrilov and Gavrilova (2011) :

"... the choice of the indicator that can affect the final result of the extrapolation process...$\mu_x$is more suitable for the fitting and the extrapolation use especially at the older ages .... '

- **Logit transformation**

Logit : from [0-1] to]-∞, $+\infty$[.

- **Linear / quadratic function**
- **Closure age constraints :**

Ultimate survival age, high age mortality constraints, or no constraint

Old age mortality models - Summary

1. GPZ: $ln(\mu_x) = ln(\alpha) + ln(\beta).xe^{\beta x}$
2. MKH: $ln(\mu_x - c) = ln(\alpha) + ln(\beta).xe^{\beta x}$
3. HP:$Logit(q_x) = \pi + \varphi.x$
4. KST: $Logit(\mu_x) = ln(\alpha) + \beta.x$
5. HPC: $Logit(\mu_x) = \theta + \Pi.x$
6. WBL: $ln(\mu_x) = ln(\alpha) + \beta.ln(x)$
7. DG : $ln(q_x) = a + bx + cx^2$
8. CK : $ln(\mu_x) = a + bx + cx^2$

Models to be compared : HP, DG, GPZ, WBL, CK and KST.

Application

- Some methodological orientations
- Model selection

1. *Goodness-of-fit and predictive capacity*
2. *Expected males females sex ratio*
3. *Coherence between single and both-sexes expected mortality rates*
4. *Expected age limit*
5. *Result's discussion*

- Model calibration under age limit assumption
- Coherence constraints

- Model evaluation / Comparison : Observed Vs Expected

Goodness-of-fit : age range [40-74], [45-75[, ... [60-75[

$$MSE_{[x-74]} = \frac{1}{t*x} \sum_{x=X}^{74} \sum_{t=1977}^{2014} [ln(\hat{q}_{xt}) - ln(q_{xt})]^2$$

Predictive-capacity: [75-84[

$$MSE_{[75-79]} = \frac{1}{5*38} \sum_{x=75}^{79} \sum_{t=1977}^{2014} [ln(\hat{q}_{xt}) - ln(q_{xt})]^2$$

Different age ranges : Adapted BIC (Schwarz, 1978; Burnham and Anderson, 1998; Hansen, 2007)

$$BIC = n.ln(MSE) + k.ln(n)$$

Application - Some methodological orientations (2)

Does Goodness of fit guarantee the Predictive Capacity? NO !

- Compare age ranges to ensure the converging of the 2 indicators ;
- Best model: If fitting quality is good, it should gives a good predictive capacity;
- Age range for model calibration ?

Application - Some methodological orientations (2)

Complementary Criteria:

- Mortality rates increasing with age :$q_{x+1} > q_x$; $\forall x \geq 75$. (Quadratic functions)
- Coherence : single and both sexes extrapolation,

for year (t) : $q_x^{both} \approx \frac{(q_x^m + q_x^f)}{2}$; $\forall x \geq 75$. or at least $q_x^b \in [q_x^f - q_x^m]$;

- Sex ratio : male mortality excess at all ages :$q_x^m \geq\ q_x^f, \forall x \geq 75$.
- Predicted surviving age limit :

For the Algerian population (MICS IV results) a maximal surviving age of 112 years/men and 110/ women (Flici et Hammouda, 2015).

Age range for calibration	*40-74*		*45-74*		*50-74*		*55-74*		*60-74*		*Models Ranking*	
Age range for evaluation	[x-74]	[75-84]	[x-74]	[75-84]	[x-74]	[75-84]	[x-74]	[75-84]	[x-74]	[75-84]	Goodness of fit	Predictive capacity
Model												
DG	-173.44	*-50.68*	-163.82	*-50.80*	-137.50	*-53.83*	-107.75	*-45.01*	-81.20	*-46.14*	2	1
HP	-173.02	*-29.85*	-165.40	*-33.75*	-138.12	*-35.92*	-109.86	*-37.36*	-89.85	*-36.63*	4	4
GPZ	-174.04	*-31.39*	-165.92	*-35.51*	-138.49	*-37.75*	-110.06	*-39.06*	-89.98	*-37.80*	3	3
WBL	-166.11	*-16.02*	-141.67	*-16.02*	-125.87	*-20.25*	-105.19	*-23.93*	-86.76	*-26.98*	6	6
CK	-178.28	*-46.95*	-157.36	*-45.61*	-131.87	*-45.91*	-102.75	*-46.90*	-75.63	*-48.42*	1	2
KST	-171.95	*-28.73*	-164.81	*-32.11*	-137.72	*-34.23*	-109.63	*-35.77*	-89.70	*-35.47*	5	5

Females

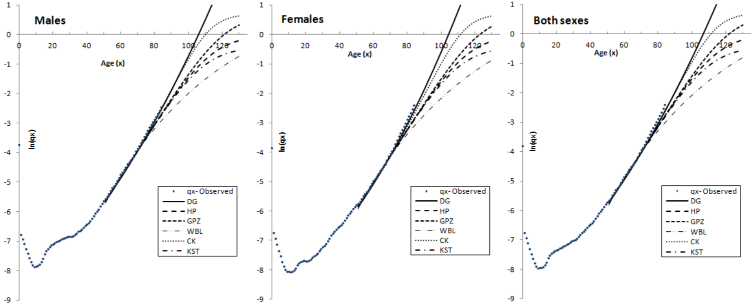
Age range for calibration	*40-74*		*45-74*		*50-74*		*55-74*		*60-74*		*Models Ranking*	
Age range for evaluation	[x-74]	[75-84]	[x-74]	[75-84]	[x-74]	[75-84]	[x-74]	[75-84]	[x-74]	[75-84]	Goodness of fit	Predictive capacity
Model												
DG	-152.12	*-24.75*	-139.67	*-23.93*	-114.48	*-32.64*	-95.36	*-29.48*	-82.05	*-31.07*	2	2
HP	-144.06	*-13.61*	-140.44	*-16.52*	-121.59	*-20.07*	-97.99	*-23.05*	-84.36	*-26.29*	4	4
GPZ	-144.81	*-14.15*	-141.07	*-17.13*	-121.98	*-20.76*	-98.20	*-23.83*	-84.44	*-27.01*	3	3
WBL	-115.92	*-3.48*	-119.29	*-7.02*	-110.25	*-11.04*	-93.31	*-15.09*	-81.85	*-19.41*	6	6
CK	-152.28	*-22.83*	-139.06	*-22.13*	-115.29	*-33.39*	-94.99	*-27.90*	-69.77	*-27.62*	1	1
KST	-143.31	*-13.05*	-139.78	*-15.97*	-121.19	*-19.34*	-97.77	*-22.35*	-84.28	*-25.58*	5	5

Both sexes

Age range for calibration	*40-74*		*45-74*		*50-74*		*55-74*		*60-74*		*Models Ranking*	
Age range for evaluation	[x-74]	[75-84]	[x-74]	[75-84]	[x-74]	[75-84]	[x-74]	[75-84]	[x-74]	[75-84]	Goodness of fit	Predictive capacity
Model												
DG	-169.38	*-36.08*	-150.69	*-35.17*	-124.04	*-35.67*	-96.69	*-36.69*	-71.77	*-37.53*	1	2
HP	-160.24	*-20.57*	-155.85	*-23.87*	-132.04	*-26.93*	-105.61	*-29.41*	-90.70	*-31.24*	4	4
GPZ	-161.23	*-21.51*	-156.64	*-24.90*	-132.54	*-28.05*	-105.89	*-30.53*	-90.85	*-32.21*	3	3
WBL	-126.52	*-7.25*	-130.85	*-11.24*	-118.59	*-15.25*	-99.95	*-19.18*	-87.13	*-23.00*	6	6
CK	-165.80	*-30.63*	-161.33	*-38.73*	-133.65	*-40.35*	-103.78	*-40.48*	-76.63	*-39.99*	2	1
KST	-159.24	*-19.71*	-155.02	*-22.96*	-131.50	*-25.92*	-105.31	*-28.33*	-90.53	*-30.30*	5	5

Best models : DG, CK and GPZ

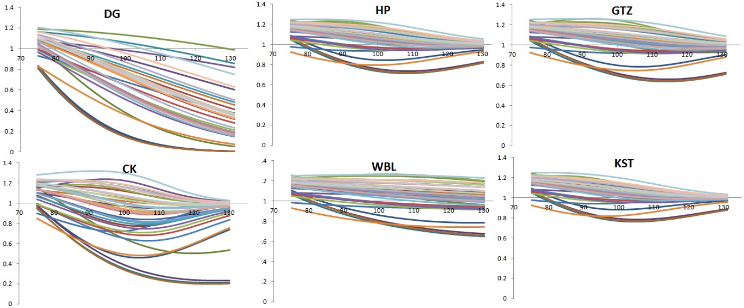
- Common age range for calibration: [50-75[



Best models : DG, CK and GPZ /

Complementary Criteria ???

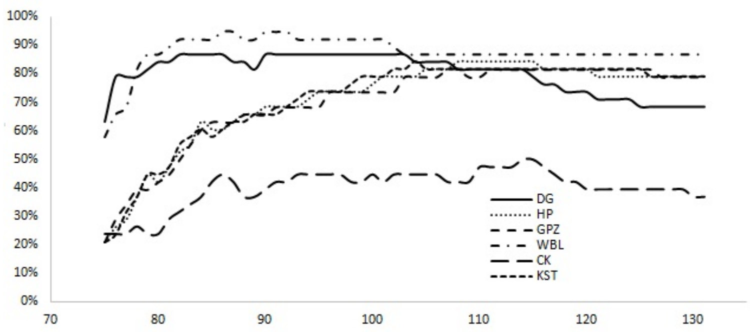
Expected males females sex ratio



KST, HP and GPZ : Acceptable results.

Coherence between unisex and both-sexes rates

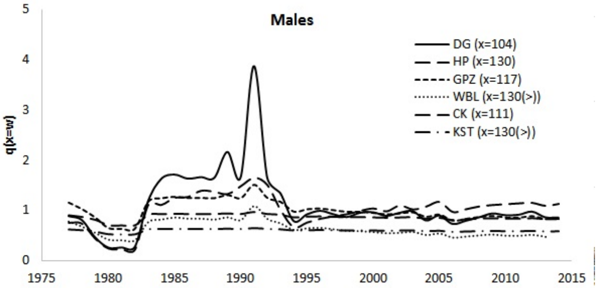
- **Failure rate** : part of cases where both-sexes expected mortality rates situated out of the interval between males and females rates.



CK model gives the less important failure ratio.

Expected Surviving age limit

- Comparison of some high ages q_x predicted by the models



- HP, WBL and KST: q (130) $<<<$ 1.
- DG : 104, 103 and 106 in average.
- CK mode: between 109 and 111 years old.
- GPZ : 117 years.

Result's discussion

Quadratic models

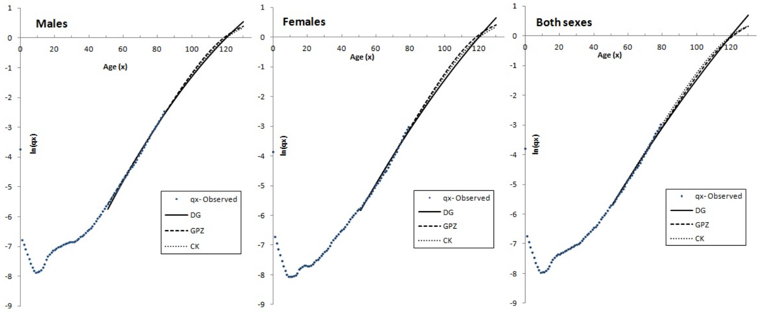
- Good quality regarding to goodness-of-fit and predictive capacity compared to the linear transformed models. (Except GPZ).
- Some incoherence regarding the sex differential or single sexes vs both-sexes extrapolated mortality rates.

Coherence : Age limit constraint (Denuit and Goderniaux, 2005) or a fixed mortality level at high age (Coale and Kisker, 1990).

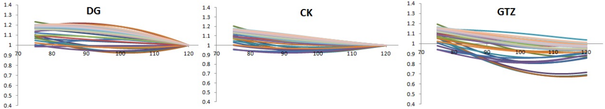
and it avoid a crossover of the males / females extrapolated mortality curves (Buettener, 2002).

We keep working with 3 models: DG, CK model with 120 as an assumed age limit, and GPZ .

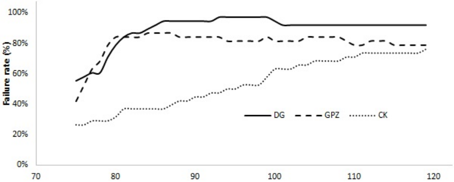
- Goodness-of-fit / Predictive Capacity



Male Female Mortality ratio



Failure ratio: single vs Both sexes extrapolation

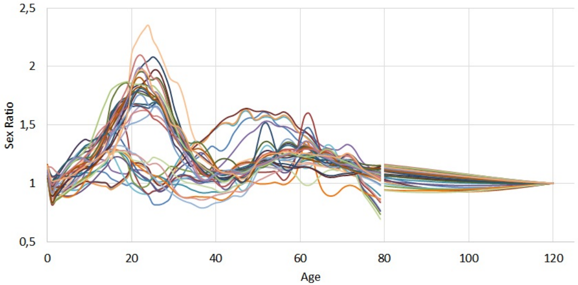


- CK >>>> DG and GPZ.

increasing trend of the failure ratio => additive constraints

Two additional constraints (CK)

- keep the females mortality under the males mortality;
- keep the both sexes expected mortality in between males and females.



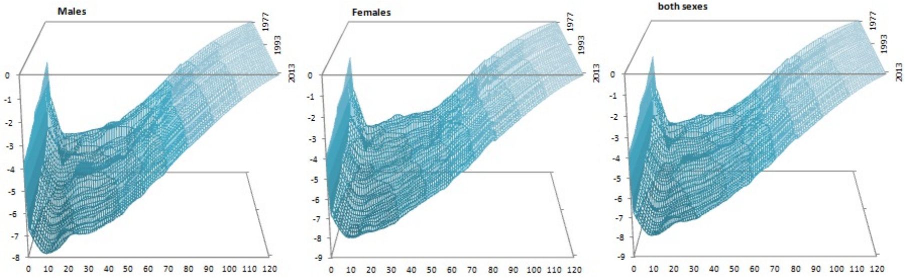
- Some difficulties were found when a female mortality excess was observed bellow age 80;

Final results

- GPZ, CK and CG : goodness-of-fit and predictive capacity (Indifference);
- Quadratic models allow more flexibility in old age mortality extrapolation;
- GPZ : a good performance regarding all selection criteria / unstable expected age limit series.
- Quadratic models : a common age limit constraints for all years.
- GPZ : suitable to extrapolate old age mortality on a well fitted mortality surface.
- CK : more coherent results compared to DG

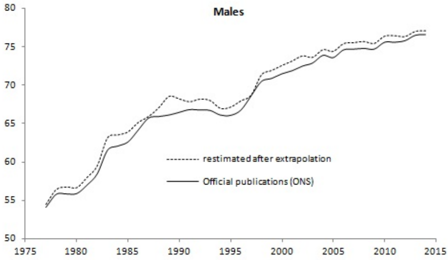
Use CK model to extend the Algerian mortality surfaces

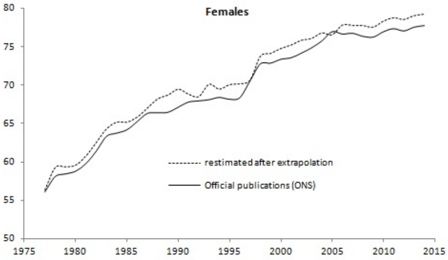
Extended mortality surface



Corrected life expectancy

- Closing out methodology and life expectancy;
- Our estimates Vs. ONS estimates





- Our estimates Vs ONS : +0.9 year for males and +1.2 year for females.
- **Flici and Hammouda (2014)** : .... some change points in the life expectancy series are explained by the closing out methodology....

Concluding Remarks

- Usefulness of the extrapolative approach Vs model life tables;
- Extending the mortality pattern untill the surviving age limit.
- Neesed in actuarial applications;
- UN life table: usefull but !!!!

Concluding Remarks (2)

- Model comparison: GPZ, WBL, KST, HP, CK and DG.
- Best model : CK, DG and GPZ.
- Goodness-of fit / predictive capacity + Complementary Criteria
- Surviving age limit;
- Coherence : Males Vs Females, Single sex Vs Both sexes;
- Additive constraintes;

Concluding Remarks (3)

- In final : a methodology to close out life tables in Algeria
- Coming works: Push-up investigation
- Compare other data sources : Health surveys, Social security ... etc.

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