International ASTIN Colloquium
Den Haag, May 2013

Reinventing Pareto

Fitting both small and large losses

Michael Fackler
Independent Actuary
Munich (Germany)

Key ideas of the paper

- Interpret Pareto as part of a **ground-up** model
- Generalize to GPD,
  using the **one** really useful parametrisation
- Spliced distributions
- 3-dimensional system of Lognormal-GPD models
- Inventory of spliced ... -GPD models
- Apply to exposure rating

Reinsurer's old love: (European) Pareto

Survival function: $\bar{F}(x) = P(X > x) = \left(\frac{\theta}{x}\right)^{\alpha}$

x > θ

More correctly: $\bar{F}(x \mid X > \theta) = \left(\frac{\theta}{x}\right)^{\alpha}$

Modelling of higher tails: $\bar{F}(x \mid X > d) = \left(\frac{d}{x}\right)^{\alpha}$

d ≥ θ

Advantages

- Parameter θ of the ground-up model does not affect high tails – no need to know it for tail modelling
- Parameter α is **common** to all tail models beyond θ

That is great!

- It enables us to compare models of the data sets we in practice have available – these have varying and sometimes unknown reporting thresholds
- We may find **market values** for α

GPD – a new love?

Generalized Pareto from Extreme Value Theory

$$\overline{F}(x|X>\theta)=\left(1+\xi\frac{x-\theta}{\tau}\right)^{-1/\xi}$$

Most interesting case: $\xi > 0$

Useful parametrisation: $\overline{F}(x|X>\theta)=\left(\frac{\theta+\lambda}{x+\lambda}\right)^{\alpha}$

$\alpha = 1/\xi > 0$

$\lambda = \alpha\tau - \theta > -\theta$

Modelling of higher tails: $\overline{F}(x|X>d)=\left(\frac{d+\lambda}{x+\lambda}\right)^{\alpha}$

$d \geq \theta$

Advantages

- Plausible shape of tail (supported by EVT)
- Parameter θ of the ground-up model does not affect high tails – no need to know it for tail modelling
- Parameters α **and** λ are common to all tail models beyond θ

That is great!

We may find market values for α **and** λ

Flexible fits: piecewise

Define separately:

r	$0 < x \le \theta$	small loss distribution ("**body**")
$1-r$	$\theta < x \le \infty$	large loss distribution (**tail**)

- Selection of very different distributions possible
- If θ known separate parameter estimation possible
- Intuitive meaning of body and tail

Potential application: single or aggregate losses

The new ***Pareto family***: a spliced model

C0 function with GPD tail:

$$\overline{F}(x) = \begin{cases} 1 - \dfrac{r}{F_1(\theta)} F_1(x) & x \leq \theta \\ (1-r)\left(\dfrac{\theta+\lambda}{x+\lambda}\right)^{\alpha} & \theta \leq x \end{cases}$$

LN-GPD-0 with 6 parameters:

$$F_1(x) = \Phi\left(\frac{\ln(x) - \mu}{\sigma}\right)$$

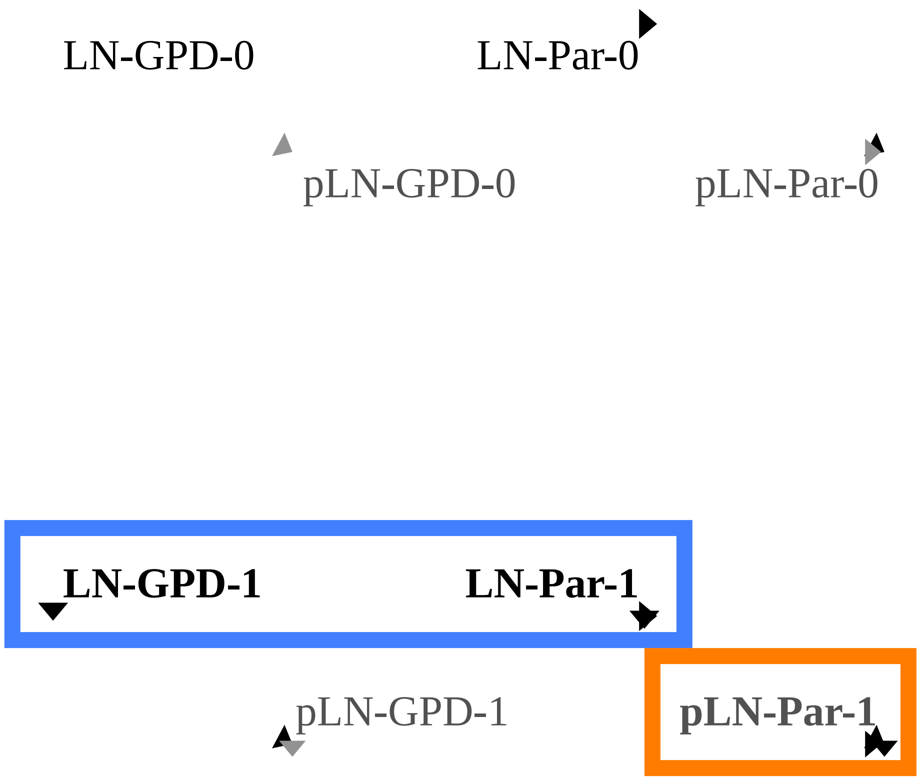
Special cases (parameter reduction)

3 obvious ways to specify:

- **tail** – GPD or Pareto: $\lambda = 0$
- **body** – distorted or proper: $r = F_1(\theta)$
- **smoothness** – C0, C1, C2, ...

- Combinations yield 3-dimensional grid

The Lognormal-GPD cube



Possible bodies for the Pareto family

- Lognormal (e.g. Czeledin)

- Exponential
- Weibull
- Power function (e.g. Double Pareto)

- discrete
- ...

Exposure rating

Based on loss severity distributions

Perfect candidates: the new Pareto family

Example: The old **Power Curve ILFs**

= Riebesell model = German method = ...

$$LEV(x) = cx^{1-\alpha}$$

Classification: ***Riebesell distribution***

- Pareto tail beyond a threshold θ
- Lots of small losses: $0 < \alpha \leq r < 1$
- Small losses concentrated just below θ

Example: C0 Power Curve-Pareto distribution with body

$$F_1(x) = \left(\frac{x}{\theta}\right)^{\alpha\frac{1-r}{r-\alpha}}$$

The End

Thanks for joining this talk.

Feedback welcome – in particular about spliced models in the literature and in practice.

michael_fackler@web.de