Stress scenario generation for solvency and risk management

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Outline

- Worst-case scenarios
- Some applications
- Results (including portfolios)
- Numerical illustrations
- Conclusion

Defining a worst-case scenario

How to **calculate (deterministic) worst-case reserves/obtain safe-side scenarios** for life insurance contracts?

Worst-case scenario

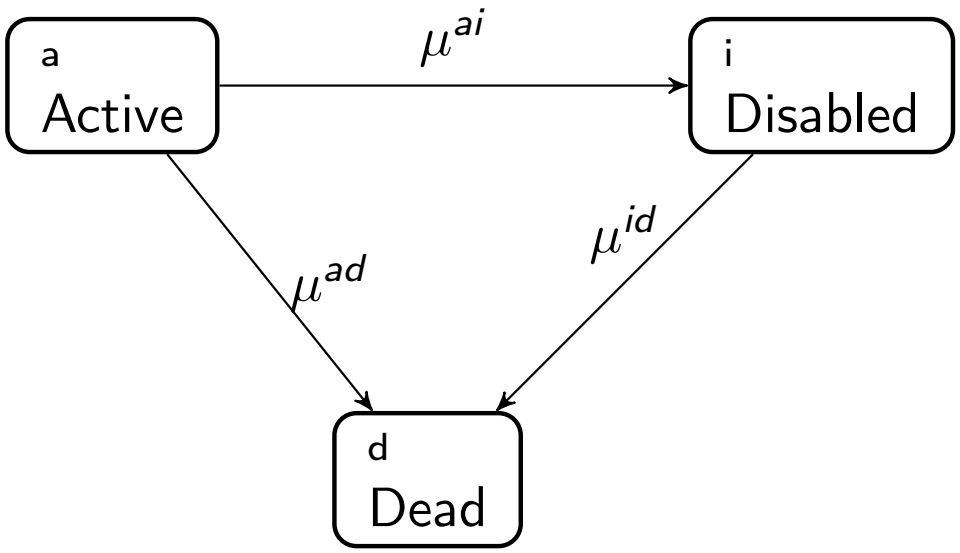
$$(\bar{\phi}, \bar{\mu}) = \underset{(\phi,\mu)\in M}{\text{argmax}} \; V_j(t, \phi, \mu)$$

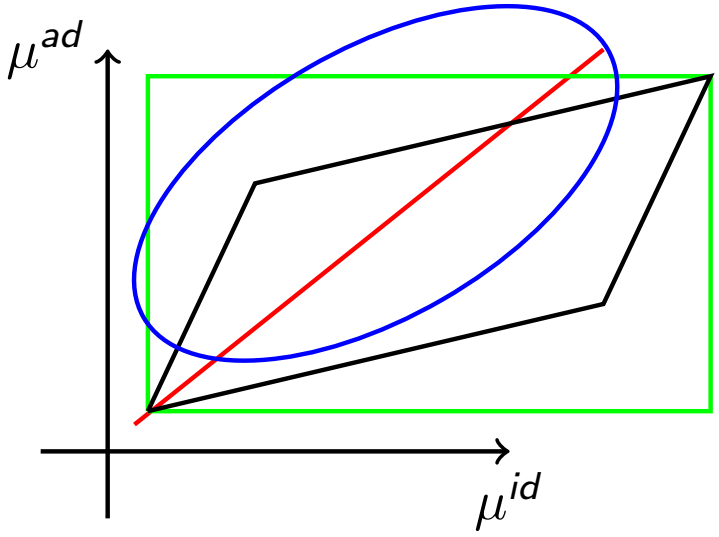
- V_j is an insurance reserve
- $(\phi, \mu) \in M$ are the possible future biometrical and interest scenarios
- $V_j(t, \bar{\phi}, \bar{\mu}) = \sup_{(\phi,\mu)\in M} V_j(t, \phi, \mu)$ if $(\bar{\phi}, \bar{\mu})$ is a worst-case scenario

About the general problem

- Transition intensities are mutually dependent
  - (Death intensities for active and disabled)
- Dependence between interest rate and intensities
  - (Interest rate and surrender intensity)
- Dependence between policyholders in a portfolio
- Approach based on deterministic optimal control

Motivating example





Want to find worst-case μ^{ad} and μ^{id}

Death risk premia:

$$\mu_{ad}(t)(b_{ad}(t) - V_a(t)) \qquad \mu_{id}(t)(b_{id}(t) - V_i(t))$$

About the set M

In general quite hard to find a set M with specific properties.

Possibilities:

- Deduce from a stochastic model
- Expert opinion

Something like a **ellipsoid** (2 dimensions, fixed time point) could be realistic for many cases

Applications of worst-case scenarios

For accounting we need **best estimate** reserves

However, worst-case/safe-side calculations can be used for:

- Risk margins
- SCR (partial internal model)
- Surrender values
- Premium settlement of traditional with-profits life insurance products

Applications of worst-case scenarios II

Solvency II/Partial internal model (assuming assets unaffected)

Let $\alpha = 0.005$:

$$P\left((\phi, \mu) \in M\right) \geq 1 - \alpha$$

$$\Rightarrow \quad P\left(\underset{(\phi,\mu)\in M}{\text{argmax}}\ V_i(t;\phi,\mu) \geq V_i(t,\phi^{\text{BE}},\mu^{\text{BE}})\right) \geq 1-\alpha.$$

$$SCR_{\text{life}} = VaR_{0.995}\left(V_i^{\mathcal{F}_{t+1}} - V_i^{\text{BE}}\right) \leq \sup_{(\phi,\mu)\in M}\left\{V_i(t;\phi,\mu) - V_i(t,\phi^{\text{BE}},\mu^{\text{BE}})\right\}$$

The worst-case scenario (single policy)

The worst-case scenario and reserves are given as the solution to the following system of equations

$$\frac{\mathrm{d}}{\mathrm{d}t}\bar{V}_j(t) = -\, b_j(t) + \bar{V}_j(t)\bar{\phi}(t) - \sum_{k\in\mathcal{S}:k\neq j}\left(b_{jk}(t) + \bar{V}_k(t) - \bar{V}_j(t)\right)\bar{\mu}_{jk}(t),$$

$$\bar{V}_j(n) = 0,$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\bar{p}(t) = -\left(\bar{\mu}(t)\right)^{\mathrm{tr}}\bar{p}(t), \quad \bar{p}(0) = \pi,$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\bar{v}(t) = -\,\bar{v}(t)\bar{\phi}(t), \quad \bar{\phi}(0) = 1,$$

$$\left(\bar{\phi}(t), \bar{\mu}(t)\right) = \underset{(f,m)\in M(t)}{\arg\sup}\left\{ -\, f\sum_{j\in\mathcal{S}}\bar{p}_j(t)\bar{V}_j(t) + \sum_{j\in\mathcal{S}}\bar{p}_j(t)\sum_{k\in\mathcal{S}:k\neq j} m_{jk}\left(b_{jk}(t) + \bar{V}_k(t) - \bar{V}_j(t)\right)\right\}.$$

The worst-case scenario (portfolio)

We have an **existence result** and **Verification lemma** (for a single contract).

Existence of a worst-case scenario is a bit **delicate** because we need compactness of $M(t)_{t\in[t,T]}$ *not only* $M(t)$.

Results can be extended to **portfolios**:

$$(\bar{\phi}(t), \bar{\mu}_J^1, \ldots, \bar{\mu}_J^L)$$

$$= \underset{(\phi,\mu_J^{\mathbf{1}},\ldots,\mu_J^L)\in M(t)}{\arg\sup} \left\{ -\phi \sum_{l=1}^{L} \sum_{j_l\in\mathcal{S}} \bar{p}_{j_l}^l(t)\bar{V}_{j_l}^l(t) \right.$$

$$\left. + \sum_{l=1}^{L} \sum_{(j_l,\tilde{j}_l)\in\mathcal{J}} \bar{p}_{j_l}^l(s,t)\mu_{j_l\tilde{j}_l}^l \left(b_{j_l\tilde{j}_l}^l(t) + \bar{V}_{\tilde{j}_l}^l(t) - \bar{V}_{j_l}^l(t) \right) \right\}$$

Numerics, initial comments

Boundary conditions in different time points

- Need iterative algorithms
- Shooting method
- Fixed point equation methods

Simplified examples:

- argmax constant wrt. transition probabilities
- Transition intensities and interest rate mutually independent

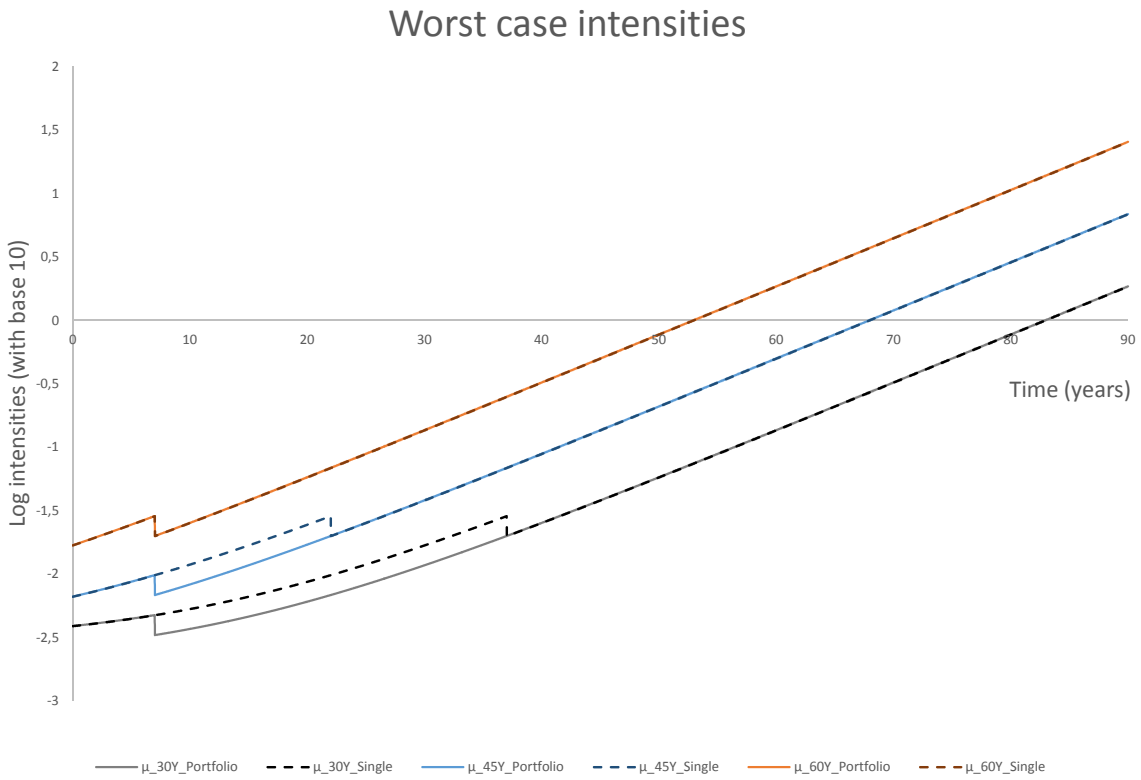
Numerical calculations, portfolio

Portfolio with 3 persons with term insurance and life annuities (2 state model)

$$M(t) = \{\phi, (\mu^1(t), \ldots, \mu^3(t)) \in \boldsymbol{R}_+^4 \big| \\ \phi \in \Phi, \mu^1(t) = \hat{\mu}^1(t)\alpha(t), \ldots, \mu^1(t) = \hat{\mu}^3(t)\alpha(t)\}$$

- Pension age: 67
- Life annuity rate: 1
- Term insurance: 15
- Interest rate: 0.02
- Age of policyholders: 30, 45 and 60
- Baseline death intensity is on GM form
- $\alpha(t) \in [0.8, 1.15]$

Numerical calculations, portfolio (cont)



Solvency II	Worst-case (PF)	Worst-case (separate)
1.59	1.83	2.19

Conclusion

- Worst-case scenarios relates to VaR and Solvency II
- Allows for mutual dependence
- Allows for inhomogeneous portfolios
- Stress-scenarios are (once calculated)
  - simple to implement
  - simple to interpret
  - simple to communicate
  - bounding the insolvency probability