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Risk-shifting and optimal asset allocation in life insurance: The impact of regulation

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Setup

Single-premium insurance policy: maturity T, maturity guarantee.

Equity holder
– provides equity E_0
– investment decision $A_0 = E_0 + L_0$

guarantee+bonus in T $\longrightarrow$

$\longleftarrow$ single premium in $t = 0$

Liability holder
– provides premium L_0

$\uparrow$

Regulator
– restricts excessive risk taking

"Optimal" risk share?

Regulatory intervention

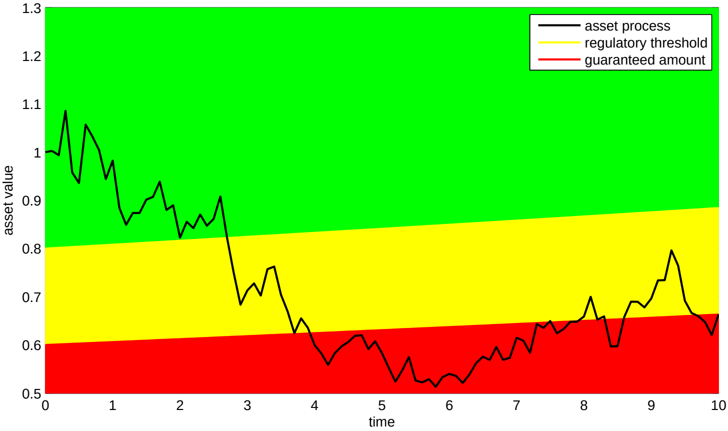
The **regulator** restricts the riskiness of the asset allocation by

- **Risk capital**: i.e. Value-at-Risk based risk capital!
- **Asset allocation constraints**: i.e. risky asset share is bounded!

Regulatory intervention

Regulator may supervise funding ratio (assets divided by liabilities).

funding ratio > 1.2	$1 <$ funding ratio ≤ 1.2	funding ratio ≤ 1
	sanctions, restructuring to avoid default	default of insurance company



Asset process, default threshold (= guaranteed amount), regulatory threshold.

Reference Portfolio

The insurance company's assets are invested in the **reference portfolio** A:

$$\mathrm{d}A_t = \big(r + \theta_1(\mu - r)\big)\, A_t\, \mathrm{d}t + \sigma\theta_1\, A_t\, \mathrm{d}W_t\,. \tag{1}$$

with **proportion in the risky asset** θ_1, risk-free interest rate r, volatility σ.

The guaranteed amount is

$$L_t = L_0\, e^{gt}\,. \tag{2}$$

where $g \leq r$ is the guaranteed return.

Payoff at Maturity T

Initial investment L_0 (liability holder), E_0 (equity holder), $A_0 = L_0 + E_0$.

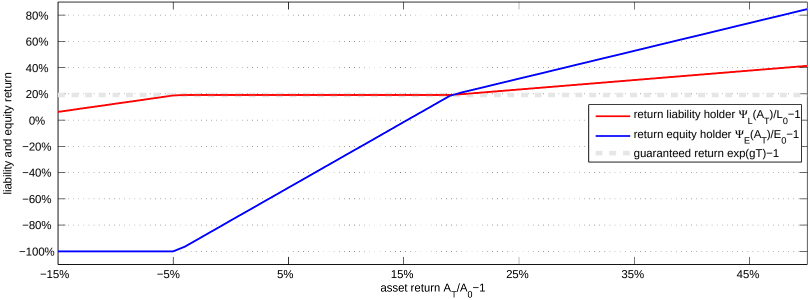
Insurance company survives maturity T, liability holder receives:

$$\Psi_L(A_T, T) = \underbrace{L_T}_{\textbf{guarantee}} + \underbrace{\delta L_0 \cdot \max\left\{\frac{A_T}{A_0} - e^{gT}, 0\right\}}_{\textbf{bonus option}},$$

equity holder receives:

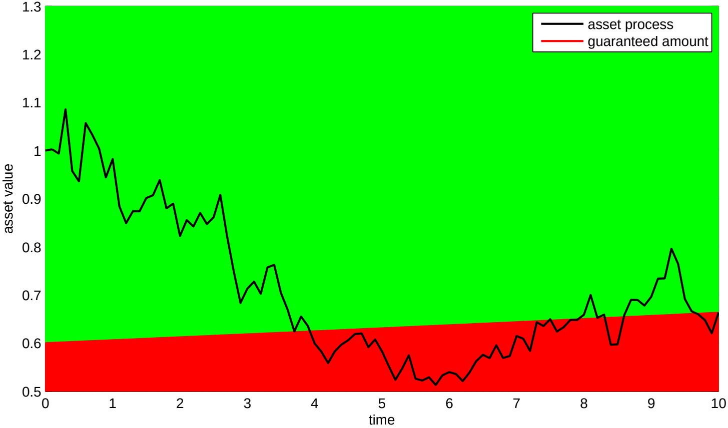
$$\Psi_E(A_T, T) = \max\left\{A_T - L_T, 0\right\} - \underbrace{\delta L_0 \cdot \max\left\{\frac{A_T}{A_0} - e^{gT}, 0\right\}}_{\textbf{bonus option}}$$

Note that $\Psi_L(A_T) + \Psi_E(A_T) = A_T$.



Return of liability and equity holder (**no premature default**, y-axis) dependent on the return $A_T / A_0 - 1$ of the reference portfolio (x-axis). ($T = 10$, $L_0 = 0.8$, $D_0 = 0.75$)

Default at Time τ



Asset process, default threshold (= guaranteed amount).

Default at Time τ

If the assets drop below

$$D_t := L_0\, e^{gt},$$

where $L_0 < A_0$, the company defaults. The default time is

$$\tau := \inf\left\{t \geq 0 \,\middle|\, A_t \leq D_t\right\}, \tag{3}$$

Payments in case of default

$$\Psi_L(A_\tau, \tau) = D_\tau,$$

$$\Psi_E(A_\tau, \tau) = 0.$$

Competitive Market

We assume that insurance policies are **fairly priced**.
The participation rate δ is set accordingly.

The asset strategy θ_1 is initially known and left unchanged (**no risk-shifting**).

Payoff evaluation

The equity holder is risk-neutral and evaluates its payoff according to expected payoff, i.e. chooses an **optimal asset allocation** θ_1

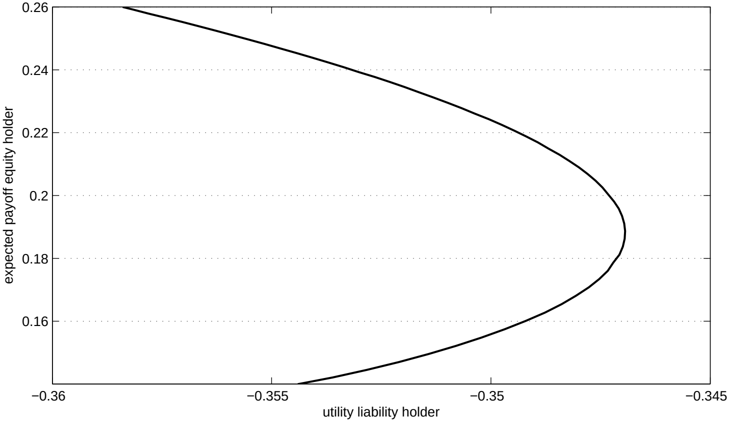
$$\max_{\theta_1} \mathbb{E}_{\mathbb{P}}\left[\int_0^T e^{r(T-\tau)}\Psi_E(A_\tau,\tau)\,\mathrm{d}\tau + \mathbb{1}_{\{\tau>T\}}\Psi_E(A_T,T)\right]. \qquad (4)$$

where θ_1 is the share in the risky asset.

The liability holder is risk-averse and optimizes payoffs according to the utility function $u_L(\cdot)$.

$$\max_{\theta_1} \mathbb{E}_{\mathbb{P}}\left[\int_0^T u_L\Big(e^{r(T-\tau)}\Psi_L(A_\tau,\tau)\Big)\,\mathrm{d}\tau + \mathbb{1}_{\{\tau>T\}}u_L\Big(\Psi_L(A_T,T)\Big)\right].$$

Optimization results



Utility liability holder (power utility, $\gamma = 3$) and expected payoff equity holder for different risky shares θ_1.

Regulatory Intervention

Regulator restricts risky asset share θ_1, i.e. by **default constraint**:

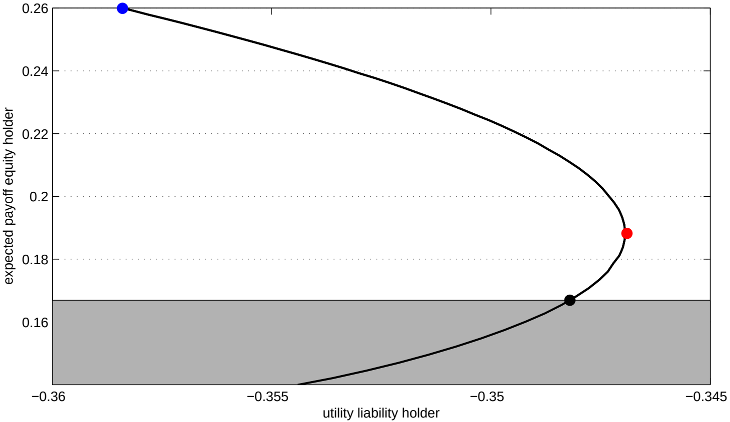
$$\mathbb{P}(\tau \leq T) \leq \epsilon \,.$$

Resulting **optimization function**:

$$\max_{\theta_1} \mathbb{E}_{\mathbb{P}}\left[\int_0^T e^{r(T-\tau)} \Psi_E(A_\tau, \tau)\, \mathrm{d}\tau + \mathbb{1}_{\{\tau > T\}} \Psi_E(A_T, T) \right]$$

s. t. **default constraint**

Optimization results



Same optimization under **default constraint**. Grey area has $\mathbb{P}(\tau \leq T) \leq 5\%$.

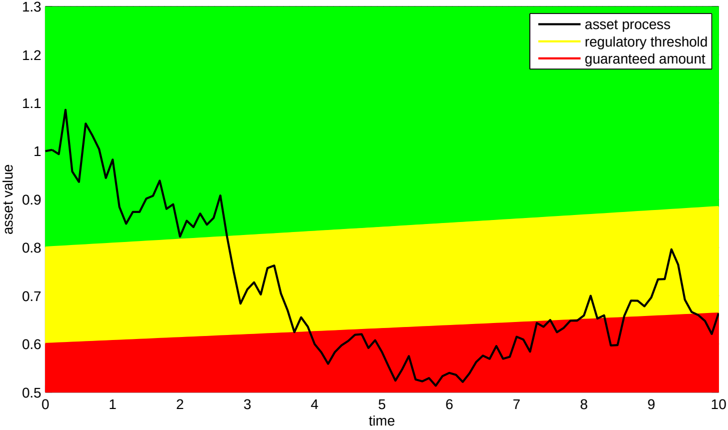
Optimal asset allocation:

- Liability holder: $\theta_1 = 0.2716$ (red dot).
- Equity holder: $\theta_1 = 1.000$ (blue dot).

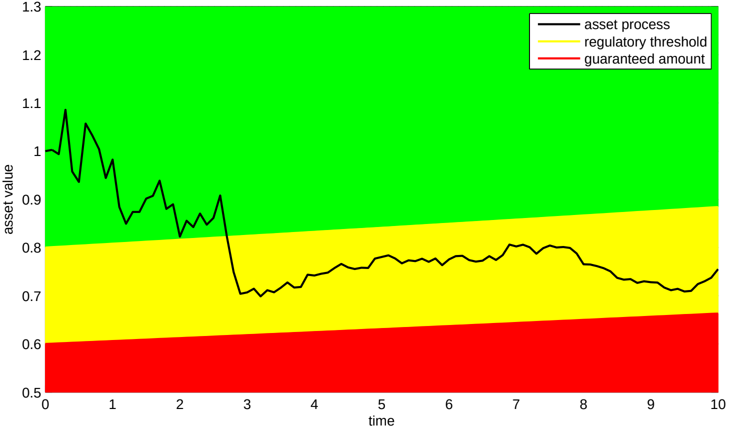
$\Longrightarrow$ **Conflict of interests!**

Regulatory default constraint: $\theta_1 \leq 0.1917$ (**black dot**). **Sub-optimal** as liability **and** equity holder would **prefer more risk**!

$\Longrightarrow$ Solve by **flexible** regulatory intervention!



Asset process, default threshold (=guaranteed amount), regulatory threshold.



Regulator intervenes at regulatory threshold.

Flexible regulatory intervention

The **time of regulatory intervention** is denoted $\hat{\tau} < \tau$. The reference portfolio then evolves as:

$$\mathrm{d}A_t = \left(r + \theta_{Z_t}(\mu - r)\right) A_t \,\mathrm{d}t + \theta_{Z_t}\sigma A_t \,\mathrm{d}W_t, \quad A_0 > 0, \tag{5}$$

where $Z_t = 1$ for $t \leq \hat{\tau}$ and $Z_t = 2$ for $t > \hat{\tau}$.

Possible strategies:

- Strategy A (**risk reduction in distress**): $\theta_2 < \theta_1$.
- Strategy B (**constant risk strategy**): $\theta_2 = \theta_1$.
- Strategy C (**risk increase in distress**): $\theta_2 > \theta_1$.

Parameters

$A_0 = 1$, $L_0 = 0.9$, $E_0 = 0.2$.

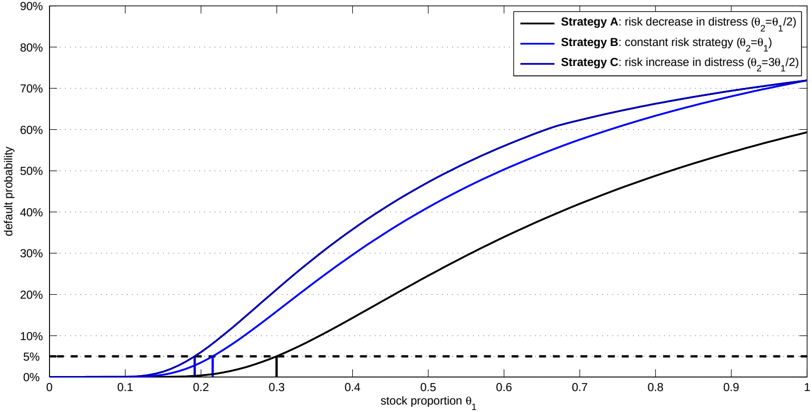
$D_0 = 0.8$ (default threshold)

$K_0 = 0.85$ (regulatory threshold)

$\mu = 6\,\%$, $r = 2.5\,\%$, $\sigma = 0.2$, $g = 1.75\,\%$.

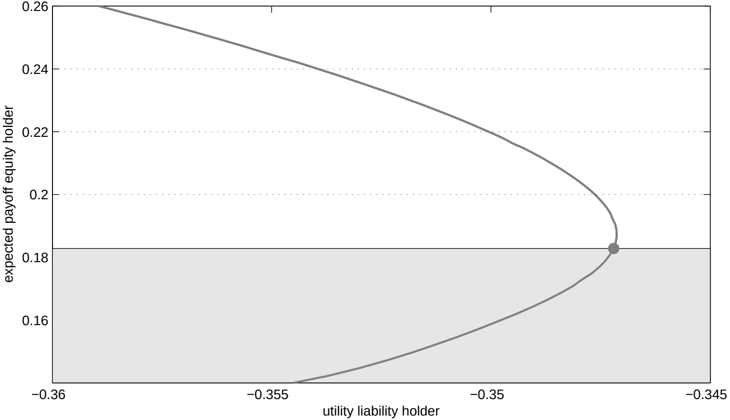
$T = 10$, $\gamma = 3$ (power utility)

Numerical results



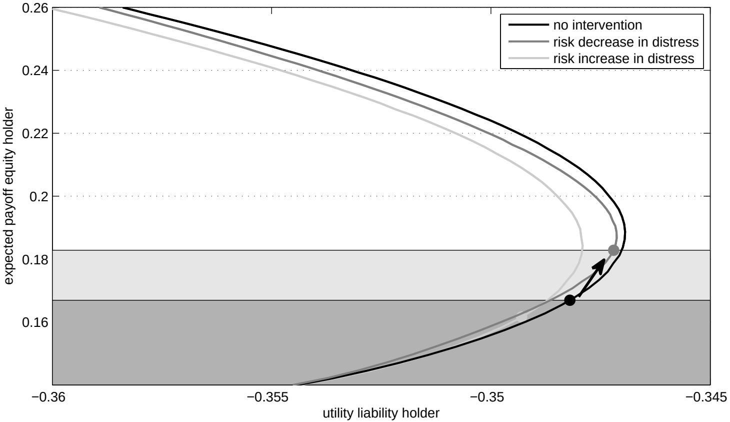
Default probability $\mathbb{P}(\tau \leq T) \leq 5\,\%$ for Strategy A, B and C.

Optimization results



Goal function of liability holder (power utility, $\gamma = 3$) and expected payoff equity holder for different risky shares θ_1 (**risk reduction if regulatory barrier is hit**). Grey area has $\mathbb{P}(\tau \leq T) \leq 5\%$.

Optimization results



Utility improvement for **both** liability holder and equity holder for constrained optimization if **risk is reduced** in distress (**black arrow**).

Model Setup and Conflict of Interests

Flexible Regulatory Intervention

Outlook

Next steps

- **Up to now**: Rather simple strategy, asset allocation is only changed once whenever the regulatory barrier is hit (**stop-loss type** strategy).
- What happens if asset allocation is changed more frequently, i.e. if a **portfolio insurance strategy is implemented (CPPI, OBPI)**?

Conclusion / further research

- Regulatory supervision $\longrightarrow$ **sub-optimal investment** decision.
- **Flexible** regulatory supervision might **increase benefits** for equity + liability holder.
- **Portfolio insurance strategies** like CPPI might be even more promising.

Literature

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