Approach to recalibrating undertaking-specific shocks parameters for the risks of incidence, recovery or non-recovery in health insurance within the framework of Solvency II

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Abstract

This article presents an operational framework for recalibrating *Undertaking Specific Parameters* for the risks of incapacity/disability incidence and non-recovery (or recovery) in a health insurance portfolio within the framework of Solvency II.

We propose an approach to calculating shocks for each of these risks, factoring in both the estimation error and the variance *process*, considered the main sources of volatility of the risks in question.

One of the difficulties in applying these methodologies (time series or Log-Normal distribution) is the availability of a sufficient volume of historical data. The data must also meet the quality criteria (completeness, accuracy, relevance) recommended by the Solvency II standard. To overcome this problem of historical representativeness, an approach based on the theory of credibility is proposed.

Like non-life USPs, this approach can serve as a basis for thinking of ways to implement healthcare USPs within the framework of the Solvency II standard for risk benefits. This approach could also be used to estimate undertaking-specific shocks for ORSA type calculations (*Own Risk and Solvency Assessment*), and provide sensitivity parameters for business plans or pricing.

Lastly, to illustrate our approach, we give an example of application of these USPs to incapacity/disability guarantee of a borrower portfolio.

Keywords: USP, risks, healthcare insurance portfolio, sick leave, incapacity, disability, incidence, keeping in force, recovery, Solvency II, *best estimate* provision, shocks, solvency capital requirement, capital requirements, time series, Log-normal, variance, estimator, Kaplan Meier, actuarial, coefficient of variation, error of *process* and estimation, credibility theory.

I. Introduction

From 2016, insurers will have to apply the standard Solvency II formula to assess their solvency capital requirements, as the said standard has been enacted in European law and implementing legislation.

Like non-life USPs, we propose a USP recalibration approach for a healthcare portfolio. The shocks on which solvency capital requirement (SCR) calculations are based cannot be modified without adopting the restrictive framework of an internal model.

So, for companies having sufficient experience and-data of quality, for non-life risks the texts recommend the use of alternative USP (*Undertaking Specific Parameters*) calibration methods, see 7. of article 104 of the Solvency II directive and section SCR.10 of the QIS5 technical specifications). This incorporates portfolio experience in SCR calculation parameters for:

- the *premium risk* and *reserve risk* of non-life insurance modules and health modules not similar to life,
- the revision risk of life and health insurance modules similar to life,
- the adjustment factor with regard to non-proportional reinsurance.

Our article presents a calibration methodology for shocks associated with SCR calculations of incapacity risks (incapacity or disability) in the Similar to Life Health (SLT Health) module. Indeed it appears that the SLT Health covers a diverse range of situations (group/individual contract, with/without selection on entry, borrower/healthcare, very variable length of commitment). Therefore the fixed-rate shock parameters as proposed by the standard formula appears unsuitable to capture the economic reality of the underlying risks.

To determine the levels of *entity specific* shock with our approach, the necessary data must meet the three requisite quality criteria (exhaustiveness, accuracy, relevance).

The proposed methods concern the risks of incidence and non-recovery (or recovery) of health insurance contracts. The **risk of incidence** corresponds to the uncertainty about the likelihood of becoming incapacitated (or disabled), and the **risk of non-recovery (**or recovery) if the uncertainty about the length of time of incapacity (or disability).

We propose an approach to calculating shocks for each of these risks, factoring in both the estimation error and the variance *process*, considered the main sources of volatility of the risks in question.

The estimation error is the difference between the estimated law and the intrinsic law, and which more particularly reflects sampling fluctuations.

The variance *process* or **process error** is the fluctuation of claims rates laws over time due to endogenous or exogenous conditions. We can note the following sources of changes:

- internal underwriting and medical selection conditions,
- claims compensation conditions,
- the general state of health,
- external and statutory rules, inasmuch as the definition of benefits and the insurer's intervention is aligned with state recognition of the policyholder's incapacity by an external body like the Social Security in France,
- the structure of the insured population (portfolio transfers, economic effects, change in the distribution target, specific marketing campaign, etc.).

We can note that the proposed method applies both to the risk of incidence and to the risk of non-recovery (or recovery) and is not called into question by the specifics of each of these risk. For instance, if we take the risk of non-recovery, even though the laws of non-recovery (incapacity and disability) have common characteristics, the fact remains that they each have their specifics. As well as differences in duration and frequency, the law clearly defines the state of disability and incapacity, in varying degrees and with a very precise classification.

So in general, whether for the risk of incidence or that of non-recovery, the proposed approach is based on an estimation of the standard deviations associated with the two sources of error (estimation and *process*). It relies on the respective **modelling of trends in incidence rates** (non-recovery or recovery rates) for the **error** of ***process*** and on determination of the **variance of the estimator** used to calculate **incidence rates** (non-recovery or recovery rates) for the **estimation error**.

The models presented in this article give a projection of the time tendency of incidence rates (non-recovery or recovery rates) over one year (t+1) then deduce from that the expected value, variance and coefficient of variation associated with this distribution. The **marginal shock for the error of *process*** is arrived at by multiplying the **coefficient of variation** estimated at t+1 by the **incidence rate** (non-recovery or recovery rate) calculated at t.

The **marginal shock corresponding to the estimation error** is defined as being the **variance of the estimator** of incidence rates (non-recovery or recovery rates) estimated at date t.

Assuming that the estimation errors can be assumed to be de-correlated from one another and from any other contingency, the **overall shock associated with the risk of incidence** (non-recovery or recovery) is arrived at by **aggregating the shocks of *process* error and estimation error**.

Finally, in a similar way to the standard formula, the proposed approach is based on applying a combined shock of incidence and non-recovery (or recovery).

In the table below we present the levels of shocks proposed by the standard formula of the Solvency II standard.

Summary table of shocks in the Standard formula:

Risk	Standard formula	USP proposal
Incidence of incapacity or disability	Year of projection t = 1: 35% Year of projection t > 1: 25%	$\Delta_{(Inc,s,USP)}$
Incapacity or disability non-recovery	If rate of non-recovery < 50% : a 20% increase in the rates of non-recovery If rate of non-recovery > 50% : a 20% increase in the rates of recovery	$\Delta_{(recov,s,USP)}$

To illustrate our approach, we give an example of these USPs applied to incapacity/disability guarantee of a borrower portfolio. This approach could also be used to calculate shocks in ORSA type calculations (Own Risk and Solvency Assessment), and provide sensitivity parameters for business plans or premium pricing.

II. Notation and assumption

We admit that neither the law of incidence nor the law of non-recovery in our approach makes a distinction between incapacity and disability. Without impairing the general nature of the model, this assumption lets us specifically disregard a law of transition rates from incapacity to disability. Therefore our article does not deal with the calibration of shocks associated with these laws of transition.

We thus consider the following three different causes of cessation of these states: recovery, death, and the contractual end of the guarantee (or cover).

Throughout this document, the term "segment" refers to the granularity of shock calibration. In general this is the LoB (*Line Of Business*). However it appears that the "SLT Health" LoB encompasses a wide variety of insurance guarantees and conditions. These differences generate very heterogeneous underlying risks. One could therefore envisage considering sub-segments of this LoB in order to model the specific levels of error. This presupposes that the risk profile is considered consistent in each sub-segment.

Furthermore, insofar as we can demonstrate with statistical surveys and decisions of loss adjusters that the risk trend is consistent between two sub-segments, the *process* error calibrated on one of the sub-segments can be used to deduce that of the other sub-segment by means of a link function. In practice this scenario arises when there is not enough history or data for a given sub-segment. This precaution is needed to preserve a degree of estimation robustness for the parameter.

If levels of volatility were calibrated on sub-segments of the LoB, we propose adopting a totally co-monotonic approach in terms of aggregation. So we do not deal with the possible effects of diversification between sub-segments.

For a LoB noted s (or a sub-segment), we analyse changes in the history of the selected observation period:

- incidence rates (incapacity or disability) noted $i(x, s, t)$ according to age x (or age bracket) and the period of estimation t,
- non-recovery rates (respectively recovery rates) note $m(x, a, s, t)$ (respectively $r(x, a, s, t)$) according to age x (or age bracket) on entry into the state, lifetime a and period t .

From a practical viewpoint, appropriate statistical tests should be run (sign tests for instance) to check that estimation errors can be considered de-correlated. We also assume that the estimation errors are de-correlated from any other sources of error, in particular the *process* error.

Lastly, the choice of the time series to adopt depends rigorously on the results of tests run on the portfolio's data. Among the ARMA (*AutoRegressive Moving-Average*) processes tested on the data of the reference portfolio, process AR(1) appears to be the most prudent one. Indeed, of all the ARMA(p,q) processes that minimize the Akaïke information criterion (AIC), AR(1) is the process that proposed the most conservative levels of shock. Tests of goodness-of-fit of the model based on the “white noise” nature of the residuals have moreover confirmed this choice. So in the rest of our paper we adopt process AR(1) as the reference model.

III. The methods

A Modelling the shocks of laws of incidence

With data meeting the aforementioned criterion of historical length, for each date $t\epsilon[t_{min}; t_{max}]$ we can calibrate laws of incidence of experience in periods $[t - T; t]$ (where $T \geq 1$ is the amplitude of estimation windows). For any given x we thus get this series of experience-based incidence rates $(i(x, s, t))_t$.

As the risk of incidence rates rises, in what follows we present the methodologies for having shocks of incidence at quantile 99.5% by incorporating the two types of error described earlier.

1. Modelling the estimation error

Depending on the structure and precision of available data, several estimators are available (Kaplan Meier, actuarial, binomial, moments of Hoem, partial quotient), each having its own limitations. These limitations are presented by F. Planchet and P. Therond (see "Duration models, Actuarial applications").

In what follows, the incidence rate estimator we use is the actuarial estimator introduced by Bömer in 1912 in an article entitled "Theorie der unabhängigen Wahrscheinlichkeiten". This estimator uses aggregated data and factors in censoring and truncation. It is easier to implement than the Kaplan Meier estimator.

Furthermore, censoring and truncation are assumed to be independent and uniformly distributed within the interval. We assume that the number of occurrences of entry into incapacity/disability at age x follows a binomial of parameters: the number of able individuals aged x at the beginning of the period and the corresponding incidence rate.

Lastly, while this estimator is suitable for large samples, the use of aggregated data leads one to speculate about its fitness for individual data and its precision.

a. Estimation of incidence rates by the actuarial estimator

In this paragraph, we recall the actuarial estimator formula used to calculate the so-called central incidence rate $i(x, s, t)$.

The incidence rate formula on segment s, at age or age bracket x and date t (year or year bracket) is as follows:

$$i(x, s, t) = \frac{\mathrm{Inc}(x, s, t)}{\mathrm{Er}(x, s, t)}$$

Where: $\mathrm{Inc}(x, s, t)$ is the number of individuals entering incapacity/disability observed for segment s, individuals of age x during year t.

Factor $\mathrm{Er}(x, s, t)$ is the risk exposure for segment s of individuals aged x during on date t.

b. *Estimating the estimation error*

The estimation error is estimated for segment s by determining the variance of the estimator used to calibrate central incidence rate $i(x,s,t)$. We note the standard deviation associated with this error as $\sigma_{\text{estim,s}}$.

If incidence rates are calibrated using the actuarial estimator, the estimation error is the variance of the actuarial estimator.

In other words, on date t_{max}, and assuming that the number of occurrences of entry into incapacity/disability obeys the following binomial distribution law:

$$\sum_{t\,=\,t_{max}-T+\,1}^{t_{max}} \text{Inc}(x,s,t) \sim Bin\big(\text{Er}(x,s,t_{max}), i(x,s,t_{max})\big)$$

We thus have: $\sigma^2_{estim,s} = \frac{i(x,s,t_{max})\times[1-i(x,s,t_{max})]}{\text{Er}(x,s,t_{max})}$

2. Modelling the process error

We want to capture the *process* error. To do so, in certain cases we need to form groupings with an aggregated granularity s' containing enough individuals to minimize the estimation error in the incidence rate calculation. For instance, this could be calculated for all sexes, grouping together age brackets and portfolios of the segment, provided the resulting sub-segment comprises homogeneous risks.

With the incidence rates $i(x,s,t)$ calculated above, we can for instance deduce aggregated incidence rates for granularity s' noted $\text{i}_{(s',t)}$ by calculating a weighted average of mean exposures (which are calculated for the entire calibration history).

a. *Approach by AR(1) modelling*

This methodology consists in tracking laws of experience incidence in order to study trends in the risk over time. Let's take the time series of experience-based incidence change rates $\left(W_{(s',t)}\right)_t$ for a given sub-segment s' of segment s, where $W_{(s',t)}$ is noted as follows:

$$\text{W}_{(\text{s}',\text{t})} = \frac{\text{i}_{(\text{s}',\text{t}+\,1)} - \text{i}_{(\text{s}',\text{t})}}{\text{i}_{(\text{s}',\text{t})}}$$

This variable is modelled by an autoregressive process of order 1 (AR(1)), the mathematical formula of which is:

$$W_{(s',t)} = \alpha_{(s')} + \beta_{(s')} \times W_{(s',t-1)} + \sigma_{(s')} \times \varepsilon_{(s',t)}$$

Where $\varepsilon_{(s',t)}$ is distributed according to standard normal distribution (noted $\text{N}(0,1)$).

Coefficients $\alpha_{(s')}$ and $\beta_{(s')}$ are estimated using the ordinary least squares estimator (or weighted by the exposures).

Factor $\sigma_{(s')}$ is the empirical standard deviation of the residuals. These residuals noted $r_{(s',t)}$ are obtained by the difference between the theoretical conditional expectations of variables $W_{(s',t)}$ and their realizations:

$$r_{(s',t)} = W_{(s',t)} - E\left\{W_{(s',t)} \middle| W_{(s',t-1)}\right\}$$

Thus:

$$r_{(s',t)} = W_{(s',t)} - \left(\alpha_{(s')} + \beta_{(s')} \times W_{(s',t-1)}\right)$$

Considering this modelling, we obtain the following relationship:

$$i_{(s',t_{\max}+1)} = i_{(s',t_{\max})} \times (1 + W_{(s',t_{\max})})$$

Thus:

$$i_{(s',t_{\max}+1)} = i_{(s',t_{\max})} \times \left(1 + \alpha_{(s')} + \beta_{(s')} \times W_{(s',t_{\max}-1)} + \sigma_{(s')} \times \varepsilon_{(s',t_{\max})}\right)$$

Where: $\varepsilon_{(s',t_{\max})} \sim N(0,1)$

From that we infer:

$$\mathrm{var}\left\{i_{(s',t_{\max}+1)}\right\} = {i_{(s',t_{\max})}}^2 \times {\sigma_{(s')}}^2$$

And:

$$E\left\{i_{(s',t_{\max}+1)}\right\} = i_{(s',t_{\max})} \times (1 + \alpha_{(s')} + \beta_{(s')} \times W_{(s',t_{\max}-1)})$$

With: $\mathrm{var}\{X\}$ variance of random variable X and $E\{X\}$ the expected value of random variable X.

The coefficient of variation associated with process error $CV_{(s')}$ is defined as follows:

$$CV_{(s')} = \frac{\sqrt{\mathrm{var}\left\{i_{(s',t_{\max}+1)}\right\}}}{E\left\{i_{(s',t_{\max}+1)}\right\}}$$

Hence:

$$CV_{(s')} = \frac{i_{(s',t_{\max})} \times \sigma_{(s')}}{i_{(s',t_{\max})} \times (1 + \alpha_{(s')} + \beta_{(s')} \times W_{(s',t_{\max}-1)})}$$

With this coefficient of variation, we can estimate on segment s and date t_{max}, the standard deviation associated with the *process* error on the incapacity/disability incidence rate $i(x, s, t_{max})$ as follows:

$$\sigma_{(process,s)} = CV_{(s')} \times i(x, s, t_{max})$$

a. Approach by log-normal modelling

The second proposed approach consists in measuring the *process* error of the risk of incidence with log-normal modelling of the incidence rate series. In this instance, for a given segment s' of segment s, we assume that the rate of incidence obeys the following logic:

$$\ln\left(\frac{i_{(s',t)}}{i_{(s',t-1)}}\right) = \alpha_{(s')} + \sigma_{(s')} \times \varepsilon_{(s',t)}$$

Where $\varepsilon_{(s',t)}$ is distributed according to standard normal distribution (noted $\mathrm{N}(0,1)$) and coefficient $\alpha_{(s')}$is estimated with the estimator of ordinary least squares (or weighted by the exposures).

Factor $\sigma_{(s')}$ is the empirical standard deviation of the residuals. These residuals $r_{(s',t)}$ are then calculated by taking the difference between the theoretical expected values (conditional) of:

$$\ln\left(\frac{i_{(s',t)}}{i_{(s',t-1)}}\right)$$

and their actual values:

$$r_{(s',t)} = \ln\left(\frac{i_{(s',t)}}{i_{(s',t-1)}}\right) - E\left[\ln\left(\frac{i_{(s',t)}}{i_{(s',t-1)}}\right)\middle|\ln\left(\frac{i_{(s',t-1)}}{i_{(s',t-2)}}\right)\right]$$

Thus:

$$r_{(s',t)} = \ln\left(\frac{i_{(s',t)}}{i_{(s',t-1)}}\right) - \alpha_{(s')}$$

Considering this modelling, we obtain the following relationship:

$$i_{(s',t_{\max}+1)} = i_{(s',t_{\max})} \times \exp(\alpha_{(s')} + \sigma_{(s')} \times \varepsilon_{(s',t_{\max})})$$

Where: $\varepsilon_{(s',t_{\max})} \sim \mathrm{N}(0,1)$ and exp: represents the exponential function.

From that we infer:

$$\mathrm{var}\left\{i_{(s',t_{\max}+1)}\right\} = i_{(s',t_{\max})}{}^2 \times \left[\exp(\sigma_{(s')}{}^2) - 1\right] \times \exp(2 \times \alpha_{(s')} + \sigma_{(s')}{}^2)$$

And:

$$E\left\{i_{(s',t_{\max}+1)}\right\} = i_{(s',t_{\max})} \times \exp(\alpha_{(s')} + \frac{\sigma_{(s')}{}^2}{2})$$

The coefficient of variation associated with *process* error $CV_{(s')}$ is defined as follows:

$$CV_{(s')} = \frac{\sqrt{\mathrm{var}\left\{i_{(s',t_{\max}+1)}\right\}}}{E\left\{i_{(s',t_{\max}+1)}\right\}}$$

hence:

$$CV_{(s')} = \frac{i_{(s',t_{max})} \times \sqrt{\left[\exp(\sigma_{(s')}^2) - 1\right] \times \exp(2 \times \alpha_{(s')} + \sigma_{(s')}^2)}}{i_{(s',t_{max})} \times \exp(\alpha_{(s')} + \frac{\sigma_{(s')}^2}{2})} = \sqrt{\exp(\sigma_{(s')}^2) - 1}$$

With this coefficient of variation, we can estimate on segment s and date t_{max}, the standard deviation associated with the *process* error on the incapacity/disability incidence rates $i(x, s, t_{max})$ as follows:

$$\sigma_{(\text{process,s})} = \text{CV}_{(\text{s}')} \times i(x, s, t_{max})$$

3. Level of incidence shock formula

Assuming non-correlation of errors of estimate and process, the proposed level of shock is defined by the following formula:

$$\Delta_{(inc,s,USP)} = \frac{\sqrt{\sigma_{(process,s)}^2 + \sigma_{(estim,s)}^2} \times Quantile_{99,5\%}(\varepsilon)}{i_{(s,t_{max})}}$$

Where: $i_{(s,t_{\max})}$: represents the rate of incapacity/disability incidence estimated on the most recent calibration history ending at $\text{t}_{\max}$ for segment s,
$Quantile_{99,5\%}(\varepsilon)$: represents the quantile at 99.5% of $\varepsilon \sim \text{N}(0,1)$.

B Modelling the shocks of non-recovery or recovery laws

We use the Kaplan Meier estimator for non-recovery rates (or recovery rates). This non-parametric estimator is used to approach the empirical form taken by the risk of exit from the state without adopting any law specification. This estimator requires one to know exactly all the dates of entry into and exit from the portfolio, including the dates of entry into incapacity/disability. This may be a drawback if the data file is unreliable.

Finally, to factor in the effect of the individual characteristics of the population studied with this method, the population must first be broken down into sub-groups that are sufficiently homogeneous with regard to these characteristics. The issues involved in implementing this model are presented in the book by Frédéric Planchet and Pierre Therond entitled “Duration models, Actuarial applications”.

In what follows, we describe only the methodology that lets one have shocks of recovery from incapacity/disability by incorporating the two requisite levels of error. As the risk of recovery rates diminishes, in what follows we present methodologies for obtaining shocks of recovery at quantile 0.5%.

Trivially, the same process applies to the calibration of shocks of non-recovery. As the risk in that case is a rise in non-recovery rates, the shocks of non-recoverys correspond to the quantile level at 99.5%.

1. Estimation of recovery rates

The duration variable of the Kaplan Meier survival function to estimate conditional recovery rates is lifetime. The mathematical formula of this estimator on segment s, for lifetime a in incapacity/disability and at date t_{max} is:

$$R_{(a,s,t_{\max})} = 1 - \frac{S_{(a,s,t_{\max})}}{S_{(a-1,s,t_{\max})}}$$

Where: $S_{(a-1,s,t_{\max})}$ and $S_{(a,s,t_{\max})}$ represent the incapacity/disability non-recovery rates estimated with the Kaplan Meier method as follows:

$$S_{(a,s,t_{\max})} = \prod_{\substack{i \\ \gamma_i \le a}} \left(1 - \frac{d_{i,x}}{n_{i,x}}\right)$$

Where:

$n_{i,x}$: represents the number of policyholders in incapacity/disability who have spent more than γ_i days in incapacity/disability for segment s,

$d_{i-1,x}$ represents the number of recoveries occurring between γ_{i-1} and γ_i days' incapacity/disability for segment s,

$n_{i,x}$: represents the risk exposure incorporating censored and truncated observations.

2. Estimating the estimation error

When Kaplan Meier is used to estimate recovery rates $R_{(a,s,t_{\max})}$, the associated estimation can be arrived at using the Greenwood variance estimator. The mathematical formula of this estimator is:

$$\sigma_{(estim,t_{\max})}{}^2 = (1 - R_{(a,s,t_{\max})})^2 \times \sum_{\substack{i \\ a < \gamma_i \le a+1}} \frac{d_{i,x,t_{\max}}}{n_{i,x,t_{\max}} \times \left(n_{i,x,t_{\max}} - d_{i,x,t_{\max}}\right)}$$

3. Modelling the *process* error

The methodology is based on monitoring the variation in recovery rates over time in order to study the trend of this risk. With a sufficiently long data history one can construct experience-based laws for each year t (calibrated on periods $[t-T;t]$ where $T \ge 1$ is the amplitude of the estimation windows). We thus obtain the series of incapacity/disability recovery rates $\left(R_{(s,t)}\right)_t$.

We suggest considering the time series of experience-based recovery change rates $\left(Z_{(s',t)}\right)_t$ for a given segment s', where $Z_{(s',t)}$ is defined as follows:

$$Z_{(s',t)} = \frac{R_{(s',t+1)} - R_{(s',t)}}{R_{(s',t)}}$$

We model this variable with an autoregressive process of order 1, the mathematical formula of which is:

$$Z_{(s',t)} = \mu_{(s')} + \alpha_{(s')} \times Z_{(s',t-1)} + \sigma_{(s')} \times \varepsilon_{(s',t)}$$

Where: $\varepsilon_{(s',t)} \sim N(0,1)$,

Coefficients $\mu_{(s')}$ and $\alpha_{(s')}$ are estimated using the ordinary least squares estimator (or weighted by the exposures).

Residuals $r_{(s',t)}$ are obtained by the difference between the theoretical conditional expectations of variables $Z_{(s',t)}$ and their realizations:

$$r_{(s',t)} = Z_{(s',t)} - E\left\{Z_{(s',t)} \middle| Z_{(s',t-1)}\right\}$$

Thus:

$$r_{(s',t)} = Z_{(s',t)} - \left(\mu_{(s')} + \alpha_{(s')} \times Z_{(s',t-1)}\right)$$

Where: $\sigma_{(s')}$ is the empirical standard deviation of the residuals.

Considering this modelling, we obtain the following relationship:

$$R_{(s',t_{\max}+1)} = R_{(s',t_{\max})} \times \left(1 + Z_{(s',t_{\max})}\right)$$

Thus:

$$R_{(s',t_{\max}+1)} = R_{(s',t_{\max})} \times \left(1 + \mu_{(s')} + \alpha_{(s')} \times Z_{(s',t_{\max}-1)} + \sigma_{(s')} \times \varepsilon_{(s',t_{\max})}\right)$$

Where: $\varepsilon_{(s',t_{\max})} \sim N(0,1)$

From that we infer:

$$\operatorname{var}\left\{R_{(s',t_{\max}+1)}\right\} = {R_{(s',t_{\max})}}^2 \times {\sigma_{(s')}}^2$$

And:

$$E\left\{R_{(s',t_{\max}+1)}\right\} = R_{(s',t_{\max})} \times (1 + \mu_{(s')} + \alpha_{(s')} \times Z_{(s',t_{\max}-1)})$$

The coefficient of variation associated with process error $CV_{(s')}$ is defined as follows:

$$CV_{(s')} = \frac{\sqrt{\operatorname{var}\left\{R_{(s',t_{\max}+1)}\right\}}}{E\left\{R_{(s',t_{\max}+1)}\right\}}$$

Hence:

$$CV_{(s')} = \frac{R_{(s',t_{\max})} \times \sigma_{(s')}}{R_{(s',t_{\max})} \times (1 + \mu_{(s')} + \alpha_{(s')} \times Z_{(s',t_{\max}-1)})}$$

The standard deviation associated with the process error on the incapacity/disability recovery rate $R_{(s,t_{\max})}$ is calculated as follows:

$$\sigma_{(process,s)} = CV_{(s')} \times R_{(s,t_{\max})}$$

4. Formula for the level of shock of recovery

Assuming non-correlation of errors of estimate and process, the proposed level of shock is defined by the following formula:

$$\Delta_{(recov,s,USP)} = \frac{\sqrt{\sigma_{(process,s)}^{2} + \sigma_{(estim,s)}^{2}} \times Quantile_{0,5\%}(\varepsilon)}{R_{(s,t_{\max})}}$$

Where: $R_{(s,t_{\max})}$ represents the rate of recovery from incapacity/disability estimated in the most recent calibration history ending at $t_{\max}$ for segment s, and $Quantile_{0,5\%}(\varepsilon)$ represents the quantile at 0.5% of a standard normal distribution ($\varepsilon \sim N(0,1)$).

C Introduction of a credibility process

We have pointed out that the pertinence of the parameters (specific to the undertaking in question) estimated using the various methods proposed in the previous paragraphs depends firstly on the quality of the data and secondly on the length of available history. So we propose a “credibility approach” to factor in undertaking-specific experience using the same logic as current non-life USPs.

From a theoretical viewpoint, the BUHLMANN theory (or credibility theory) based on the greatest accuracy credibility theory seems to be the most comprehensive one and is preferable. However, current data from most undertakings is not detailed enough to enable us to use this approach.

We propose a standardized variant of the limited fluctuation credibility theory, also known as "American credibility". The latter proposes a full credibility criterion based on the size of the portfolio. Full credibility means that only the results of the portfolio should be used and industry-wide data should be discarded. Moreover, it provides a special methodology for determining partial credibility, whereby the results of the portfolio and those of the industry as a whole are weighted.

In our framework the credibility theory is applied to factor in the parameters from both the Standard Formula and the specific parameters in the calculation of final shocks.

In this particular case, and as for current non-life USPs, we use a minimum history for each risk (incidence or non-recovery vs. recovery), defining the minimum length required for specific calibration of shocks.

With the constraint of available history length (greater than or equal to the minimum length), we assume that the final overall level of shock will be the sum of the experience parameter weighted by a credibility factor and the

standard formula parameter proposed by Solvency II weighted by a minus credibility factor. The estimation assumptions for credibility factors are inspired by the limited fluctuation credibility theory, on the strength of which we obtain the final overall shock by selecting a deviation parameter r $(r > 0)$ and a level of likelihood p $(0 < p < 1)$, with the result that the difference between the final overall shock and the parameter standard formula is small.

By applying this approach to available data for the LoB under study (with more than 15 years' history), for $r = 3\%$ and $p = 90\%$, we obtain the first credibility factors that formed the basis for business discussion and analysis. After they were submitted for expert opinion, these initial results have been refined and the credibility factor are summarized in the tables below according to the risk of incidence or non-recovery (vs. recovery) and the length of data history.

Credibility factors for years of available history: shocks of incidence											
Year	3	4	5	6	7	8	9	10	11	12	13 and +
C	20%	35%	50%	60%	70%	75%	80%	85%	90%	95%	100%

Credibility factors for years of available history: shocks de non-recovery vs. recovery											
Year	5	6	7	8	9	10	11	12	13	14	15 and +
C	30%	40%	50%	60%	70%	75%	80%	85%	90%	95%	100%

Where:
Year: number of years' history available,
C: credibility factors.

IV. Application

A Presentation of the data and the portfolio

The scope of our work comprises portfolios of loan insurance contracts of a particular insurance company distributed by several partners. These contracts offer several guarantees, including one covering policyholders against the risk of incapacity and disability.

1. Description of the study databases

The data comes from a qualified and certified data centre originating from operative management systems. From this data centre we had:

- detailed databases on individual policyholders (the length of history varying according to the partner, some less than 5 years),
- databases aggregated by policyholder age bracket (usually with a longer history),
- detailed one-to-one data on claimants with a significant length of history (more than 20 years for the oldest contract),
- a significant volume of data on policyholders and claimants.

To estimate the laws of incidence, we need databases for both the policyholders and the claimants. The best approximation of the laws is achieved with detailed databases (head by head). Even if the history is at times shallow for certain partners (in the region of 3 to 5 years), it suffices for most of the portfolios. However, the decision to use the actuarial estimator to calculate incapacity/disability incidences rates depends on the fact that

the policyholder and claimant databases are not interlinked, in the absence of a common ID key for a given policyholder.

As we pointed out earlier, the lack of historical length is another limit for calibrating shocks associated with the risk of incidence. The models proposed to evaluate estimation and variance *process* errors (see paragraph III) require us to have a data history long enough to capture fluctuations and trends in the risk over time. To overcome this limitation, we implemented a credibility-based approach.

Furthermore, to compensate for the lack of sufficient history, the proposed alternatives are:

- either to keep the parameters of the standard formula,
- or to merge this portfolio with a larger one having the same structure, as long as we can prove that the risks are homogeneous.

Lastly, we should note that the risk of non-recorvery in incapacity/disability did not pose any particular problems, either to estimate the law of non-recovery (or recovery) and to calibrate the related shocks. The modelling is done only on the claimants database, which what is more detailed and has a long history.

B Estimation of shocks of incidence

Case with 5 years' history (c = 50%):

	SHOCKS - MODELLING THE PROCESS ERROR WITH THE AR(1) APPROACH						
	Estim. sigma	**Process sigma**	**Specific shock**	**FS shock (year 1)**	**FS shock (year > 1)**	**Specific shock with credibility factor (year 1)**	**Specific shock with credibility factor (year > 1)**
20	0.07%	0.36%	49%	35%	25%	42%	37%
38	0.01%	0.02%	9%	35%	25%	22%	17%
59	0.01%	0.03%	7%	35%	25%	21%	16%

	SHOCKS - MODELLING THE PROCESS ERROR WITH THE LOG-NORMAL APPROACH						
	Estim. sigma	**Process sigma**	**Specific shock**	**FS shock (year 1)**	**FS shock (year > 1)**	**Specific shock with credibility factor (year 1)**	**Specific shock with credibility factor (year > 1)**
20	0.06%	0.38%	86%	35%	25%	61%	56%
38	0.01%	0.02%	15%	35%	25%	25%	20%
59	0.01%	0.03%	14%	35%	25%	24%	19%

Whichever method is used, we observe a big difference between the shocks resulting from undertaking-specific calibration and the shocks proposed by the Standard Formula. Unlike the Standard Formula shocks, which are constant irrespective of age, the shocks resulting from undertaking-specific calibration depend on age and have sharp fluctuations between the levels of shock applied to different ages. We also see that the levels of shock vary according to the method used. They are more conservative with the Log-normal method than with the AR(1) approach in our example.

With the AR(1) method, the levels of shock observed for younger ages (at 20 years) are the highest and higher than the shocks of the Standard Formula. If the credibility factor is applied to this case, this corrects the effect of an over-estimation of levels of shock relating to the size of the population. The final credible level of shock falls from 49% to 42% in the 1st year and to 37% thereafter. However the populations to which these shocks are applied are relatively small, which induces little impact on the SCR calculation. The specific parameters in this case are therefore still interesting.

Case with 16 years' history (c = 100%):

	SHOCKS - MODELLING THE PROCESS ERROR WITH THE AR(1) APPROACH						
	Estim. sigma	**Process sigma**	**Specific shock**	**FS shock (year 1)**	**FS shock (year > 1)**	**Specific shock with credibility factor (year 1)**	**Specific shock with credibility factor (year > 1)**
20	0.07%	0.50%	68%	35%	25%	68%	68%
38	0.01%	0.03%	12%	35%	25%	12%	12%
59	0.01%	0.07%	18%	35%	25%	18%	18%

	SHOCKS - MODELLING THE PROCESS ERROR WITH THE LOG-NORMAL APPROACH						
	Estim. sigma	**Process sigma**	**Specific shock**	**FS shock (year 1)**	**FS shock (year > 1)**	**Specific shock with credibility factor (year 1)**	**Specific shock with credibility factor (year > 1)**
20	0.06%	0.53%	111%	35%	25%	111%	111%
38	0.01%	0.03%	17%	35%	25%	17%	17%
59	0.01%	0.07%	22%	35%	25%	22%	22%

When the length of history is greater and the credibility factor is 100%, the shocks originating from undertaking-specific calibration are still more interesting. Even if we observe very high levels of specific shock for young ages (ages for which the groups are small), they are still lower than the shocks of the Standard Formula at ages at which exposure is the highest in the portfolio under study.

C Estimation of non-recovery and recovery shocks

Case with 23 years' history (c = 100%):

	Recovery shock for the first year of lifetime				
	Estim. sigma	**Process sigma**	**Specific shock**	**FS shock**	**Specific shock with credibility factor**
20-30	0.4%	0.6%	+ 2% on the non-recovery rate	+ 20% on the non-recovery rate	+ 2% on the non-recovery rate
45-50	0.4%	0.4%	+ 3% on the non-recovery rate	+ 20% on the non-recovery rate	+ 3% on the non-recovery rate
55-65	0.4%	0.3%	- 3% on the recovery rate	- 20% on the recovery rate	- 3% on the recovery rate

	Recovery shock for the third year of lifetime				
	Estim. sigma	**Process sigma**	**Specific shock**	**FS shock**	**Specific shock with credibility factor**
20-30	2.5%	1.4%	+ 15% on the non-recovery rate	+ 20% on the non-recovery rate	+ 15% on the non-recovery rate
45-50	0.8%	0.8%	- 10% on the recovery rate	- 20% on the recovery rate	- 10% on the recovery rate
55-65	0.8%	0.5%	- 13% on the recovery rate	- 20% on the recovery rate	- 13% on the recovery rate

	Recovery shock for the fifth year of lifetime				
	Estim. sigma	**Process sigma**	**Specific shock**	**FS shock**	**Specific shock with credibility factor**
20-30	3.6%	0.9%	+ 55% on the non-recovery rate	+ 20% on the non-recovery rate	+ 55% on the non-recovery rate
45-50	0.7%	0.5%	- 24% on the recovery rate	- 20% on the recovery rate	- 24% on the recovery rate
55-65	2.4%	0.9%	- 35% on the recovery rate	- 20% on the recovery rate	- 35% on the recovery rate

The first main difference with the shocks proposed by the Standard Formula is that we see sharp fluctuations between the levels of shock applied to different lifetimes and age brackets. The levels of shock observed for the longest lifetimes are higher than the shocks of the Standard Formula. However the rates to which they are applied are relatively small, which induces little impact on the SCR calculation.

Case with 7 years' history (c = 50%):

	Recovery shock for the first year of lifetime				
	Estim. sigma	**Process sigma**	**Specific shock**	**FS shock**	**Specific shock with credibility factor**
20-30	0.4%	0.8%	+ 2% on the non-recovery rate	+ 20% on the non-recovery rate	+ 11% on the non-recovery rate
45-50	0.4%	0.6%	+ 2% on the non-recovery rate	+ 20% on the non-recovery rate	+ 11% on the non-recovery rate
55-65	0.4%	0.4%	- 2% on the recovery rate	- 20% on the recovery rate	- 11% on the recovery rate

	Recovery shock for the third year of lifetime				
	Estim. sigma	**Process sigma**	**Specific shock**	**FS shock**	**Specific shock with credibility factor**
20-30	2.5%	1.1%	+ 7% on the non-recovery rate	+ 20% on the non-recovery rate	+ 14% on the non-recovery rate
45-50	0.8%	0.6%	- 3% on the recovery rate	- 20% on the recovery rate	- 11% on the recovery rate
55-65	0.8%	0.4%	- 2% on the recovery rate	- 20% on the recovery rate	- 11% on the recovery rate

	Recovery shock for the fifth year of lifetime				
	Estim. sigma	**Process sigma**	**Specific shock**	**FS shock**	**Specific shock with credibility factor**
20-30	3.6%	0.5%	+ 9% on the non-recovery rate	+ 20% on the non-recovery rate	+ 15% on the non-recovery rate
45-50	0.7%	0.3%	- 2% on the recovery rate	- 20% on the recovery rate	- 11% on the recovery rate
55-65	2.4%	0.5%	- 6% on the recovery rate	- 20% on the recovery rate	- 13% on the recovery rate

We see that the standard deviation associated with the *process* error is smaller in this case than when we consider a longer history. The small amount of data leads one to estimate an AR(1) model that better fits the observed data. The standard deviation associated with the AR(1) model that stems from this estimation is thus all the smaller.

In our example, introducing the credibility factor offsets this unwarranted effect. Overall, the levels of shock obtained with the shortest history are more severe than those obtained with a long history but are nonetheless still an alternative to the advantageous Standard Formula in the proposed illustration.

V. Conclusion

The lifetime of cases in the portfolio and their volume should enable certain firms to have very accurate modelling of the risks they cover. A thorough knowledge of these risks would be likely to protect firms by underlining potential deviations very early on. The proposed models could serve as a basis for reflection on shock levels specific to each undertaking for the risk of incidence and non-recovery (or recovery) in health insurance portfolio with regard to USPs, an internal model and ORSA calculations. They will also help accurately evaluate capital requirements and study the earning power of cases when preparing business plans or premium pricing.

Unfortunately, the business models of firms are at times such that, in the past, the data historization has only been partially addressed. Firms have often already done much to put things right. This study illustrates the benefits such action could offer and confirms the need to continue investing in this area.

Lastly, to make up for the lack of a long history, the proposed credibility approach is a good alternative and produces satisfactory results. We suggest that this approach be improved to offer more refined coefficients.

VI. Main references

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