A two-account life insurance model for scenario-based valuation including event risk

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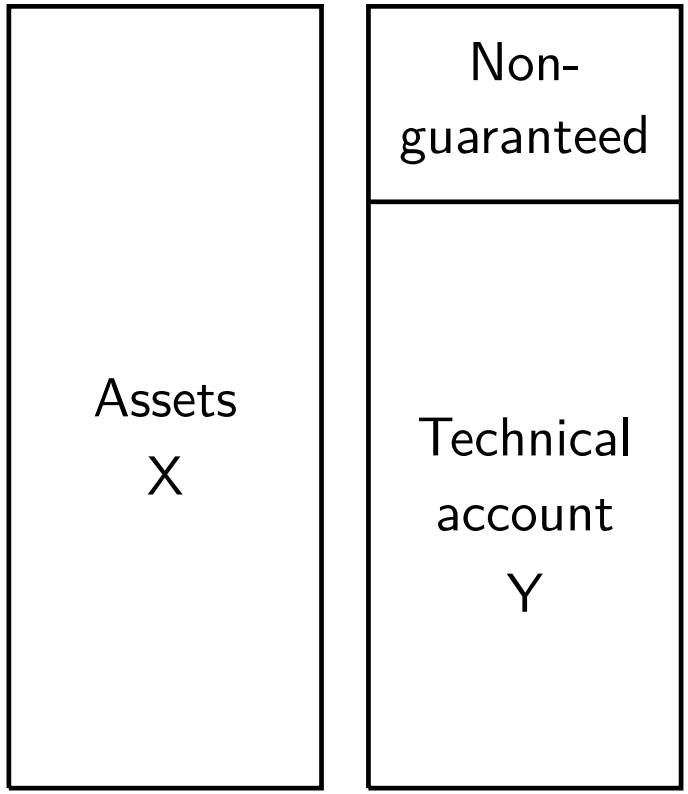
Main contributions

- Two-account model with event risk. Inspired by Steffensen and Waldstrøm (2009).
- Two interacting accounts: a technical account Y and one describing the assets X.
- Focus on valuation of non-guaranteed payments via introduction of economic scenarios.
- Common framework for valuation of participating life (PL) and unit-linked (UL) insurance policies.



Valuation in life insurance/pensions

	Participating life insurance	Unit-linked insurance
Guaranteed payments	Classical life insurance mathematics	Unit-linked guarantees
Non-guaranteed payments	Bonus	Pure unit-linked

- Payment streams

$$\mathrm{d}B^{PL}(t) = k^{(\varepsilon(t))}\,\mathrm{d}B^{u}(t) + \mathrm{d}B^{f}(t) - \mathrm{d}C(t)\ ,$$
$$\mathrm{d}B^{UL}(t) = X(t-)\,\mathrm{d}B^{p}(t) + \mathrm{d}B^{f}(t) - \mathrm{d}C(t)\ .$$

Account projections – Participating life

$$\mathrm{d}X(t) = \overbrace{X(t-)\,\mathrm{d}R_X(t)}^{\text{investment returns}} + \overbrace{\mathrm{d}\varsigma(t) - \mathrm{d}\beta^f(t) - k^{(\varepsilon(t))}\,\mathrm{d}\beta^u(t)}^{\text{expected premiums and benefits}} \text{ (upscaling factor)}$$
$$+ \underbrace{g(t)}_{\text{guarantee injection}}\,\mathrm{d}\varepsilon(t) - \underbrace{\pi_g(t)}_{\text{guarantee fee}}\,\mathrm{d}\varepsilon(t)\ ,$$

$$\mathrm{d}Y(t) = Y(t)\,\overbrace{r^{*(\varepsilon(t))}\mathrm{d}t}^{\text{technical interest rate}} + \mathrm{d}\varsigma(t) - \mathrm{d}\beta^f(t) - k^{(\varepsilon(t))}\mathrm{d}\beta^u(t)$$
$$+ \underbrace{d(t)}_{\text{bonus}}\,\mathrm{d}\varepsilon(t) + \underbrace{\alpha\left(t, r^{*(\varepsilon(t))}, k^{(\varepsilon(t))}\right)}_{\text{surplus from intensities}}\mathrm{d}t\ .$$

Additional benefits

- Upscaling factor $k^{(\varepsilon(t))}$ is determined by

$$d(t) = \left(k^{(\varepsilon(t))} - k^{(\varepsilon(t-))}\right) V^{u,*,m,+}(t-) \ ,$$

where

$$V^{u,*,m,+}(t) = \sum_{j \in \mathcal{J}} p_{0j}^{m}(0,t)\, V_j^{u,*,+}(t, r^*)$$

$$= \text{market expected technical reserve}$$

with state-wise technical reserves

$$V_j^{u,*,+}(t, r^*) = \mathbb{E}^* \left[\int_t^T e^{-r^*(s-t)} \, \mathrm{d}B^u(s) \middle| Z(t) = j \right] \ .$$

Account projections – Unit-linked

$$\mathrm{d}X(t) = \overbrace{X(t-)\,\mathrm{d}R_X(t)}^{\text{investment returns}} + \overbrace{\mathrm{d}\varsigma(t) - \mathrm{d}\beta^f(t) - X(t-)\,\mathrm{d}\beta^p(t)}^{\text{expected premiums and benefits}}$$
$$+ \underbrace{(Y(R-) - X(R-))^+}_{\text{final guarantee}}\,\mathrm{d}\varepsilon_R(t) - \underbrace{\pi_g(t)}_{\text{guarantee fee}}\,\mathrm{d}\varepsilon(t)\ ,$$

$$\mathrm{d}Y(t) = Y(t)\,\overbrace{r^*\mathrm{d}t}^{\text{technical interest rate}} + \mathrm{d}\varsigma(t) - \mathrm{d}\beta^f(t) - X(t-)\,\mathrm{d}\beta^p(t)$$
$$+ \underbrace{u(t)}_{\text{guarantee upgrade}}\,\mathrm{d}\varepsilon(t)\ ,\ t \le R\ ,$$
$$Y(t) = 0\ ,\ t > R\ .$$

Overlapping generations example – Participating life

- Two policy holders aged 25 enter 20 years apart.
- Product:
  - Term insurance of 1 upon death before T.
  - Pure endowment of 3 upon survival until T.
  - Continuous premium payment of 0.04614 while active.
- Guarantee fee is a constant fraction of the yield:

$$\pi_g = \theta_3[R_X(t)X(t-1)]^+.$$

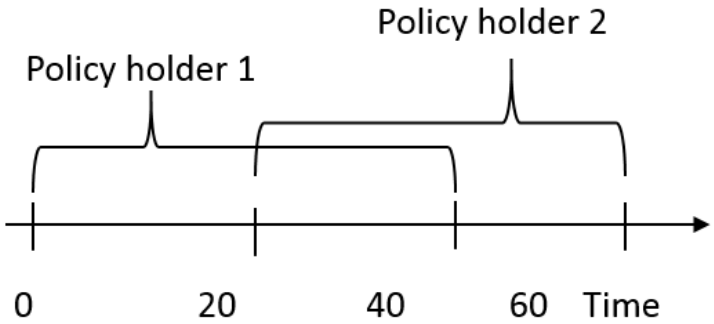


Figure: Overlapping generations.

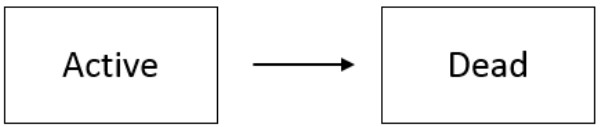


Figure: Life death model.

Overlapping generations example – continued

- Market value of portfolio is $W(0) = \sum_{i=1,2} W_i(0)$ where
$$W_i(0) = V^i(0) + \mathbb{E}^Q\left[\int_0^T e^{-\int_0^s r(v)dv}\left(k_i^{(\varepsilon(s))} - 1\right)\mathrm{d}\beta^u(s)\right].$$
- Fairness on portfolio level since $W_1(0) + W_2(0) = 0$...
- ... but unfair since $W_1(0) < 0$ and $W_2(0) > 0$.

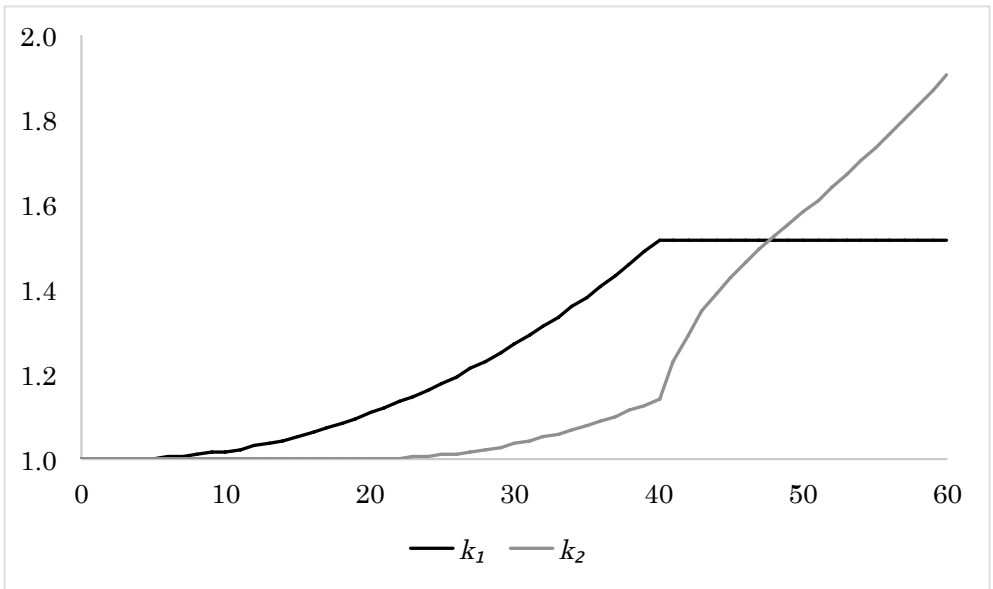


Figure: k_2 ends higher than k_1.

Single-policy example – Unit-linked

- Same death sum and premium as PL, but different guarantee.
- The size of endowment is

$$\frac{\text{"Asset value at time } R\text{"}}{\text{"Probability of surviving to time } R\text{"}} \ .$$

- The guarantee upgrade is

$$u(t) = \theta_1 \left[X(t-) - \pi_g(t) - Y(t-) \right]^+ \ .$$

- At expiration $[Y(R-) - X(R-)]^+$ is added to the assets.

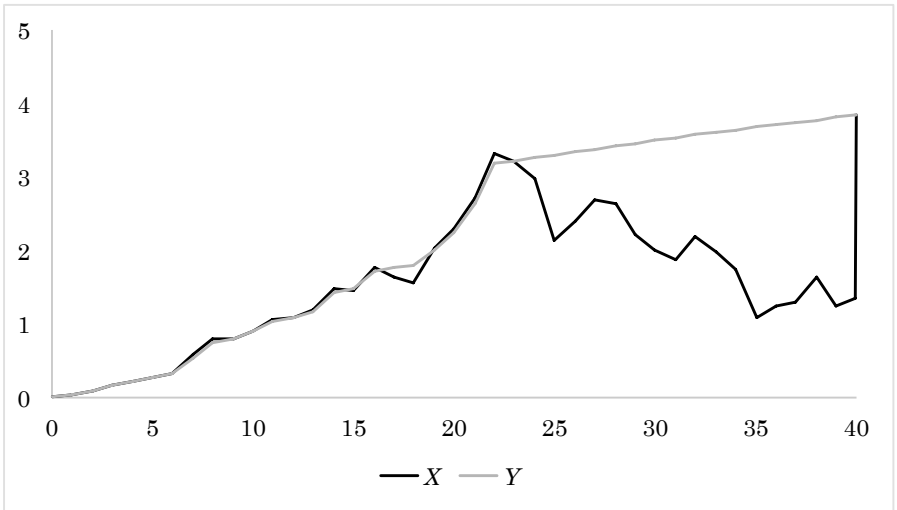


Figure: Sample paths for assets and guarantee.

Participating life vs. Unit-linked

- By construction the unit-linked and participating life product have the same average cash flow.
- However, the products differ in riskiness.

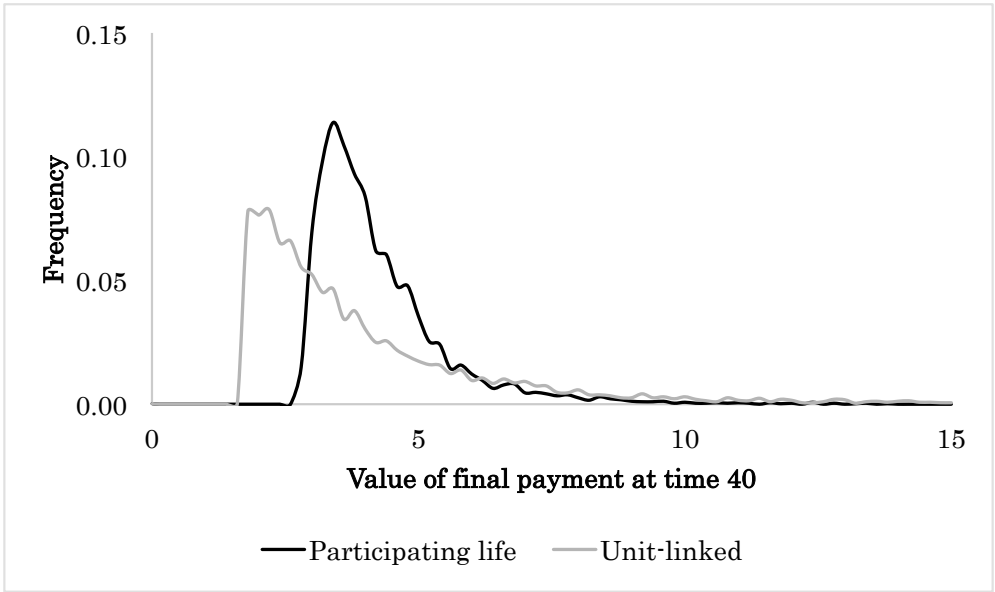


Figure: Unit-linked has bigger up- and downside.

Summing up

- Two-account model **with event risk**.
- Focus on valuation of non-guaranteed payments.
- Common valuation framework for PL and UL.

- **Questions?**

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