Risk Management of Investment Guarantees in Life Insurance

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Outline

- Investment Guarantees
- Risk management considerations
- Conditional lower bound approximation
- Asset-based charges
- Hedging

Investment Guarantees

Brief background to VA & UL business

Tax	Deferred annuities served as tax-deferred vehicle with pure risk benefits, e.g. endowment with fixed benefits.
Investment Upside	Policyholders became more aware of investment potential and demanded more investment upside exposure and investment choice.
Marketability	Guarantees were offered to transfer downside risk back to insurers and to improve the marketability of VA / UL products
Mispricing	Guarantees that were considered to be far OTM were offered at very low and sometimes without charges in early days. Risk management considerations not integral to pricing and product development.

Investment Guarantees

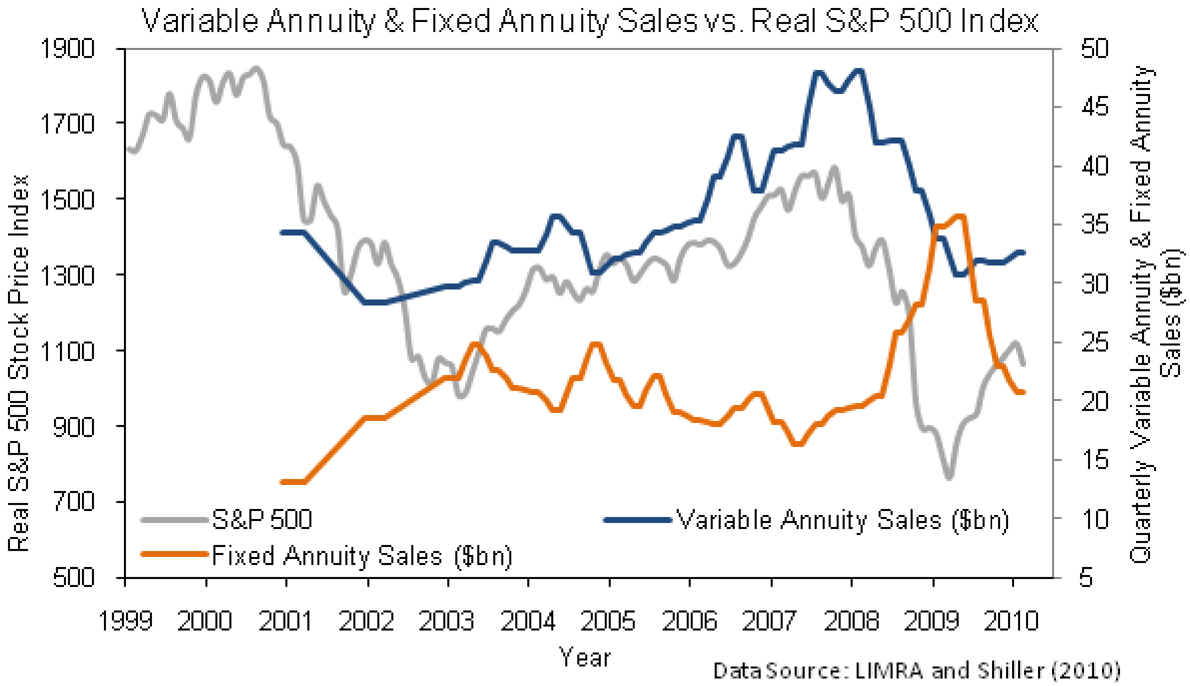
Types of Investment Guarantees

GMDB (1980)	Guarantees a minimum amount payable on death
GMIB / GAO (1996)	Guarantees a minimum rate of interest at conversion of a lump sum to an immediate annuity
GMMB / GMAB (2002)	Guarantees a minimum amount payable at maturity or at fixed intervals over the policy term
GMWB (2002)	Guarantees withdrawals to at least the value of the original premium over a specified term
GLWB (2004)	Guarantees a percentage withdrawal over the remaining lifetime of the policyholder

Investment Guarantees

Policyholder preferences and the market

- VA sales positively correlated, while fixed annuity sales negatively correlated
- Policyholders buy VAs at market highs and fixed annuities at market lows

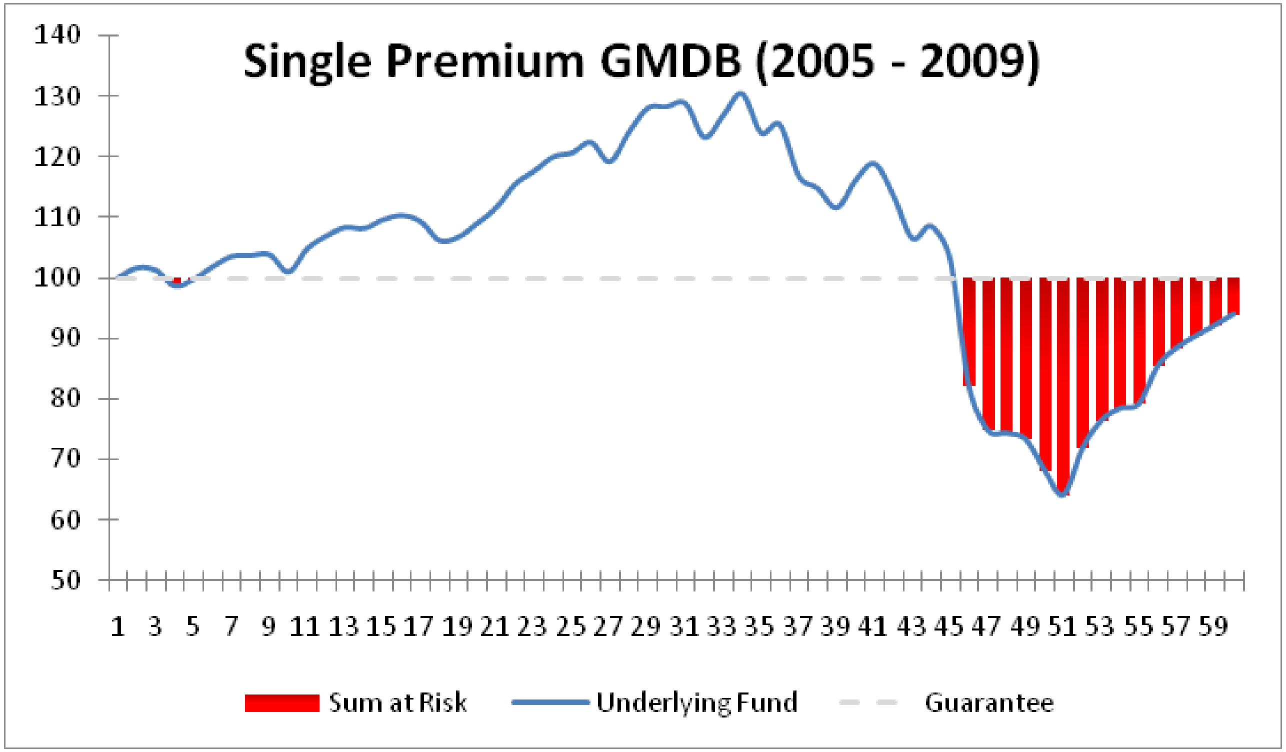


Data Source: LIMRA and Shiller (2010)

Investment Guarantees

S&P 500 Examples: Single Premium GMDB

- GMDB guarantees ROC - single premium of 100 over 5 years
- No tax, fees or other deductions considered



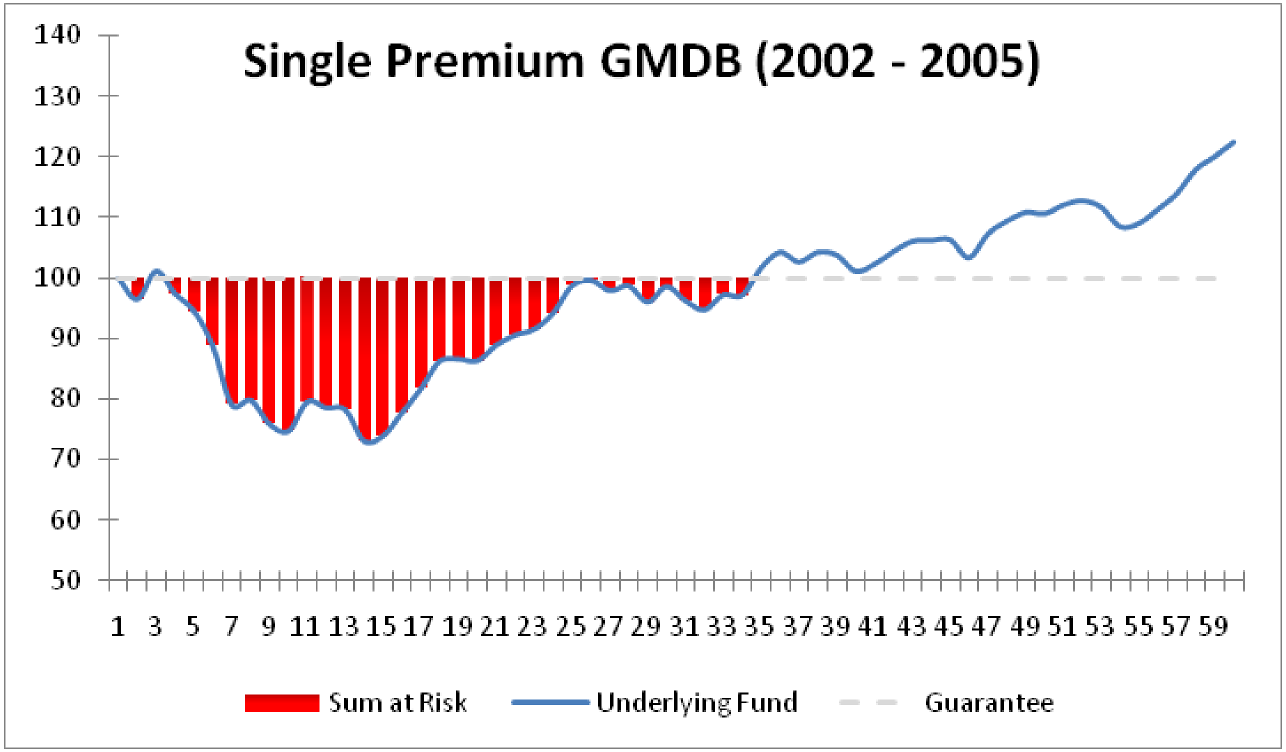
Investment Guarantees

Cape Town Afrique du Sud

iCA 2010 CiA

S&P 500 Examples: Single Premium GMDB

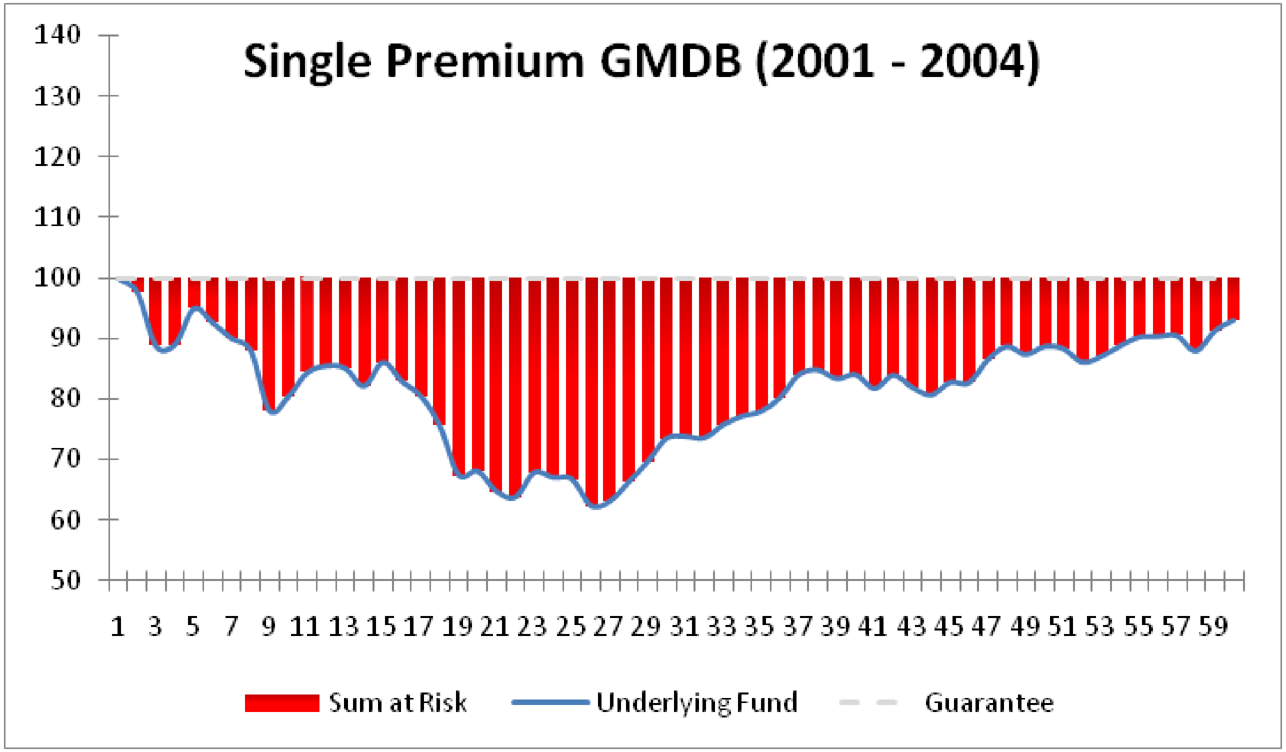
- Guarantee very sensitive to initial value of the underlying fund
- Phasing-in of single premiums can mitigate risk



Investment Guarantees

S&P 500 Examples: Single Premium GMDB

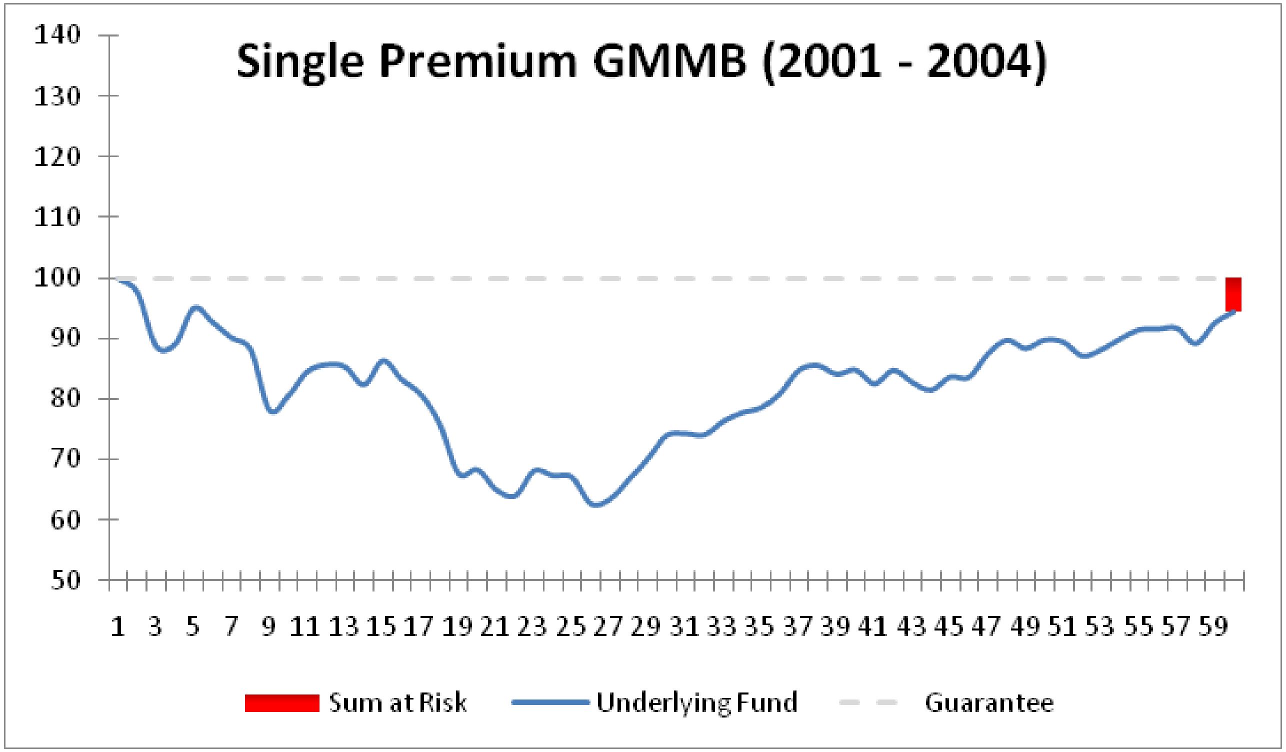
- One year difference (one year earlier) can lead to drastically different results!



Investment Guarantees

S&P 500 Examples: Single Premium GMMB

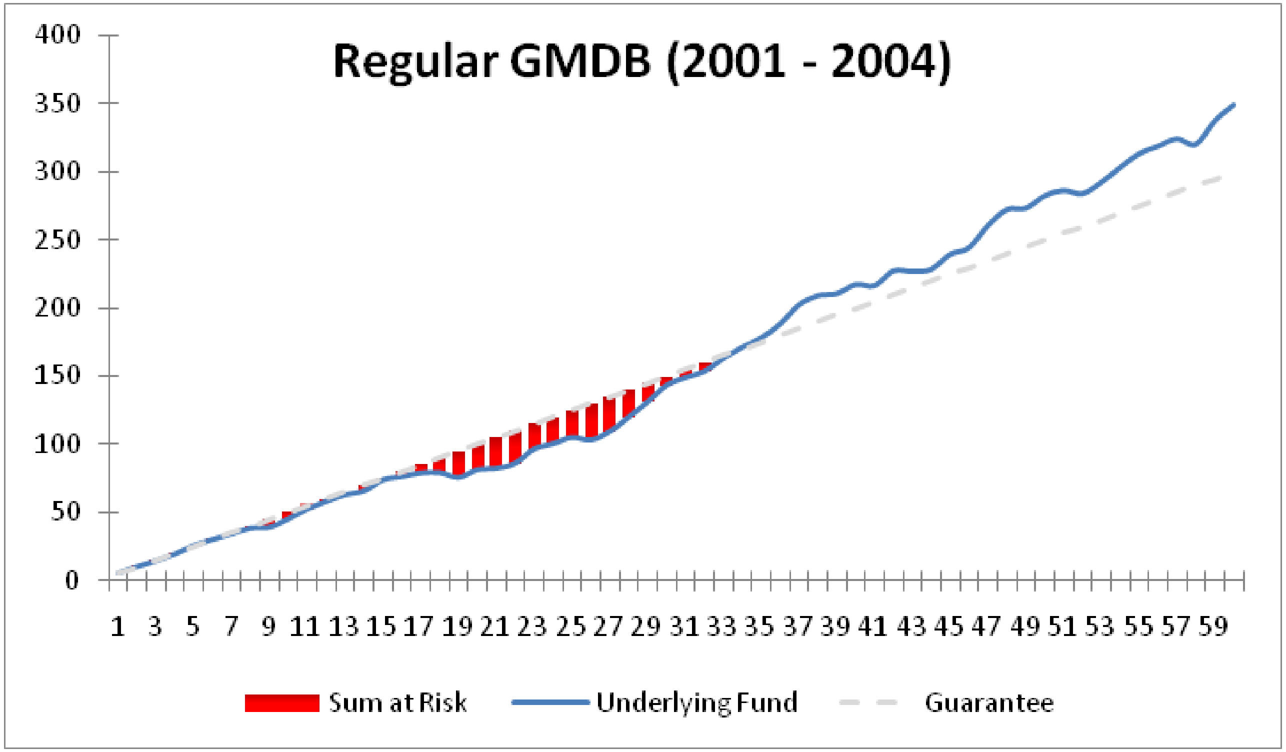
- GMMB guarantees ROC – single premium after 5 years



Investment Guarantees

S&P 500 Examples: Regular Premium GMDB

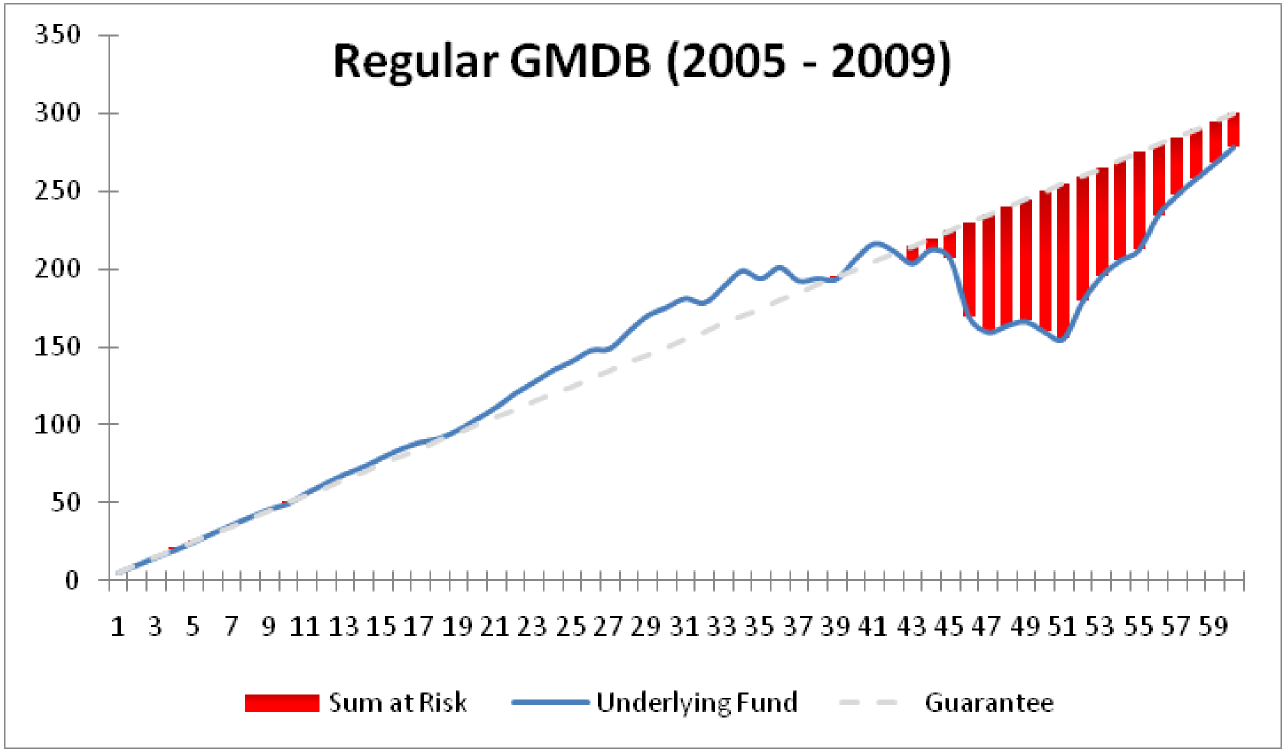
- GMDB guarantees ROC – regular premium of 5 over 5 years
- Dollar cost-averaging significantly reduces risk (cf. phasing-in of single premium)



Investment Guarantees

S&P 500 Examples: Regular Premium GMDB

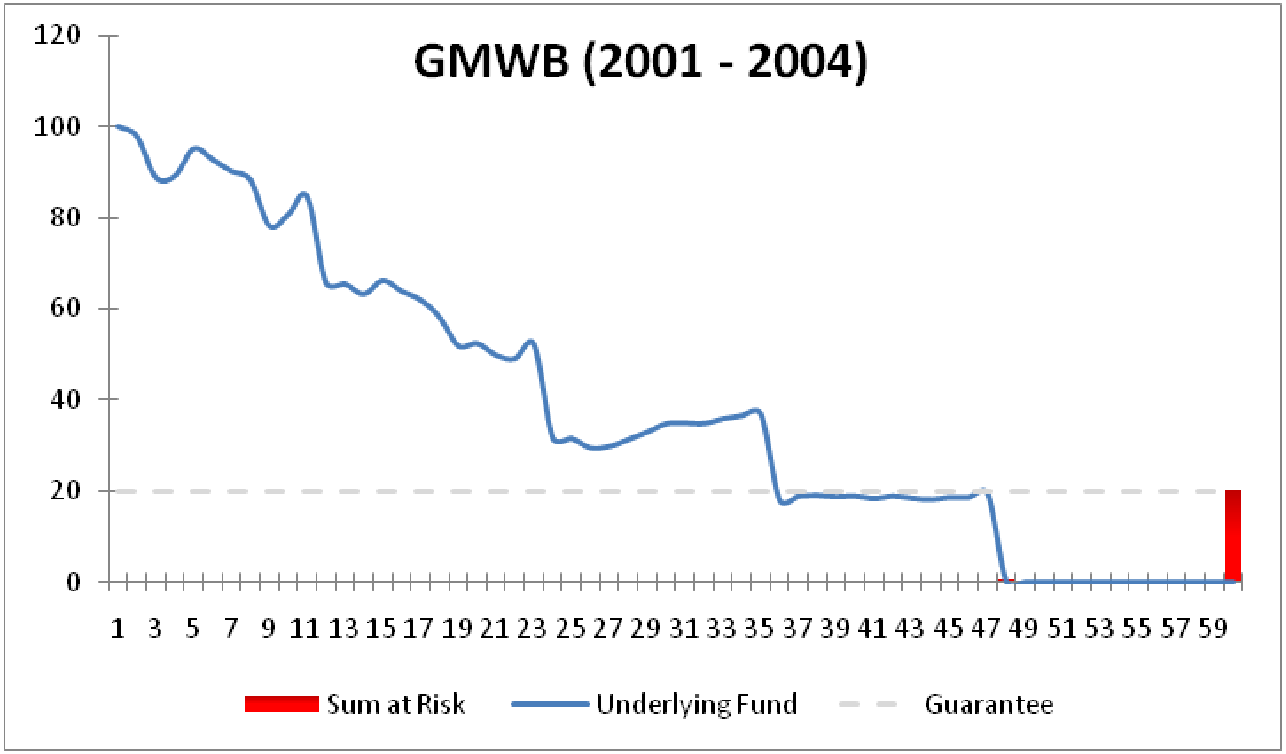
- Unfortunately, crises do happen (quite regularly!)
- Risk mitigating strategies such as hedging necessary



Investment Guarantees

S&P 500 Examples: GMWB

- GMWB guarantees 20% withdrawals for annually for 5 years
- Absorbing barrier at zero



Risk Management Considerations

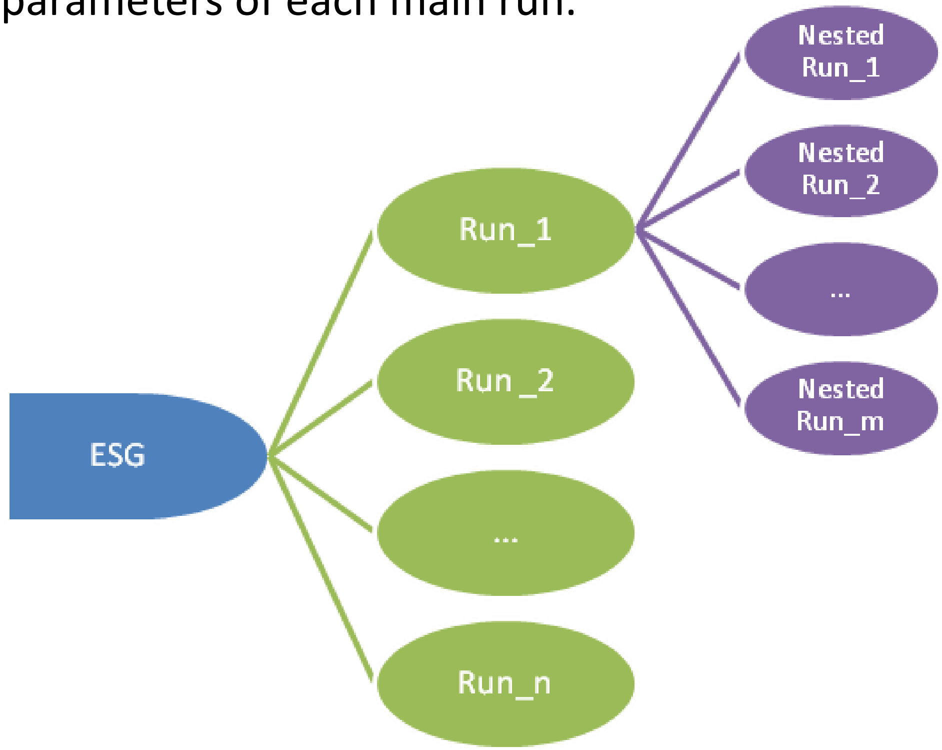
Some key challenges

Dependency	Guarantees of individual policies largely dependent, i.e. should one guarantee bite, others will probably follow
Combined Market	Real world independence ***not*** necessarily maintained under change of measure from physical probability *P* to equivalent martingale measure *Q.* Even more complex issue if combined market consist of policyholder preferences, e.g. GMWB
Initial Strain	Typically, guarantee charges are asset-based, i.e. the initial value needed to set up a replicating portfolio is recouped over time.
Complexity	Nested runs needed to determine value of guarantee in scenario-based valuation

Risk Management Considerations

ESG valuation of investment guarantees

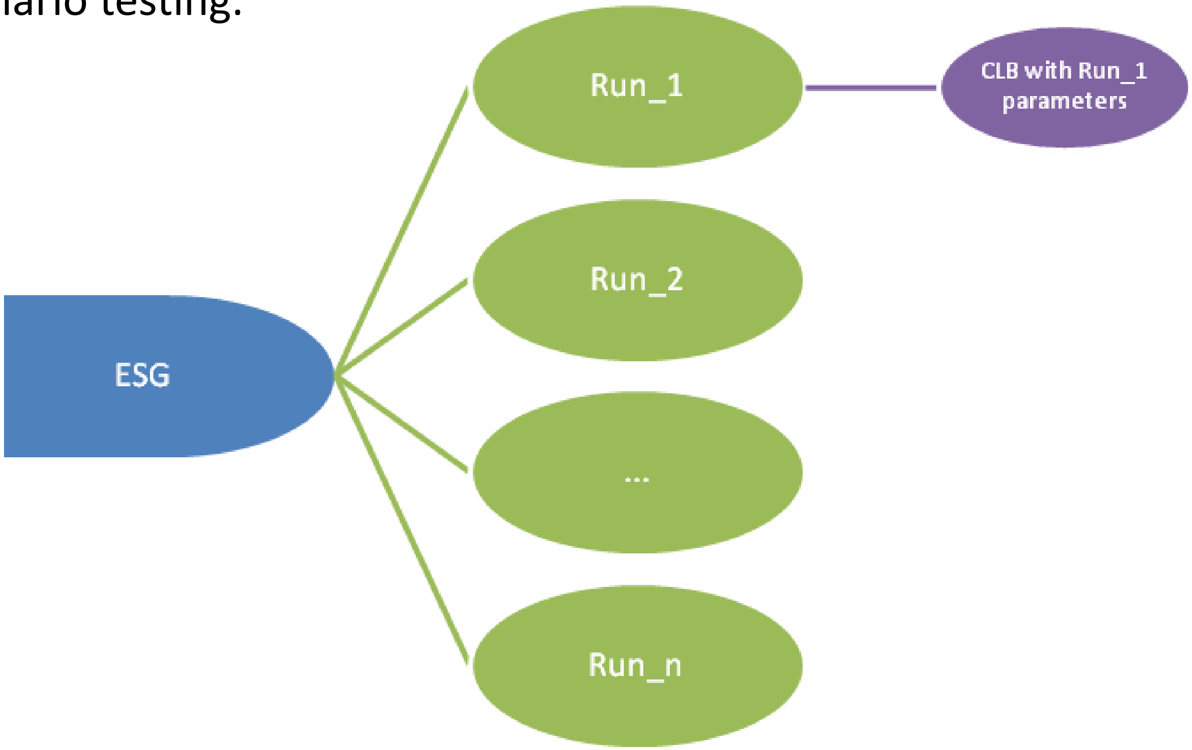
- Investment guarantee value needs to be solved via nested simulation runs based on the economic parameters of each main run.



Risk Management Considerations

ESG valuation of investment guarantees

- Instead, an approximation (CLB) can be used for interim valuations and economic capital scenario testing.



Risk Management Considerations

Complexity of guarantees

- Single premium contracts allow a simple application of financial economics, e.g. a GMMB without mortality – a ROC guarantee of the invested premium at policy maturity *n:*

$$B_n = \max\left(\pi_0^{(s)} \frac{F_n}{F_0}, b_n \right)$$

- Regular premiums complicate the application of financial economics, even in a simplified setting.

$$B_n = \max\left(\sum_{k=0}^{n-1} \pi_k^{(s)} \frac{F_n}{F_k}, b_n \right)$$

Risk Management Considerations

Embedded arithmetic Asian options

No analytical solution	Several approximations available, e.g. based on geometric Asian options, but no analytical solutions – even in simplified settings.
Conditional lower bound	Dhaene et al. (2002) derived accurate bounds for arithmetic Asian options in Black-Scholes-Merton setting. The conditional lower bound proved exceptionally accurate and will be used as an approximation.
Further outline	Conditional lower bound (CLB) is derived for simple GMMB and GMDB products. Asset-based charges are determined for these products. CLB will be used for possible hedging strategy.

Conditional Lower Bound (CLB)

Steps in deriving CLB

Conditional Lower Bound (CLB)	The CLB is stochastic lower bound to true fund value: $V_n^{-(l)} = \sum_{k=0}^{n-1} \pi_k^{(s)} E\left[\frac{F_n}{F_k} \middle\| \Lambda\right] \leq_{cx} V_n = \sum_{k=0}^{n-1} \pi_k^{(s)} \frac{F_n}{F_k}$
Optimise the CLB	The CLB is maximised by maximising its variance.
Value of embedded options	Value of embedded options can then be found by exploiting the fact that the expectation is a sum of functions of the same r.v. *Z*, i.e. $V_n^{-(l)} = \sum_{k=0}^{n-1} \pi_k^{(s)} e^{\left(\delta - \frac{1}{2}\sigma^2 r_k^2\right)(n-k) + \sigma r_k \sqrt{n-k}\, Z}$

Conditional Lower Bound (CLB)

Comonotonicity as a tool

- One condition for a random vector $\underline{X}$ to be comonotonic is if there exist a random variable *Z* and non-decreasing functions f_i such that:

$$\underline{X} =_d \left(f_1(Z), f_2(Z), \ldots, f_n(Z) \right)$$

- In the case of a comonotonic sum S^c, we have that the stop-loss premiums of the sum is equal to the sum of the comonotonic components X_i:

$$E\left[\left(S^c - d\right)_+\right] = \sum_{i=1}^{n} E\left[\left(X_i - d_i\right)_+\right]$$

where

$$d_i = F_{X_i}^{-1}\left(F_{S^c}(d)\right)$$

Conditional Lower Bound (CLB)

Determining the value of the embedded call option

- The value of the embedded call option payoff is :

$$E\left[\left(V_n^{-(l)} - b_n\right)_+\right]$$

$$= \sum_{k=0}^{n-1} \pi_k^{(s)} e^{\delta(n-k)} \Phi\left[\sigma r_k \sqrt{n-k} - \Phi^{-1}\left(F_{V_n^{-(l)}}\left(b_n\right)\right)\right]$$

$$-b_n\left(1 - F_{V_n^{-(l)}}\left(b_n\right)\right)$$

where

$$\sum_{k=0}^{n-1} \pi_k^{(s)} e^{\left(\delta - \frac{1}{2}\sigma^2 r_k^2\right)(n-k) + \sigma r_k \sqrt{n-k}\Phi^{-1}\left(F_{V_n^{(l)}}\left(b_n\right)\right)} - b_n = 0$$

Conditional Lower Bound (CLB)

Determining the value of the embedded put option

- Similarly, the value of the put option payoff is :

$$E\left[\left(b_n - V_n^{-(l)}\right)_+\right]$$

$$= -\sum_{k=0}^{n-1} \pi_k^{(s)} e^{\delta(n-k)} \Phi\left[-\sigma r_k \sqrt{n-k} + \Phi^{-1}\left(F_{V_n^{-(l)}}\left(b_n\right)\right)\right]$$

$$+ b_n F_{V_n^{-(l)}}\left(b_n\right)$$

where as before

$$\sum_{k=0}^{n-1} \pi_k^{(s)} e^{\left(\delta - \frac{1}{2}\sigma^2 r_k^2\right)(n-k) + \sigma r_k \sqrt{n-k}\,\Phi^{-1}\left(F_{V_n^{(l)}}\left(b_n\right)\right)} - b_n = 0$$

Conditional Lower Bound (CLB)

Determining the value of the GMMB & GMDB (put option)

- We can use the previous results for the GMMB case:

$$P_0 = {}_n p_x e^{-\delta n} E\left[\left(b_n - V_n^{-(l)}\right)_+\right]$$

- For the GMDB case, we have:

$$P_0 = \sum_{k=0}^{n-1} {}_k p_x q_{x+k} e^{-\delta(k+1)} E\left[\left(b_{k+1} - V_{k+1}^{-(l)}\right)_+\right]$$

Conditional Lower Bound (CLB)

Accuracy of the conditional lower bound

- Consider again the pure financial contract that guarantees ROC at maturity.
- Monte Carlo (MC) estimates and their standard errors (s.e.) were computed as “true” values for differing values of the risk-free rate and σ = 20%:

	Guarantee	CLB	MC	s.e.
δ=1%	500	1.9299	2.0269	0.00191
	750	31.1708	31.3591	0.00872
	1000	120.7156	120.8753	0.01294
	1250	266.7567	266.8974	0.00837
	1500	449.5724	449.7517	0.00658
δ=5%	500	0.2899	0.3191	0.00061
	750	7.6583	7.7911	0.00368
	1000	39.3632	39.5205	0.00924
	1250	104.2183	104.3376	0.01103
	1500	198.393	198.5049	0.00816
δ=10%	500	0.0178	0.0218	0.00012
	750	0.9215	0.9665	0.00105
	1000	7.0577	7.1558	0.00343
	1250	24.3875	24.5078	0.00643
	1500	56.0633	56.1616	0.00947

Conditional Lower Bound (CLB)

Accuracy of the conditional lower bound (cnt'd)

- Consider again the pure financial contract that guarantees ROC at maturity.
- Monte Carlo (MC) estimates and their standard errors (s.e.) were computed as "true" values for differing values of volatility and δ = 5%:

	Guarantee	CLB	MC	s.e.
σ=20%	500	0.2899	0.3191	0.00061
	750	7.6583	7.7911	0.00368
	1000	39.3632	39.5205	0.00924
	1250	104.2183	104.3376	0.01103
	1500	198.393	198.5049	0.00816
σ=30%	500	4.6067	4.9362	0.00299
	750	30.2476	30.7541	0.00824
	1000	84.6857	85.1418	0.01132
	1250	164.6151	164.9986	0.01243
	1500	264.0077	264.3668	0.00794
σ=40%	500	15.6902	16.722	0.00561
	750	60.3649	61.5619	0.01058
	1000	131.4565	132.5241	0.01172
	1250	222.2414	223.1759	0.01005
	1500	327.2443	328.0961	0.00796

Conditional Lower Bound (CLB)

Accuracy of the conditional lower bound - GMMB

- Consider now a GMMB that guarantees ROC at maturity.
- Values compared to a pure endowment (absolute upper bound)

Risk-free Rate	Guarantee		Volatility		Pure Endowment
		σ=20%	σ=30%	σ=40%	
δ=1%	50%	1.92602	14.25033	36.38255	451.51378
	75%	31.10845	76.21130	125.15746	677.27067
	100%	120.47413	189.48745	255.44786	903.02756
	125%	266.22310	340.61672	413.57425	1128.78445
	150%	448.67315	516.84348	590.24730	1354.54135
δ=5%	50%	0.28930	4.59746	15.65882	302.65874
	75%	7.64299	30.18714	60.24417	453.98811
	100%	39.28447	84.51634	131.19351	605.31748
	125%	104.00982	164.28582	221.79691	756.64685
	150%	197.99618	263.47966	326.58979	907.97622
δ=10%	50%	0.01779	0.93749	4.96337	183.57180
	75%	0.91966	8.15758	22.27280	275.35771
	100%	7.04358	26.95710	53.07222	367.14361
	125%	24.33875	58.58643	95.34859	458.92951
	150%	55.95119	101.86759	146.77681	550.71541

Conditional Lower Bound (CLB)

Accuracy of the conditional lower bound - GMDB

- Consider now a GMDB that guarantees ROC at death.
- Values compared to a term life contract (absolute upper bound)

Risk-free Rate	Guarantee	Volatility			Term Life Benefit
		σ=20%	σ=30%	σ=40%	
δ=1%	50%	0.00117	0.01055	0.02975	0.53091
	75%	0.02652	0.06959	0.11833	0.79636
	100%	0.12424	0.19430	0.26201	1.06180
	125%	0.29966	0.37195	0.44508	1.32730
	150%	0.52294	0.58396	0.65421	1.59270
δ=5%	50%	0.00020	0.00384	0.01436	0.40142
	75%	0.00790	0.03203	0.06497	0.60213
	100%	0.05105	0.10244	0.15504	0.80284
	125%	0.14871	0.21352	0.27642	1.00350
	150%	0.29074	0.35470	0.42017	1.20430
δ=10%	50%	0.00002	0.00097	0.00548	0.28707
	75%	0.00143	0.01128	0.02968	0.43060
	100%	0.01481	0.04367	0.07855	0.57414
	125%	0.05646	0.10254	0.14973	0.71767
	150%	0.12967	0.18433	0.23837	0.86121

Asset-Based Charges

Fair value approach

- The asset-based charge for the guarantee can be solved for by equating the present value of the benefits to the present value of the contributions.
- For the pure financial contract, we therefore have:

$$\sum_{k=0}^{n-1} \pi_k^{(s)} e^{-\delta k} = e^{-\delta n} E\left[\max\left(V_n^{-(l)}, b_n\right)\right]$$

$$= e^{-\delta n} E\left[V_n^{-(l)}\right] + P_0$$

where

$$P_0 = -\sum_{k=0}^{n-1} \pi_k^{(s)} e^{-\delta k} \left(1-e\right)^{n-k} \Phi\left[-\sigma r_k \sqrt{n-k} + \Phi^{-1}\left(F_{V_n^{-(l)}}\left(b_n\right)\right)\right]$$

$$+ e^{-\delta n} b_n F_{V_n^{-(l)}}\left(b_n\right)$$

Asset-Based Charges

Fair value approach – GMMB & GMDB

- Similarly, we can solve the asset-based charges for the GMMB:

$$\sum_{k=0}^{n-1} {}_k p_x \pi_k^{(s)} e^{-\delta k} = {}_n p_x e^{-\delta n} E\left[\max\left(V_n^{-(l)}, b_n\right)\right]$$

- For the GMDB, we have:

$$\sum_{k=0}^{n-1} {}_k p_x \pi_k^{(s)} e^{-\delta k} = \sum_{k=0}^{n-1} {}_k p_x q_{x+k} e^{-\delta(k+1)} E\left[\max\left(V_{k+1}^{-(l)}, b_{k+1}\right)\right]$$

Asset-Based Charges

Numerical Example

- Consider again the pure financial contract that guarantees ROC at death - annual premium of 100 over 10 years.
- Values computed for differing values of the risk-free rate and volatility.

	Guarantee	σ=20%	σ=30%	σ=40%
δ=1%	500	0.3664	2.8282	7.5429
	750	6.9304	18.7686	32.8233
	1000	51.1506	86.564	120.8808
δ=5%	500	0.06095	0.9881	3.4656
	750	1.6931	7.187	15.1582
	1000	10.5377	25.1368	41.4655
	1250	48.1448	81.8928	114.5406
δ=10%	500	0.00425	0.2254	1.2115
	750	0.2218	2.0356	5.7732
	1000	1.7803	7.3267	15.2321
	1250	6.9371	18.4198	31.8342
	1500	20.2496	40.713	61.8769

Hedging

Mitigating risk through hedging

Some possible solutions	• Reinsurance (limited) • Structured solutions • Super-hedging • Risk-minimising strategies
Static or Dynamic?	Only 1 out of 16 responded utilised a pure static hedging approach according to a Milliman survey (2008). Other respondents used dynamic strategies or a combination.
Greeks	The three main Greeks on which respondents to the Milliman survey focused were delta, rho and vega.
Further outline	The CLB is used to calculate Greeks for the GMMB and GMDB products. Greeks of CLB approximation facilitate reasonableness checks to more complex scenario based models.

Hedging

Calculating the Greeks

- The delta can be analytically determined.
- The delta of the embedded put option for the pure financial contract is given by:

$$\Delta_t = \frac{\partial P_t}{\partial F(t)}$$

$$= -\sum_{k=0}^{\lceil t \rceil - 1} \frac{\pi_k^{(s)}}{F(k)} \Phi\left[-\sigma r_t \sqrt{n-t} + \Phi^{-1}\left(F_{V_n^{-(l)}}(b_n)\right)\right]$$

where *t* is the time of valuation such that $0 \le t \le n$.

Hedging

Calculating the Greeks – GMMB & GMDB

- The delta of the embedded put option for the GMMB is given by:

$$\Delta_t = \frac{\partial P_t}{\partial F(t)} \, {}_t p_x P_t$$

- The delta of the embedded put option for the GMDB is given by:

$$\Delta_t = \frac{\partial P_t^{j+1}}{\partial F(t)} \sum_{j=\lceil t \rceil - 1}^{n-1} {}_j p_{x+t} q_{x+t+j} P_t^{j+1}$$

Hedging

Calculating the Greeks

- The other Greeks can be easily determined by the centre differencing approach, i.e. the rho is given by:

$$\rho_t = \frac{\partial P_t}{\partial \delta} \approx \frac{P_t(\delta + \Delta\delta) - P_t(\delta - \Delta\delta)}{2\Delta\delta}$$

and the vega is given by:

$$\mathrm{Vega}_t = \frac{\partial P_t}{\partial \sigma} \approx \frac{P_t(\sigma + \Delta\sigma) - P_t(\sigma - \Delta\sigma)}{2\Delta\sigma}$$

- Higher order Greeks such as gamma, and other sensitivity measures can be determined in a similar way.

Hedging

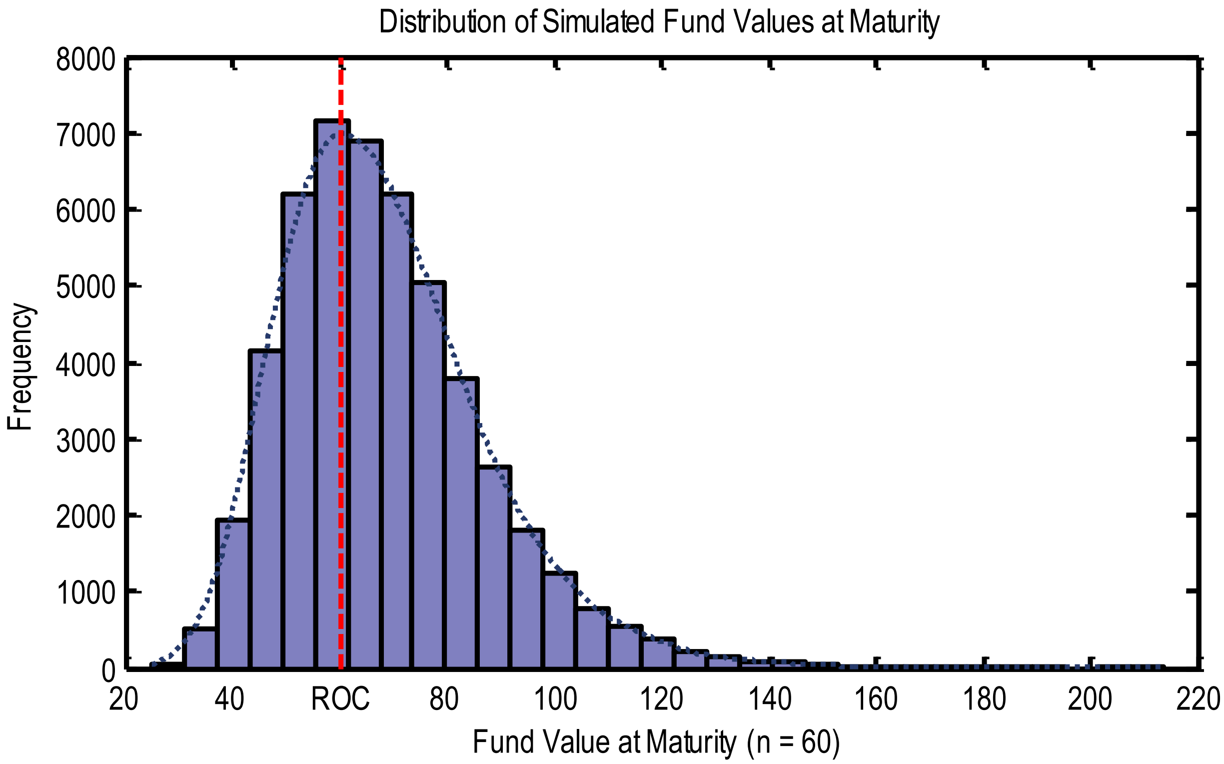
Illustrative Example

Pure Financial Contract	Guarantees ROC at maturity A payment of 1 is made monthly, i.e. ROC = 60.
Monthly Rebalancing	Each month, the delta is held in the underlying fund and the remainder in the risk-free bond.
Parameters	Assume a risk-free rate of 5% and volatility of 20%.
MC paths	Single Monte Carlo simulation paths were used to assess the delta hedging strategy.

Hedging

Illustrative Example Results

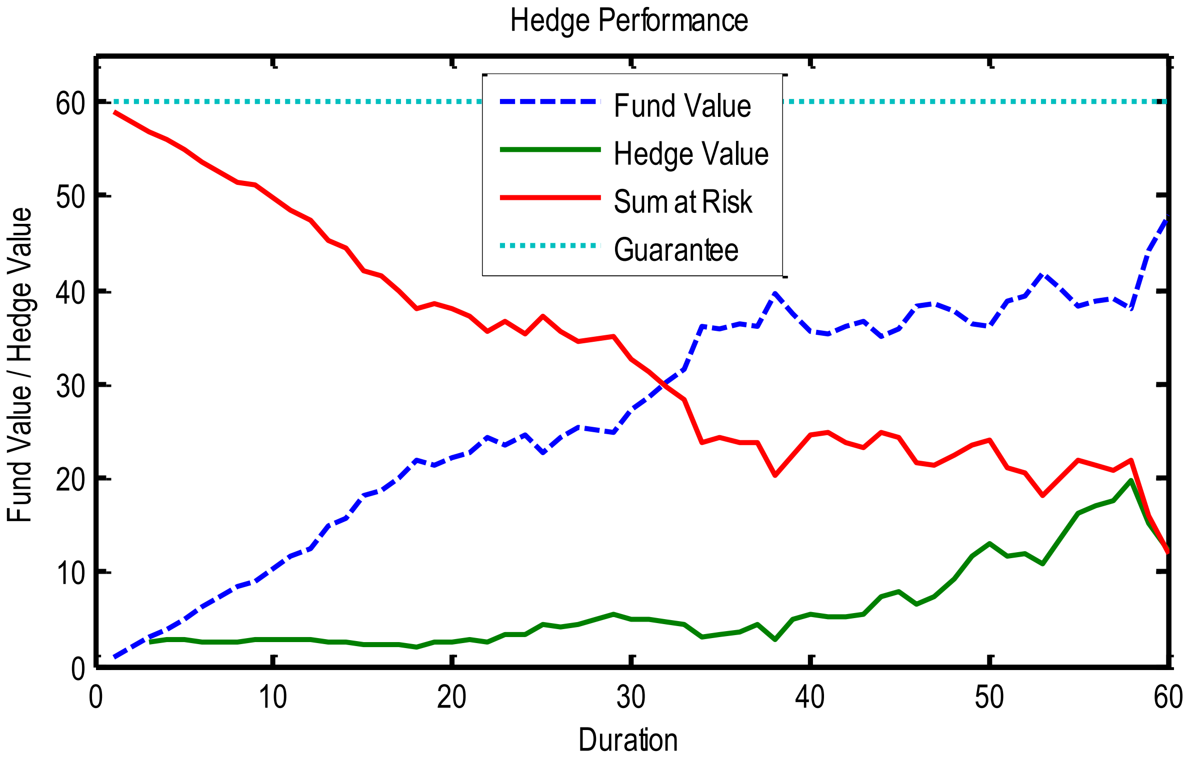
- The distribution of the fund value at maturity is as follows:



Hedging

Illustrative Example Results

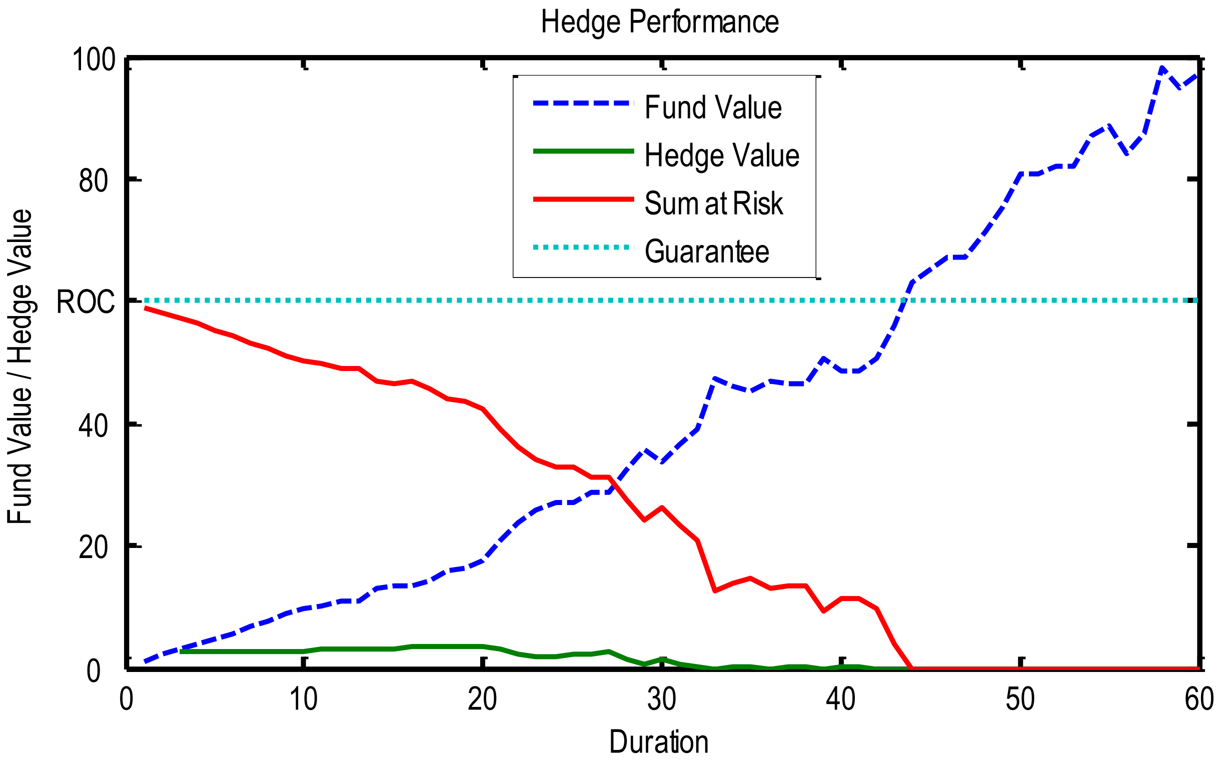
- In the event of poor market performance, a likely delta hedging strategy might resemble the following:



Hedging

Illustrative Example Results

- In the event of good market performance, a likely delta hedging strategy might resemble the following:



Conclusions

Integrated risk management	Risk management should be a first consideration in product development, i.e. IF YOU CAN’T MANAGE IT, DON’T SELL IT.
CLB a complement	The CLB could act as a pragmatic complement to existing, more resource intensive, risk management processes in: • Pricing • Reserving, and • Hedging

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Good luck Bafana Bafana!