

# An actuarial approach to pricing Mortgage Insurance considering simultaneously mortgage default and prepayment

Alan Elizondo  
Valeria Álvarez  
Sergio Valladares

SHF – Sociedad Hipotecaria Federal

# Mortgage Insurance (MI)



MI is an insurance created to transfer default risk of a mortgage loan granted by a financial institution. Its objective is to pay a percentage of the outstanding balance varying between 5% to 35% upon loan to value ratio (LTV).

MI provides protection against losses upon defaults resulting from falling house prices or borrower's credit risk.

MI is typically purchased by mortgage investors, such as banks, savings and loan associations, and government sponsored enterprises. Title insurance provides protection against losses created by defective property titles.

# Analogy to Life Insurance



Life insurance is a contract between the insurer and the **policy owner** whereby a **benefit** is paid to the designated **beneficiaries** if an insured event occurs. The policy owner agrees to pay a stipulated amount called a **premium**.

In mortgage insurance the **policy owner** is the financial institution that provides the mortgage loan, the **benefit** is an amount of money expressed as a percentage of the outstanding balance of the loan at the moment of default and the **insured event** is the failure from the borrower to pay his loan obligations.

The policy owner agrees to pay a **premium** that is paid either as a fixed amount at the beginning of the loan or periodically during the life time of the loan.

# MI in emerging markets



Many countries in development stage are experimenting an important transition in their mortgage market going from public to private sector. Government as well as private lenders face the challenge to attract enough capital with competitive conditions from private resources so that the house industry can continue its development.

MI is fundamental in mortgage development. It helps to expand credit access by offering risk protection specially for risky population segments which also are the ones with higher financing requirements and needs. Its achievement is to diminish LTV so that the risk may be acceptable to the lenders.

MI diminishes credit risk making it accessible even with low down payments.

# Benefits of MI



- Improves mortgage origination standards promoting competitive prices and quality (it does not relax origination policy)
- Promotes generation, analysis and acknowledge of high quality credit information.
- Promotes the integrity and standardization of the information and documentation of house valuation.
- Facilitates the accessibility to mortgage loans by granting credit to population with low down payments.
- Promotes market liquidity assuring to international investors quality of the mortgage portfolio. Attracts private capital to the housing market.
- By transferring risk to a financial institution, investors have the possibility to offer mortgage; besides it benefits and encourages the mortgage backed securities market.

# Challenges of MI



- MI differs from other types of insurance, making difficult to adopt techniques already developed in insurance industry.
- The insurance benefit is not a fixed amount as it depends on the outstanding balance.
- Claim risk decreases rather than increases over time due to amortization schedule.
- Default and prepayment are mortgage termination possibilities but they also are competitive risks.
- Default and prepayment probabilities change frequently. Long term decrement tables would not be convenient.
- Economic cycles and crisis are correlated with default and prepayment probabilities.

# Data



- The empirical analysis is based upon a set of mortgage lending data from a housing government bank –Sociedad Hipotecaria Federal- which concentrates information of most of the financial intermediaries operating mortgage credit.
- The information goes through the period of years 2003 – 2008.
- The dataset allows to study mortgage borrower prepayment and default behavior at loan level.
- The ability of loan level data allowed to account for the dynamics of mortgage behavior based on payment history of individual mortgage loans.

# Competing risks of mortgage termination



- Managing the risk of mortgage portfolios requires analytical tools that are able to consider a variety of factors that affect the cash flows of the loans.
- It is a well documented fact that borrowers face two mutually exclusive alternatives when paying a mortgage that affect programmed cash flows, either to prepay on the loan or to default on it.
- For the MI provider, prepayment means cessation of cash flows in the form of any mortgage insurance premium and thereafter eliminates possibility of default. Defaulting implies incurring in costs associated with the MI contract.
- One cannot calculate accurately the economic value of the cost of the insurance (premium) without considering default and prepayment risks.
- Prepayment and default are considered as mortgage competing risks.

# Models for competing risks of mortgage termination



- The theoretical and empirical analysis to estimate the competing risks of mortgage prepayment and default is based upon a discrete-time multinomial logistic model of mortgage termination.
- The three mutually exclusive choices leading to mortgage termination are considered in this work:
  - Default
  - Prepayment
  - Successful completion
- The multinomial logistic regression model estimates conditional probabilities of prepayment and default depending on common factors so that there is some inherent simultaneity in the decisions of borrowers.

## Model structure

- Assuming there exist “p” covariates and a constant term:

$$\begin{aligned}
 g_1(x_1, \dots, x_p) &= \ln \left( \frac{P(Y = 1 / x_1, \dots, x_p)}{P(Y = 0 / x_1, \dots, x_p)} \right) \\
 &= \beta_{10} + \beta_{11} \cdot x_1 + \dots + \beta_{1p} \cdot x_p \\
 &= \sum_{j=0}^p \beta_{1j} \cdot x_{1j}
 \end{aligned}$$

$$\begin{aligned}
 g_2(x_1, \dots, x_p) &= \ln \left( \frac{P(Y = 2 / x_1, \dots, x_p)}{P(Y = 0 / x_1, \dots, x_p)} \right) \\
 &= \beta_{20} + \beta_{21} \cdot x_1 + \dots + \beta_{2p} \cdot x_p \\
 &= \sum_{j=0}^p \beta_{2j} \cdot x_{2j}
 \end{aligned}$$

- It follows that the conditional probabilities of each outcome category given the covariate vector are:

**Default**

$$P(Y = 1 / x_1, \dots, x_p) = \frac{e^{g_1(x_1, \dots, x_p)}}{1 + e^{g_1(x_1, \dots, x_p)} + e^{g_2(x_1, \dots, x_p)}}$$

**Prepayment**

$$P(Y = 2 / x_1, \dots, x_p) = \frac{e^{g_2(x_1, \dots, x_p)}}{1 + e^{g_1(x_1, \dots, x_p)} + e^{g_2(x_1, \dots, x_p)}}$$

**Successful completion**

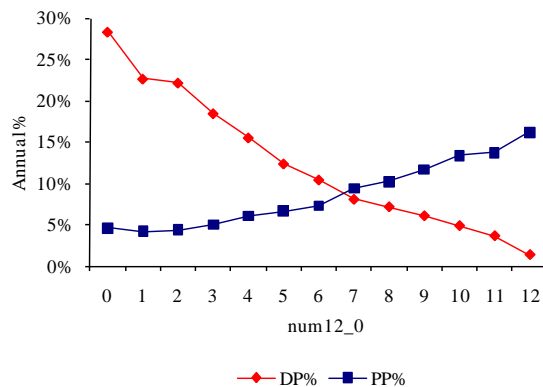
$$P(Y = 3 / x_1, \dots, x_p) = \frac{1}{1 + e^{g_1(x_1, \dots, x_p)} + e^{g_2(x_1, \dots, x_p)}}$$

# Model covariates

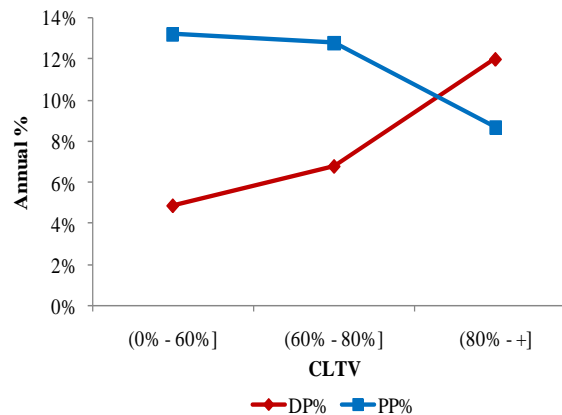
To calibrate de model 5 types of covariates were introduced

1. Financial Characteristics of the loan.
2. Borrower information
3. Mortgage Behavior \*
4. Financial Environment \*
5. Payment history \*

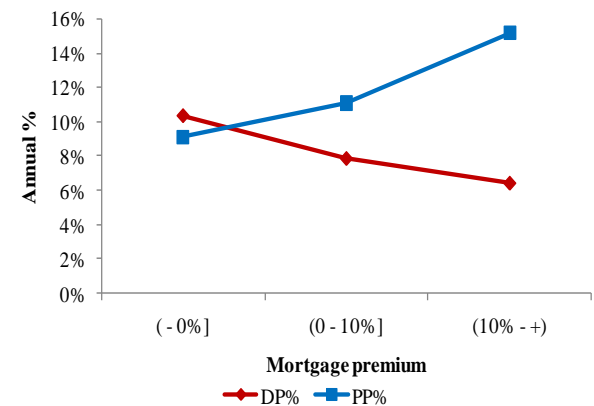
**Default vs Prepayment Probabilities**



**Default and Prepayment probabilities**



**Default and Prepayment probabilities**



# Model covariates



The estimation of the model leads to the following selected significant variables:

Analysis of Maximum Likelihood Estimates					
Parameter		Function	Estimate	Standard Error	Pr > ChiSq
Intercept		Default	-1.0656	0.0361	<.0001
		Prepayment	-3.0583	0.038	<.0001
Mortgage Premium	(- 0%]	(0 - 10%] Default	-0.3054	0.0258	<.0001
		Prepayment	0.1929	0.0233	<.0001
	(10% - +)	Default	-0.2552	0.0321	<.0001
		Prepayment	0.4745	0.0242	<.0001
Current Loan to Value	(0% - 60%]	(60% - 80%] Default	0.2203	0.0376	<.0001
		Prepayment	0.0361	0.025	0.149
	(80% - mayor]	Default	0.5718	0.0337	<.0001
		Prepayment	-0.2086	0.0251	<.0001
Num12_0		Default	-0.2236	0.00296	<.0001
		Prepayment	0.1087	0.00301	<.0001

# MI Pricing Expected Loss



The expected value of losses (EL) to the insurance company under MI subject to multiple causes of termination of the contract is calculated as follow:

$$EL = \sum_{j=1}^J \sum_{k=1}^{term} B_k^j p_0^{(\tau)} q_k^j v^k$$

“J” is proposed to be equal to 3 where j=1 is the event of default, j=2 is the event of prepayment of the loan and j=3 is the successful completion of the contract. The benefit for the policy owner for **the event of default** is:

$$B_k^1$$

Where  $B_k^1$  is for first loss mortgage insurance an amount equal to a percentage of the outstanding balance of the loan at the moment of default, therefore:

$$B_k^1 = c * OB_k$$

# MI Pricing

## Expected Income



The expected value of income for the insurance company under MI subject to multiple causes of termination of the contract when the premium is expressed as a fixed percentage of the outstanding balance of the mortgage is calculated as follow:

$$EI = \sum_{k=1}^{term} ps \cdot OB_k \cdot {}_kP_0^{(\tau)} \cdot v^k$$

It is important to note that  ${}_kP_0^{(\tau)} < {}_kP_0^{(j=1)}$  which implies that for the same premium,  $EI < EI^{j=1}$ . This means that if other causes of termination different than default are not considered, the insurance company may overestimate which in turn would lead to subsequent losses for the mortgage insurance product.

Hence the ***mortgage insurance reserving*** is defined as:

$$MIR_k^{(\tau)} = EL_k - EI_k$$

# Mortgage default and prepayment mortality conventions



## Default measures

- **Conditional default rate (CDR)**

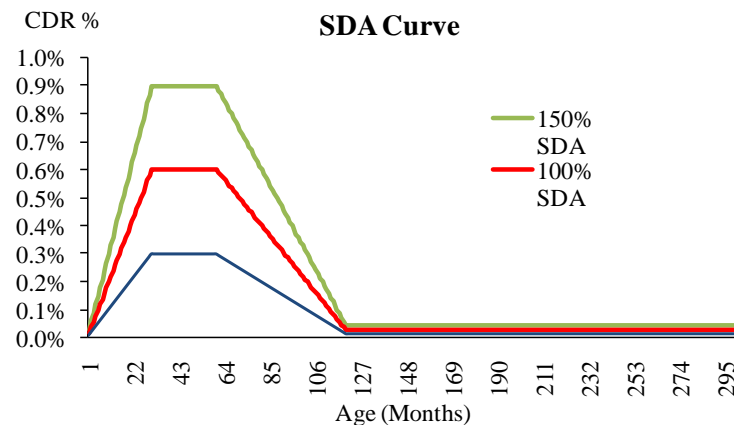
Represents the monthly default rate expressed as annual percentage.

$$MDR_t = \frac{\text{Default loan balance}_{month t}}{\text{Beginning Balance}_{month t} - \text{Scheduled Principal Payment}_{month t}}$$

$$CDR = 1 - (1 - MDR_t)^{12}$$

- **Standard Default Assumption (SDA)**

It is a reference measure to default in a mortgage pool of 30 years fixed rate loans upon the age of the loan. It is expressed in terms of the CDR.



# Mortgage default and prepayment mortality conventions



## Prepayment measures

- **Conditional prepayment rate (CPR)**

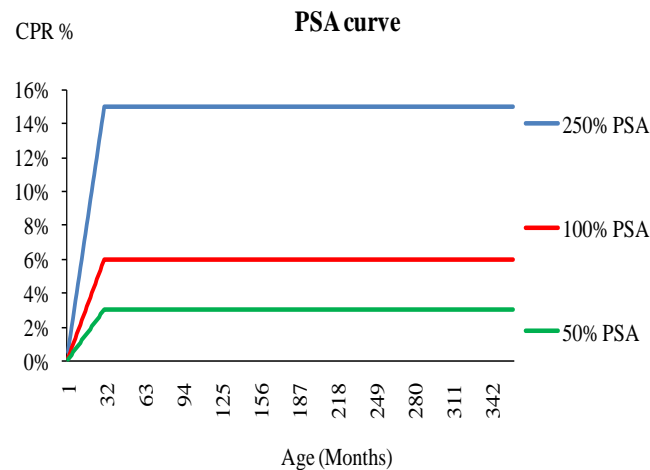
Is a constant probability of prepayment rate in a mortgage portfolio.

$$SMM = \frac{\text{Total payments, including prepayments} - \text{Scheduled interest payment} - \text{Scheduled principal payment}}{\text{Unpaid principal balance} - \text{Scheduled principal payment}}$$

$$(1 - CPR) = (1 - SMM)^{12}$$

- **Public Securities Association (PSA)**

It is built through a series of annualized prepayment rates changing upon the mortgage age in months.





## MI (multiple decrements)

In life insurance the population of new born individuals incurs with certainty in the insured event (**death**) during a finite period ( **$e$  years**) which means that  ${}_e q_0 = 1$ .

However, a number of events may lead to the termination of the mortgage contract which in turn may represent costs to the policy owner

- **Default, prepayment and successful completion of the contract**

All three events lead to the termination of the contract which completes the analogy with life insurance whereby  ${}_e q_0 = 1$ .

Each of the insured events leads to a different insured amount for the insurance company. Each cause of contract termination is considered a “**decrement**” which shows a different probability of occurrence.

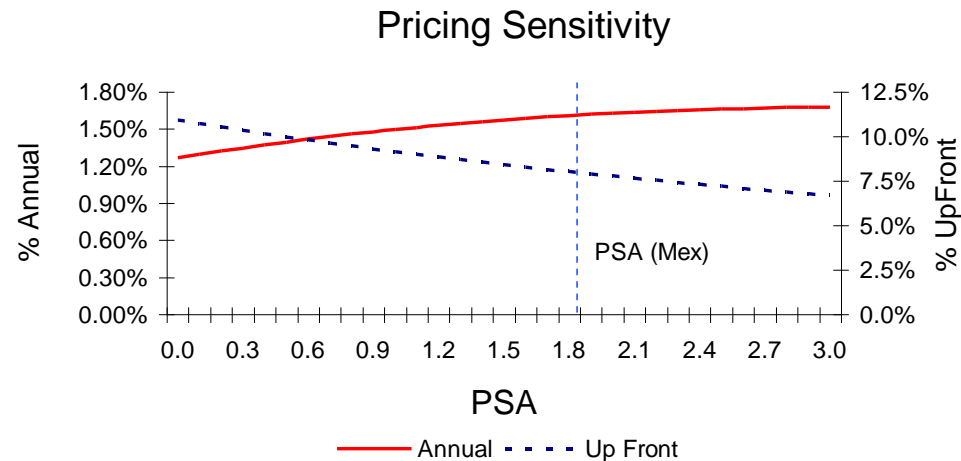
# Multiple decrement table

- $MDR_k = q_k^1 = \frac{d_k^1}{l_k}$  both CPR and CDR can be translated to actuarial terminology assuming that loan amounts are the same as follow:  $SMM_k = q_k^2 = \frac{d_k^2}{l_k}$

Age Months	$q_k^{(1)}$	$q_k^{(2)}$	$q_k^{(3)}$	$q_k^{(\tau)}$	$l_k^{(\tau)}$	$d_k^{(1)}$	$d_k^{(2)}$	$d_k^{(3)}$	$d_k^{(\tau)}$	$d_k^{(\tau)} / l_0^{(\tau)}$	${}_k P_0^{(\tau)}$	${}_k P_0^{(\tau)} \cdot q_k^{(1)}$	${}_k P_0^{(\tau)} \cdot q_k^{(2)}$	${}_k P_0^{(\tau)} \cdot q_k^{(3)}$	${}_k P_0^{(\tau)} \cdot q_k^{(\tau)}$
1	0.00%	0.00%	0.00%	0.00%	100,000	0	0	0	0	0.00%	100.00%	0.00%	0.00%	0.00%	0.00%
2	0.00%	0.00%	0.02%	0.02%	100,000	0	2	17	18	0.02%	100.00%	0.00%	0.00%	0.02%	0.02%
3	0.00%	0.00%	0.03%	0.04%	99,982	0	3	33	37	0.04%	99.98%	0.00%	0.00%	0.03%	0.04%
4	0.00%	0.01%	0.05%	0.06%	99,945	0	5	50	55	0.06%	99.94%	0.00%	0.00%	0.05%	0.06%
5	0.00%	0.01%	0.07%	0.07%	99,890	0	7	67	74	0.07%	99.89%	0.00%	0.01%	0.07%	0.07%
6	0.00%	0.01%	0.08%	0.09%	99,816	0	8	84	92	0.09%	99.82%	0.00%	0.01%	0.08%	0.09%
7	0.00%	0.01%	0.10%	0.11%	99,724	0	10	100	110	0.11%	99.72%	0.00%	0.01%	0.10%	0.11%
8	0.00%	0.01%	0.12%	0.13%	99,614	0	12	117	129	0.13%	99.61%	0.00%	0.01%	0.12%	0.13%
9	0.00%	0.01%	0.13%	0.15%	99,486	0	13	134	147	0.15%	99.49%	0.00%	0.01%	0.13%	0.15%
10	0.00%	0.02%	0.15%	0.17%	99,339	0	15	150	165	0.17%	99.34%	0.00%	0.01%	0.15%	0.17%
11	0.00%	0.02%	0.17%	0.18%	99,174	0	17	167	183	0.18%	99.17%	0.00%	0.02%	0.17%	0.18%
12	0.00%	0.02%	0.19%	0.20%	98,990	0	18	183	202	0.20%	98.99%	0.00%	0.02%	0.18%	0.20%
13	0.00%	0.02%	0.20%	0.22%	98,789	0	20	200	220	0.22%	98.79%	0.00%	0.02%	0.20%	0.22%
14	0.00%	0.02%	0.22%	0.24%	98,569	0	21	216	238	0.24%	98.57%	0.00%	0.02%	0.22%	0.24%
15	0.00%	0.02%	0.24%	0.26%	98,332	0	23	232	255	0.26%	98.33%	0.00%	0.02%	0.23%	0.26%
16	0.00%	0.03%	0.25%	0.28%	98,076	0	25	249	273	0.27%	98.08%	0.00%	0.02%	0.25%	0.27%
17	0.00%	0.03%	0.27%	0.30%	97,803	0	26	265	291	0.29%	97.80%	0.00%	0.03%	0.26%	0.29%
18	0.00%	0.03%	0.29%	0.32%	97,512	0	28	281	308	0.31%	97.51%	0.00%	0.03%	0.28%	0.31%
19	0.00%	0.03%	0.31%	0.34%	97,204	0	29	297	326	0.33%	97.20%	0.00%	0.03%	0.30%	0.33%
20	0.00%	0.03%	0.32%	0.35%	96,878	0	31	312	343	0.34%	96.88%	0.00%	0.03%	0.31%	0.34%
21	0.00%	0.03%	0.34%	0.37%	96,535	0	32	328	360	0.36%	96.54%	0.00%	0.03%	0.33%	0.36%
22	0.00%	0.04%	0.36%	0.39%	96,175	0	34	343	377	0.38%	96.17%	0.00%	0.03%	0.34%	0.38%
23	0.00%	0.04%	0.37%	0.41%	95,798	0	35	359	394	0.39%	95.80%	0.00%	0.04%	0.36%	0.39%
24	0.00%	0.04%	0.39%	0.43%	95,404	0	37	374	410	0.41%	95.40%	0.00%	0.04%	0.37%	0.41%
25	0.00%	0.04%	0.41%	0.45%	94,994	0	38	389	427	0.43%	94.99%	0.00%	0.04%	0.39%	0.43%
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356	0.00%	0.00%	0.51%	0.52%	16,634	0	0	86	86	0.09%	16.63%	0.00%	0.00%	0.09%	0.09%
357	0.00%	0.00%	0.51%	0.52%	16,548	0	0	85	86	0.09%	16.55%	0.00%	0.00%	0.09%	0.09%
358	0.00%	0.00%	0.51%	0.52%	16,463	0	0	85	85	0.09%	16.46%	0.00%	0.00%	0.08%	0.09%
359	0.00%	0.00%	0.51%	0.52%	16,378	0	0	84	85	0.08%	16.38%	0.00%	0.00%	0.08%	0.08%
360	99.48%	0.00%	0.51%	100.00%	16,293	16,209	0	84	16,293	16.29%	16.29%	16.21%	0.00%	0.08%	16.29%

100.00%

# MI pricing sensitivity



- A higher value of PSA affects the cash flows of the product and generates a lower value of the **upfront premium**. This effect is explained by the fact that a lesser number of loans default which in turn generates lesser expenses to the insurance company.
- In contrast, the same effect works differently for the **annual premium** as it rises when prepayment rises. This in turn is explained by the fact that a higher rate of prepayment generates a lesser amount of premium charged during the life time of the portfolios of loans and hence requires a larger premium to compensate.

# Conclusions



- The importance of considering prepayment for mortgage insurance pricing is described as prices not only shift importantly according to the prepayment assumption taken into consideration but for the case of the annual premium move in opposed direction to the upfront premium which implies hidden second round effects that the insurance company needs to take into consideration in its pricing policy.
- Valuing the risks of default and prepayment together is highly recommended at two instances, first the pricing formulas which should consider default and prepayment but second, and most important, at the parameter estimation level where it was shown that a simultaneous estimation of default and prepayment provides different parameters and probabilities estimation which are considered to contain more information than their single risk parameter estimation counterparts.
- This document applies one of the multiple approaches proposed in the extensive literature on the topic of simultaneous estimation of prepayment and default risk and adds to the growing field of pricing mortgage insurance by proposing a decrement table actuarial approach using mortgage market PSA and SDA conventions. It also represents a first reference for mortgage insurance pricing with Mexican experience.

# Contact Details



- **Jesús Alan Elizondo Flores**
  - Comisión Nacional Bancaria y de Valores (CNBV)
  - [aelizondo@cnbv.gob.mx](mailto:aelizondo@cnbv.gob.mx)
  
- **Valeria Álvarez Navarro**
  - Sociedad Hipotecaria Federal (SHF)
  - [valvarez@shf.gob.mx](mailto:valvarez@shf.gob.mx)
  
- **Israel Sergio Valladares Cedillo**
  - Sociedad Hipotecaria Federal (SHF)
  - [ivalladares@shf.gob.mx](mailto:ivalladares@shf.gob.mx)