Reinsurance Pricing: Pareto extrapolation downward

Marc Bagarry
Léa Nefussi
PartnerRe

Introduction

- Practical issue of pricing event exposures using a Pareto model.
- Pareto model consists of an Observation Point (OP), a Return Period (RP) and an Alpha parameter.
- Often calibrated to fit large event losses to price above the OP, issue for the part below the OP.
- A concern for P&C treaties with low deductibles exposed to small event losses with a large frequency.

Current pricing issues

- Issues and choice of alpha when extrapolating scenarios up and down with the Pareto model.
- How to price bottom layers when we have an event scenario above?
- Can we extrapolate downward?
  1. Is it right?
  2. How should we do it?

Plan

- Scope of the Analysis
- The Pareto model
- Is it right to extrapolate downward?
- Alpha selection and sensitivity
- Pricing approaches of per event exposures
- Gonu Scenario Example
- Recommendations
- References

Scope of analysis

- Short tail non-proportional per event
- Short tail proportional event loss ratio and large loss ratio

The Pareto model

Definition: losses above the observation point (OP) are Pareto-distributed if the loss distribution is:

$f(x) = \alpha \cdot OP^{\alpha} \cdot x^{-\alpha-1}$ Depends on alpha and OP

Severity: expected value of a single loss in the layer

$$\text{Expected loss} = \frac{\text{Deductible}}{1-\alpha} \cdot \left(RL^{1-\alpha} - 1\right) \quad \text{for } \alpha \neq 1\text{, or}$$

$$= \text{Deductible} \cdot \ln(RL) \quad \text{for } \alpha = 1$$

Where RL is the relative length of the layer: $RL = \frac{\text{Cover} + \text{Deductible}}{\text{Deductible}} = \frac{\text{Exit Point}}{\text{Deductible}}$

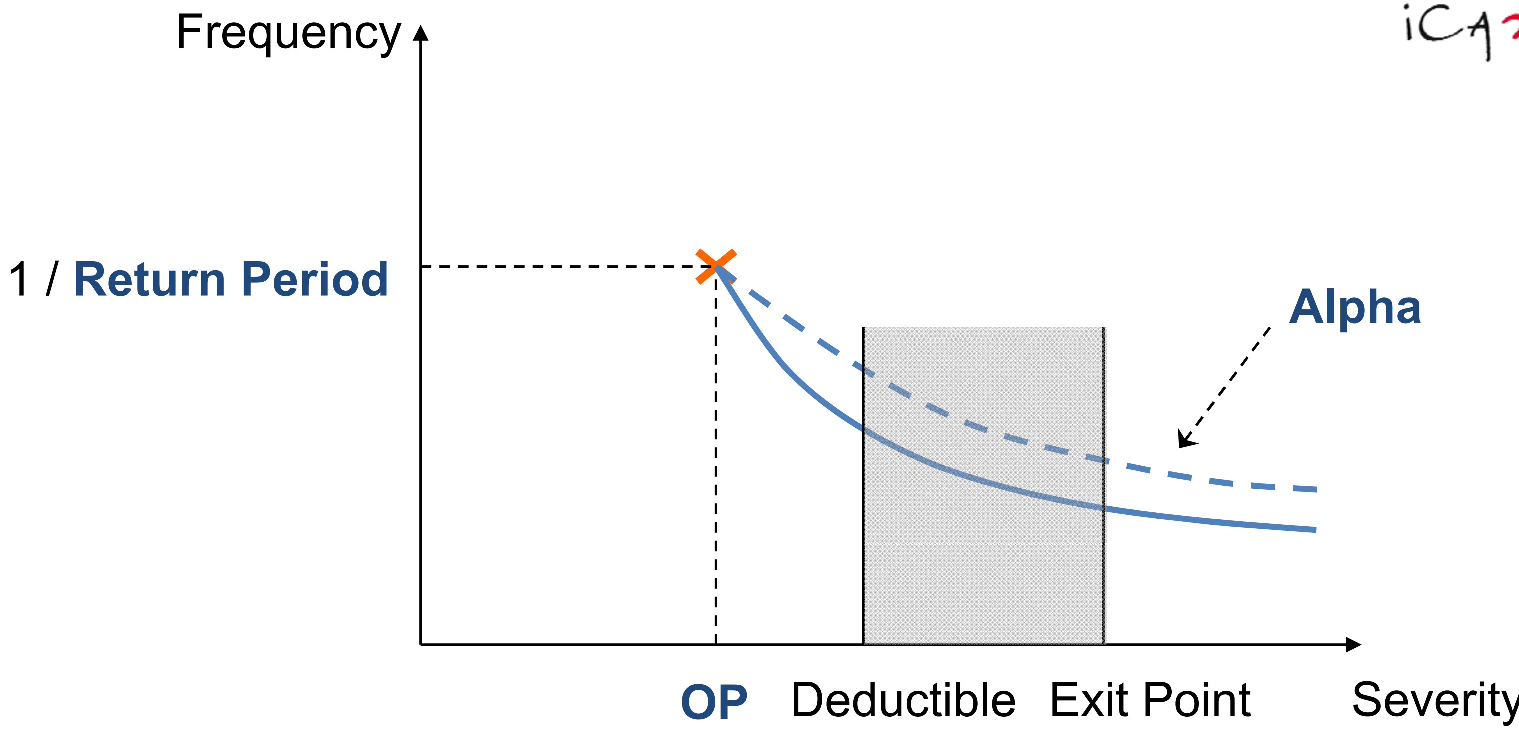
Frequency: assume to be known at OP, at an arbitrary point x (above OP):

$$\text{Frequency}(x) = \text{Frequency}(OP) \cdot \left(\frac{OP}{x}\right)^{\alpha}$$

Risk Premium: combine severity and frequency

$$\text{Risk Premium} = \text{Frequency}(\text{Deductible}) \cdot \text{Expected loss}$$

The Pareto model cont'd



A Pareto scenario is defined by 3 parameters:

- Return Period
- Observation Point
- Alpha: sets the convexity of the curve above the OP

Is it right to extrapolate downward?

If X follows Pareto (OP,α):

$$\frac{freq(x)}{freq(OP)} = \left(\frac{OP}{x}\right)^{\alpha}$$

We can also calculate frequency at Deductible from the frequency at Event, with min(Event,Ded)≥OP, as:

$$freq(Ded) = \left(\frac{Event}{Ded}\right)^{\alpha} freq(Event)$$

Then, the expected loss to layer depends only on: α, deductible and limit.

So, it seems that we can define a scenario based on any event, “no matter” what the OP is…

Yes, supposing that we can define an OP !

Is it right to extrapolate downward?

Assumptions

N number of losses

X_i for $i = 1..n$ loss amount for the i[th] loss, iid and independent of N

With $N_u = \sum_{i=1}^{N} I_{(X_i > u)}$ as number of losses that exceed a given amount

We can define the excess frequency at u :

$$freq(u) = E[N_u] = E(N).E(I_{X_i > u}) = E(N).P(X > u)$$

Yes, as conditional probabilities are used

While using Pareto (OP,α), we calculate probabilities knowing that X ≥ OP

$$P(X \geq x / X \geq OP) = \frac{P(X \geq x)}{P(X \geq OP)} = \frac{freq(x)}{E(N)} \cdot \frac{E(N)}{freq(OP)} \quad \text{for } x \geq OP$$

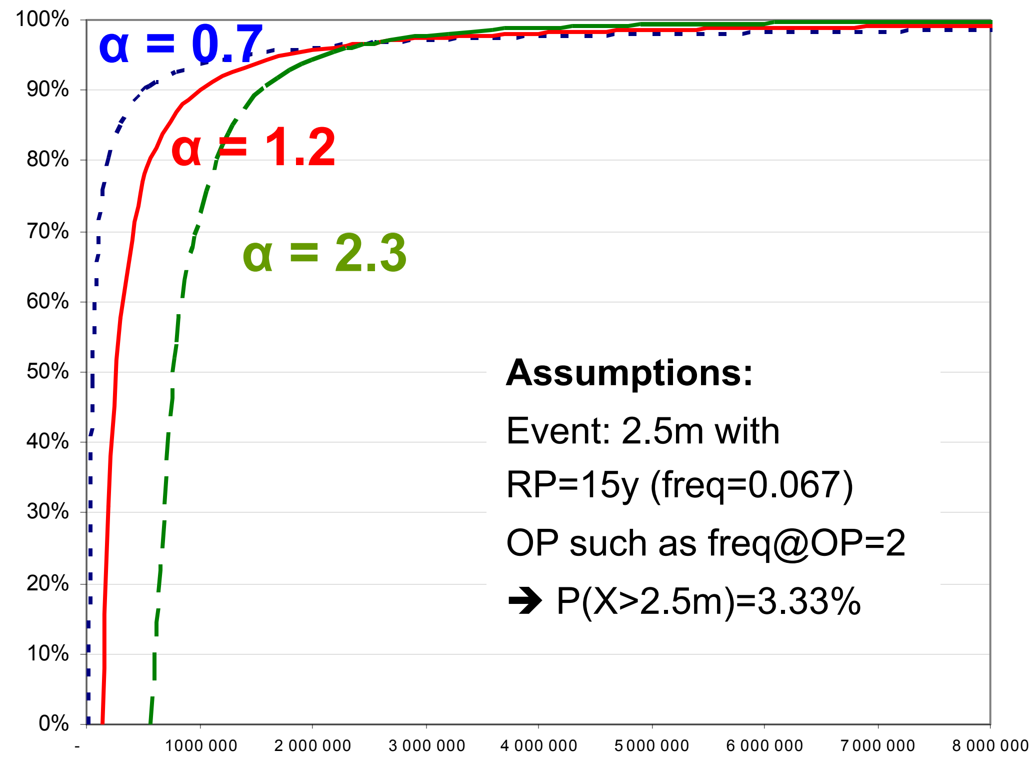
$$P(X \geq x / X \geq OP) = \frac{freq(x)}{freq(OP)}$$

Alpha selection

The choice of alpha depends on:

1. The kind of event:
   - Cat: α<1 (event with infinite expected value)
   - Fire : 1<α<2
2. The kind of approach: conservative or not ?

Pareto distributions



- Upward extrapolation : α↓ = conservative
- Downward extrapolation : α↑ = conservative

Sensitivity to alpha

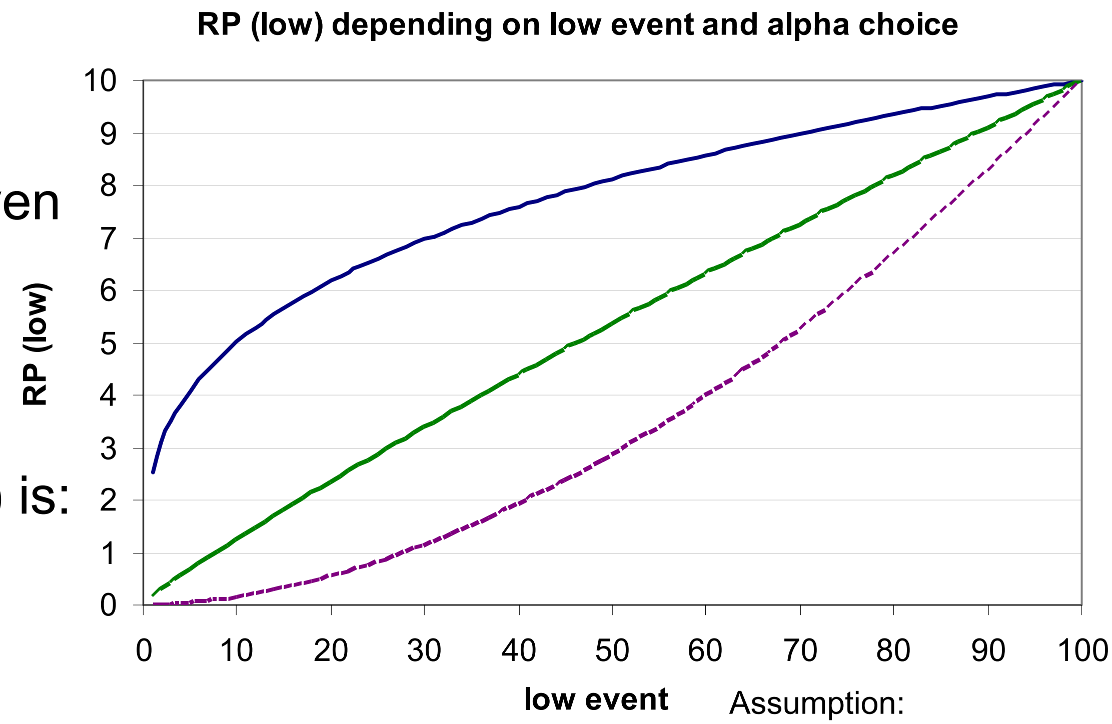
Assuming we define a big event with a fixed RP=RP(big).

The RP of a low event is given by:

$$RP(low) = RP(big).\left(\frac{low}{big}\right)^{\alpha}$$

So, the deviation of RP(low) is:

$$\frac{\Delta RP(low)}{\Delta RP(big)} = \left(\frac{low}{big}\right)^{\alpha}$$



Assumption:
Big Event = 100 with RP=10

The choice of the interval of study is crucial !

Pricing approaches of per event exposures

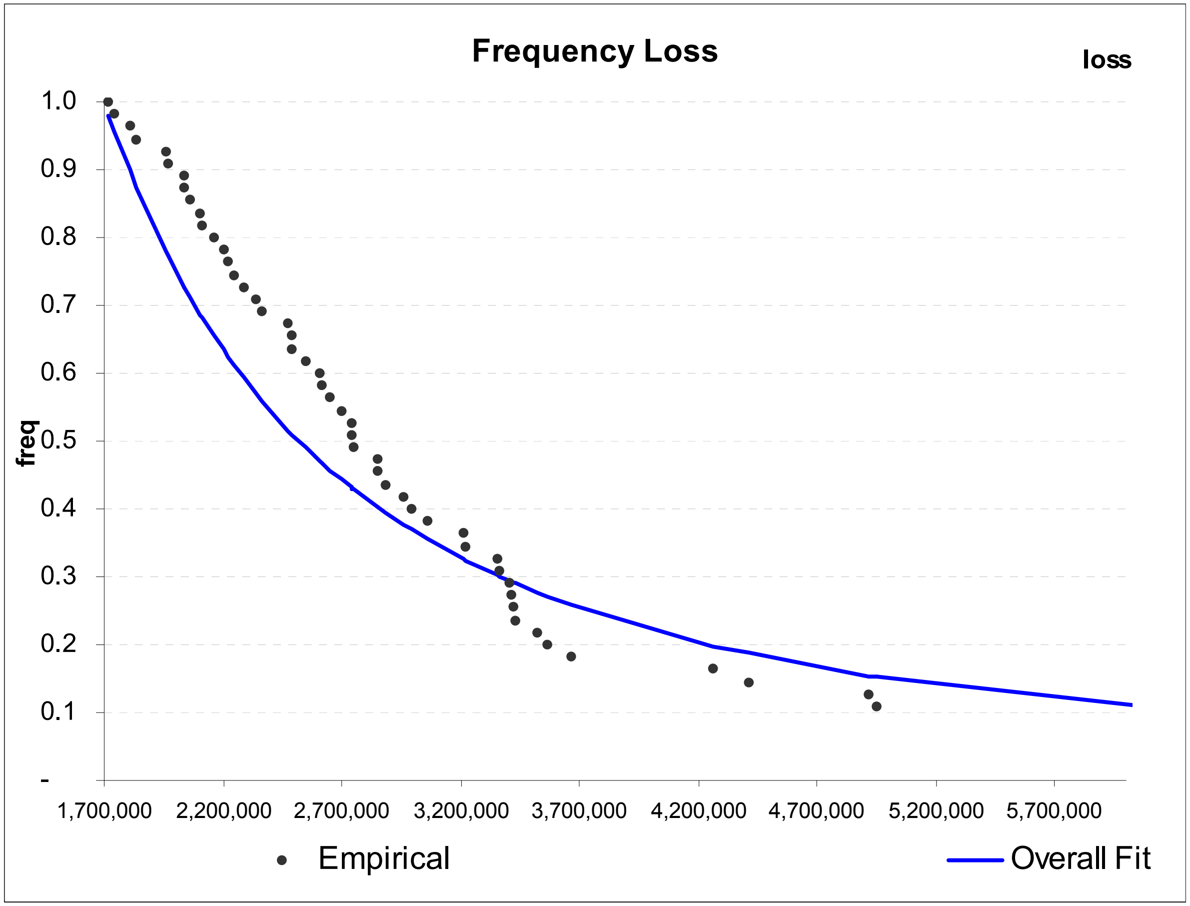
1. One scenario

- One curve "fits" the whole program
- Up Fit + straight extrapolation of the event scenario

2. Two scenarios

- Scenarios fixed independently, one bottom and one up
- Set a **(RP,OP)** for the lower part + compute the implicit alpha from another **event scenario** / a given **RROL**

One curve fits all approach

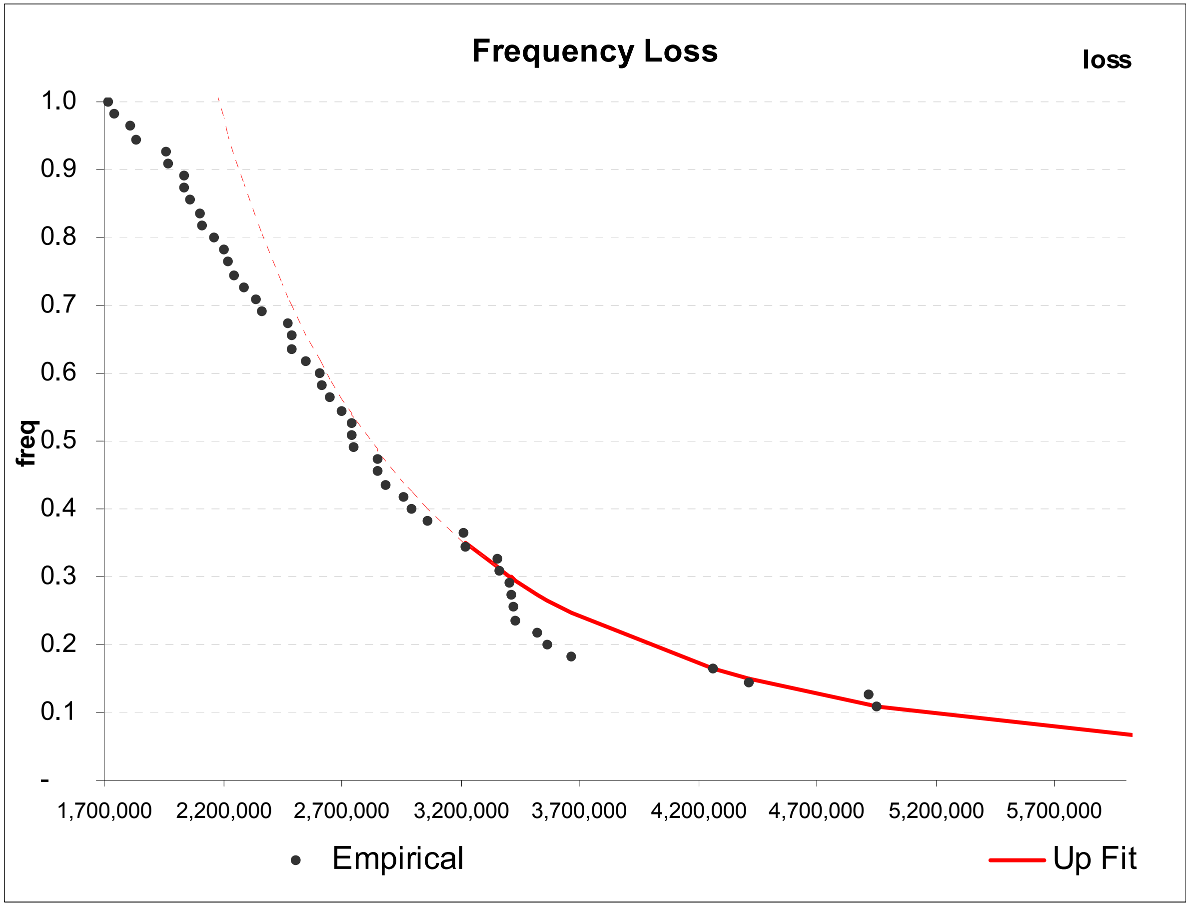


Overall Fitted Alpha=1.76

- too cheap below 3.3m,
- too expensive over 3.3m

➔ Try to be closer to the experience

Up Fit approach

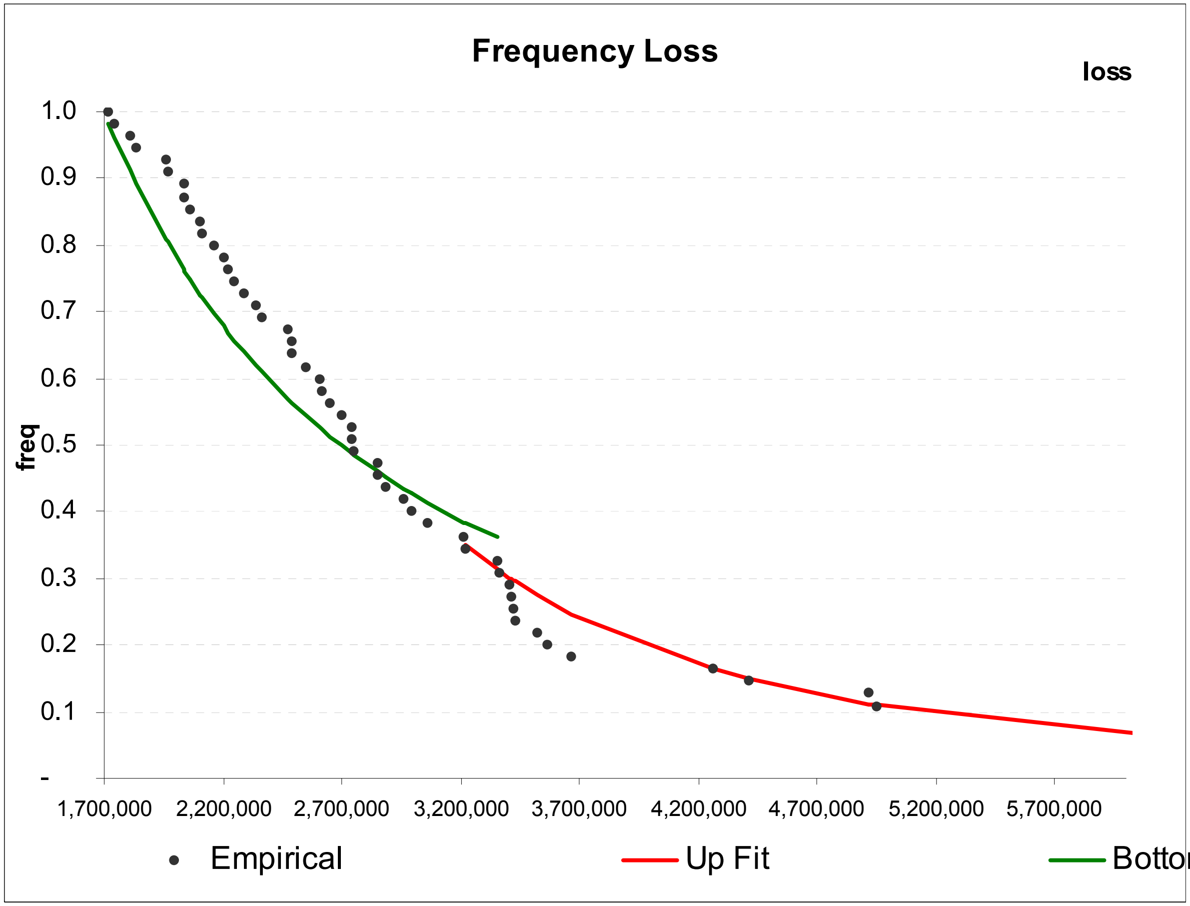


Right Tail Fitted
Alpha=2.7

- OK above 3.3m,
- too expensive below

➔ Need a better fit for the lower part

Two scenarios: one bottom and one up

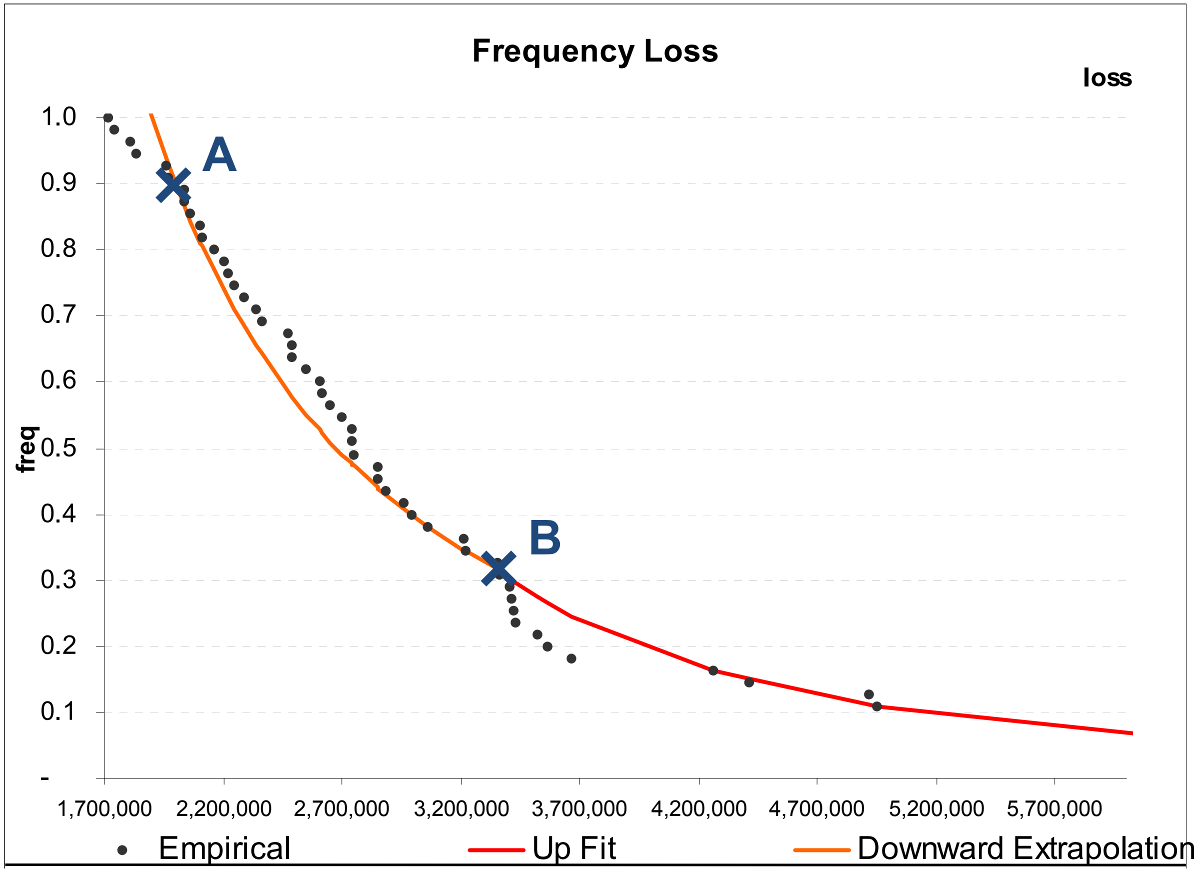


Left Tail Fit
Alpha=1.5

- OK below 3.3m,
- Gap at 3.3m

➔ 2 scenarios set up independently leads to inconsistency

Downward extrapolation approach



Two Scenarios:

(2m, Alpha 2.02, Freq 0.9) to price downward **(equivalent to 3.3m, Alpha 2.02, Freq 0.33)**

(3.3m, Alpha 2.7, Freq 0.33) to price upward

Compute the implicit Alpha to connect A and B

Alpha=2.02

- Ensure continuity at 3.3m
- Best Fit

➔ Can further refine between A and B

A priori RROL guidance

If constraints on RROL of the layers

e.g. min RROL on top layers

→ Select α with 2 scenarios:

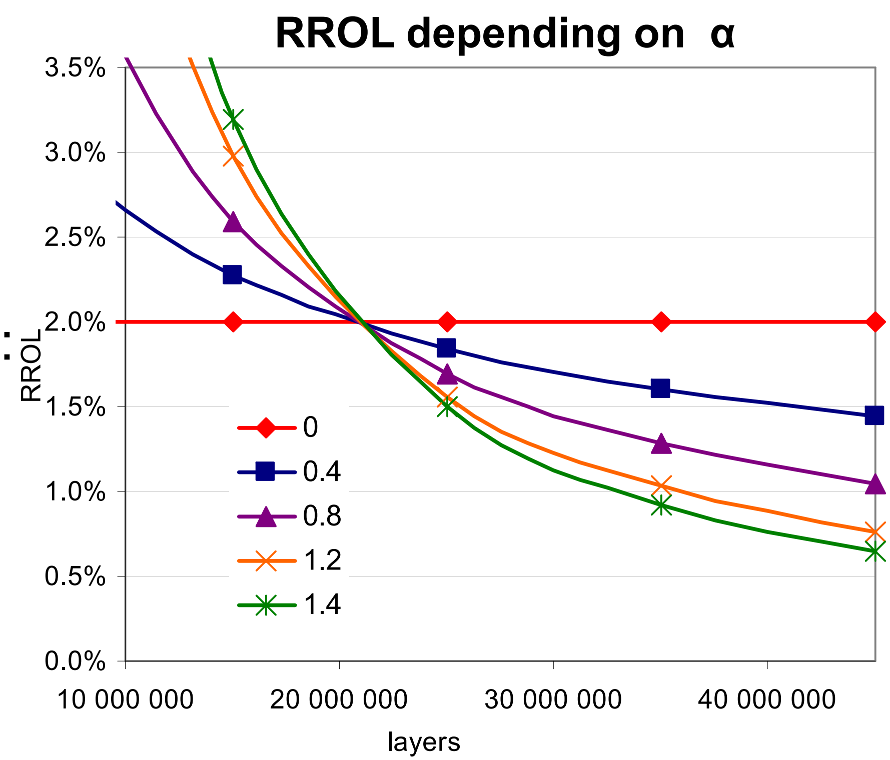
- point 1 (event1, freq1)
- point 2 (ded2, lim2, RROL2) with:

$$\alpha = -\frac{\ln RROL_2 - \ln freq_1}{\ln GLM_2 - \ln event_1}$$

Where GLM2 is the Geometrical Layer Mid point given by:

$$freq(GLM_2) \approx RROL_2$$

$$GLM_2 = \sqrt{ded_2(ded_2 + lim_2)}$$



For Min RROL > 1% for all layers

→ choose α < 0.8

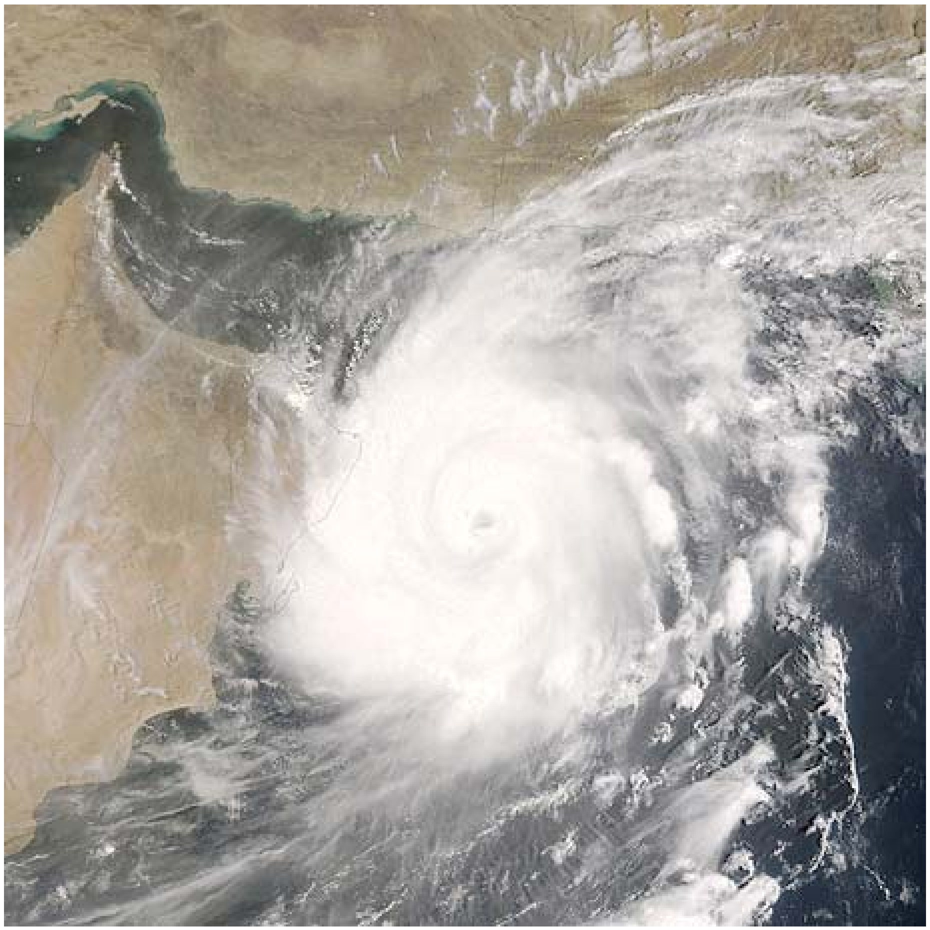
It's the same !

Pricing approaches of per event exposures

1. One scenario
   - One curve "fits" the whole program
     - Overall right, simple and pragmatic
     - BUT Possibly far from reality for specific segments
   - Up Fit + straight extrapolation of the event scenario
     - Robust a priori alpha and consistent scenarios to price up
     - BUT Marginal change up → exponential impacts down
2. Two scenarios
   - Scenarios fixed independently, one bottom and one up
     - Inconsistent and possible arbitrage opportunities
   - Set a **(RP,OP)** for the lower part + compute the implicit alpha from another **event scenario** / a given **RROL**
     - Realist model given the a priori RP and OP
     - Continuity between the two scenarios

Cylcone Gonu
Oman June 5, 2007

Cape Town Afrique du Sud
iCA 2010 CiA



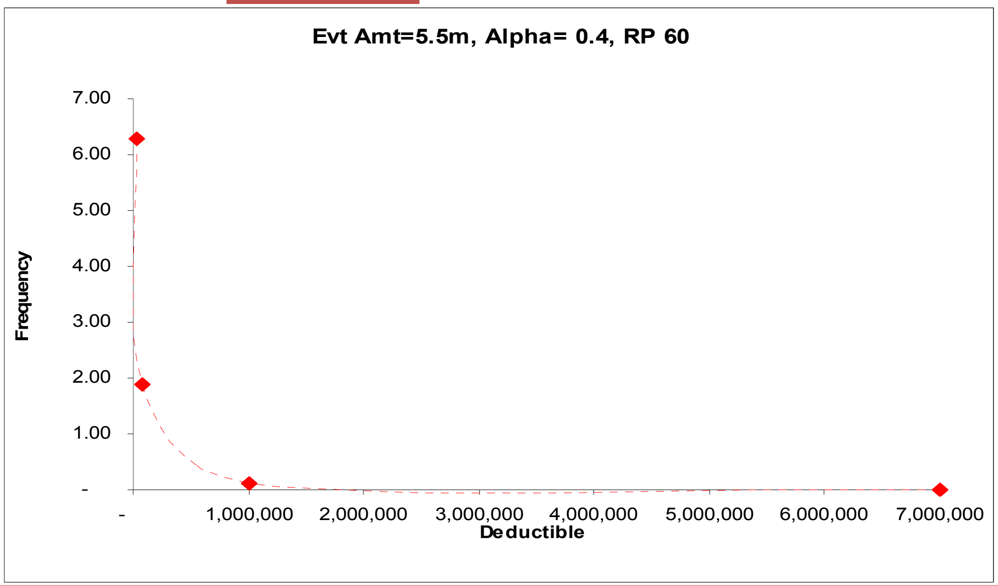
- **Strongest** cyclone in the Arabian Peninsula **in 60 years**[1]
- Record keeping started in 1945[1]
- **Extremely rare** in this part of the world, usually tend to be small and dissipate quickly.
- Caused Flooding and **heavy damage** in Oman, around $4 billion (2007 USD), worst natural disaster on record in Oman. [2]

1. Associated Press (2007). Jun. 5-Strongest Cyclone in 60 Years Hits in Arabian Sea
2. Middle East North Africa Financial Network (2007). "Oman suffers $4b from Cyclone Gonu". http://www.menafn.com/qn_news_story_s.asp?StoryId=1093156726. Retrieved 2007-06-18.

Image courtesy NASA/Jeff Schmaltz, MODIS Rapid Response Team, Goddard Space Flight Center

Gonu Scenario Example – RP at low Deductible issue

Layer	L1	L2	L3	L4
Limit	50,000	225,000	6,000,000	10,000,000
Deductible	25,000	75,000	1,000,000	7,000,000
Return Period	60	60	60	60
Alpha	1.10	1.10	1.10	1.10
OP (GONU)	5,500,000	5,500,000	5,500,000	5,500,000
FQ(Deductible)	6.288	1.878	0.109	0.013
Return period	0.2	0.5	9.2	78.2



Gonu Scenario: Amt 5.5m, RP 60, α 1.1

Induce 25k (USD 65k) Wind/Flood loss every 2 ½ months in Oman.

A priori 1.1 alpha set-up to extrapolate up.

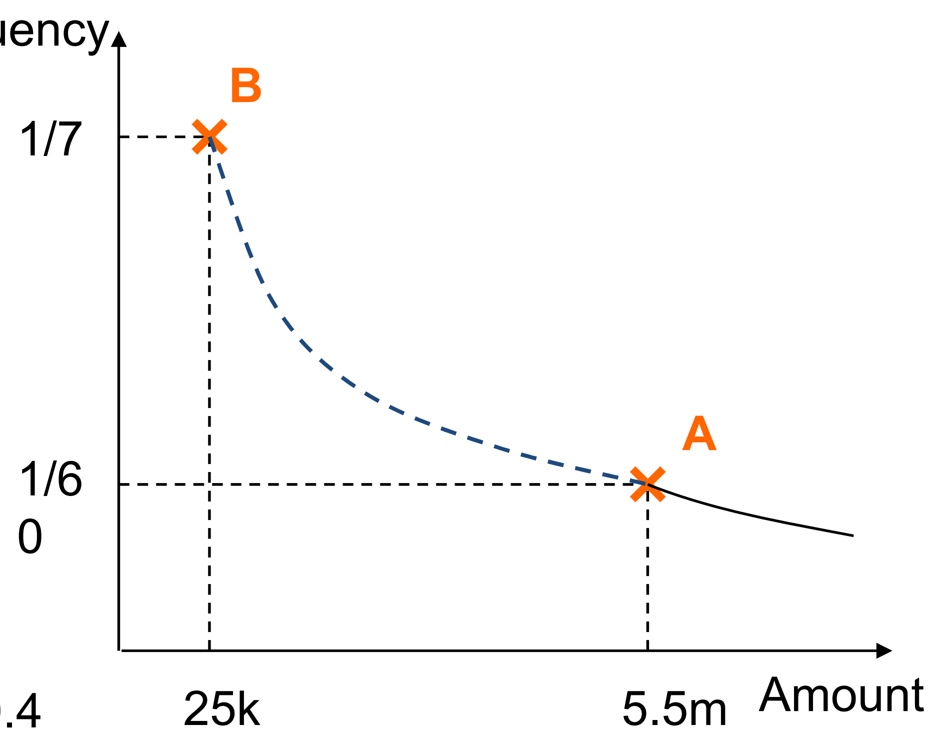
➔But we want to assume a RP of 7 yrs at 25k

Gonu Scenario Example – How to connect 2 scenarios?

Objective: find the right alpha when two points A and B (Return Period, Event Amount) are provided.

$$\alpha = \frac{\ln\left(\frac{\text{Return Period A}}{\text{Return Period B}}\right)}{\ln\left(\frac{A}{B}\right)}$$

Amount A	5,500,000
Return Period A	60
Amount B	25,000
Return Period B	7
Implied Alpha	0.40

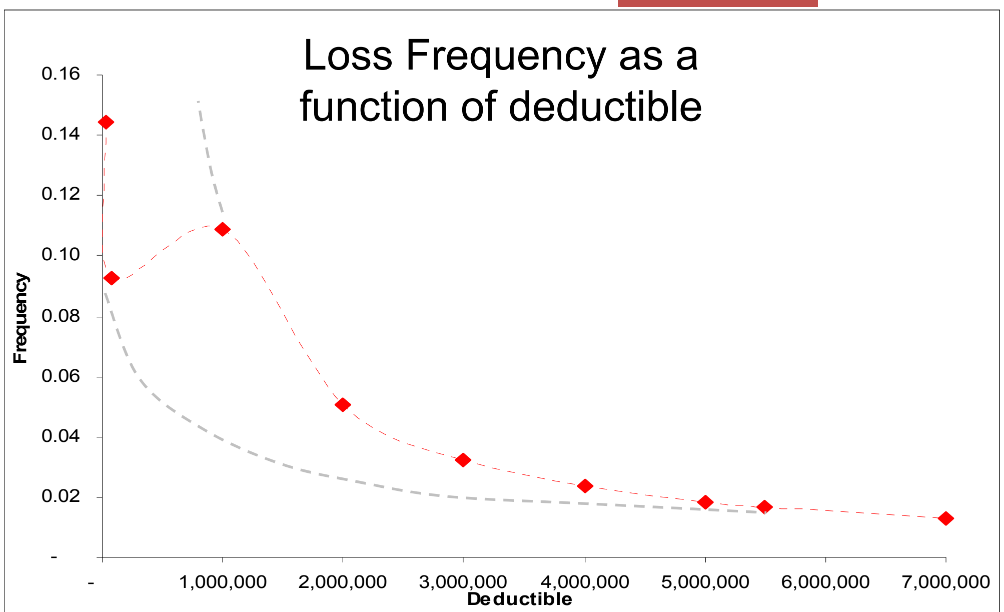


Alpha = ln(5500000/25000)/ln(60/7) = 0.4

Scenarios (5.5m, RP 60, Alpha 0.4) ⇔ (25k, RP 7, Alpha 0.4)

Gonu Scenario Example – RP at low Deductible issue

Layer	L1	L2	L3	L4
Limit	50,000	225,000	6,000,000	10,000,000
Deductible	25,000	75,000	1,000,000	7,000,000
Return Period	60	60	60	60
Alpha	0.40	0.40	1.10	1.10
OP (GONU)	5,500,000	5,500,000	5,500,000	5,500,000
FQ(Deductible)	0.144	0.093	0.109	0.013
Return Period	6.9	10.8	9.2	78.2



Event amount lies in the 3rd Layer

2 scenarios:

- Amt 5.5m, RP 60, α 0.4
- Amt 5.5m, RP 60, α 1.1

Intersection @ Freq 1/60 and Amt 5.5m

Low scenario set-up to price below Evt Amt.

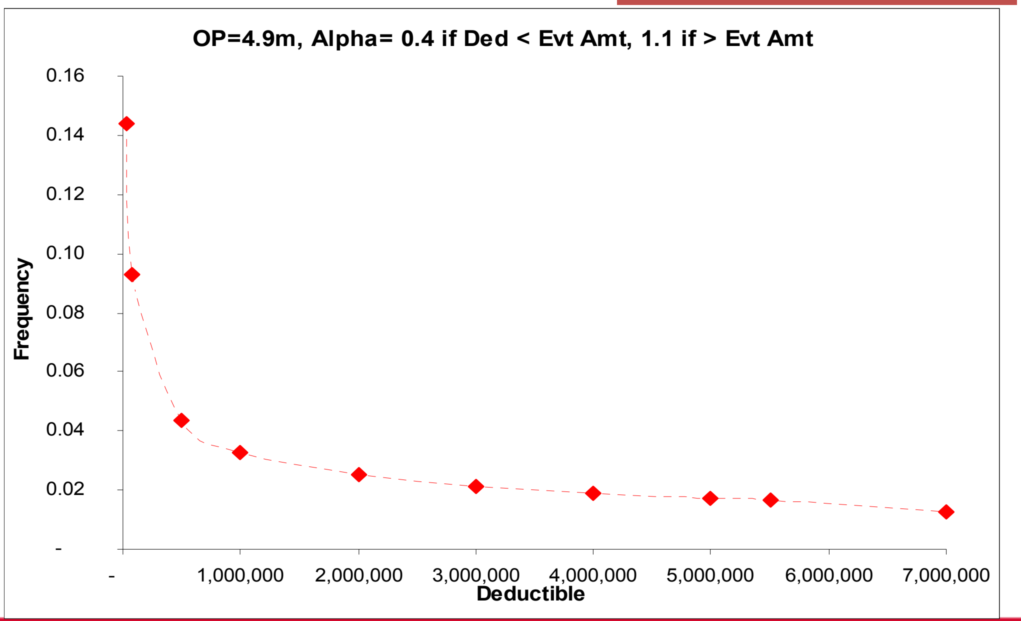
Apply it coherently !

Gonu Scenario Example – Intermediate Coherence

Layer	L1	L2	L3 A	L3 B	Cat L2
Limit	50,000	225,000	4,500,000	1,500,000	10,000,000
Deductible	25,000	75,000	1,000,000	5,500,000	7,000,000
Return Period	60	60	60	60	60
Alpha	0.40	0.40	0.40	1.10	1.10
OP (GONU)	5,500,000	5,500,000	5,500,000	5,500,000	5,500,000
FQ(Deductible)	0.1442	0.0929	0.0330	0.0167	0.0128
Return Period	6.9	10.8	30.3	60.0	78.2

In case the Event amount lies in a layer

Split it in 2!



Caution

•Theoretically right to extrapolate downward assuming that a lower OP exists.

•But the implied frequency at OP should be monitored as the event scenario with an a priori alpha is fitted to extrapolate up.

•It's right to pick different Pareto curves depending on the layers but it's a must to ensure their continuity.

Recommendations

Realism: Monitor the Return Period implied by the Pareto Scenario, Think if the event is a hit or miss or if small losses might occur, Come-up with an a priori on RP and OP for the bottom part.

Continuity: Connect the bottom scenario(s) to the upper one(s) by computing the implied Alpha to avoid arbitrage opportunities.

Coherence: Split a layer in case an OP would lie into it.

Consistency: Apply the same a priori bottom (RP,OP) across pricing of one market segment.

Suggestions:

Plot a Frequency vs. Severity chart for each scenario

Check the return period at deductible for each scenario and layer

Contact Details

Marc Bagarry

marc.bagarry@partnerre.com

Phone +33 1 44 01 68 83

Fax +33 1 44 01 17 03

Léa Nefussi

lea.nefussi@partnerre.com

Phone +33 1 44 01 68 56

Fax +33 1 44 01 68 63

References

- J. Blondeau, C. Partrat (2003), La Réassurance- Approche technique, Economica
- P. Antal (2003), “Quantitative Methods in Reinsurance”
- H. Schmitter, P. Bütikofer (1998) “Estimating property excess of loss risk premiums by means of the Pareto model”
- M. Schmutz, R. Doerr (1998), “The Pareto model in property reinsurance”

Appendix

A priori RROL guidance

Method 1

By solving the equation (excel,…): $\frac{1}{\text{lim.RP}_1} \cdot \frac{\text{event}_1^{\alpha}(\text{RL}^{1-\alpha}-1)}{\text{ded}^{\alpha-1}(1-\alpha)} = \text{RROL}$

Method 2

By approximating RROL

For every layer, we have: freq(EP) < RROL < freq(ded)

As frequencies decrease at increasing deductibles, it exists x such as: RROL=freq(x)

x is approximated by the “geometrical layer mid-point”:

$$x \approx \text{GLM} = \sqrt{\text{ded.EP}} = \sqrt{\text{ded.(lim} + \text{ded)}}$$

Thus, $\text{freq(GLM)} \approx \text{RROL}$

Then, we can use the formula: $\alpha = \frac{\ln(\text{freq(A)}) - \ln(\text{freq(B)})}{\ln B - \ln A}$

Then, assuming: ded_1, lim_1; ded_2, lim_2 and their RROL: RROL_1 , RROL_2, the parameter α is given by:

$$\alpha = \frac{\ln \text{RROL}_2 - \ln \text{RROL}_1}{\ln\left(\frac{\sqrt{\text{ded}_1(\text{ded}_1 + \text{lim}_1)}}{\sqrt{\text{ded}_2(\text{ded}_2 + \text{lim}_2)}}\right)}$$