



Models of Development of Losses in the Worst Condition by Kinds with Long Settlement - a modification method of the nearest neighbour

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Abstract



The work is devoted to construct and investigate a new method of calculating a size of losses by kinds with long settlement: a modification method of the nearest neighbour. The proposition method is to define a size of losses in the worst condition of system and is based on using of so-called model of numbers tree. By using this method for model, data showed and carried out some computational experiments.



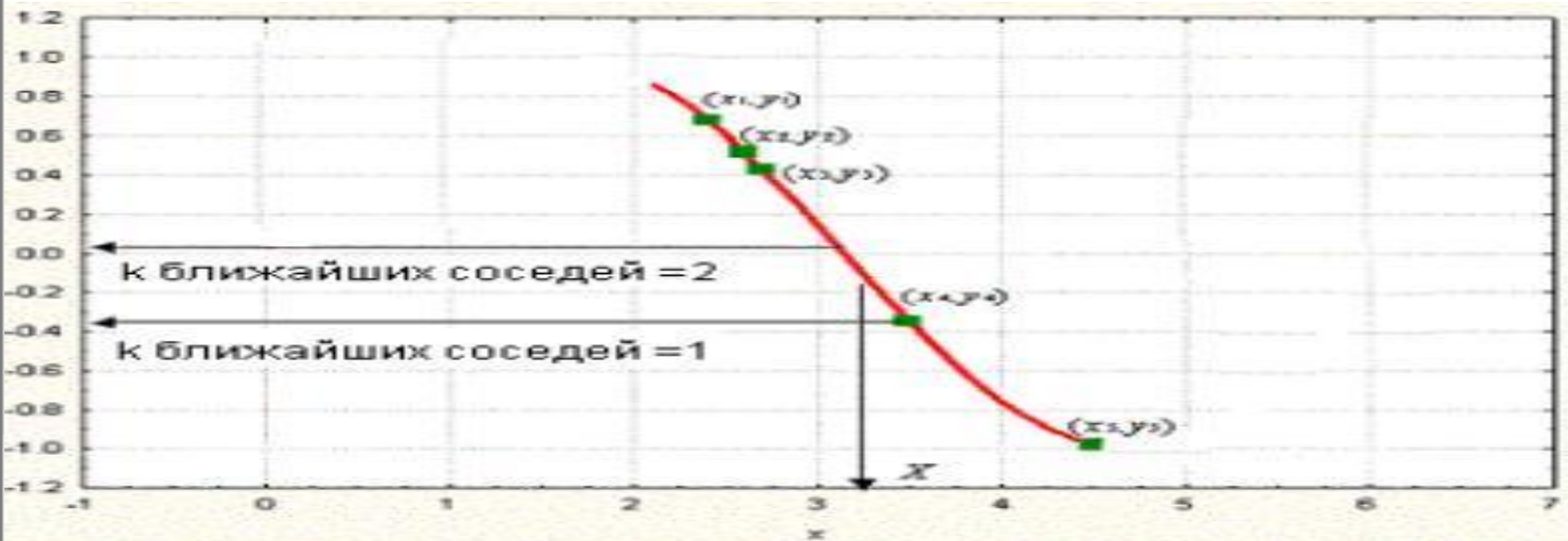
Introduction

- Method of the nearest neighbour (NN - Nearest Neighbor). It is the most widespread method of comparison and extraction of precedents. It rather easily allows to calculate a degree of similarity of the current problem situation and precedents from BP (Libraries of precedents - the saved up experience) systems. With the purpose of definition of a degree of similarity on set of the parameters used for the description of precedents and the current situation, the certain metrics is entered. Further, according to the chosen metrics, the distance from the target point appropriate to the current problem situation up to the points representing precedents from BP is defined, and the nearest gets out to target a point.
- The method of definition of the nearest neighbour (the nearest neighbours) also is widely applied to the decision of problems of classification, clusters, regresses and recognition of images.

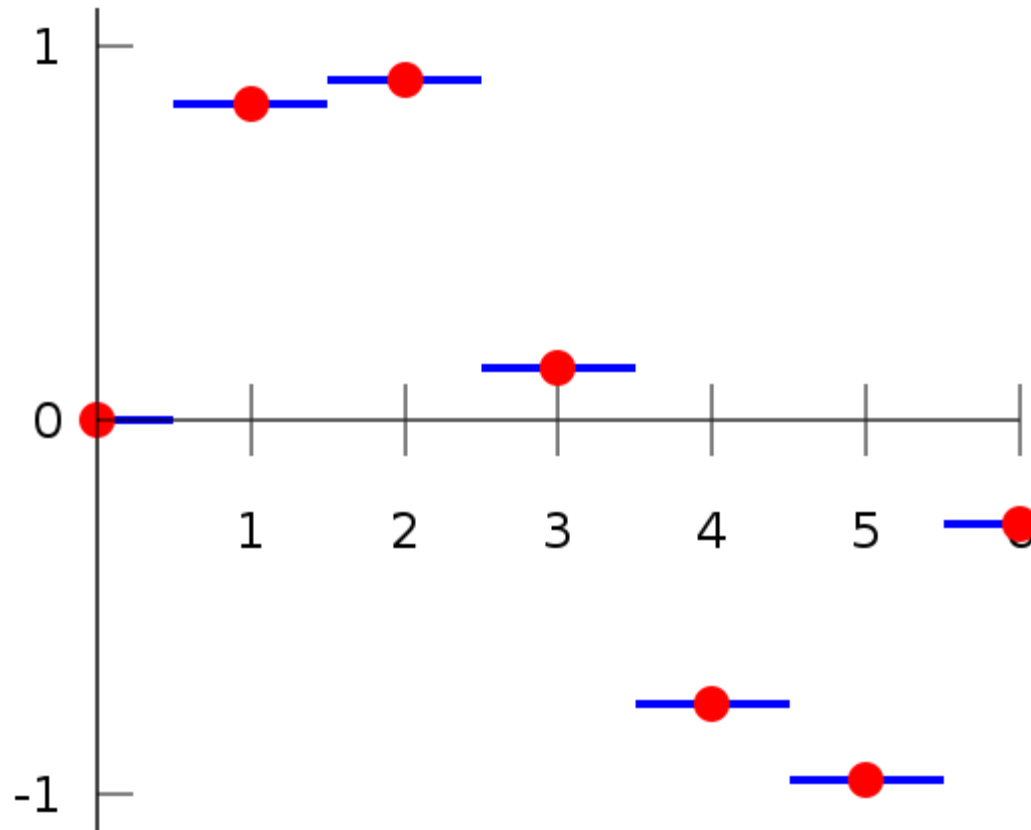
From all the methods, the method of the nearest neighbour is most distributed. This method is known also under the name, a method of single connection.

- Cross-check - a known method of reception of estimation of unknown parameters of model. The basic idea - division of the sample of data on v "Warehouse part". V " Warehouse parts " here essence in the casual image allocated and isolated samples. Method of " the nearest neighbour ". An estimation of parameter k , a method of cross-check

Method of " the nearest neighbour ".
The decision of a problem of forecasting.



The method of the nearest neighbour - the most simple method of interpolation of function of one or several variables. As the interpolated value the nearest known value of function gets out.



In most general view algorithm the nearest neighbours is

$$a(u) = \operatorname{argmax}_{y \in Y} \sum_{i=1}^m [x_{i;u} = y] w(i, u),$$

$w(i, u)$ Is given weight function which estimates a degree of importance

$x_{i;u}$ is object of training sample which

u

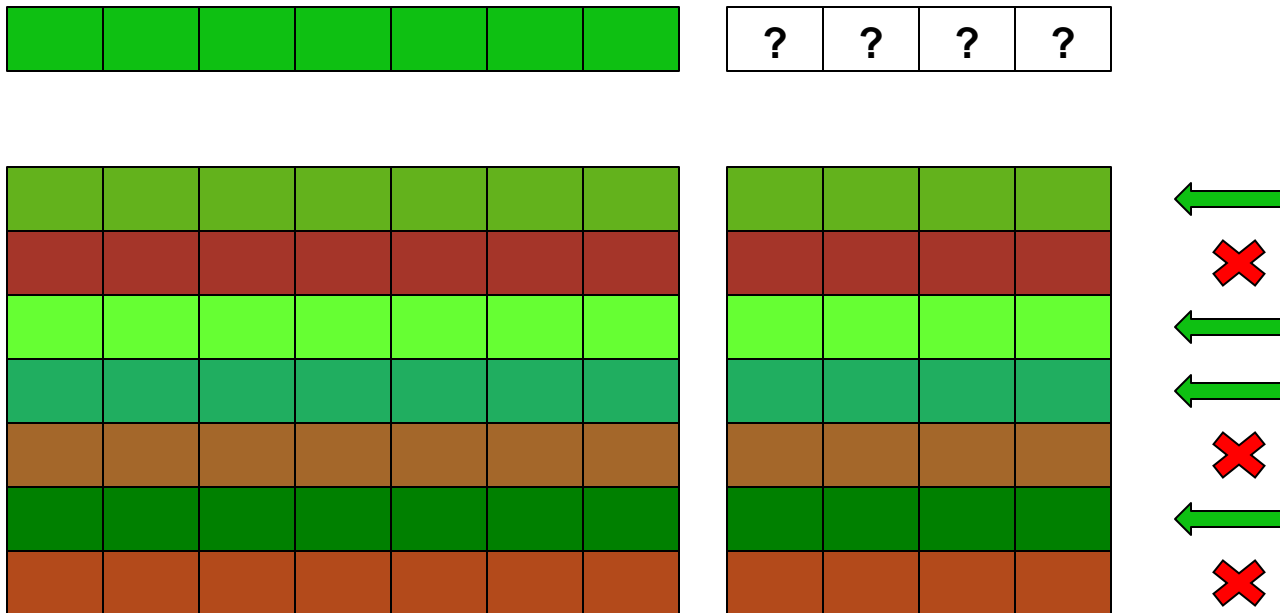


As is known, the forecast of the future losses, using observably average development of the nearest neighbours, usually define under the following circuit:

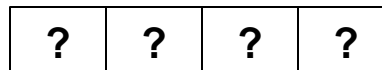
Irena Kaiser Instructions of the Eurounion on compensation of the harm caused to life and health and tariffing OSAGO. V the international conference « Obligatory insurance of a civil liability of owners of vehicles in the Russian Federation: tariffing and regulation. The first actuaries congress of the CIS » November, 27-28, 2008, Moscow.

Methods for development of losses by kinds with long settlement method of the nearest neighbour

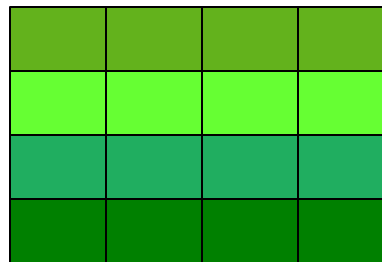
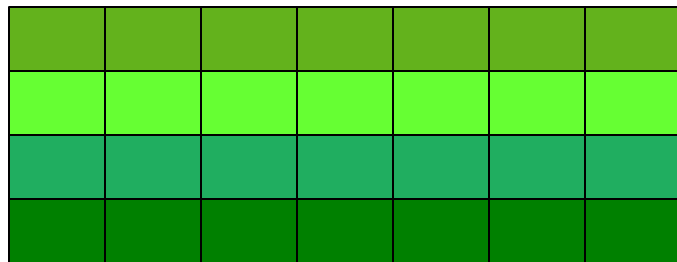
The forecast of individual losses by direct comparison with the observably tendency of losses. Identification k, similar losses given supervision. Use of a method of the nearest neighbour for development of the considered loss



- *The forecast of the future losses using observably average development of the nearest neighbours*



Y



Y_1

\vdots

Y_k

$$Y = w_1 \cdot Y_1 + \dots + w_k \cdot Y_k \quad \text{for} \quad w_1 + \dots + w_k = 1$$

On the basis of given to the circuit the model of the nearest neighbour assumes the following

$$Y = \sum_{i=1}^k \omega_i Y_i, \text{ where}$$

$$\omega_i \geq 0, i = 1, \dots, k; \quad \sum_{i=1}^k \omega_i = 1 \quad (1)$$

We shall consider more general model, than model (1):

$$Y_k = \left(\sum_{i=1}^k \omega_i Y_{ik}^s \right)^{1/s} \quad \sum_{i=1}^k \omega_i^{\frac{n}{n-s}} = 1, \quad n > s > 0, \quad ; k=2,3, \dots \quad (2)$$

Model of the worst development of losses

$$Y_k = \max_{\omega \in M} \left(\sum_{i=1}^k \omega_i Y_{ik}^s \right)^{1/s} \quad (4)$$

$$M = \left\{ \omega : 0 \leq \omega_j \leq 1, \sum_{j=1}^k \omega_j^{\frac{n}{n-s}} = 1, n > s, s > 0, k > 1 \right\}$$

The equation (4), we shall call “*Model of the worst development of losses*”.

Theorem 1



The equation (4):
and the equation

$$Y_k = \max_{\omega \in M} \left(\sum_{i=1}^k \omega_i Y_{ik}^s \right)^{1/s}$$

$$Y_k^n = \sum_{i=1}^k Y_{ik}^n \quad (5)$$

are equivalent.

Algorithm of solution of the equation of worst development of losses:



Let $X' = \mu_t X$ which is connected with discrete model equations

$$\sum_{i=1}^m X_{im}^n = Z_m^n,$$

where

$$X' = (X'_{1m}, \dots, X'_{mm}, Z'_m), \quad Z'_m = \left(\sum_{i=1}^m X'_{im}{}^{1/t} \right)^t,$$

$$X = (X_{1m-1}, X_{2m-1}, \dots, X_{m-1m-1}, Z_{m-1}, Z_{m-1}) \quad Z_{m-1} = \left(\sum_{i=1}^{m-1} X_{im-1}{}^{1/t} \right)^t$$

$$\mu_t = \begin{pmatrix} x^t & \mathbf{O} & \dots & \mathbf{O} & \mathbf{O} & \mathbf{O} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \mathbf{O} & \mathbf{O} & \vdots & x^t & \mathbf{O} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} & \vdots & \mathbf{O} & y^t & \mathbf{O} \\ \mathbf{O} & \mathbf{O} & \vdots & \mathbf{O} & \mathbf{O} & z^t \end{pmatrix}$$

Theorem 2



For any natural $n > 1$ between to the set of solutions (4),
i.e. under $m = k - 1$ and $m = k$ it takes place next presentation:

$$\tilde{x}_{jk} = \tilde{x}_{12} \tilde{x}_{jk-1}, \quad \tilde{x}_{kk} = \tilde{x}_{22} \tilde{z}_{k-1}, \quad \tilde{z}_k = \tilde{z}_2 \tilde{z}_{k-1}, \quad j = 1, 2, \dots, k - 1$$

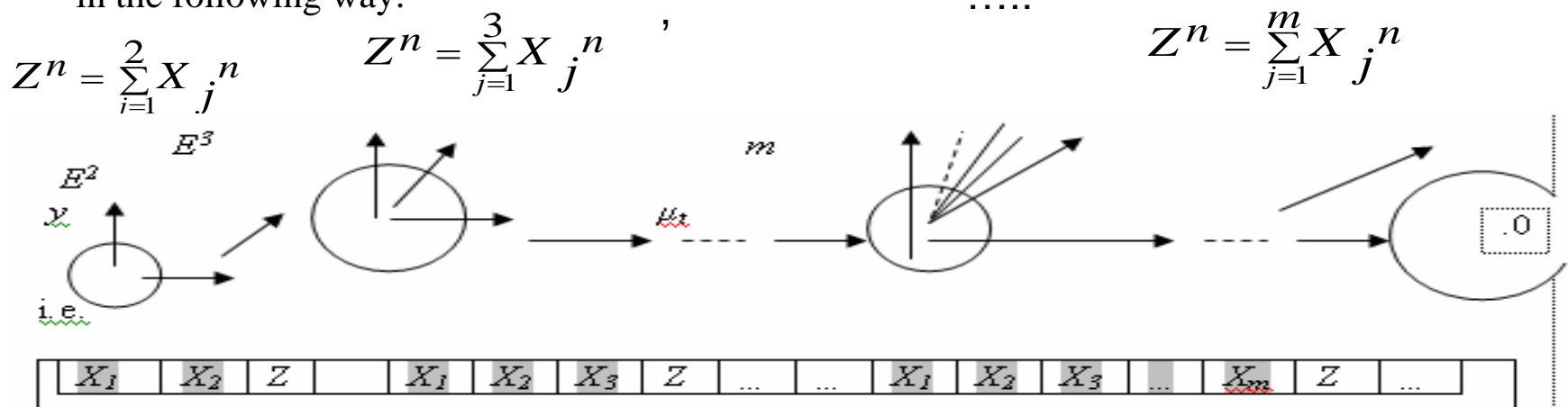
This transformation may be written in the form of

$$\tilde{X}'_m = \mu_t \tilde{X}_m$$

Proposition

- The transformation of μt transfers arbitrary point E_{m-1} and into some corresponding points from E_m metrics for all $m=3,4,5,\dots$ and has semi-group properties: $0 \leq t \leq tk$, $0 \leq s \leq tk$, and it is linear, uniformly bounded and uniformly continued. Besides, the engine - values of μt are represented in the form of: $\lambda_j = xt$, $j=1,2,\dots,k-1$; $\lambda_k = yt$; $\lambda_{k+1} = zt$; $\|\mu t\| = zt$; $\|\mu t^{-1}\| < \infty$ 2)
- The infinitesimal generating operator $A = \lim_{t \rightarrow 0} t^{-1}(\mu_t - I)$ is diagonal matrix and represented in

the following way: $a_{ii} = \ln x$, $i=1,\dots,k-1$, $a_{kk} = \ln y$, $a_{k+1,k+1} = \ln z$, and what is more $R(m,A) = (qI - A)^{-1}$, $\mu t = e^{tA}$, $A = \ln \mu t / t$. 3). The transformation $M = \mu t / t$ is also transferred E_{k-1} into E_k under corresponding condition $x1/t + y1/t = z1/t$, $0 \leq t \leq tk$, $tk < \infty$, 4). The transformations μt , μt^{-1} may be used for coding and decoding of corresponding input and output information. It is characterized in the following way:



From the developed method follows, that if we know any solution of the equation (4)

the appropriate decisions at $k=m$ is defined under the formula $X' = K X$,

where $X' = (X'_{1m}, \dots, X'_{mm}, Z'_m)$, $Z'_m = \left(\sum_{i=1}^m X'_{im} \right)^{1/n}$ $Z_{m-1} = \left(\sum_{i=1}^{m-1} X_{im-1}^n \right)^{1/n}$

$X = (X_{1m-1}, X_{2m-1}, \dots, X_{m-1m-1}) \quad m=2,3,\dots,$

$$K = \begin{bmatrix} x & 0 & 0 & \dots & \dots & \dots & 0 & 0 & 0 \\ 0 & x & 0 & \dots & \dots & \dots & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & \dots & \dots & x & 0 & 0 \\ 0 & 0 & 0 & \dots & \dots & \dots & 0 & y & 0 \\ 0 & 0 & 0 & \dots & \dots & \dots & 0 & 0 & z \end{bmatrix}$$

Such any representation of type (4), (5), (6) with help of transformation (9) is transferred to the form

$$Y_m^n = \left(x^{m-1} \right)^n + \sum_{i=2}^m \left(yx^{m-i} z^{i-2} \right)^n$$

x, y, z - are the solutions of the equation $z^n = x^n + y^n$. For example, in a case $n=2$ at any $m=2,3,4,\dots$, knowing the decision, equation $Y_1^2 + Y_2^2 = Y^2$ on the basis of transformation (9), (10), (11) gradually we shall define the decisions of the equations $Y_1^2 + Y_2^2 + \dots + Y_m^2 = Y^2$. At $m=2$ the equation $Y_1^2 + Y_2^2 = Y^2$ we have obvious decisions such as [1]:

Y_1	Y_2	Y	Y_1	Y_2	Y
3κ	4κ	5κ	$4m$	$15m/2$	$17m/2$
5κ	12κ	13κ	$6m$	$35m/2$	$37m/2$
7κ	24κ	25κ	$8m$	$63m/2$	$65m/2$
9κ	40κ	41κ	$10m$	$99m/2$	$101m/2$
...

Here κ - natural number, $m = 2\kappa$.

At $m = 3, 4, 5, \dots$, for $Y_1 = 3, Y_2 = 4, Y = 5$ the appropriate solutions Y_1, Y_2, \dots, Y_m, Y are given in the following tables:

$m = 3$	$m = 4$	$m = 5$
$Y_1 = 9$	$Y_1 = 27$	$Y_1 = 81$
$Y_2 = 12$	$Y_2 = 36$	$Y_2 = 108$
$Y_3 = 20$	$Y_3 = 60$	$Y_3 = 180$
$Y = 25$	$Y_4 = 100$	$Y_4 = 300$
	$Y = 125$	$Y_5 = 500$
		$Y = 625$

$Y_1 = 9$ $Y_2 = 12$ $Y_3 = 20$ $Y = 25$	$Y_1 = 27$ $Y_2 = 36$ $Y_3 = 60$ $Y_4 = 100$ $Y = 125$	$Y_1 = 81$ $Y_2 = 108$ $Y_3 = 180$ $Y_4 = 300$ $Y_5 = 500$ $Y = 625$
<i>m=9</i>	<i>m=10</i>	<i>m=11</i>
$Y_1 = 6561$ $Y_2 = 8748$ $Y_3 = 14580$ $Y_4 = 24300$ $Y_5 = 40500$ $Y_6 = 67500$ $Y_7 = 112500$ $Y_8 = 187500$ $Y_9 = 312500$ $Y = 390625$	$Y_1 = 19683$ $Y_2 = 26244$ $Y_3 = 43740$ $Y_4 = 72900$ $Y_5 = 121500$ $Y_6 = 202500$ $Y_7 = 337500$ $Y_8 = 562500$ $Y_9 = 937500$ $Y_{10} = 1562500$ $Y = 1953125$	$Y_1 = 59049$ $Y_2 = 78732$ $Y_3 = 131220$ $Y_4 = 218700$ $Y_5 = 364500$ $Y_6 = 607500$ $Y_7 = 1012500$ $Y_8 = 1687500$ $Y_9 = 2812500$ $Y_{10} = 4687500$ $Y_{11} = 7812500$ $Y = 9765625$

The Application of Model of Numbers Tree to Models of Development of Losses.

Let's enter definition a tree of numbers. Let N - some natural number. We shall tell, that the number N forms a tree of numbers if there will be natural numbers $n, m \geq 2$ and integers a_1, a_2, \dots, a_m for which

$$N^n = a_1^n + a_2^n + \dots + a_m^n, \quad (12)$$

and in turn some a_j (or all) represented as

$$a_j^n = a_{1j}^n + a_{2j}^n + \dots + a_{m_jj}^n, \quad m_j \leq m \quad (13)$$

and some a_{ij} of (13) also can be submitted as

$$a_{ij}^n = a_{1ij}^n + \dots + a_{m_2ij}^n, \quad m_2 \leq m_1, \dots,$$

and at last decomposition takes place

$$a_{i_1 \dots i_{m_k}}^n = a_{1i_1 \dots i_{m_k}}^n + a_{2i_1 \dots i_{m_k}}^n, \quad (14)$$

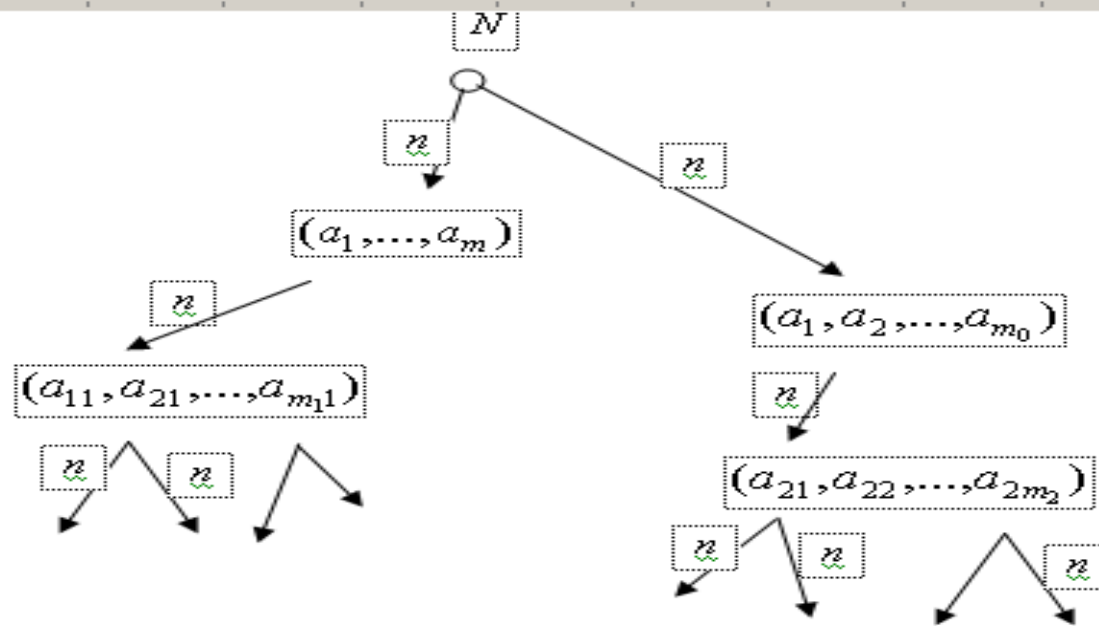


Fig.1. Conceptual Model of Numbers Tree in general case

So last a level the tree consists of the sum such as (14). base elements can enter into each level a tree, therefore from each level we take only those elements, which not представимы as (12), (13).

Then in result the number N is uniquely represented as

$$N^n = \sum_{j_0} k_{j_0} a_{\overline{w}_1 \dots j_0}^n, \quad (15)$$

where k_{j_0} - number of occurrence of a basic element $a_{\overline{w}_1 \dots j_0}$ in a tree of numbers.



We shall consider a problem for number N and a vector $a = (a_1, \dots, a_k)$, $k \geq 2$ from the equation $N = \max_{\alpha \in M} (\alpha, a)$, where

$M = \left\{ (\alpha_1, \dots, \alpha_k) = \alpha; \sum_{j=1}^k \alpha_j^{\frac{k}{n-s}} = 1, n > s > 0, 0 < \alpha_j < 1 \right\}$. The set M nd represents measured curvilinear spheroid and at $s=1$, $n=2$ turns in usual m-a measured spheroid. Using theorems 1 from work [2], we shall receive:

$$\begin{cases} N^n = a_1^n + \dots + a_k^n, \\ N_k^n = a_{1k}^n + \dots + a_{kk}^n, \quad k \geq 2 \end{cases} \quad (16)$$

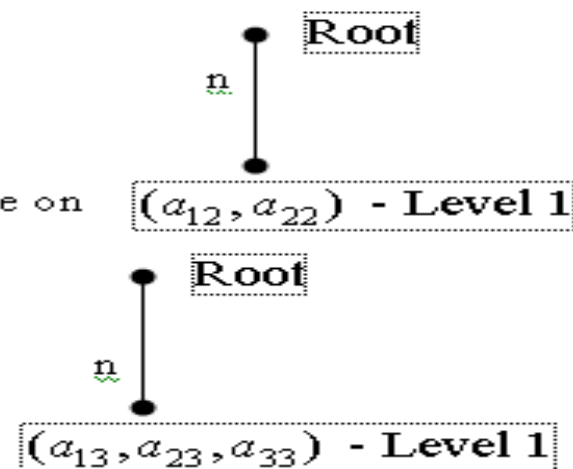
Thus, this equation is optimum in sense (16), and the tree of numbers appropriate to this equation represented on rice. 1 also is an optimum tree.

From the theorem 2 follows, that if we have a tree to the appropriate equation

$$N_2^n = a_{12}^n + a_{22}^n,$$

Then there is transformation K which to translate the given tree on

$$N_3^n = a_{13}^n + a_{22}^n + a_{33}^n,$$



etc. and the number of elements at top to the appropriate level 1 is increased by unit and therefore, the received trees are "growing", that corresponds to value $n = 2$.

At $n \geq 3$, most likely, it cannot be approved as the equation (16) integer numbers not solving. But here, we can instead of N_k^n take any natural number \tilde{N}_k and on the basis of Warring's theorem to receive representation [2]

$$\tilde{N}_k = a_{1k}^n + a_{2k}^n + \dots + a_{kk}^n, \quad (17)$$

and numbers such as « a root - level 1, a level 2, a level 3, ..., the level m » in this case can not exist. But there is a decision of a problem. For trees, such as « a root, a level 1 » in a case $n \geq 3$ we can make pasting with trees of numbers at $n = 2$ (see last tree).

We shall consider the equation

$$N_2 = a_{12}^n + a_{22}^n \quad (18)$$

and

$$N_m = a_{1m}^n + a_{2m}^n + \dots + a_{mm}^n, \quad m \geq 2, n \geq 2.$$

The Theorem 3. Transformation

$$\begin{cases} a_{im} = x a_{i,m-1}, & i = 1, m-1 \\ a_{mm} = y^n \sqrt{N_{m-1}} \\ N_m = z N_{m-1}, & \text{where } x^n + y^n = z^n \end{cases} \quad (19)$$

translates the solution of the equation

$$\sum_{i=1}^{m-1} a_{im-1}^n = N_{m-1} \quad (20)$$

on the solution of the equation

$$\sum_{i=1}^m a_{im}^n = N_m \quad (21)$$

Really, we shall increase both parts of the equation (10) on x^z with the account $x^z + y^z = z$ we have

$$\sum_{i=1}^{m-1} (a_{im-1}, x)^z = (z - y^z) N_{m-1} .$$

From here, valid (9) we shall receive (11) and more over we have

$$\begin{aligned} N_m &= z^{m-1}, \\ a_{1m} &= x^{m-1}, \quad a_{2m} = yx^{m-2}, \\ a_{im} &= yz^{\frac{i-2}{z}} x^{m-i}, \quad i = 1, \quad m = 3, \dots, m., \end{aligned}$$

Such any representation of type (18) with help of transformation (19) is transferred to the form

$$N_m = (x^{m-1})^z + \sum_{i=2}^m \left(yx^{m-i} z^{\frac{i-2}{z}} \right)^z \quad (22)$$

It should be out that transformations (17), (19) are described the process of numbers tree grows and formula (22) is formula for definition of the size Losses in the Worst Condition by Kinds with Long Settlement

Some examples of Numbers Tree.

Now we shall consider now examples of Numbers Tree for different numbers.

1). Let $N = 25$, $n = 2$, then we have

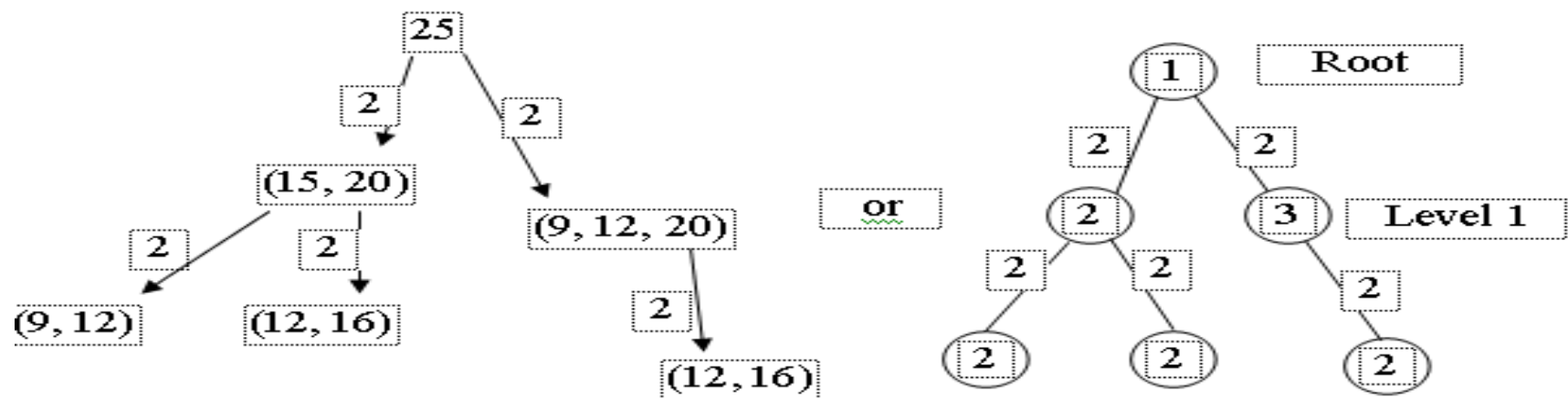


Fig.2. Model of Numbers Tree for $N = 25$, $n = 2$

\textcircled{k} - means quantity(amount) of an element of the given top of a tree, and number on edges saw decomposition. From here follows, that representation (4) takes the following kind

$$25^2 = 9^2 + 2 \cdot 12^2 + 16^2.$$

2). Now we shall consider number $N = 50$.

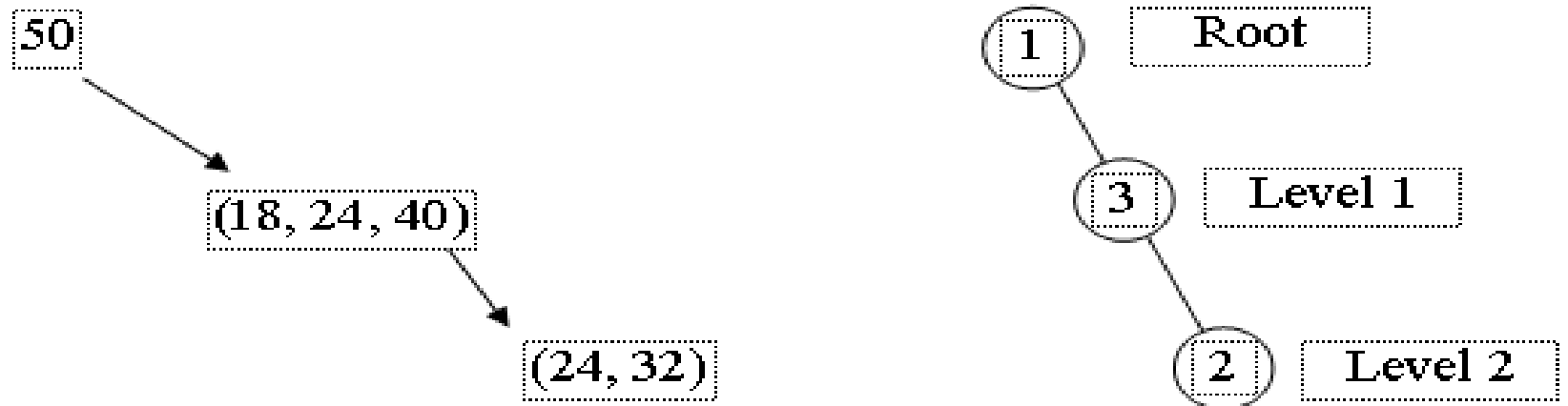


Fig.3. Model of Numbers Tree for $N = 50$

Hence $50^2 = 18^2 + 2 \cdot 24^2 + 32^2$

3). $N = 75$.



Fig.4. Model of Numbers Tree for $N = 75$

t.e. $75^2 = 27^2 + 2 \cdot 36^2 + 42^2$

5). $N = 125$.

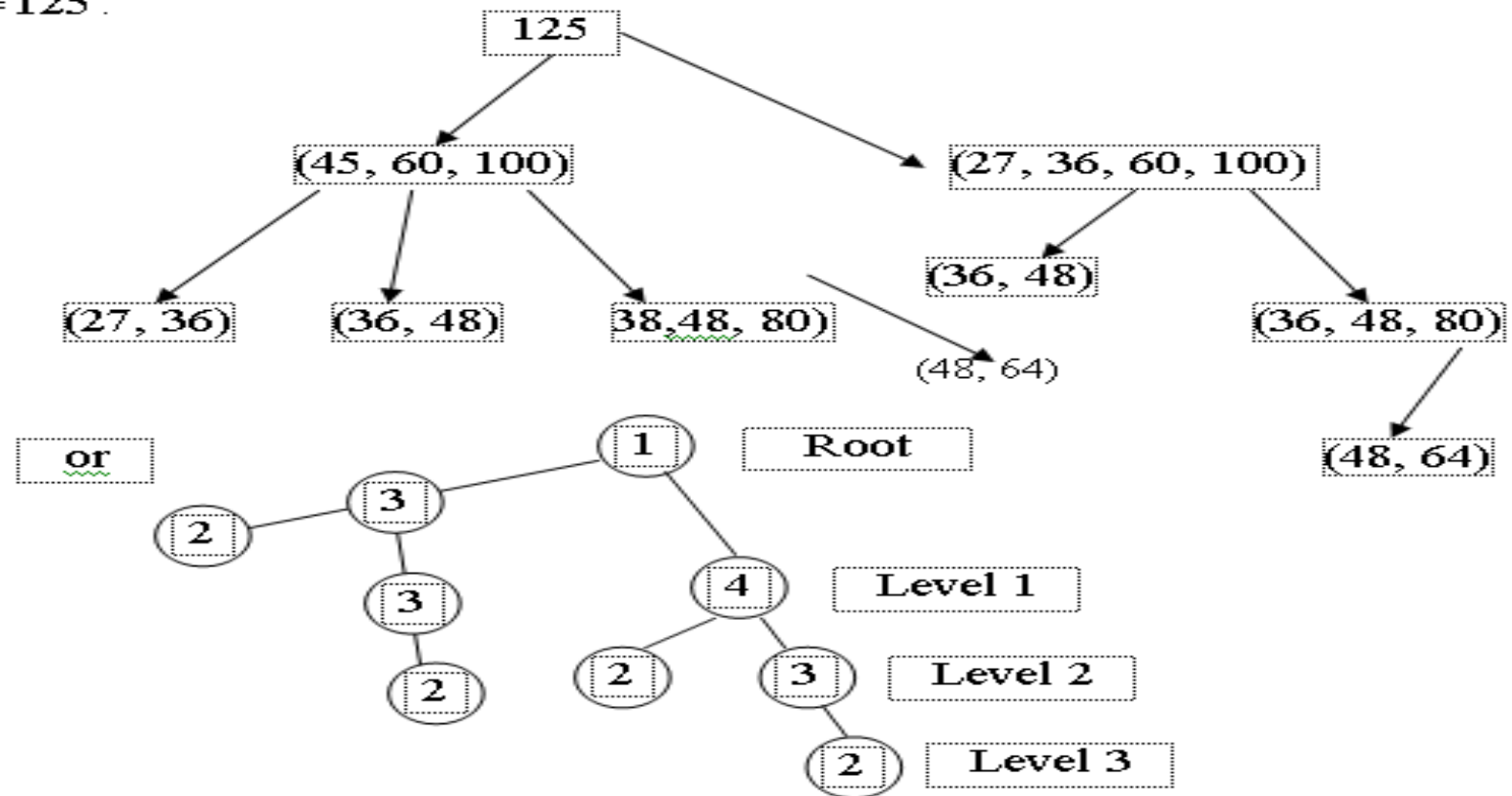


Fig.6. Model of Numbers Tree for $N = 125$

Then $125^2 = 27^2 + 3 \cdot 36^2 + 3 \cdot 48^2 + 64^2$

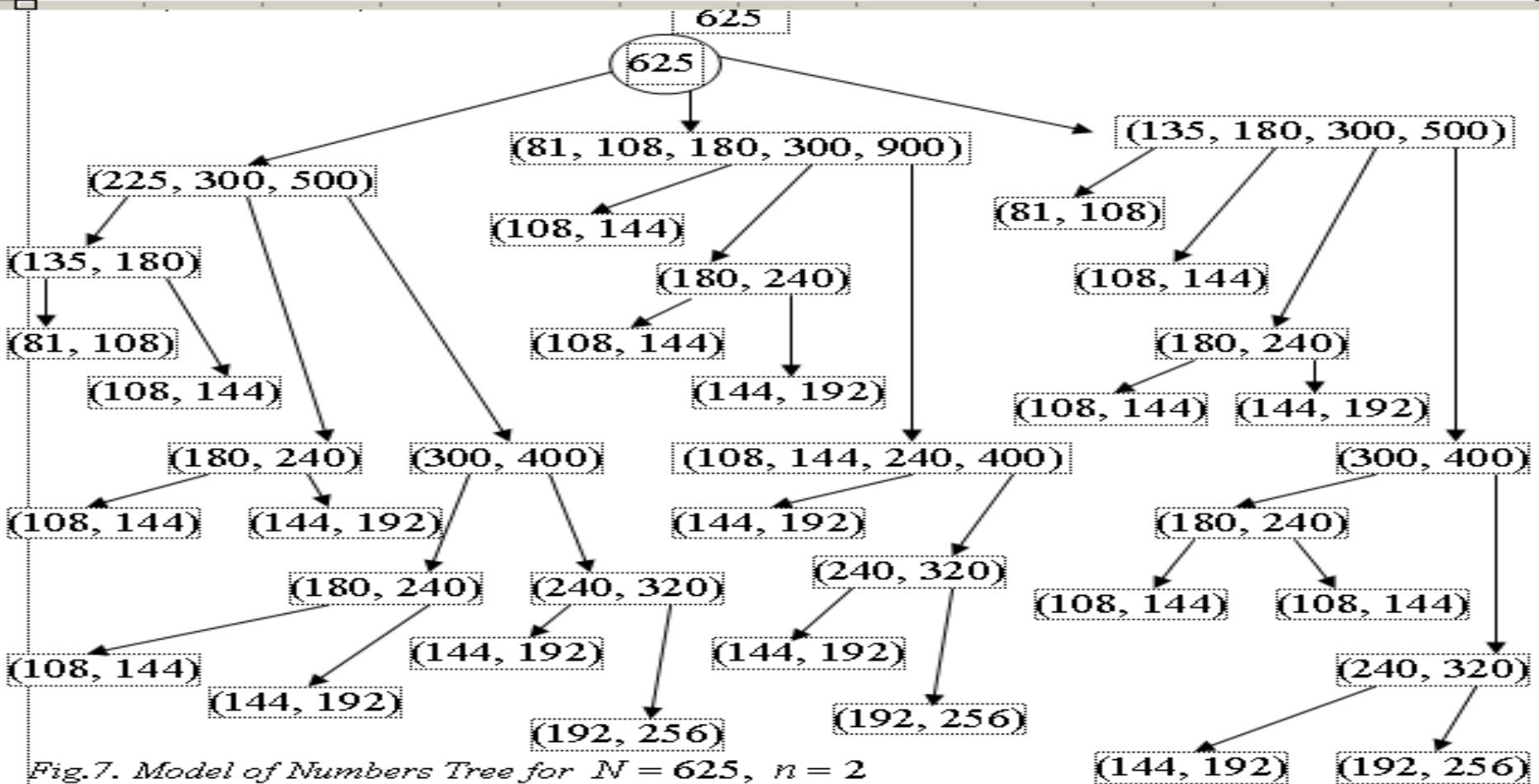
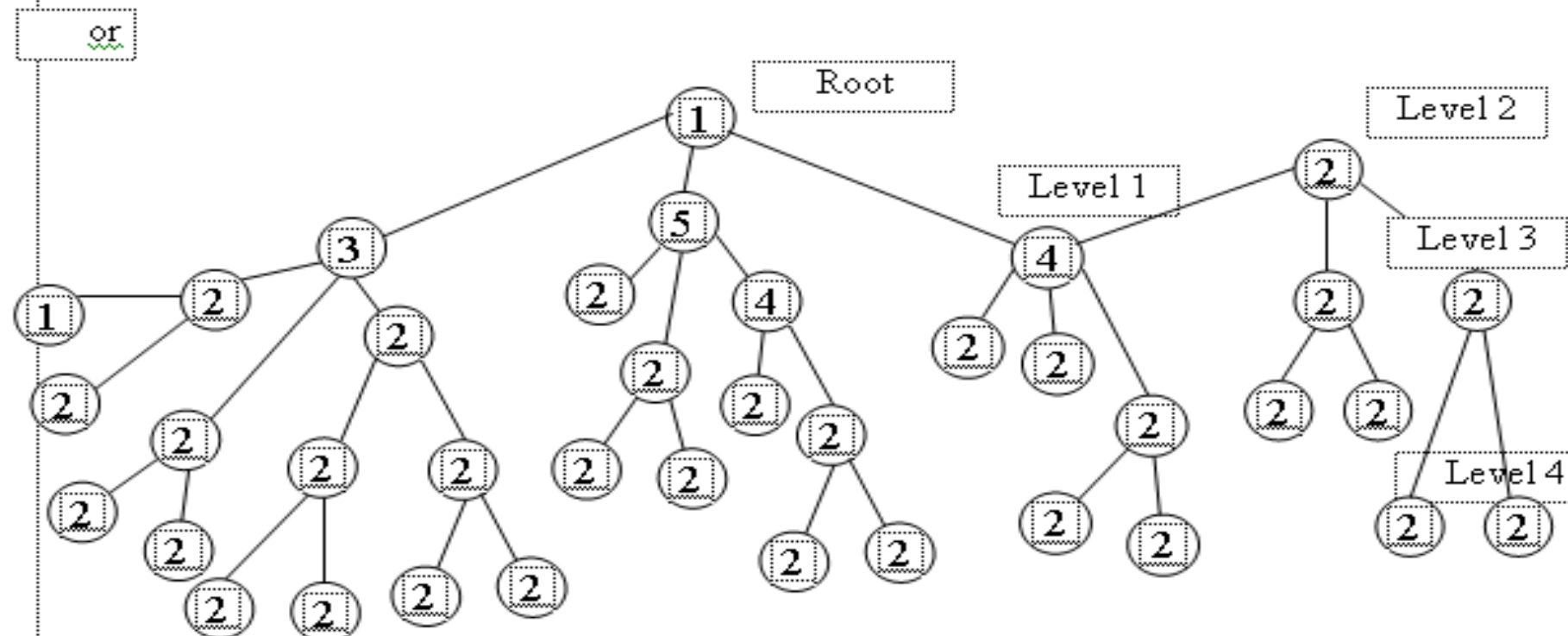


Fig.7. Model of Numbers Tree for $N = 625$, $n = 2$

Fig.7. Model of Numbers Tree for $N = 625$, $n = 2$



Hence $625^2 = 81^2 + 4 \cdot 108^2 + 6 \cdot 144^2 + 4 \cdot 192^2 + 256^2$.

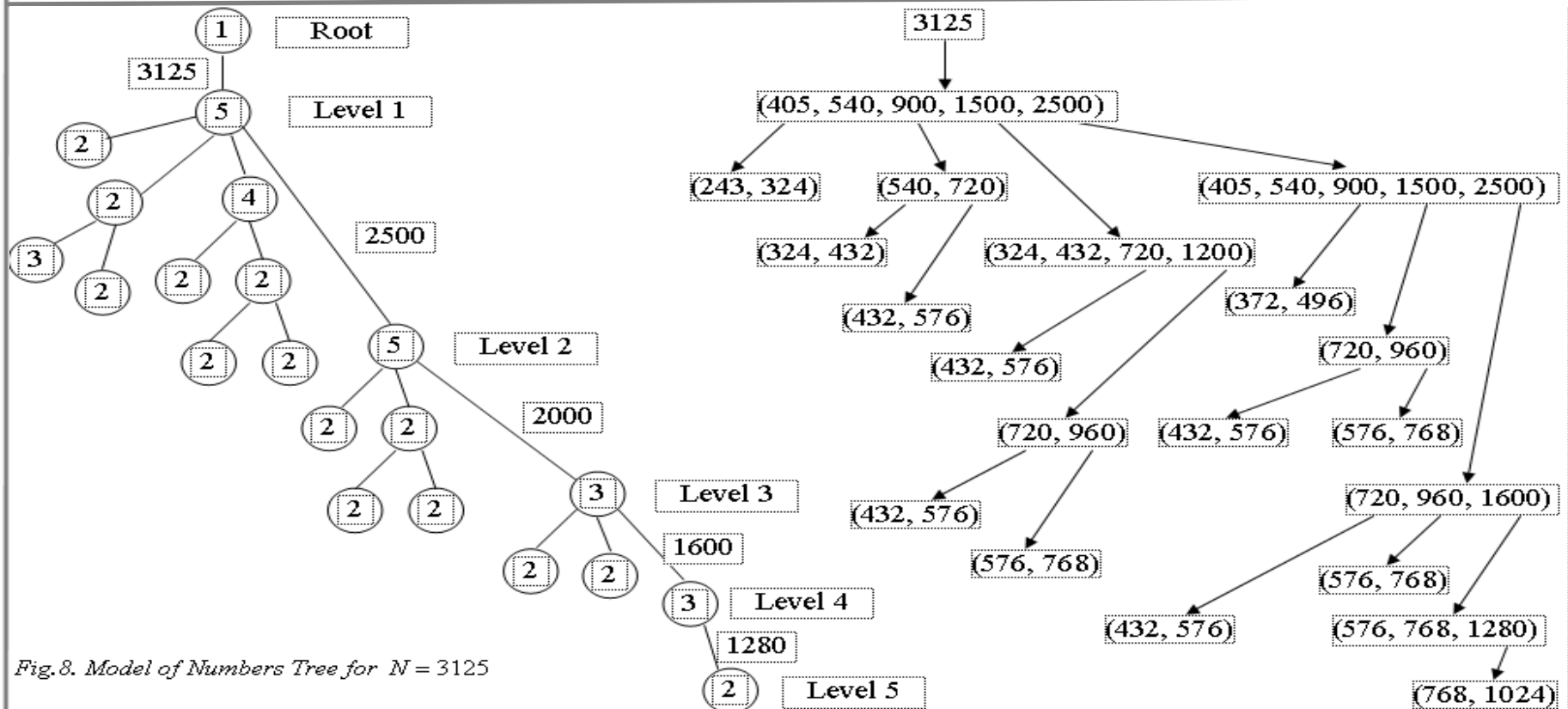
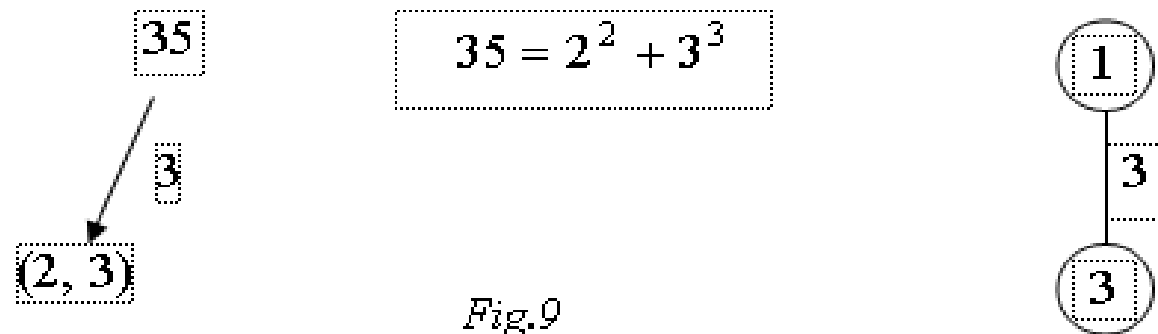


Fig.8. Model of Numbers Tree for $N = 3125$

and $3125^2 = 243^2 + 4 \cdot 324^2 + 372^2 + 8 \cdot 432^2 + 496^2 + 9 \cdot 576^2 + 5 \cdot 768^2 + 1024^2$.

For cases when $n > 2$ not always it is possible to construct graceful examples. We shall present one example. Let $N = 35$, $n = 3$, then



From the given example follows, that we have not received a tree, but only it(him) berry. We shall name such "trees" not growing trees. For growth of such tree it is necessary to make "cuttings" from another (for example) $n = 2$ a tree and to insert them into not growing trees. For example

From the given example follows, we have not received a tree, but received only its branches. We shall name such "trees" not growing trees. For growth of such tree it is necessary to make "cuttings" from another (for example) $n = 2$ a tree and to insert them into not growing trees. For example

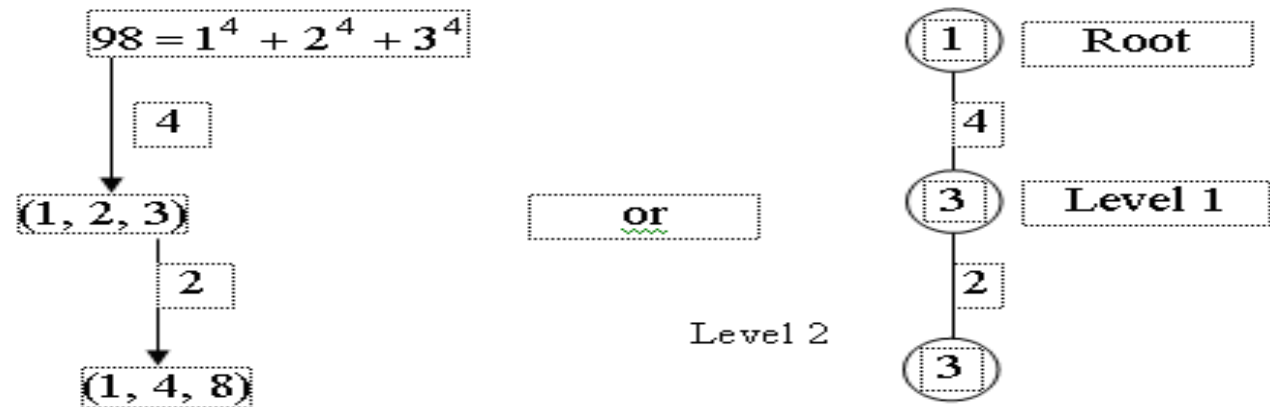


Fig.10.

i.e., $98 = 1^4 + 2^4 + 1^2 + 4^2 + 8^2$.

Conclusion: The propose method basis on numbers tree model, is a simple and universal method for definition of value of Losses in the Worst Condition by Kinds with Long Settlement and it is easy programmed on all computer languages. Receiving formulas (4), (5), (14), (22) are controlled our calculations.



Actuaries mathematics in Tajikistan

- Development actuaries mathematics in Tajikistan for last 5 years was connected as to efforts of the persons working at national university, and with activity practising actuaries.
- Rates actuaries mathematics were read on branch of computer science of 2001. In 2004-2009 specialization in modelling actuaries processes there was created within the framework of a speciality of "computer science" which operates on present time. Preparation of students within the framework of this specialization has allowed to sate initial demand on actuaries in the insurance companies. Work on preparation of modern textbooks on various sections actuaries the mathematics, reflecting experience of reading of the specified rates now proceeds.



- Besides on the basis of institute of natural sciences the laboratory which is engaged in modelling of economic processes in extreme modes is created.
- Experience of teaching of bases actuaries mathematics was distributed on activity actuaries— other educational institutions and the establishment specializing on preparation of experts for insurance business. Subsequently actuaries rates began to be read and at some other universities of city.
- On faculty of computer science and in laboratory modelling of institute of natural sciences of national university the international scientific - practical conferences devoted to the most actual problems of development of mathematical models will traditionally be carried out.

- As the methodology actuaries calculations uses the probability theory, given and the long-term statistical data, financial calculations that on faculty are in full read to a demography rates connected with
- System mathematical and the statistical regularities establishing mutual relation between the insurer and the insurant. They reflect as mathematical formulas the mechanism of formation(education) and an expenditure of insurance fund in long-term insurance operations. To them also carry calculations of tariffs on any kind of insurance: lifes, pensions, from accidents, property, work capacity. The methodology actuaries calculations uses the probability theory, given to a demography and the long-term statistical data, financial calculations.



- By means of the last in tariffs the income which is received by the insurer from use as credit resources of the accumulated payments of insurants is taken into account. Except for that rates on actuaries are read to calculations which is connected to one of the widespread problems(tasks) of such -statistical calculations connected to definition of norms and conditions of insurance, is, that the sum of insurance payments minus relying payments guaranteed reception by insurance firm (or the state organization) expected results.

Main results:

$$Y_m^n = \left(x^{m-1} \right)^n + \sum_{i=2}^m \left(yx^{m-i} z^{i-2} \right)^n$$

$$z^n = x^n + y^n$$

$$Y_m^n = \sum_{j=1}^m X_j^n$$

$$Y_m = \left(x^{m-1} \right)^n + \sum_{i=2}^m \left(yx^{m-i} z^{\frac{i-2}{n}} \right)^n$$

$$z = x^n + y^n$$

$$Y_m = \sum_{j=1}^m X_j^n$$

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