Models of Development of Losses in the Worst Condition by Kinds with Long Settlement - a modification method of the nearest neighbour

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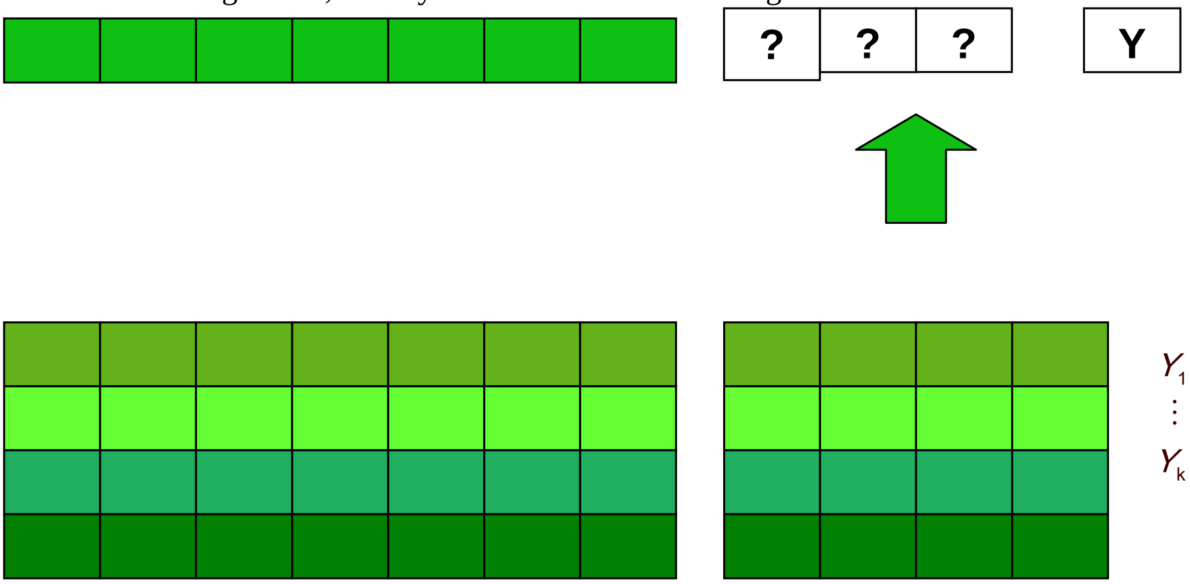
Abstract. The work is devoted to construction and investigation of a new method to calculate a size of losses by kinds with long settlement: a modification method of the nearest neighbour. The proposition method is to define a size of losses in the worst condition of system and is based on using of so-called model of numbers tree. By using this method for model, data showed and carried out some computational experiments.

Mathematical Subject classification (2000): 65 Y20, 68 Q17, 68Q25.

Kew words: *Model, Development of Losses, numbers tree, optimal tree, analyses, Long Settlement.*

Introduction. Method of the nearest neighbour (NN - Nearest Neighbor***)*** for solution different problems of some our everyday life is given in works [1-11]. It is the most widespread method of comparison and extraction of precedents. It rather easily allows calculating a degree of similarity of the current problem situation and precedents from BP (Libraries of precedents - the saved up experience) systems. With the purpose of definition of a degree of similarity onset of the parameters used for the description of precedents and the current situation, the certain metrics is entered. Further, according to the chosen metrics, the distance from the target point appropriate to the current problem situation up to the points representing precedents from BP is defined, and the nearest gets out to target a point. The method of definition of the nearest neighbour (the nearest neighbours) also is widely applied to the decision of problems of classification, clasters, regresses and recognition of images. From all the methods, the method of the nearest neighbour is most distributed. This method is known also under the name, a method of single connection. Cross-check - a known method of reception of estimation of unknown parameters of model. The basic idea - division of the sample of data on v "Warehouse part". V «Warehouse parts " here essence in the casual image allocated and isolated samples. Method of " the nearest neighbour " as a method of cross-check use for estimation of parameters and solution of a problem of forecasting and interpolation of function of one or several variables. In this paper we proposed new variant of the method of the nearest neighbor and it is also used in different branches of science and technology [11-22]. This variant may be used under modeling so-called a tree of numbers which is arising at the analysis of the numerical and corresponding text, economical biological and technological information. It is shown that many questions of mathematical analysis of problems and their applications are reduced to construction of models in extremes regimes and connected with it construction of Model Numbers Tree. Extreme regimes of such physical processes are arising in the case when the values of their parameters are chaining in some given set. They can be an accumulation of the warmth, particles, wave energy in some areas where the considered physical processes are arising. Applications of this idea were also used in investigations of some models of biological populations (with regard to time-age-spaces) and economical systems.

As is known, the forecast of the future losses, using observably average development of the nearest neighbours, usually define under the following circuit



On the basis of given to the circuit the model of the nearest neighbour assumes the following

$$Y = \sum_{i=1}^{k} \omega_i Y_i, \text{ where } \omega_i \geq 0, i = 1,\dots,k; \ \sum_{i=1}^{k} \omega_i = 1. \qquad (1)$$

We shall consider more general model, than model (1):

$$Y_k = \left(\sum_{i=1}^{k} \omega_i Y_{ik}^{\ s} \right)^{1/s}, \text{ where } \sum_{i=1}^{k} \omega_i^{\frac{n}{n-s}} = 1,\ n > s > 0,\ \omega_i \geq 0,\ i = 1,\dots,k;\ k=2,3,\ . \qquad (2)$$

Having entered a designation $X_{ik} = \omega_i^{1/s} Y_{ik}$ model (2) we shall copy as $Y_k = \left(\sum_{i=1}^{k} X^{s}{}_{ik} \right)^{1/s}$ or

$$Y_k^{\ s} = \sum_{i=1}^{k} X^{s}{}_{ik} \quad \text{and} \quad \sum_{i=1}^{k} \left(\frac{X_{ik}}{Y_{ik}} \right)^{\frac{ns}{n-s}} = 1, \quad n>s>0,\ k=2,3,4\dots\dots \qquad (3)$$

The equation (3) is the equation of a degree s with $k+1$ unknown and has infinite number of decisions. For allocation of the necessary decisions, it is necessary that we use echo of the equation (2).

Model of the worst development of losses: For everything $\sum_{i=1}^{k} \omega_i^{\frac{n}{n-s}} = 1,\ n > s > 0$ $\omega_i \geq 0,\ i = 1,\dots,k$ right part of the equation (2) has the maximal value that there corresponds the worst condition of system, i.e.

$$Y_k = \max_{\omega \in M} \left(\sum_{i=1}^{k} \omega_i Y_{ik}^{\ s} \right)^{1/s} \qquad (4)$$

where $M = \left\{ \omega : 0 \leq \omega_j \leq 1,\ \sum_{j=1}^{k} \omega_j^{\frac{n}{n-s}} = 1,\ n > s,\ s > 0,\ k > 1 \right\}$.

The equation (4) we shall call "*Model of the worst development of losses*".

Theorem 1. The equation (4) and the equation

$$Y_k^{\ n} = \sum_{i=1}^{k} Y^n{}_{ik} \tag{5}$$

are equivalent.

The Proof: *The necessity***.** We shall introduce a designation $Z = Y_k$, $X_j = Y_{jk}$, $j = \overline{1,k}$. Let the condition (5) takes place then

$$Z^n = \sum_{j=1}^{m} X_j^{\ n} \tag{6}$$

Let's show, validity (4) i.e.

$$Z = \max_{\omega \in M} \left(\sum_{j=1}^{k} \omega_j X_j^{\ s} \right)^{1/s} \tag{7}$$

Let $(X_1, X_2, ..., X_k, Z)$ is the decision of the equation (6), then having entered a designation $\omega_j = \left(\frac{X_j^{\ n}}{Z^n} \right)^{\frac{n-s}{n}}$, from (6) we have the following system:

$$\omega_1 X_1^s + ... + \omega_k X_k^s - Z^s = 0, \ X_j^s - \alpha_j^{\frac{s}{n-s}} Z^s = 0 , \tag{8}$$

As $(X_1, X_2, ..., X_k, Z)$ is the decision (6), i.e. (4), it is easy to see $\sum_{j=1}^{k} \omega_j^{\frac{n}{n-s}} = \frac{\sum_{j=1}^{k} X_j^{\ n}}{Z^n} = 1$, as, hence, determinant of system (8) is equal to zero $\sum_{j=1}^{k} \omega_j^{\frac{n}{n-s}} - 1 = 0$. Really, we apply a method of a mathematical induction, and as $\Delta_2 = \omega_1^{\frac{n}{n-s}} - 1, \ \Delta_3 = \omega_1^{\frac{n}{n-s}} + \omega_2^{\frac{n}{n-s}} - 1$. It can be assumed, that $\Delta_k = \sum_{j=1}^{k-1} \omega_j^{\frac{n}{n-s}} - 1$, *k=2,3,4…*. Let us show it's validity at $k+1$, is valid, decomposing determinant on *k+1* elements of a line is received:

$$\Delta_{k+1} = \begin{vmatrix} -1 & \omega_1 & \omega_2 \ \cdots & \omega_{km-1} & \omega_m \\ -\omega_1^{\frac{s}{n-s}} & 1 & 0 \ \ldots & 0 & 0 \\ -\omega_2^{\frac{s}{n-s}} & 0 & 1 \ \ldots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ -\omega_{m-1}^{\frac{n}{n-s}} & 0 & 0 \ \ldots & 1 & 0 \\ -\omega_m^{\frac{s}{n-s}} & 0 & 0 \ \ldots & 0 & 1 \end{vmatrix} =$$

$$= (-1)^{k+2} \cdot \left(-\alpha_k^{\frac{s}{n-s}} \right) \begin{vmatrix} \omega_1 & \omega_2 & \cdots & \omega_{k-1} & \omega_k \\ 1 & 0 & \ldots & 0 & 0 \\ 0 & 1 & \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & 1 & 0 \end{vmatrix} +$$

$$+ (-1)^{2m+2} \Delta_m = (-1)^{k+3} \cdot \alpha_m^{\frac{s}{n-s}} \cdot (-1)^{m+1} \cdot \alpha_m \begin{vmatrix} 1 & 0 & \ldots & 0 \\ 0 & 1 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & 1 \end{vmatrix} +$$

$$+ \sum_{j=1}^{k-1} \omega_j^{\frac{s}{n-s}} - 1 = (-1)^{2k+4} \cdot \omega_k^{\frac{n}{n-s}} + \sum_{j=1}^{k-1} \omega_j^{\frac{n}{n-s}} - 1 = \sum_{j=1}^{k} \omega_j^{\frac{n}{n-s}} - 1 \ .$$

As it was shown and as $\omega \in M$, where $\sum_{j=1}^{k} \omega_j^{\frac{n}{n-s}} = 1$, i.e. it means $\Delta_{k+1} = 0$. From 1st equation (6) we have:

$Z^s = \left(\sum_{j=1}^{k} \omega_j X_j^s \right)$ и $\omega_j = \left(\dfrac{X_j^n}{Z^n} \right)^{\frac{n-s}{n}}$. Hence, as, $\sum_{j=1}^{k} \omega_j X^s \le \left(\sum_{j=1}^{k} \omega_j^0 X_j^s \right) =$ $\sum_{j=1}^{k} \dfrac{X_j^n}{Z^{n-s}}$ т.е. $Z^s \cdot Z^{n-s} = \sum_{j=1}^{k} X_j^n$, from here for any $\omega \in M$, $Z^n = \sum_{j=1}^{k} X_j^n$,

$$\omega_j^0 = \left(\frac{X_j^n}{\sum_{j=1}^{k} X_j^n} \right)^{\frac{n-s}{n}}$$

. And in equality (1) the maximum is reached. Thus

$$Z = \max_{\omega \in M} \left(\sum_{j=1}^{k} \omega_j X^s \right)^{1/s} .$$

The sufficiency: The equation (4) let takes place. Let's prove validity (6). Let's designate $Z=\mu(\omega)=\left(\sum_{j=1}^{k}\omega_j X_j^s\right)^{1/s}$, $\omega\in M$. It is easy to see, that from a condition

$\frac{\partial\mu}{\partial\omega_j}=0$ the system of the equations follow $X_j^s-\omega_k^{-\frac{s}{n-s}}\cdot\omega_j^{\frac{s}{n-s}}\cdot X_k^s=0,\ j=\overline{1,k}$

and from here $\omega_k^{-\frac{s}{n-s}}\cdot\omega_j^{\frac{s}{n-s}}=\frac{X_j^s}{X_k^s}$ or

$\omega_k^{-\frac{n}{n-s}}\cdot\omega_j^{\frac{n}{n-s}}=\frac{X_j^n}{X_k^n}$. To sum the last equality on *j* from up *1* to *k*, we have,

$\omega_k^{-\frac{n}{n-s}}=\frac{\sum_{j=1}^{k}X_j^n}{X_k^n}$. Then $\omega_j^{0\,\frac{n}{n-s}}=\frac{X_j^n}{\sum_{j=1}^{k}X_j^n}$ is a point of a maximum of function $\mu(\omega)$, $\omega\in M$ as $(\mu''_{\omega\omega}<0)$. Let's calculate the value of function $\mu(\omega^0)$.
It is easy to see, that

$$Z^s=\sum_{j=1}^{k}\omega_j X_j^s\le\sum_{j=1}^{k}\omega_j^0 X_j^s=\sum_{j=1}^{k}\left(\frac{X_j^n}{Z^n}\right)^{\frac{n-s}{n}}\cdot X_j^s=\sum_{j=1}^{k}\left(\frac{X_j^n X_j^{\frac{sn}{n-s}}}{Z^n}\right)^{\frac{n-s}{n}}=$$

$$=\sum_{j=1}^{k}\left(\frac{X_j^{1+\frac{s}{n-s}}}{Z}\right)^{n-s}=\sum_{j=1}^{k}\frac{X_j^n}{Z^{n-s}}$$ i.e. $Z^s\cdot Z^{n-s}=\sum_{j=1}^{k}X_j^n$. And hence

$Z^n=\sum_{j=1}^{k}X_j^n$; $\forall\omega\in M$. It is easy to see that

$$Z=\mu(\omega_0)=\left(\sum_{j=1}^{k}\left(\frac{X_j^n}{\sum_{j=1}^{k}X_j^n}\right)^{\frac{n-s}{n}}\cdot X_j^s\right)^{1/s}=\left(\sum_{j=1}^{k}\left(\frac{X_j^n\cdot X_j^{\frac{sn}{n-s}}}{\sum_{j=1}^{k}X_j^n}\right)^{\frac{n-s}{n}}\right)^{1/s}=\left(\sum_{j=1}^{k}\frac{X_j^n}{Z^{n-s}}\right)^{1/s},$$

and $Z^s = \mu^s(\omega_0) = \frac{1}{Z^{n-s}} \sum_{j=1}^{k} X_j^n,$ from here $Z = \mu(\omega_0) = \left(\sum_{j=1}^{k} X_j^n \right)^{1/n}$, i.e.

it takes place (6). Thus $Z = \mu(\omega_0) = \left(\sum_{j=1}^{k} \left(\frac{X_j^n}{\sum_{j=1}^{k} X_j^n} \right)^{\frac{n-s}{n}} \cdot X^s \right)^{1/s} = \left(\sum_{j=1}^{k} X_j^n \right)^{1/n}$.

The theorem is proved. Now we write the algorithm of solution of the equation of worst development of losses.

Algorithm of solution of the equation of worst development of losses.

<u>The Proposition 1</u> Let the tree of numbers to the appropriate equations (5) let is given then there is transformation K which translates the solutions (5) at $k = m-1$ on the solution (5) at $k = m$, i.e. $Y = KX$, where

$$K = \begin{pmatrix} x & 0 & \cdots & 0 & 0 & 0 \\ 0 & x & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & x & 0 & 0 \\ 0 & 0 & \cdots & 0 & y & 0 \\ 0 & 0 & \cdots & 0 & 0 & z \end{pmatrix}, \quad x^n + y^n = z^n ,$$

$X = (a_{1,m-1}, ..., a_{m-1,m}, N_{m-1}, N_{m-1}),\quad N_{m-1} = \left(\sum_{j=1}^{m-1} a_{im-1}^n \right)^{\frac{1}{n}}$, $Y = (a_{1m}, a_{2m}, ..., a_{1m}, N_m),\quad N_m = \left(\sum_{j=1}^{m} a_{im}^n \right)^{\frac{1}{n}}$.

<u>The proof</u>. Let (x, y, z) is the solution $x^n + y^n = z^n$ $n \geq 2$. Transformation K we shall copy as

$$a_{im} = xa_{im-1},\quad a_{mm} = yN_{m-1},\quad N_m = zN_{m-1},\quad i = 1,2,...,m-1;\quad k = 2,3...$$

Let $(a_{im-1}, ..., a_{m-1m-1}, N_{m-1})$ is the solution (5) at $k = m-1$. As $\sum_{j=1}^{m-i} a_{jm-1}^n = N_{m-1}^n$

Multiplying on x^n we shall receive: $x^n \sum_{j=1}^{m-1} a_{jm-1}^n = x^n N_{m-1}^n$. From here

$\sum_{j=1}^{m-1} (xa_{jm-1})^n = (z^n - y^n) N_{m-1}^n$ and therefore $\sum_{j=1}^{m} a_{jm}^n = N_m^n$ .That it was required to prove.

Let X' = μ_tX which is connected with discrete model equations

$\sum_{i=1}^{m} X_{im}^{n} = Z_m^n$, where $Z_{m-1} = \left(\sum_{i=1}^{m-1} X_{im-1}^{1/t} \right)^t$ and $X' = (X'_{1m}, ..., X'_{mm}, Z'_m)$, $Z'_m = \left(\sum_{i=1}^{m} X'^{1/t}_{im} \right)^t$,

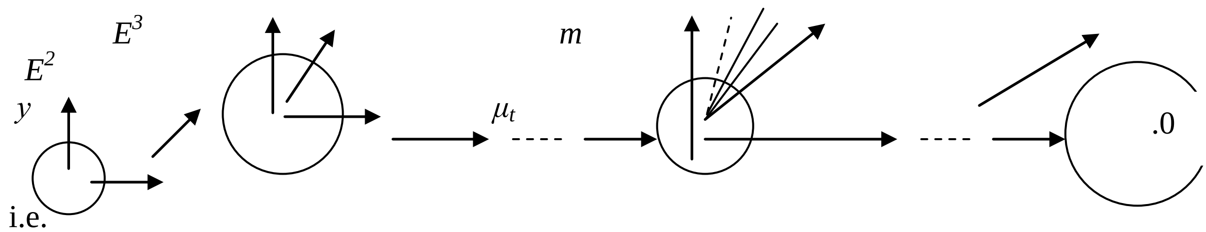
m=1, 2... ;μ_t is diagonal matrix of *m+1*-th order (see *K*)

$$\mu_t = \begin{pmatrix} x^t & O & \ldots & O & O & O \\ \vdots & & \ddots & \vdots & \vdots & \vdots \\ O & O & \vdots & x^t & O & O \\ O & O & \vdots & O & y^t & O \\ O & O & \vdots & O & O & z^t \end{pmatrix}$$

where *x,y* is some point of E^2 with metrics $z=x+y$ and $0<x<e^q$, $0<y<e^q$,$0<z<2e^2$, and $0 \le t \le t_k$, $t_k<\infty, q=const>0$.

Proposition 2: The transformation of μ_t transfers arbitrary point $(X_{1m-1}, X_{2m-2}, ..., X_{m-1m-1})E^{m-1}$ with metrics Z'_{m-1} and into some corresponding points $(X'_{1m}, ..., X'_{mm})$ from E^m with metrics Z'_m for all m=3,4,5.... and has semi-group properties: $\mu_{t+s} = \mu_t \mu_s$, $0 \le t \le t_k$, $0 \le s \le t_k$, and it is linear, uniformly bounded and uniformly continued. Besides, the engine - values of μ_t are represented in the form of: $\lambda_j = x^t$, j=1,2...,k-1; $\lambda_k = y^t$; $\lambda_{k+1} = z^t$; $\|\mu_t\| = z^t$; $\|\mu_t^{-1}\| < \infty$ 2).The infinitesimal generating operator $A = \lim_{t \to 0} t^{-1}(\mu_t - I)$ is diagonal matrix and represented in the following way: $a_{ii} = \ln x$, $i=1,...,k-1$, $a_{kk} = \ln y$, $a_{k+1k+1} = \ln z$, and what is more $R(m,A) = (qI-A)^{-1}$, $\mu_t = e^{tA}$, $A = \ln \mu_t^{1/t}$. 3). The transformation $M = \mu_t^{1/t}$ is also transferred E^{k-1} into E^k under corresponding condition $x^{1/t} + y^{1/t} = z^{1/t}$, $0 \le t \le t_k$, $t_k < \infty$, 4). The transformations μ_t, μ_t^{-1} may be used for coding and decoding of corresponding input and output information. It is characterized in the following way:

$$Z^n = \sum_{j=1}^{2} X_j^n, \quad Z^n = \sum_{j=1}^{3} X_j^n, \quad \ldots, \quad Z^n = \sum_{j=1}^{m} X_j^n \quad \ldots\ldots\ldots$$



i.e.

X_1	X_2	Z		X_1	X_2	X_3	Z	...	...	X_1	X_2	X_3	...	X_m	Z	...

It is true for next theorem:

Theorem 2. For any natural *n>1* between to the set of solutions (4), i.e. under *m=k-1* and *m=k* it takes place next presentation:

$$\tilde{x}_{jk} = \tilde{x}_{12}\tilde{x}_{jk-1}, \quad \tilde{x}_{kk} = \tilde{x}_{22}\tilde{z}_{k-1}, \quad \tilde{z}_k = \tilde{z}_2\tilde{z}_{k-1}, \quad j=1,2,..k-1 \qquad (9)$$

where k=3, 4…, $(\tilde{x}_{12}, \tilde{x}_{22})$ is some point of the special Plane with distance $\tilde{z}_2 = \sqrt[n]{\tilde{x}_{12}^n + \tilde{x}_{22}^n}$. The transformation of (9) may be written in the form of $\tilde{X}_m = \mu_t \tilde{X}_m$, where the group transformations are

$$\tilde{\mu}_t = \begin{pmatrix} \tilde{x}^t & 0\ldots\ldots & 0 & 0 & 0 \\ 0 & \tilde{x}^t\ldots\ldots & 0 & 0 & 0 \\ \ldots\ldots\ldots & & & & \\ 0 & 0\ \ldots\ldots & \tilde{x}^t & 0 & 0 \\ 0 & 0\ \ldots\ldots & 0 & \tilde{x}^t & 0 \\ 0 & 0\ \ldots\ldots & 0 & 0 & \tilde{z}^t \end{pmatrix}, \qquad (10)$$

where $\tilde{z} = \tilde{x} + \tilde{y},\ t = 1/n$, transfer any points $\left(\tilde{x}_{1m-1} \cdots \tilde{x}_{m-1m-1}\right) \in \tilde{M}^{m-1}$ with metrics $\tilde{z}_{m-1}$ into some corresponding point properties of $\left\{\tilde{\mu}_t\right\}$ are given in [6]. The Plane of $\tilde{M}^2$ is special plane with all points of the model spaces $\tilde{M}^n(\alpha)$, $\alpha \in M_n^s$ that are depending from points of this Plane. From the developed method follows, that if we know any solution of the equation (4), i.e. $\sum_{i=1}^{k} X_{ik}^n = Z_k^n$, at *k=m-1*, the appropriate decisions at *k=m* is defined under the formula **X′ =K X,** where $X' = \left(X'_{1m},\ldots,X'_{mm},Z'_m\right)$, $Z'_m = \left(\sum_{i=1}^{m} X'^n_{im}\right)^{1/n}$, $Z_{m-1} = \left(\sum_{i=1}^{m-1} X_{im-1}^n\right)^{1/n}$, $X = \left(X_{1m-1}, X_{2m-1},\ldots, X_{m-1m-1}\right)$, *m=2,3,...*, and *x, y, z* - are the solutions of the equation $z^n = x^n + y^n$. Such any representation of type (4), (5), (6) with help of transformations *K* and μ_t are transferred to the form

$$Y_m^n = \left(x^{m-1}\right)^n + \sum_{i=2}^{m}\left(yx^{m-i}z^{i-2}\right)^n \qquad (11)$$

It should be out that transformations (9), (11) are described the process of numbers tree grows and formula (11) is formula for definition of the size Losses in the Worst Condition by Kinds with Long Settlement in general case.

For example, in a case $n = 2$ at any $m = 2,3,4\ldots..$, knowing the decision, equation $Y_1^2 + Y_2^2 = Y^2$ on the basis of transformation (9), (10), (11) gradually, we shall define the decisions of the equation $Y_1^2 + Y_2^2 + \ldots. + Y_m^2 = Y^2$. At $m = 2$ the equation $Y_1^2 + Y_2^2 = Y^2$ we have obvious decisions such as [1]:

Y_1	Y_2	Y
3κ	4κ	5κ
5κ	12κ	13κ
7κ	24κ	25κ
9κ	40κ	41κ
	...	...

Y_1	Y_2	Y
$4m$	$15m/2$	$17m/2$
$6m$	$35m/2$	$37m/2$
$8m$	$63m/2$	$65m/2$
$10m$	$99m/2$	$101m/2$
...	...	...

Here k - natural number, $m = 2\kappa$. At $m = 3,4,5...$, for $Y_1 = 3, Y_2 = 4, Y = 5$

The appropriate solutions $Y_1, Y_2, ..., Y_m$, Y are given in the following tables:

$m=3$	$m=4$	$m=5$
$Y_1 = 9$ $Y_2 = 12$ $Y_3 = 20$ $Y = 25$	$Y_1 = 27$ $Y_2 = 36$ $Y_3 = 60$ $Y_4 = 100$ $Y = 125$	$Y_1 = 81$ $Y_2 = 108$ $Y_3 = 180$ $Y_4 = 300$ $Y_5 = 500$ $Y = 625$
m=9	*m=10*	*m=11*
$Y_1 = 6561$ $Y_2 = 8748$ $Y_3 = 14580$ $Y_4 = 24300$ $Y_5 = 40500$ $Y_6 = 67500$ $Y_7 = 112500$ $Y_8 = 187500$ $Y_9 = 312500$ $Y = 390625$	$Y_1 = 19683$ $Y_2 = 26244$ $Y_3 = 43740$ $Y_4 = 72900$ $Y_5 = 121500$ $Y_6 = 202500$ $Y_7 = 337500$ $Y_8 = 562500$ $Y_9 = 937500$ $Y_{10} = 1562500$ $Y = 1953125$	$Y_1 = 59049$ $Y_2 = 78732$ $Y_3 = 131220$ $Y_4 = 218700$ $Y_5 = 364500$ $Y_6 = 607500$ $Y_7 = 1012500$ $Y_8 = 1687500$ $Y_9 = 2812500$ $Y_{10} = 4687500$ $Y_{11} = 7812500$ $Y = 9765625$

The Application of Model of Numbers Tree to Models of Development of Losses. Let's enter definition a tree of numbers. Let N - some natural number. We shall tell, that the number N forms a tree of numbers if there will be natural numbers $n, m \geq 2$ and integers $a_1, a_2, ..., a_m$ for which

$$N^n = a_1^n + a_2^n + ... + a_m^n \,, \tag{12}$$

and in turn some a_j (or all) represented as

$$a_j^n = a_{1j}^n + a_{2j}^n + ... + a_{m_1 j}^n \,, \; m_1 \leq m \tag{13}$$

and some a_{ij} of (13) also can be submitted as

$$a_{ij}^n = a_{1ij}^n + ... + a_{m_2 ij}^n \,, \;\; m_2 \leq m_1, \ldots,$$

and at last decomposition takes place

$$a_{ijj_1 \dots j_{m_k}}^n = a_{1ijj_1 \dots j_{m_k}}^n + a_{2ijj_1 \dots j_k}^n \,, \tag{14}$$

In which members of the right part (14) can not beat are submitted as the final sum composed n-th degrees of some integers so-called by a branches of a tree. Conceptual Model of Numbers Tree in general case is given in fig. 1.

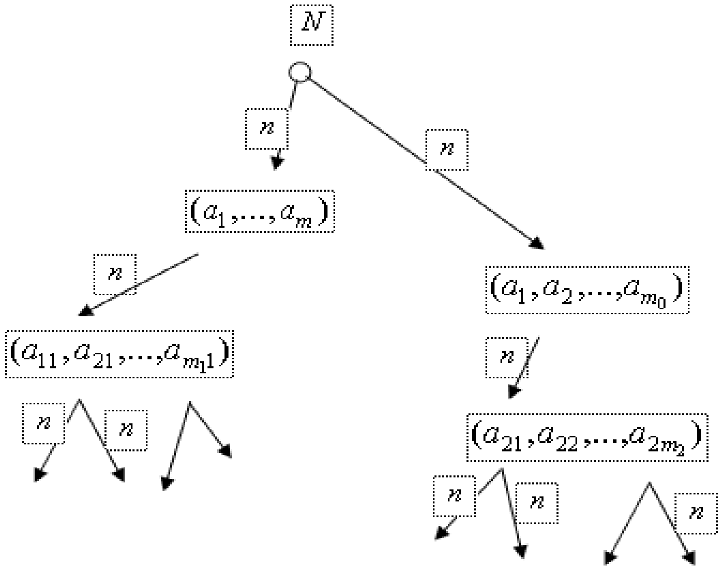


Fig.1. Conceptual Model of Numbers Tree in general case

So, at last, the level of the tree consists of the sum such as (14) base elements can enter into each level of a tree, therefore from each level, we take only those elements, which are not presented as (12), (13).

Then in result the number N is uniquely represented as

$$N^n = \sum_{j_q} k_{j_q} a^n_{ijj_1 \dots j_q}, \tag{15}$$

where k_{j_q} *- number of occurrence of a basic element* $a_{ijj_1 \dots j_q}$ *in a tree of numbers.*

We shall consider a problem for number N and a vector $a = (a_1, \dots a_k),\ k \ge 2$ from the equation $N = \max_{\alpha \in M} (\alpha, a)$, where

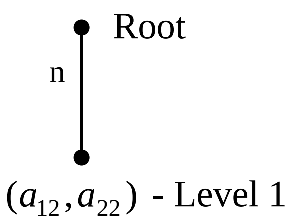
$M = \left\{ (\alpha_1, \dots, \alpha_k) = \alpha;\ \sum_{j=1}^{k} \alpha_j^{\frac{h}{n-s}} = 1,\ n > s > 0,\ 0 < \alpha_j < 1 \right\}$. The set M represents a measured curvilinear spheroid and at $s = 1,\ n = 2$ turns in usual m-a measured spheroid. Using theorem 1 from work [2], we shall receive:

$$\begin{cases} N^n = a_1^n + \dots + a_k^n, \\ N_k^n = a_{1k}^n + \dots + a_{kk}^n, \quad k \ge 2 \end{cases} \tag{16}$$

Thus, this equation is optimum in sense (16), and the tree of numbers appropriate by this equation represented on rice. 1 also is an optimum tree.

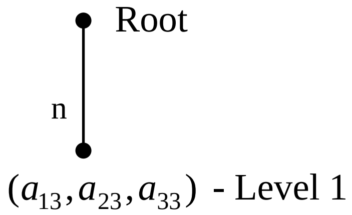
From the theorem 2 follows, that if we have a tree to the appropriate equation

$$N_2^n = a_{12}^n + a_{22}^n \quad ,$$



Then there is transformation K which is to translate the given tree on

$$N_3^n = a_{13}^n + a_{22}^n + a_{33}^n \quad ,$$



etc. and the number of elements at top to the appropriate level 1 is increased by unit and therefore, the received trees are "growing", that corresponds to value $n = 2$.

At $n \geq 3$, most likely, it cannot be approved as the equation (16) integer numbers not solving. But here, we can instead of N_k^n take any natural number $\tilde{N}_k$ and on the basis of Warring's theorem to receive representation [2]

$$\tilde{N}_k = a_{1k}^n + a_{2k}^n + ... + a_{kk}^n \quad , \tag{17}$$

and numbers such as « *a root - level 1, a level 2, a level 3, …, the level m* » in this case can not exist. But there is a decision of a problem. For trees, such as « a root, a level 1 » in a case $n \geq 3$ we can make pasting with trees of numbers at $n = 2$ (see last tree).

We shall consider the equation

$$N_2 = a_{12}^n + a_{22}^n \tag{18}$$

,...

and

$$N_m = a_{1m}^n + a_{2m}^n + ... + a_{mm}^n \quad , \qquad m \geq 2, \ n \geq 2 \ .$$

<u>The Theorem 3</u>. Transformation

$$\begin{cases} a_{im} = x a_{im-1}, \quad i = \overline{1, m-1} \\ a_{mm} = y \sqrt[n]{N_{m-1}} \\ N_m = z N_{m-1}, \quad where \ x^n + y^n = z^n \end{cases} \tag{19}$$

translates the solution of the equation

$$\sum_{i=1}^{m-1} a_{im-1}^n = N_{m-1} \tag{20}$$

on the solution of the equation

$$\sum_{i=1}^{m} a_{im}^n = N_m \tag{21}$$

Really, we shall increase both parts of the equation (10) on x^n with the account $x^n + y^n = z$ we have

$$\sum_{i=1}^{m-1} (a_{im-1}, x)^n = (z - y^n) N_{m-1} \ .$$

From here, valid (9) we shall receive (11) and more over we have

$$N_m = z^{m-1}, \ a_{1m} = x^{m-1}, \ a_{2m} = y x^{m-2},$$

$$a_{im} = y z^{\frac{i-2}{n}} x^{m-i}, \quad i = 1, \quad m = 3, ..., m.,$$

Such any representation of type (18) with help of transformation (19) is transferred to the form

$$N_m = \left(x^{m-1}\right)^n + \sum_{i=2}^{m} \left(y x^{m-i} z^{\frac{i-2}{n}} \right)^n \qquad (22)$$

It should be out that transformations (17), (19) are described the process of numbers tree grows and formula (22) is the second formula for definition of the size Losses in the Worst Condition by Kinds with Long Settlement.

Some examples of Numbers Tree. Now, we shall consider examples of Numbers Tree for different numbers.

1). Let $N = 25, \ n = 2$, then we have

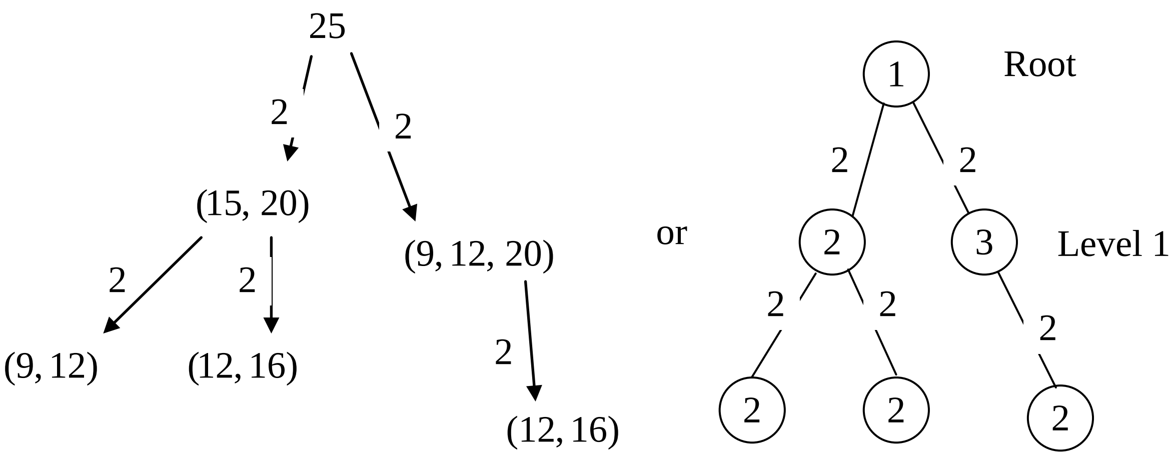


Fig.2. Model of Numbers Tree for $N = 25, \ n = 2$

(k) - means quantity (amount) of an element of the given top of a tree, and number on edges are paw decomposition. From here follows, that representation (4) takes the following kind

$$25^2 = 9^2 + 2 \cdot 12^2 + 16^2 .$$

2). Now we shall consider number $N = 50$.

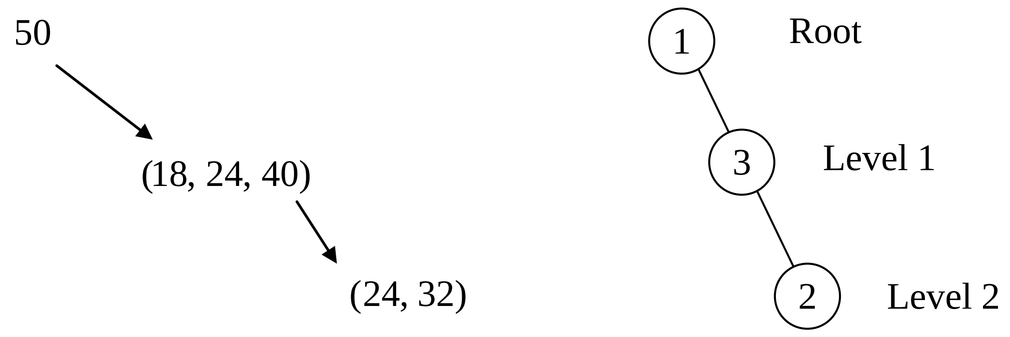


Fig.3. Model of Numbers Tree for $N = 50$

Hence $50^2 = 18^2 + 2 \cdot 24^2 + 32^2$

3). $N = 75$.

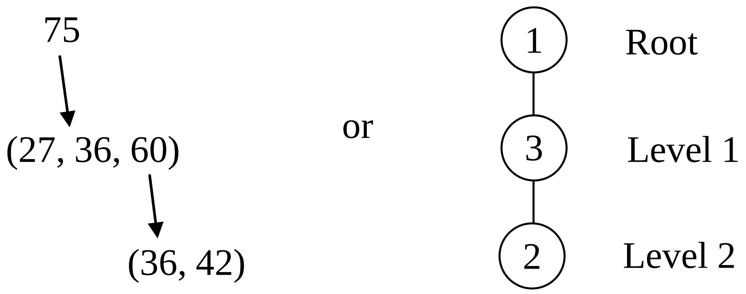


Fig.4. Model of Numbers Tree for $N = 75$

т.е. $75^2 = 27^2 + 2 \cdot 36^2 + 42^2$

4). $N = 100$.

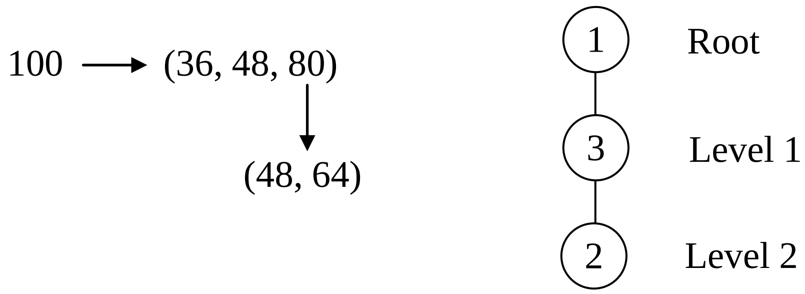


Fig.5. Model of Numbers Tree for $N = 100$

And therefore, $100^2 = 36^2 + 2 \cdot 48^2 + 64^2$.

5). Consider the number $N = 125$. In this case we received next number tree.

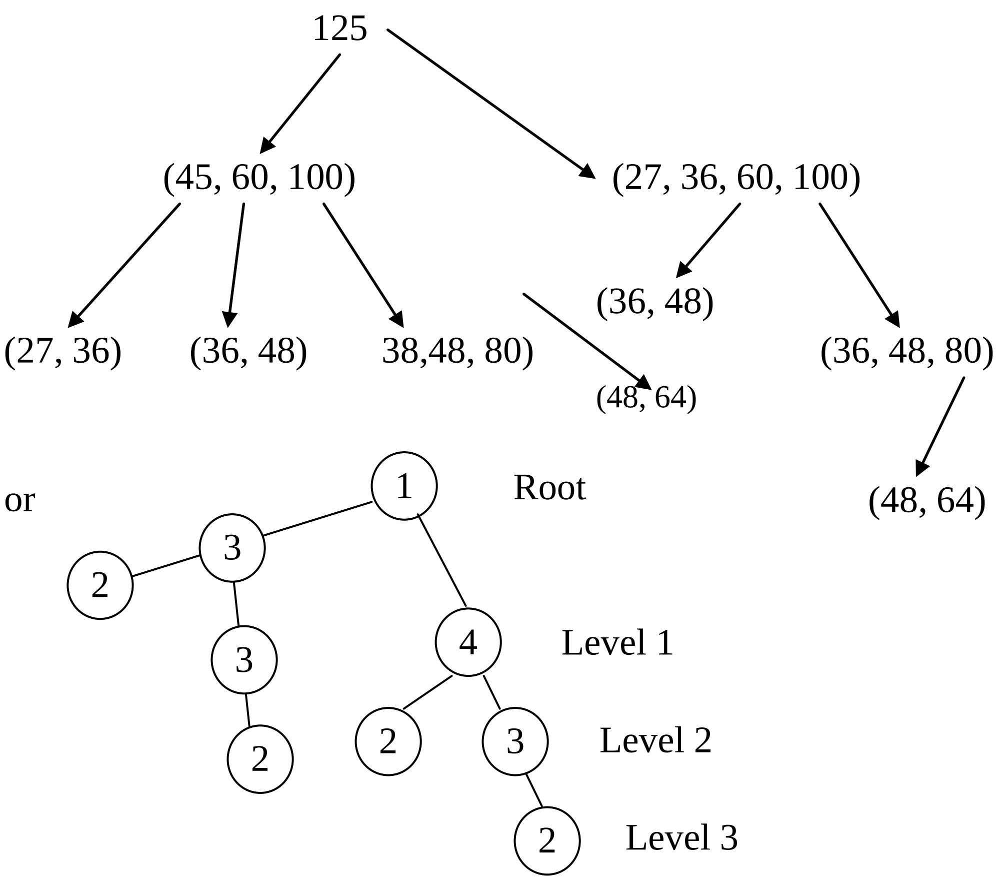


Fig.6. Model of Numbers Tree for $N = 125$

Then we have $125^2 = 27^2 + 3 \cdot 36^2 + 3 \cdot 48^2 + 64^2$

6). $N = 625,\ n = 2$.

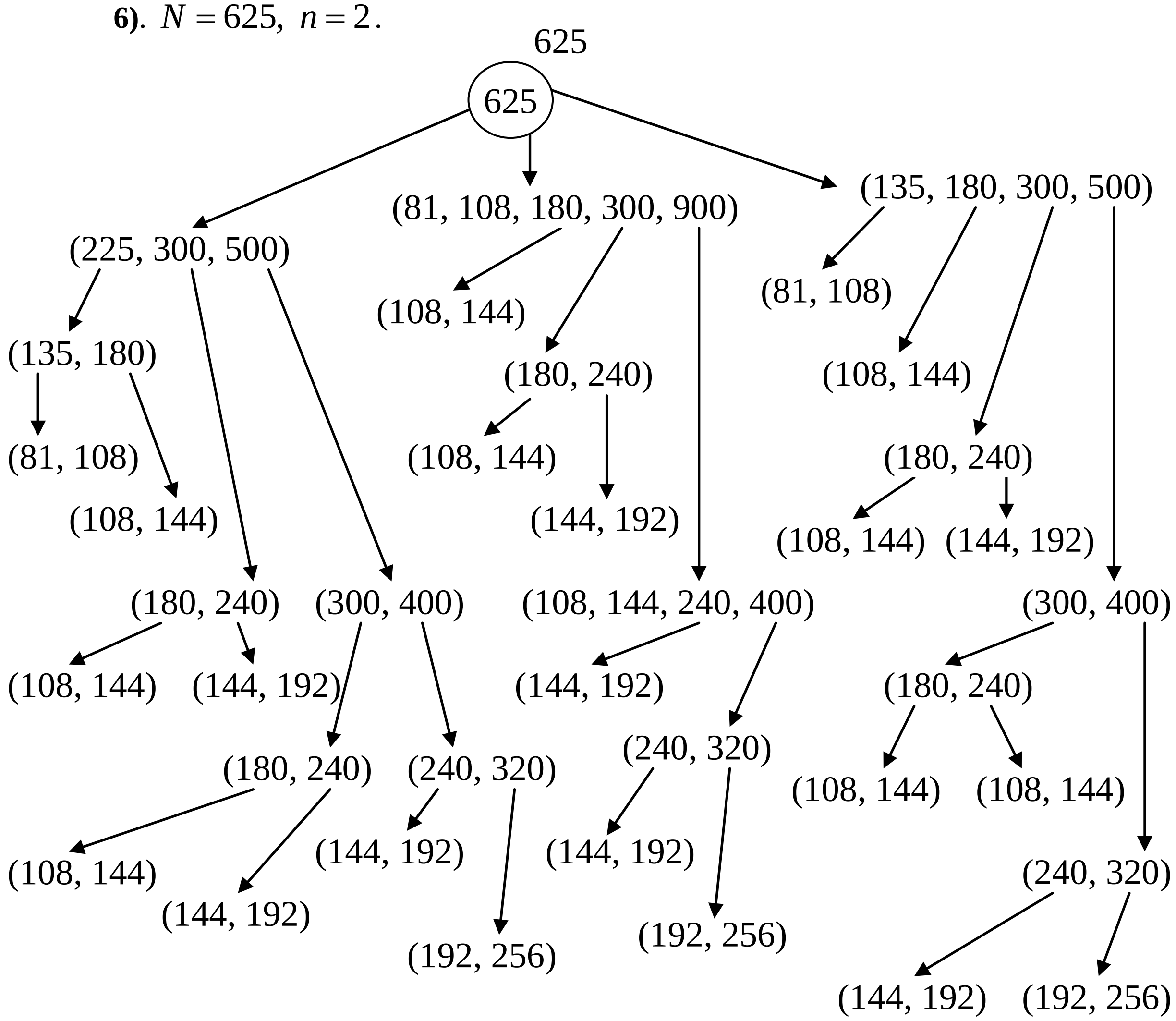
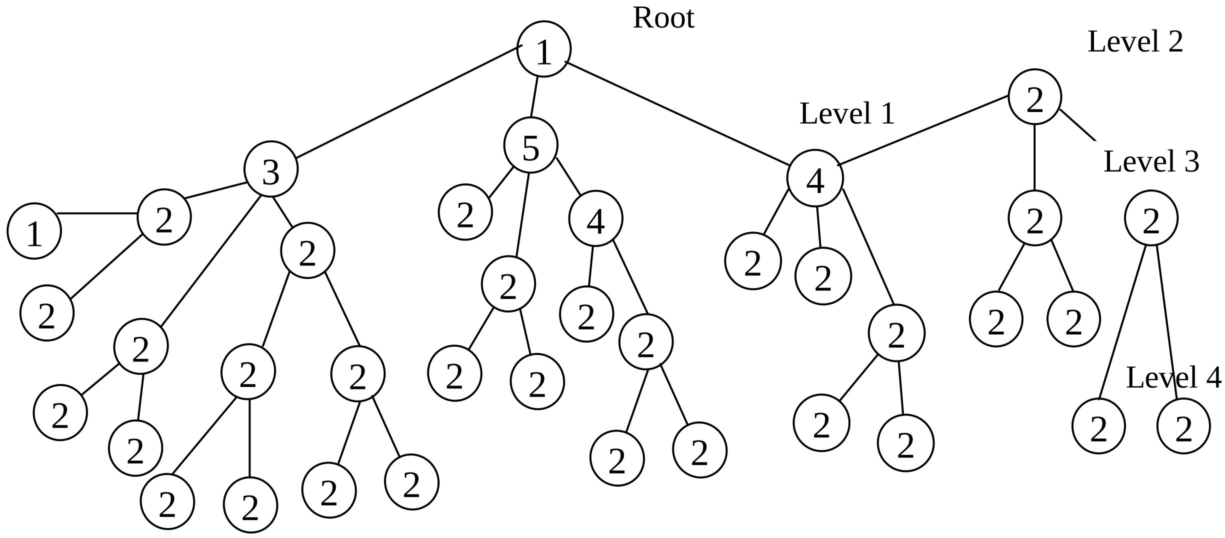


Fig.7. Model of Numbers Tree for $N = 625,\ n = 2$

or



Hence $625^2 = 81^2 + 4\cdot 108^2 + 6\cdot 144^2 + 4\cdot 192^2 + 256^2$.

7). Similarly, for $N = 3125$ we shall receive representation

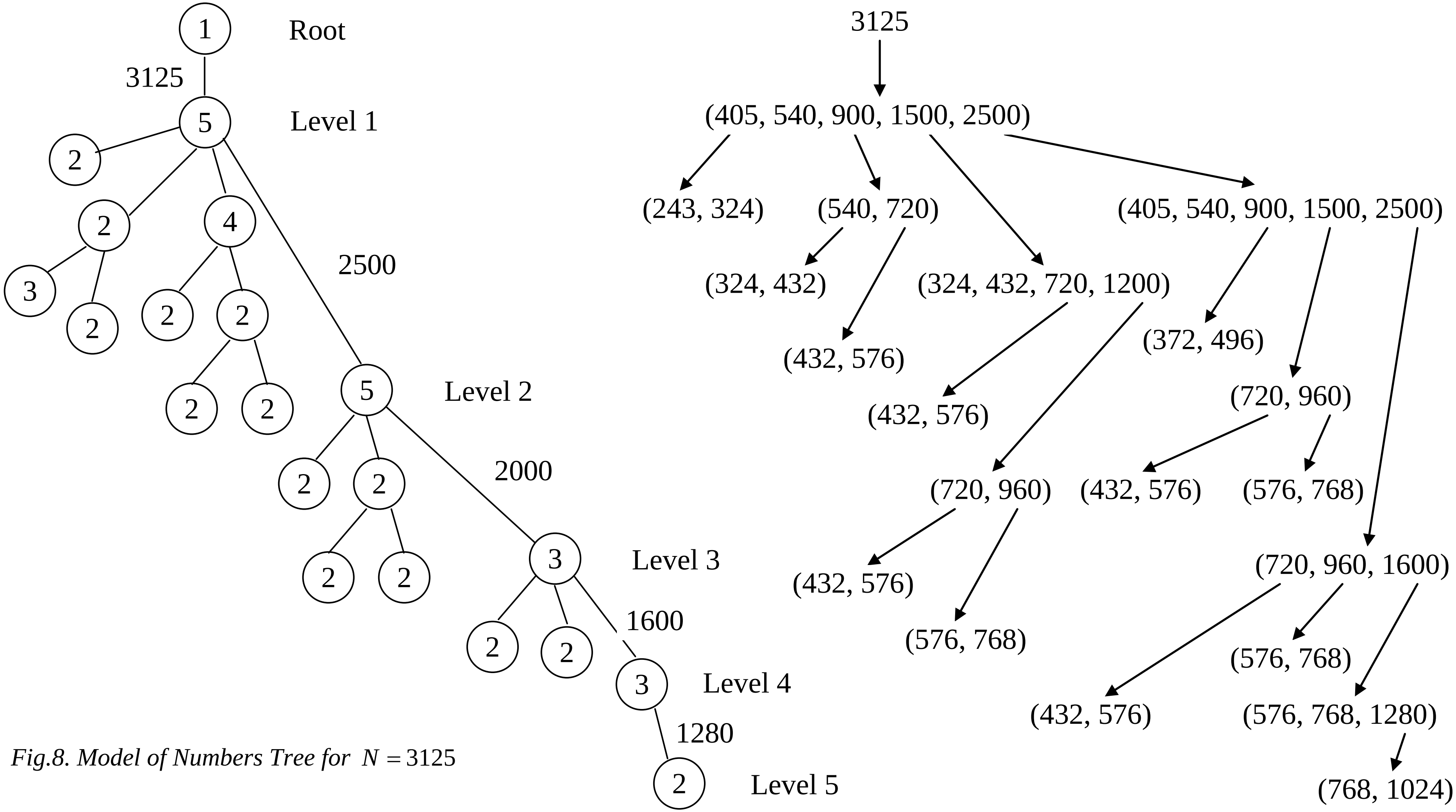


Fig.8. Model of Numbers Tree for $N = 3125$

and $3125^2 = 243^2 + 4 \cdot 324^2 + 372^2 + 8 \cdot 432^2 + 496^2 + 9 \cdot 576^2 + 5 \cdot 768^2 + 1024^2$.

For cases, when $n > 2$ not always it is possible to construct graceful examples. We shall result one example. Let, $N = 35, \quad n = 3$, then

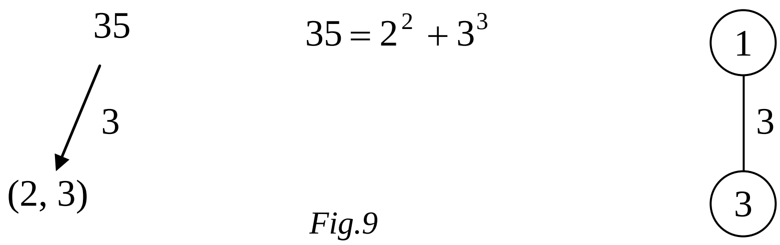


Fig.9

From the given example follows, we have not received a tree, but received only its branches. We shall name such "trees" not growing trees. For growth of such tree it is necessary to make "cuttings" from another (for example) $n = 2$ a tree and to insert them into not growing trees. For example

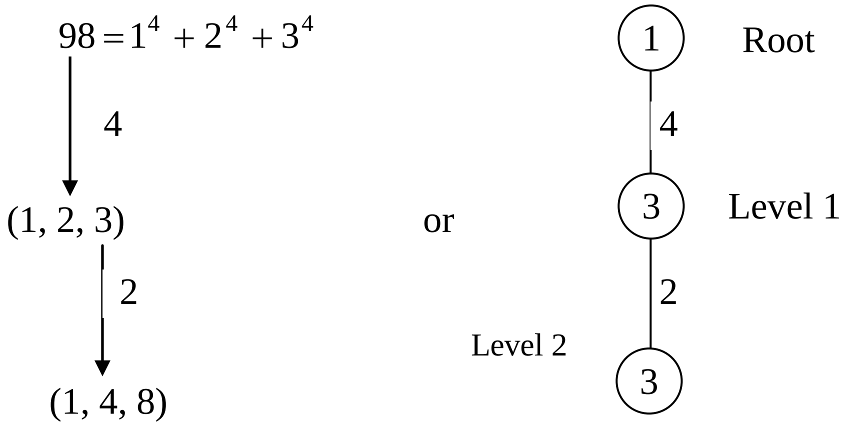


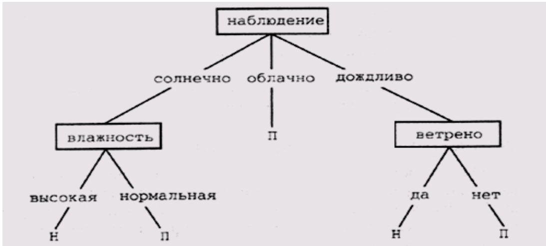
Fig.10.

i.e.. $98 = 1^4 + 2^4 + 1^2 + 4^2 + 8^2$.

Structure of a tree of decisions. The tree of decisions represents one of ways of splitting of set of the data on classes or categories. The root of a tree implicitly contains all classified data, and leaves - the certain classes after performance of classification. Intermediate units of a tree represent items(points) of decision making on a choice or performance of testing procedures with attributes of elements of the data which serve for the further division of the data in this unit. Usually] the tree of decisions is determined as structure which consists from

Units - leaves, each of which represents the certain class;

Units of acceptance of decisions, специфицирующих the certain test procedures which should be executed in relation to one of values of attributes; the unit of acceptance of decisions is left with branches which quantity corresponds to quantity(amount) of possible(probable) outcomes of testing procedure. It is possible to consider a tree of decisions and from other point of view: intermediate units of a tree correspond to attributes of classified objects, and arches - to possible alternative values of these attributes. The example of a tree is submitted on figure. On this tree intermediate units represent attributes supervision, humidity, is windy. Leaves of a tree are marked by one of two classes P or H. It is possible to count, that P corresponds to a class of positive copies consultation, and H - to a class negative. For example, P can represent a class " to leave on walk ", and H - the class " to sit at home ". Though it is obvious, that the tree of decisions is the way of representation which is distinct from inducing rules, the tree can compare the certain rule of classification which gives for each object having the appropriate set of attributes (it(he) is submitted by set of intermediate units of a tree), the decision to what from classes to attribute(relate) this object (a set of classes is submitted by set of values of leaves of a tree). In the given example the rule will carry objects to class P or H. Directly it is possible to broadcast a tree in a rule shown below:

If supervision = is cloudy v Supervision = sun & Humidity = it is normal v Supervision = rain & It is windy = there is no that P	

The reason on which the preference is sometimes given trees of decisions, instead of to inducing rules, is, that there are rather simple algorithms of construction of a tree of decisions during processing training sample, and the constructed trees can be used further for correct classification of the objects which have been not submitted in training sample.

Algorithms of extraction of precedents with use of various metricss and the account of factors of importance of parameters of object. On the basis of the modified method of definition of the nearest neighbour described above (the nearest neighbours) the appropriate algorithms of extraction of the precedents, using various metrics for definition of a degree of similarity of precedents with the current problem situation and taking into account factors of importance of parameters of object were developed. Let's consider algorithm of extraction of precedents from SP with use general Euclidian metrics. The entrance data: current situation T (i.e. the values of the parameters describing a usual situation), CL - nonempty set of precedents (SP), w_1 ..., w_n - weights (factors of importance) parameters, m - quantity of considered(examined) precedents from SP and threshold value of a degree of similarity K. The target data: Set of precedents SC (Set of Cases) which have a degree of similarity (affinity) more or equal threshold value K. The intermediate data: Auxiliary variables i, j (loop variables).

1. SC=Æ, j=1 also we pass to the following step.
2. If j≤m we choose precedent Cj from set CL ($C_{j\hat{\imath}CL}$) and we pass to a step 3, differently all precedents from SP are considered and we pass to a step 6.
3. We consider distance in евклидовой to the metrics between chosen precedent Cj and current situation T (dCjT) in view of factors of importance of parameters:

$$Y_k = \max_{\omega \in M} \left(\sum_{i=1}^{k} \omega_i Y_{ik}{}^{s} \right)^{1/s}, \text{ where } M = \left\{ \omega : 0 \le \omega_j \le 1, \sum_{j=1}^{k} \omega_j{}^{\frac{n}{n-s}} = 1,\ n > s,\ s > 0,\ k > 1 \right\},$$

$d_{CjT} = \arg Y_k$, $Y_{ik} = (x_{iC} - x_{iT})$. In case of absence of value of parameter x_{iCj} in the description of precedent Cj we spend calculation of distance d_{CjT}, taking into account, that $xi_{Cj} = x_{iT}$, and for a case when there is no value of parameter x_{iT} in the description of current situation T calculation of distance d_{CjT} is carried out, believing xiT = max {(x_{iCj} - $x_{i нач}$), (xiкон - x_{iCj})}. Further we pass to the following step.

4. On this step we calculate a degree of similarity $S_{(Cj, T)} = 1 - d_{CjT}/d_{MAX}$ or in percentage $S_{(Cj, T)} = (1 - d_{CjT}/d_{MAX}) * 100$ % if threshold value K is given in percentage, (at calculation dMAX weights of parameters are taken into account) and we pass to a step 5.
5. If S (Cj, T) ≥K given precedent Cj is added in resulting set $SC_{(Cj\hat{I}SC)}$, i.e. the given precedent from БП is taken. After check j=j+1 also we pass to a step 2.
6. If SC=Æ precedents for the current problem situation are not found and we pass to a step 7 with distribution of the message for user about necessity of reduction of threshold value K, differently precedents for the current situation are successfully taken and we pass to the following step.
7. The end (end of algorithm).

In the result, the found precedents can be ordered on decrease of values of their degree of similarity to the current situation and are given to user.

Difference of other metric algorithms from considered consists that on the third step the distance is calculated with the help of other metrics. Realization of the generalized algorithm of extraction of precedents is in the long term possible (in the long term probable) on the basis of various metrics. Use of various metric algorithms of extraction of precedents in systems of expert diagnosing a technical condition of complex objects and, in particular, subsystems of the power unit provides more flexible work of mechanisms of search of the decision on the basis of precedents. At user there is an opportunity to consider various metrics for extraction of precedents from SP systems that provides a choice of more adequate metrics, capable to take into account specificity of a concrete decided(solved) problem) of expert diagnosing. It is necessary to note, that in algorithms of extraction of precedents for the account of factors of importance of parameters of object it can be carried out preliminary a stage (the Step 0) updating of values of borders of ranges of parameters and parameters that excludes necessity for the subsequent account of factors of importance at extraction of some precedents.

Economical interpretation: If tops and other levels " leaves "of tree corresponding numerical characteristics (cost, time of realization, probability of end in time etc., necessary for their realization) as a result of processing model, the best variants of performance of the initial purpose, we can be found some other realization of our initial models, i.e. model of tree numbers. As criteria of a choice the minimal cost, an expenditure of any other resources, time, probability of failure of plans etc can serve. Proceeding from the big number of the purposes of the enterprise, their individual character and complex interrelations for their analysis the special model - model of a tree of the purposes is used. Besides the similar model allows taking into account and existence of hierarchy of the purposes. It means that between the purposes, besides conflicts, usually there are also other communications. They are attitudes or relations of submission (for realization of the purpose and realization of purposes B is necessary, C etc. which name purposes - means) and presentation (up to purpose D it is necessary to execute purpose E). Besides between the purposes, there can be attitudes or relations of joint submission at which they are details parts or predecessors of same more global purpose. For construction of such model of the formulation of the purposes should consist of the following elements:

· *The maintenance (contents) of the purpose* (that should be achieved?);
· *Scale of the purpose* (in what volume the purpose should be achieved?);
· Term of performance of the purpose (for what time the purpose should be achieved?).

The model of a tree of the purposes can be described with the help *of the messenger focused treelike the column* with tops are the purposes of a various degree of detailed elaboration, and edges - communications (connections) between them. These communications (connections) consist that for performance of some purpose (top the column), it is necessary and to execute even its part sub purposes (the tops subordinated to it) enough.

The column is understood as "connectivity", that it cannot be broken even on two, the purposes completely independent from each other system. "Orient ability" means, that for two elements connected among themselves and is correct only one of statements such as» For performance of the purpose and is necessary to execute the purpose in " or, on the contrary,". For performance of the purpose In, it is necessary to execute the purpose and ".

The model of a tree of the purposes only in part corresponds to strict definition of concept "tree" of the theory graph and has the following features:

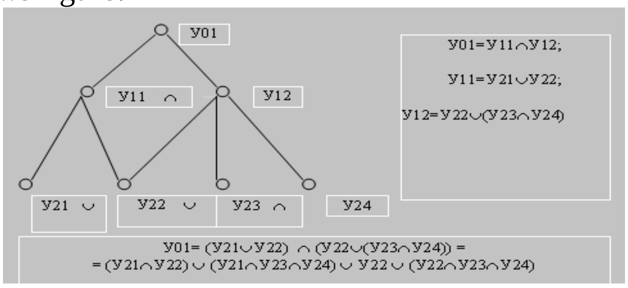
1). There is a unique top - "root" of a tree which is not sub top any other top. It is an overall objective, and the others only detail, open her;

2). At all levels, except for the first to which there corresponds (meets) a root; there can be the tops which are not having sub top - "leaves" of a tree. Leaves - the finest, private (individual) purposes (purposes - means or actions), not subject to the further decoding proceeding from the chosen degree of detailed elaboration;

First two requirements are completely borrowed from the theory graph for objects such as "tree".

1. The same top can be sub top several tops. It means that the same event or purpose can be required for realization of the different purposes more a high level. Introduction of such difference from a classical tree of the theory graph reflects existence of effect c-energy (for example, - reduction of expenses due to multi-purpose use of the same element);

2. For realization of any top, which are not being "sheet", it can be necessary and enough performance only for its part sub top (alternative group). Practically it means that there are various ways of performance of the same problem, and each way can be opened as more detailed recipe. Among themselves tops in alternative group are connected by a parity logic and, between groups the parity (ratio) logic operates. The typical model of a tree of the purposes of the enterprise looks as follows figure**:**



Here at its top level there is the unique fictitious purpose (У01) consisting in performance of all purposes subordinated to it (У11 and У12), describing various fields of activity. The following levels detail these purposes to the chosen classification attributes. It is sub purpose (У21, У22 etc.). Elements of a tree of the purposes, realizations revealing a way sub purpose are purposes - means, and "leaves" - concrete actions. The algorithm of processing of similar model includes consecutive substitution of the expressions appropriate to the description sub-purpose in the formulas for the purposes of the previous level, and then their disclosing by rules of logic addition and multiplication. In connection with that by definition for performance of the top, which are not being a sheet, is necessary and enough realization of any of sub-ordinates for it of the alternative groups, received variants of realization of the initial purpose can be simplified. First, it is possible to exclude those variants in which contain not only the same sub-purpose, as in any other alternative groups from the subsequent consideration, but also additional elements. So, in the given example variant У21^У22 obviously is less effective, than У22 as demands not smaller expenses, but results besides to result. Second, anyone sub-purpose should meet in each alternative group no more than once, i.e. (certainly, within the framework of one group) it is possible to exclude repeating elements. Bookkeepers frequently face with problems which result them in the quantitative approach of acceptance of decisions. Therefore in the report the quantitative analysis will be stated. However you should remember, that usually the decision grows out applications both quantitative, and subjective approaches. The subjective approach is expressed in the various purposes and characters of people managing the company.

So, to find the good decision, follows:

- To define, the purpose of the decision.
- To define possible variants of the decision of a problem.

- To define possible outcomes of each decision.
- To estimate each outcome (to count up the possible income).
- To choose the optimum decision on the basis of an object in view.

As you can see, search of the decision begins with transfer of possible(probable) variants and their outcomes, then the estimation of each outcome is made. The circuit of reasoning is those at realization of the quantitative analysis. The technique based on construction of tables of incomes illustrates the unique decision, "tree" shows set of decisions and their outcomes, therefore for consideration we shall choose it.

Example a)

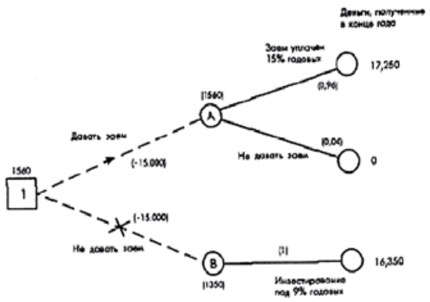
For financing the project the businessman needs to borrow(occupy) 15000 rubles for the period of one year. The bank can borrow(lend) it(him) this money under 15 % annual or enclose in business with 100 %-s' return of the sum, but under 9 % annual. From the last experience the banker knows, that 4 % the loan do not return such (0,04) clients. What to do? To give it a loan whether or not? We shall take advantage of "tree" of decisions.

The decision. If the loan was given and returned, the net profit will make:

The net profit = ((15000 + 15 % from 15000) - 15000) = 2250 rubles.

If the loan was not given, and enclosed in business the net profit will make:

The net profit = ((15000 + 9 % from 15000) - 15000) = 1350 rubles.



Example b)

Let's consider a situation more complex(difficult), than in the previous example, namely: The bank solves the problem whether to check competitiveness of the client before giving out a loan. The auditor firm takes from bank 80 rubles for each check. As a result of it before bank there are two problems:

The first - to carry out(spend) or to not inspect,

The second - to give out after that a loan whether or not. Solved the first problem, the bank checks correctness of data given out by auditor firm. For this purpose get out 1000 person which were checked up also which subsequently loans were given out:

Table 1. Recommendations of auditor firm and return of the loan

Recommendations after check of credit status	Actual result: The Client the loan has returned	the Client the loan has not returned	All
To grant the loan	735	15	750
To not grant the loan	225	25	250
In total	960	40	1000

What decision the bank should accept?

Stage 1. Let's construct "tree", as shown below. Probabilities are put down according to a stage 2.

Stage 2. Using given tab. 1, we shall calculate probability of each outcome:

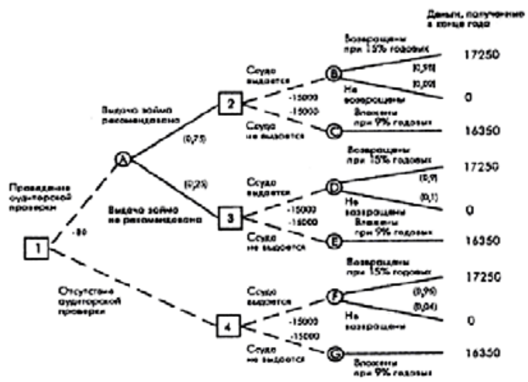
P (the client will return the loan; the firm recommended) = 7,35/750 = 0,98;

P (the client will not return the loan; the firm recommended) = 15/750 = 0,02;

P (the client will return the loan; the firm did not recommend) = 225/250 = 0,9;

P (the client will not return the loan; the firm did not recommend) = 25/250 = 0,1.

Stage .3. At this stage from left to right we shall put down monetary outcomes of each of "units", using the end results calculated earlier. Any meeting charges it is deducted from expected incomes. Thus we count up all "tree", leaning(basing) on earlier received results. After пройдены squares of "decisions", the "branch" conducting to greatest of possible(probable) at given decision expected income gets out. Other "branch" is crossed out, and the expected income is put down above a square of the decision.



Conclusion: The propose method basis on numbers tree model, is a simple and universal method for definition of value of Losses in the Worst Condition by Kinds with Long Settlement and it is easy programmed on all computer languages. Receiving formulas (4), (5), (14), (22) are controlled our calculations. As the methodology actuaries calculations uses the probability theory, given and the long-term statistical data, financial calculations that on faculty are in full read to a demography rates connected with System mathematical and the statistical regularities establishing mutual relation between the insurer and the insurant. They reflect as mathematical formulas the mechanism of formation (education) and an expenditure of insurance fund in long-term insurance operations. To them also carry calculations of tariffs on any kind of insurance: life's pensions, from accidents, property, work capacity. The methodology actuary's calculations use the probability theory, given to demography and the long-term statistical data, financial calculations. By means of the last in tariffs the income which is received by the insurer from use as credit resources of the accumulated payments of insurants is taken into account. Except for that rates on actuaries are read to calculations which is connected to one of the widespread problems(tasks) of such -statistical calculations connected to definition of norms and conditions of insurance, is, that the sum of insurance payments minus relying payments guaranteed reception by insurance firm (or the state organization) expected results. Making "tree" of decisions, it is necessary to draw "trunk" and the "branches" displaying structure of a problem. "Trees" from left to right settle down."Branches" designate possible alternative decisions which can be accepted, and the possible outcomes arising as a result of these decisions. On the circuit we use two kinds of "branches": The first - the dashed lines connecting squares possible(probable) decision, the second - the continuous lines connecting circles of possible(probable) outcomes. Square "units" designate places where it is made a decision, Round "units" - occurrence of outcomes. As accepting the decision can not influence occurrence of outcomes, it needs to calculate probability of their occurrence only. When all decisions and their outcomes are specified on "tree", each of variants is counted, and in the end its monetary income is put down. All charges caused by the decision, are put down appropriate "branch".

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