THE INTERVAL ANALYSIS IN MARKOV MODELS USING FOR ACTUARIAL CALCULATIONS

Spivak Semyen
Bashkir State University
Frunze str. 32, Ufa, 450074, Russia
Phone: +7 3472-736162
Fax: +7 3472-736680
Email: s.spivak@bashnet.ru

DIRECT AND INVERSE PROBLEM

Two mutually opposite problems arise
when using Markov processes for modeling.

DIRECT PROBLEM

is to calculate probabilities of corresponding states
and other characteristics of process.
It is assumed that parameters of model are available.

INVERSE PROBLEM

is to evaluate parameters of model
by using experimental output data.

In Markov model input parameters are forces of transition from state to state. When dealing with queuing problems or insurance models these values are unknown in advance. Meanwhile, statistical information about output results for some models can be found in special literature.

ESTIMATING THE PROBABILITIES

As stated above, many traditional problems of actuarial analysis can be considered in terms of multi-state processes. It is assumed that at any time the individual is in one of a number of states. The current state of the individual or movement (transition) from one state to another may have some financial impact. The task is to quantify this impact. That is, we need to estimate the probabilities of being in a particular state.

Markov process is a very convenient tool for calculation the occurrence probabilities of any events.

Figure 2 represents the scheme including three possible states: "active", "disabled" and "dead" commonly used in modeling disability insurance. In this case premiums are payable while the insured is in state 1, and benefits are payable while he is in state 2.

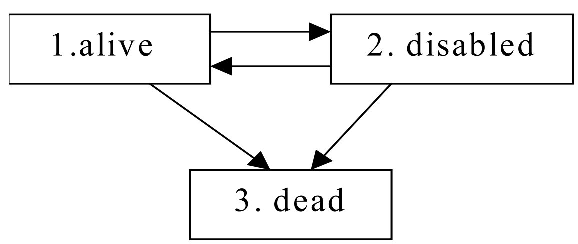


Figure 2.
Model with three states.

THE PARAMETERS ESTIMATION

One more assumption, which would make a solution much simpler, is that we have the complete information about each observable individual's history. In [Industrial Accident Insurance: Actuarial Foundations. Ed. by V.N. Baskakov, Moscow, Academia, 2001] the model of many states is applied to modeling the process of disability insurance. The estimates of transition forces are carried out by the maximum likelihood method with the complete data – all individual's transition states and time spent in them are assumed known.

However, it is very rare when we have the complete data. In the problems of life insurance experimental information is likely to be represented in so-called mortality tables only. From them one can obtain the information on values of probability of death in a particular age for individuals of a particular group.

One of the most common methods of statistical estimation is

the least squares method.

The idea being is that the unknown parameters are estimated by minimizing the standard error vector.

The two-sided interval in the inverse problem of the estimation parameters of Markov models.

Kantorovich L.V. About new points of view in the problems of measurements analysis // Russian: Siberian Mathematical Journal,, 1962, v.3, №5, p. 701-709.

The calculation and measurements proximity:

$$\left| p_j^m - p_j^c \right| \leq \varepsilon \qquad (1)$$

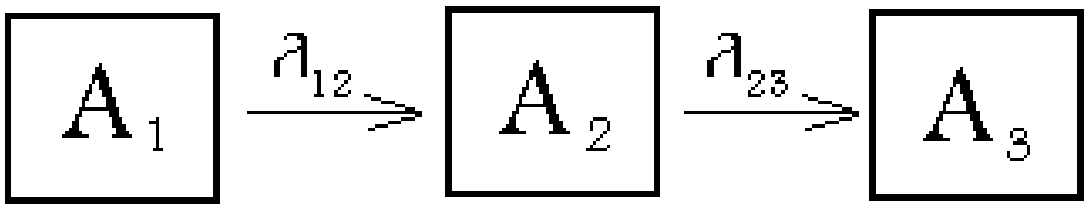
Where p_j^m – measurement, p_j^c – calculation,
ε - the vector of possible measurements error.

Problem:
The definition [min λ_s ; max λ_s]
by condition (1)

We must decide 2s Mathematical Programming Problems.

Mathematical Model of Tubercular Process.

We consider the following graph for Tubercular Process:



A1 – is well;
A2 – is tubercular patient;
A3 – is died.

Where λ is the intensity of transfer between states.

Spivak S.I., Raimanova G.K. Mathematical model of Tubercular Process. // *Russian: The Control Systems and Information Technologies, 2009, №2.2(36), p.293-297;*

The Colmogoroff equations

The system of Differential Equations:

$$\begin{cases} \frac{dp_1(t)}{dt} = -\lambda_{12} p_1(t) \\ \frac{dp_2(t)}{dt} = \lambda_{12} p_1(t) - \lambda_{23} p_2(t) \\ \frac{dp_3(t)}{dt} = \lambda_{23} p_2(t) \end{cases}$$

.

$p_i(t)$ $(i = 1..3)$ – the probability of A_i.

.

The initial condition:

$p_1(0) = 0{,}9987645$, $p_2(0) = 0{,}0012355$, $p_3(0) = 0$

The Normalization: $p_1(t) + p_2(t) + p_3(t) = 1$

The Analysis of real situation in Bashkortostan Republic

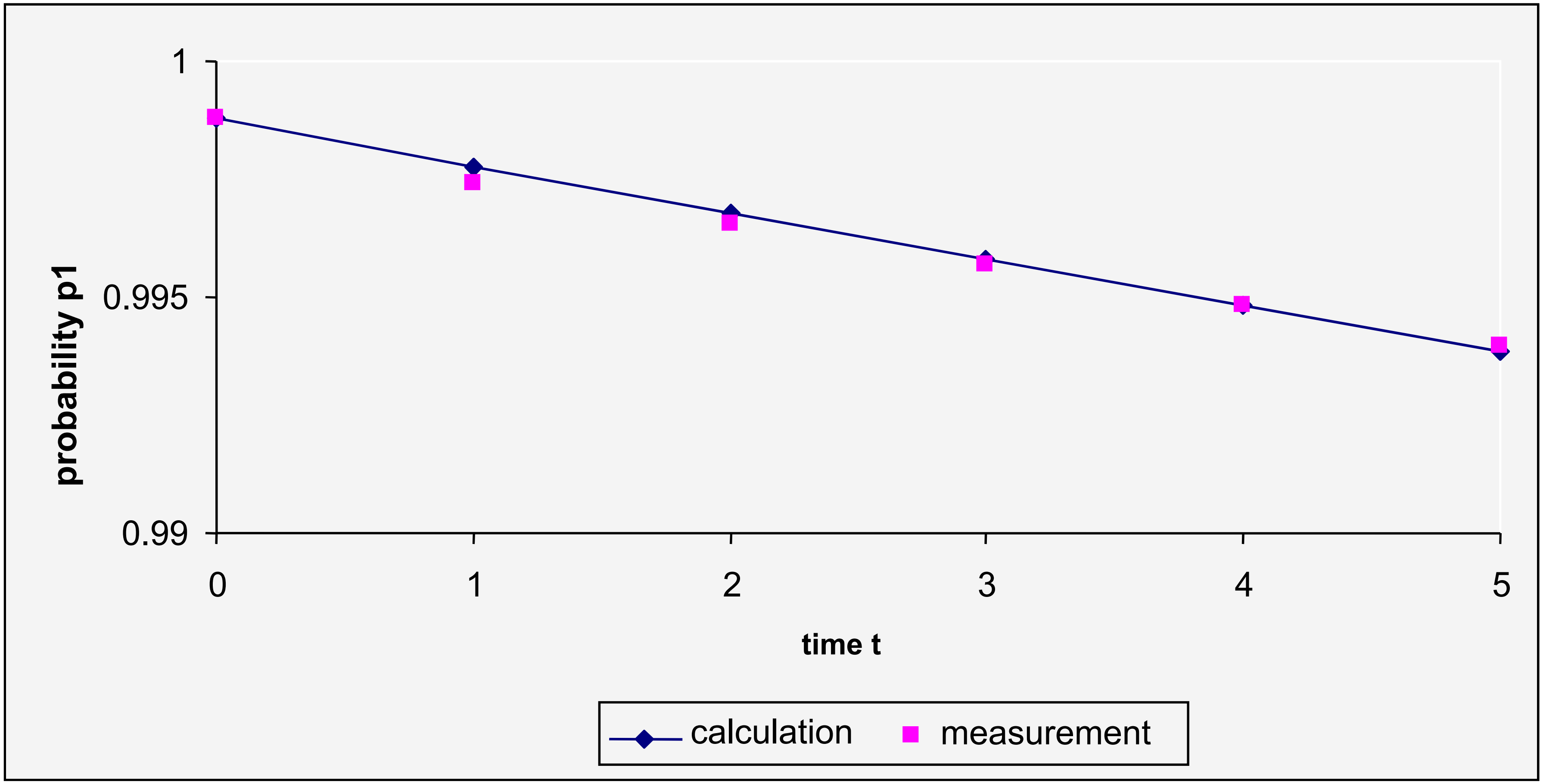
The Number of People in Corresponding States

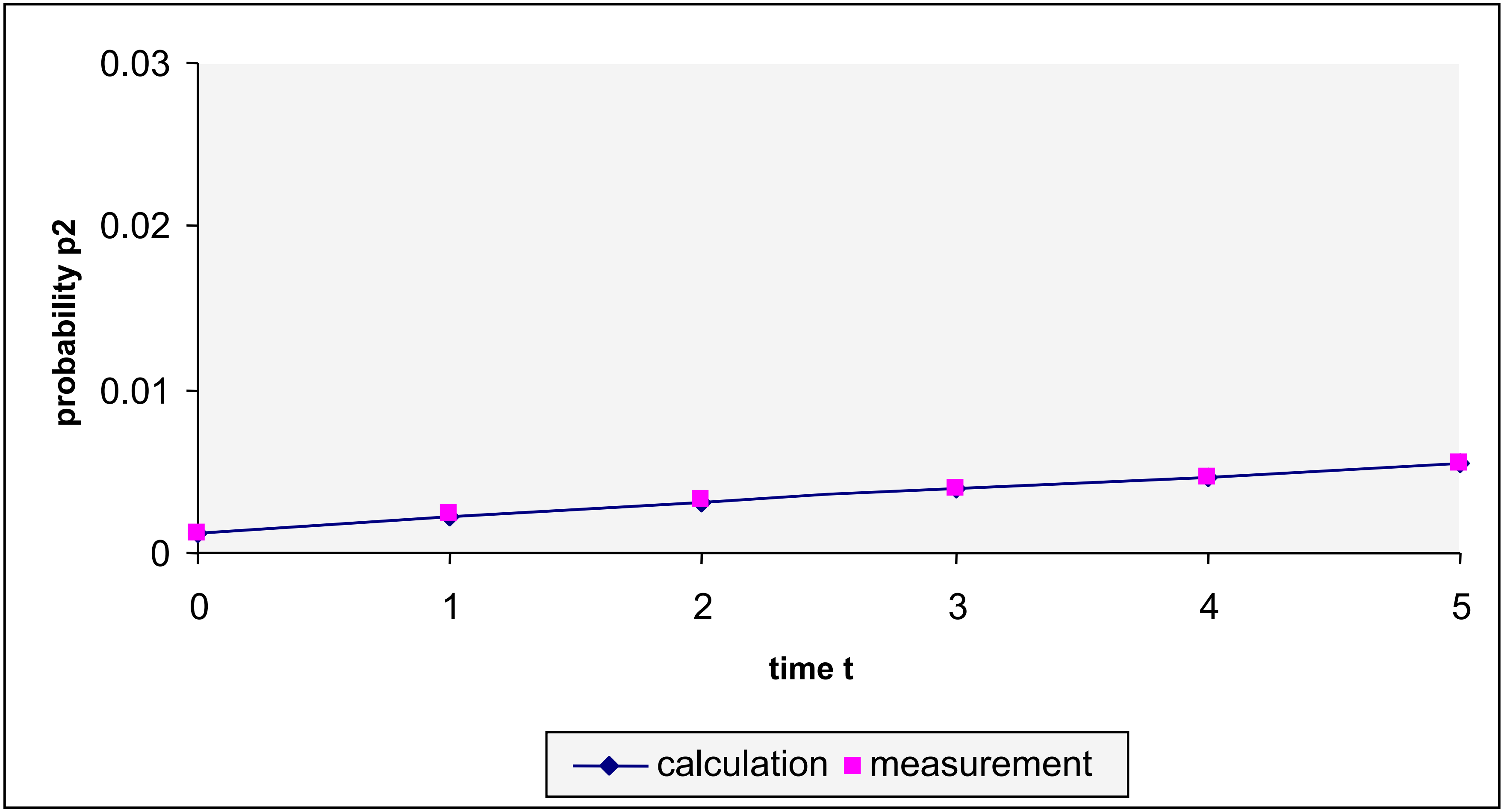
	2001	2002	2003	2004	2005	2006
time	0	1	2	3	4	5
is well	3995058	3989765	3986276	3982784	3979308	3975789
tubercular	4942	9770	12788	15791	18784	21822
died	0	465	936	1425	1908	2389

The Probability of Corresponding States

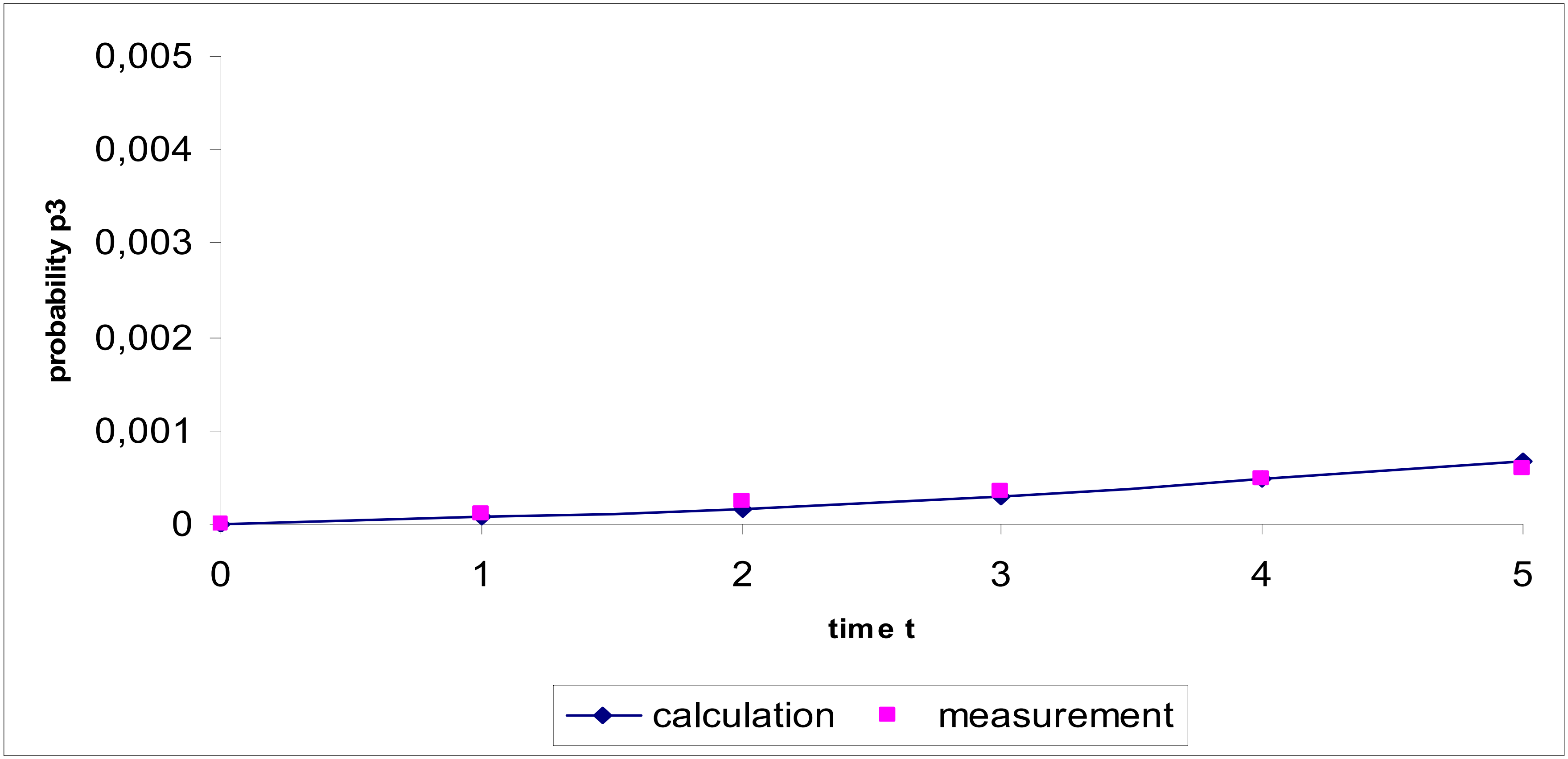
	2001	2002	2003	2004	2005	2006
t	0	1	2	3	4	5
p_1	0,9987645	0,99744125	0,996569	0,995696	0,994827	0,99394725
p_2	0,0012355	0,0024425	0,003197	0,00394775	0,004696	0,0054555
p_3	0	0,00011625	0,000234	0,00035625	0,000477	0,00059725

The Calculation and Measurements:





0.03
0.02
0.01
0
probability p2
0
1
2
3
4
5
time t
calculation
measurement


0,005
0,004
0,003
0,002
0,001
0
probability p3
0
1
2
3
4
5
time t
calculation
measurement

The intervals of intensity of transfer are the solution of inequality systems

$$\left| p_1^m - p_1^c \right| \leq 0.01$$

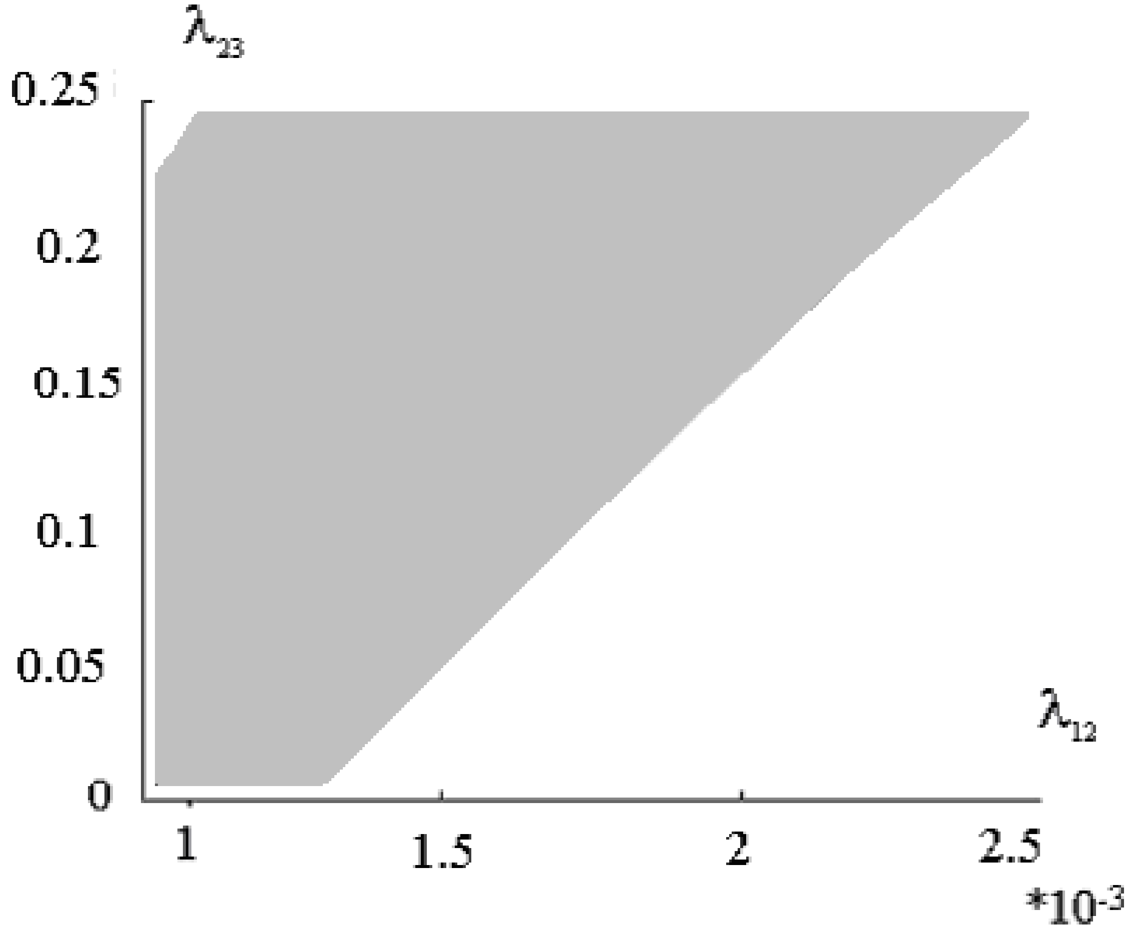
$$\left| p_2^m - p_2^c \right| \leq 0.002$$

The results of calculation:

$$\lambda_{12} = [0{,}001;0{,}0029]$$

$$\lambda_{23} = [0{,}00001;0{,}2462]$$

The Uncertainty Domain



The real and calculation interval of expenses to medical aid.

The course of treatment for tubercular patient is approximate 259000 Russian rubles in one year. The interval of transfer is equal [140 000.; 383 000].

From expectation of patient number we have the interval of expenses to medical aid for tubercular patients [140 000 rub. *4942; 383 000 rub.* 21822]=[691 880 000rub.; 8 357 826 000 rub.]=[6,9188*10^8 ; 83,57826*10^8].

We can calculate the interval of expenses to medical aid for tubercular patients using our model:

$$\lambda_{12} = [0{,}001;0{,}0029]$$

$$[0{,}001 * 259000 * 4000000;0{,}0029 * 259000 * 4000000] =$$
$$= [1036000000;3004400000] = [10{,}36 * 10^8;30{,}044 * 10^8]$$

<u>Thus, the calculation interval is belonging to real interval.</u>