A nonlinear reinsurance treaty *versus* piecewise linear treaties*

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Maximization of the adjustment coefficient

This paper follows

- Guerra, M. and Centeno, M. L. (2008). Optimal reinsurance policy: The adjustment coefficient and the expected utility criteria, *Insurance: Mathematics and Economics* 42, 529-539.
- Guerra, M. and Centeno, M. L., Optimal reinsurance for variance related premium calculation principles, *ASTIN Bulletin*, to appear.
- Centeno, M. L. and Guerra, M. ,The optimal reinsurance strategy - the individual claim case, Insurance: mathematics and Economics, forthcoming.

Problem: Find among "all possible" reinsurance schemes, the reinsurance policy which maximizes the adjustment coefficient of the retained risk.

Notation and Assumptions

- Per-claim (or per event) reinsurance
- N - insurer's number of claims in a given period of time (one year); (Special case $N\equiv 1$)
- Y_i - value of the ith claim occurred;
- $Z(y)$ - value ceded to the reinsurer under the reinsurance policy Z for a claim size y;
- $0\leq Z(y)\leq y$

Notation and assumptions

- $\hat{Y}$ - aggregate claim amount,

$$\hat{Y} = \sum_{i=0}^{N} Y_i, \quad Y_0 \equiv 0$$

- $\{Y_i\}_{i=1,2,\ldots}$ are i.i.d and independent of *N*
- $E[Y^2] < +\infty$
- N belongs to the $(a,b,0)$ family
- c – gross (of reinsurance) premium

Notation and assumptions

$\hat{Z} = \sum_{i=0}^{N} Z(Y_i)$ - ceded aggregate claims

$\hat{Y} - \hat{Z} = \sum_{i=0}^{N} (Y - Z(Y_i))$ - retained aggregate claims

$P(Z)$ – reinsurance premium (on the aggregate claims)

Main Assumptions

$$P(Z) = E[\hat{Z}] + g(Var[\hat{Z}])$$

g is an increasing function

$$g(0) = 0,\ g'(x) > 0,\ \forall\ x > 0$$

$$\frac{g''(x)}{g'(x)} \geq -\frac{1}{2x}$$

$$L_Z = c - P(Z) - (\hat{Y} - \hat{Z}) =$$

$$= c - P(Z) - \sum_{i=0}^{N} (Y_i - Z_i)$$

L_Z - net result

Problem

- The adjustment coeficient is the only positive root of

$$E[e^{-RL_Z}] = 1$$

- Problem:

$$Find\,(R^*, Z^*)$$

such that

$$R^* = R_{Z^*} = \max\{R_Z\}$$

Optimal Treaty

$$y = Z(y) + \frac{1}{R}\ln\left(\frac{Z(y)+\alpha}{\alpha}\right)$$

$$\alpha = \frac{1}{2g'(Var(\hat{Z}))} - \left(1 - \frac{Var(N)}{E(N)}\right)E[Z]$$

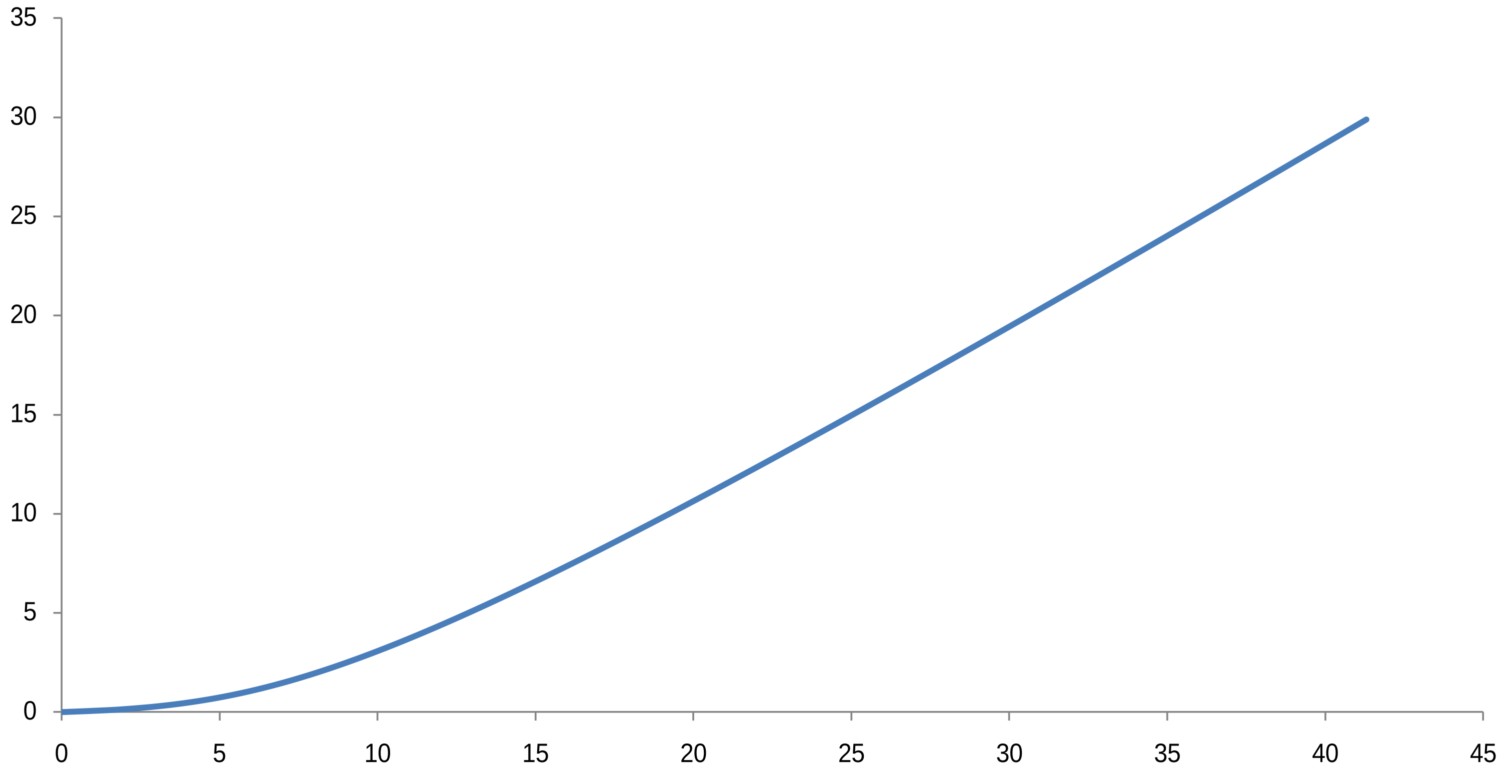
$$E[e^{-RL_Z}] = 1$$

Optimal Treaty

- Note that the distribution function of Z is easily calculated as

$$\Pr\{Z_{R,a} \le z\} = \Pr\left\{ Y \le z + \frac{1}{R}\ln\left(\frac{z+a}{a}\right)\right\}$$

Optimal Treaty



Piecewise Linear Policies

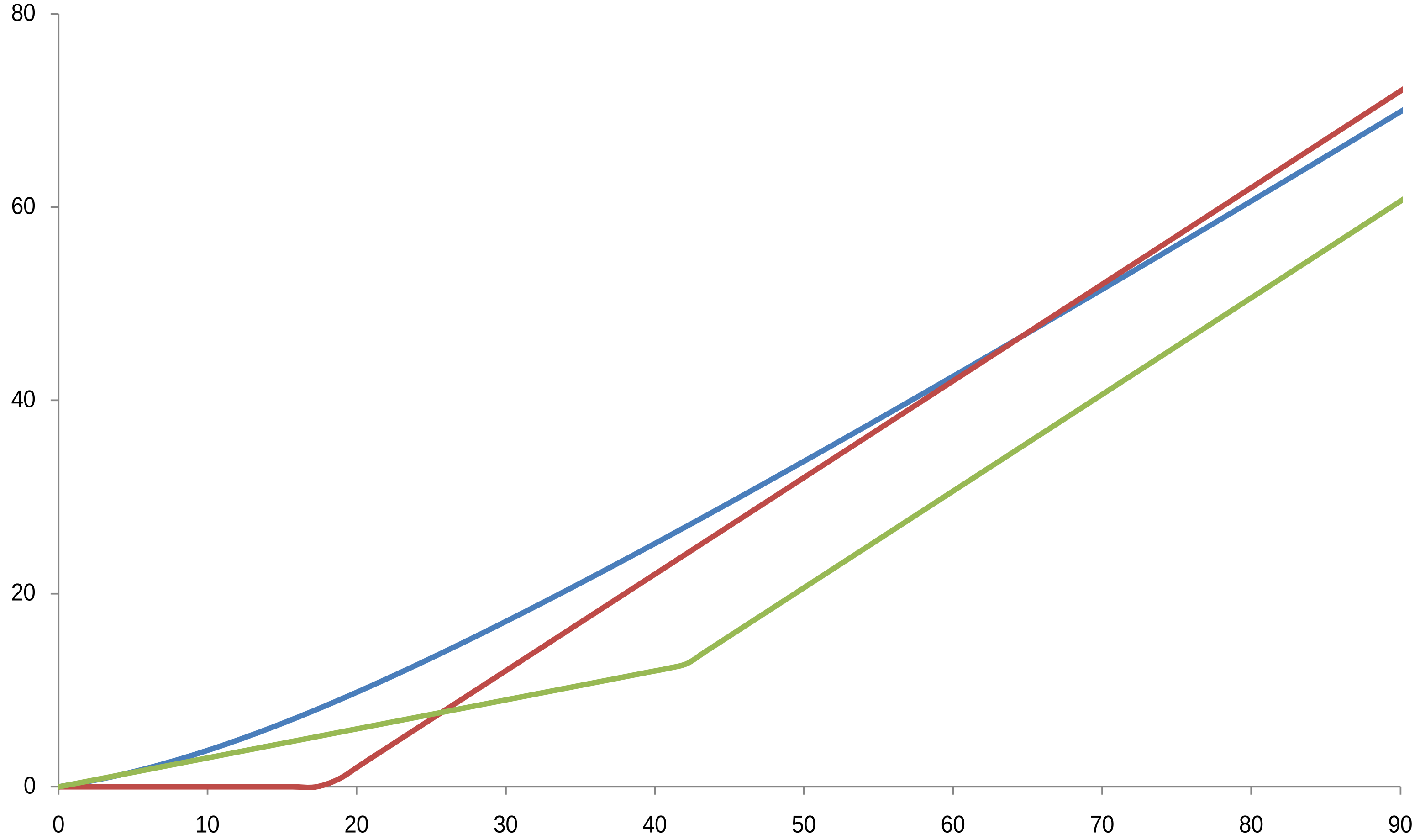
- Excess of Loss Treaty

$$Z(y) = \max(y - M, 0)$$

- Combination of quota-share with excess of loss

$$Z(y) = \begin{cases} \alpha y & y \le M \\ y - (1-\alpha)M & y > M \end{cases}$$

Optimal Treaty versus Piecewise linear



VaR and CTE

$$\hat{T}_Z = \sum_{i=0}^{N} (Y_i - Z_i) + P(Z)$$

- total risk exposure of the insurer in the presence of reinsurance

$$VaR_{\hat{T}(Z)}(p) = \inf\left\{x : \Pr\left\{\hat{T}(Z) > x\right\} \le p\right\}$$

and

$$CTE_{\hat{T}(Z)}(p) = E\left[\hat{T}(Z) \mid \hat{T}(Z) \ge VaR_{\hat{T}(Z)}(p)\right]$$

VaR and CTE

- From the translation invariance property

$$VaR_{\hat{T}(Z)}(p) = VaR_{\hat{Y}-\hat{Z}}(p) + P(Z)$$

and

$$CTE_{\hat{T}(Z)}(p) = CTE_{\hat{Y}-\hat{Z}}(p) + P(Z)$$

Example

- N is either Poisson or Negative Binomial with E[N]=10
- Y is Pareto with

$$\Pr\{Y > y\} = \left(\frac{2}{2+y}\right)^3, y > 0$$

E[Y]=1 and Var[Y]=3

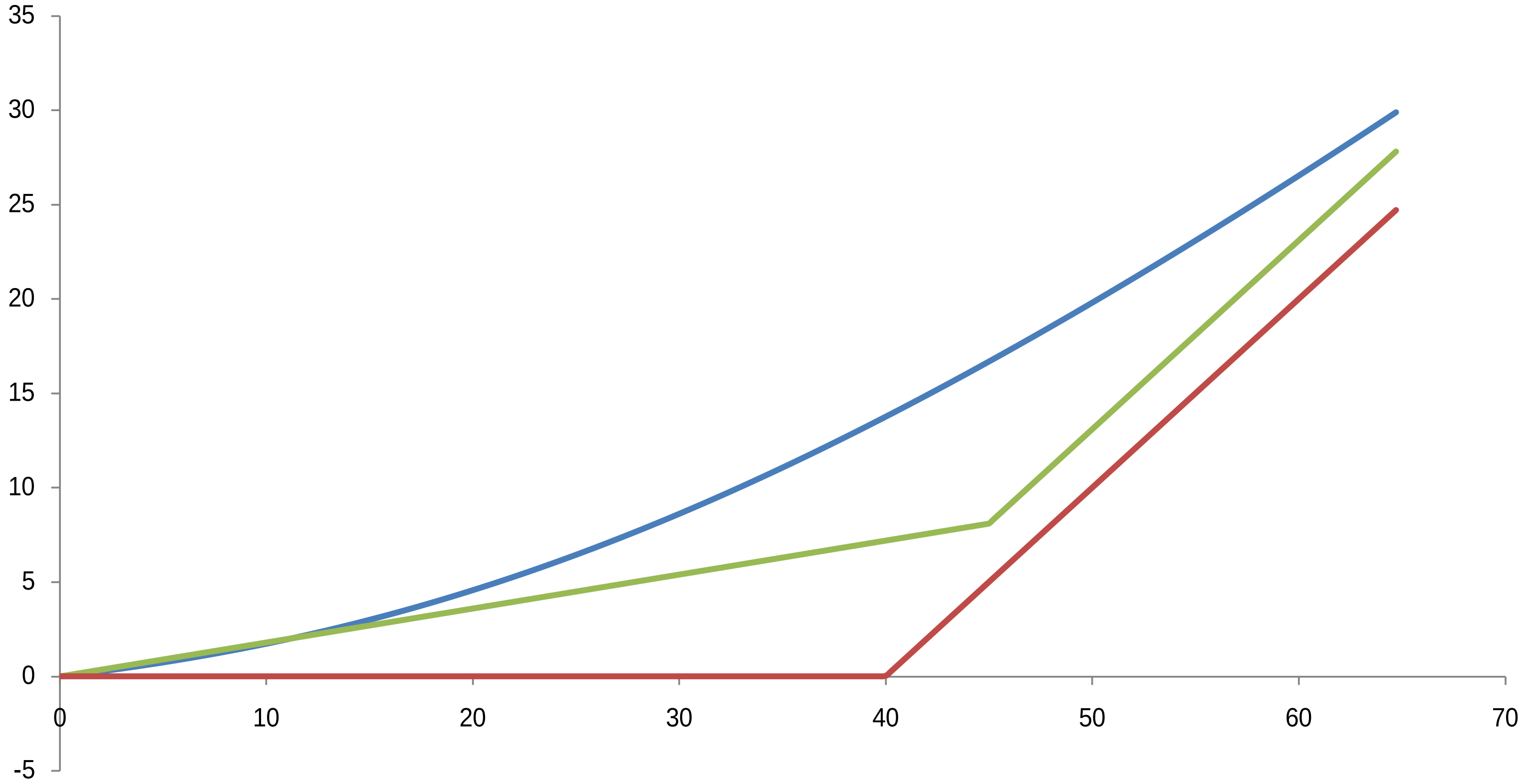
$$P(Z) = E[\hat{Z}] + b\sqrt{Var[\hat{Z}]}$$

$$c = E[\hat{Y}] + 0.5\sqrt{Var[\hat{Y}]}$$

Example

		Variance gr. ag. cl.	b	Optimal treaty		Best XL		Best (QS-XL)	
				P(Z)	R	P(Z)	R	P(Z)	R
1	Poisson	10	0.6	9.03	0.207	3.18	0.099	9.33	0.199
2			1	3.21	0.083	1.41	0.070	3.61	0.079
3	Neg. binomial	40	0.6	10.28	0.160	8.24	0.096	10.40	0.157
4			1	3.67	0.071	1.32	0.060	3.91	0.069

Example



Example

Case	Treaty	R/R*	E[L_Z]	Variance	VaR(0.05)	CTE(0.05)
1	Optimal	1	0.491	3.911	16.374	17.931
	XL	0.480	1.013	18.458	19.987	22.547
	QS-XL	0.963	0.480	3.865	16.407	18.202
2	Optimal	1	1.405	24.838	20.973	25.611
	XL	0.837	1.780	36.270	22.365	28.491
	QS-XL	0.953	1.376	24.628	20.786	25.871
3	Optimal	1	0.647	6.437	18.454	20.570
	XL	0.600	0.672	12.216	20.139	22.836
	QS-XL	0.981	0.640	6.414	18.487	20.728
4	Optimal	1	2.177	44.726	24.907	30.874
	XL	0.852	2.882	66.583	26.928	34.625
	QS-XL	0.972	2.137	44.385	24.790	31.903

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