



# Teaching life insurance mathematics: from the $l_x$ 's to the Risk Management approach

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# Agenda

- Introduction
- The RM process in life insurance
- Risk assessment
- Managing longevity risk in a life annuity portfolio
- Conclusions

*Paper based on research work jointly with Annamaria Olivieri (University of Parma) and teaching experience in both undergraduate and graduate courses*



# Introduction

- Traditional *deterministic approach*: justified (to some extent) by advantages provided by “large” portfolio sizes (J. Tetens, 1786: role of  $\sqrt{n}$ , with  $n$  = portfolio size)
- *Stochastic approach*: focus on random lifetimes and functions of the random lifetime (B. de Finetti, 1950, 1957; E. Sverdrup, 1952)
- *Process risk* captured: random fluctuations only
- *Uncertainty risk* may affect the choice of the survival function (*model risk, parameter risk*)

- Mortality: a *source* of risk, with *components*
  - Process risk
  - Uncertainty risk
  - Catastrophe risk
- Other sources of risk:
  - Market
  - Expenses
  - ...
- How to assess these risks ?
- What actions to face them ?

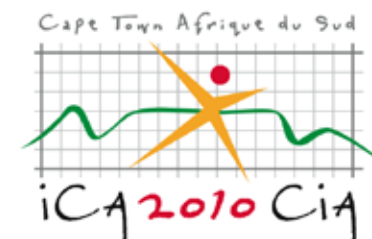
- Shift to a RM approach
- Actuarial mathematics
  - an early example of rigorous approach to risk assessment
  - actually, one of the first applications of RM (Tapiero, 2004)
- “Formal” approach to life insurance technique inspired to RM principles: a recent issue
- A leading issue (e.g. in Solvency 2)

# The RM process in life insurance

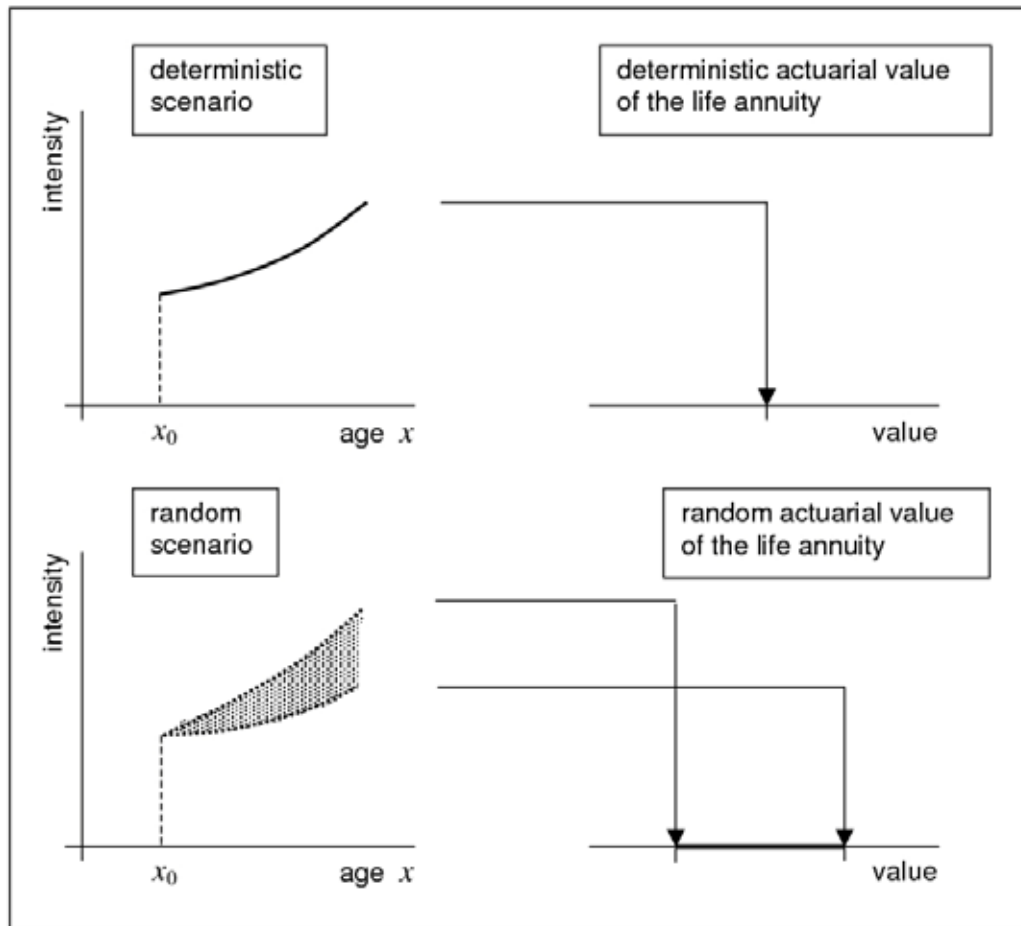


- Risk identification
  - Risk sources and risk components
- Risk assessment
  - Quantifying the impact of risks on results of interest (cash flows, profits, Net Asset Value, ...)
- Managing risks
  - Actions: product design, pricing, capital allocation, reinsurance, ART
- Monitoring

# Risk assessment



- A “job” for life insurance mathematics
- We focus on two issues:
  - Allowing for uncertainty in future mortality scenarios (dramatic interest when dealing with long-term products, in particular life annuities)
  - Designing a comprehensive modeling framework



## *Mortality scenarios and actuarial values of life annuities*

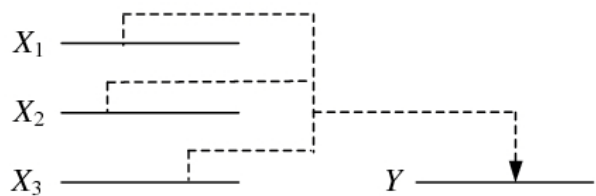
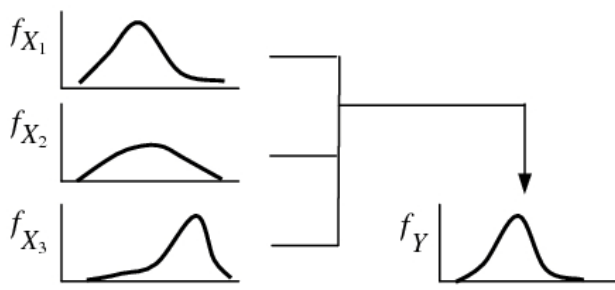
# *A modeling framework*

From purely deterministic models to stochastic models allowing for:

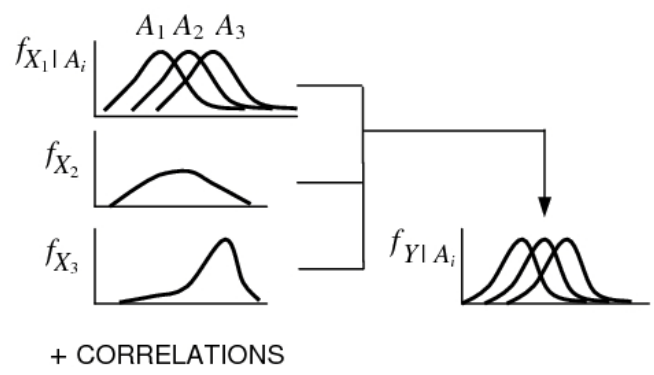
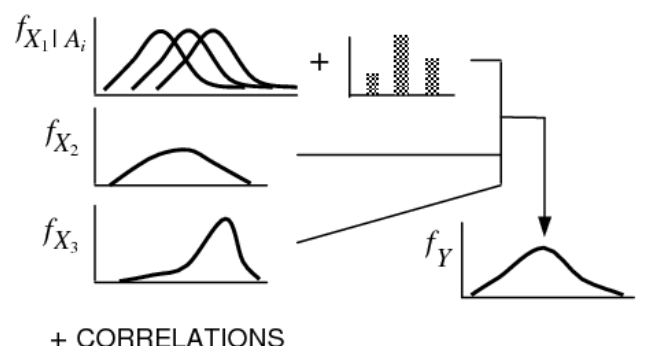
- § random fluctuations (only)
- § both random fluctuations and systematic deviations

Main issue: computational tractability

Need for stochastic simulation (a stimulating approach under a teaching perspective)

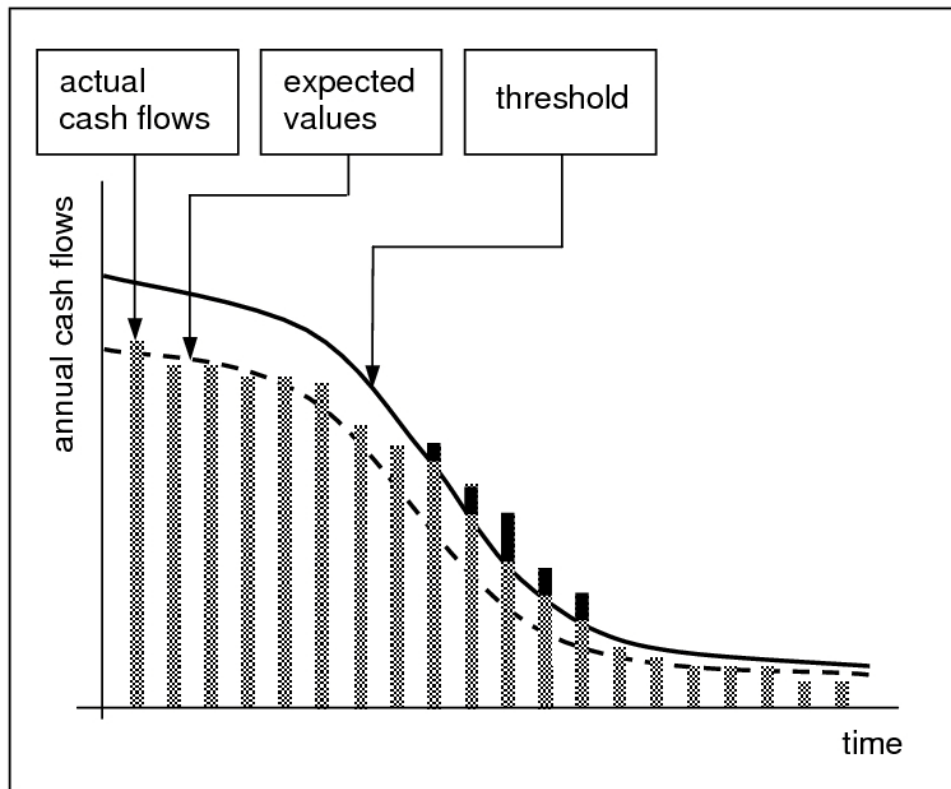
	INPUT	OUTPUT	IMPLEMENTATIONS	EXAMPLES
1		$Y$	<b>a</b> - single <hr/> <b>b</b> - iterative	<ul style="list-style-type: none"> <li>- traditional actuarial approach e.g. Embedded Value</li> <li>- stress testing e.g. Solvency 2</li> <hr/> <li>- scenario testing</li> <li>- sensitivity testing</li> </ul>
2	 <p>+ CORRELATIONS</p>	$f_Y$	<b>a</b> - analytical <hr/> <b>b</b> - analytical approx <hr/> <b>c</b> - numerical <hr/> <b>d</b> - simulation	<p>assessment of process risk, for</p> <ul style="list-style-type: none"> <li>- pricing</li> <li>- reserving</li> <li>- capital allocation</li> <li>- reinsurance</li> </ul>

## Modeling approaches (1)

	INPUT	OUTPUT	IMPLEMENTATIONS	EXAMPLES
3	 <p>+ CORRELATIONS</p>	 <p>+ CORRELATIONS</p>	<p>a - analytical</p> <hr/> <p>b - analytical approx</p> <hr/> <p>c - numerical</p> <hr/> <p>d - simulation</p>	<p>assessment of process risk and scenario testing for uncertainty risk</p>
4	<p>simulation</p>	<p>assessment of process risk and uncertainty risk</p>		

## Modeling approaches (2)

# Managing longevity risk in a life annuity portfolio



***Annual cash flows in a life annuity portfolio (one cohort)***

A range of actions available (at least in principle), in order to

- q raise the threshold
- q modify the cash flow profile (reduce, stabilize, etc)

In terms of RM actions:

## § Loss control

### § Loss prevention

- Pricing: (1) → (a)

### § Loss reduction

- Pricing: (1) → (a)
- Profit participation: (4) → (b)
- Reduction of the annuity benefit: (5) → (b)

## § Loss financing

### § Risk transfer

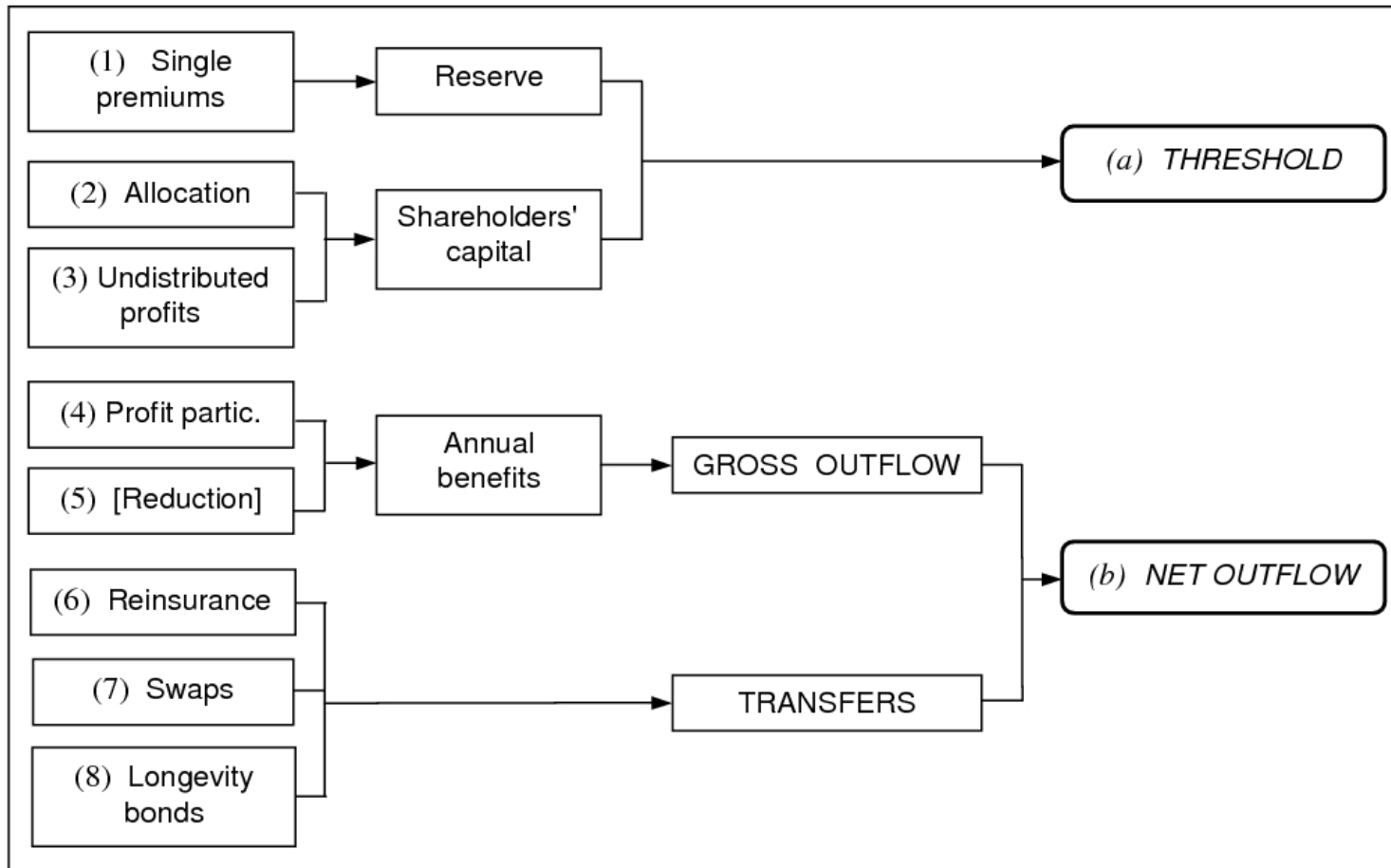
- Traditional reinsurance arrangements: (6) → (b)
- Swap-like reinsurance: (7) → (b)
- ART (longevity bonds): (8) → (b)

### § Capital allocation

- Shareholders' capital: (2) → (a), (3) → (a)

### § Natural hedging

- Across LOBS
- Across time



## ***Portfolio strategies for risk mitigation***

# Conclusions



- In recent decades: evolution of life insurance mathematics and related teaching
  - Multistate Markov (and semi-Markov) models: a powerful modeling tool for life insurance and other insurances of the person (J. Hoem, 1969, 1988)
  - Numerical approaches and stochastic simulation methods
  - Contributions from Finance (portfolio immunization, ALM, hedging opportunities, option pricing, etc )
  - ...

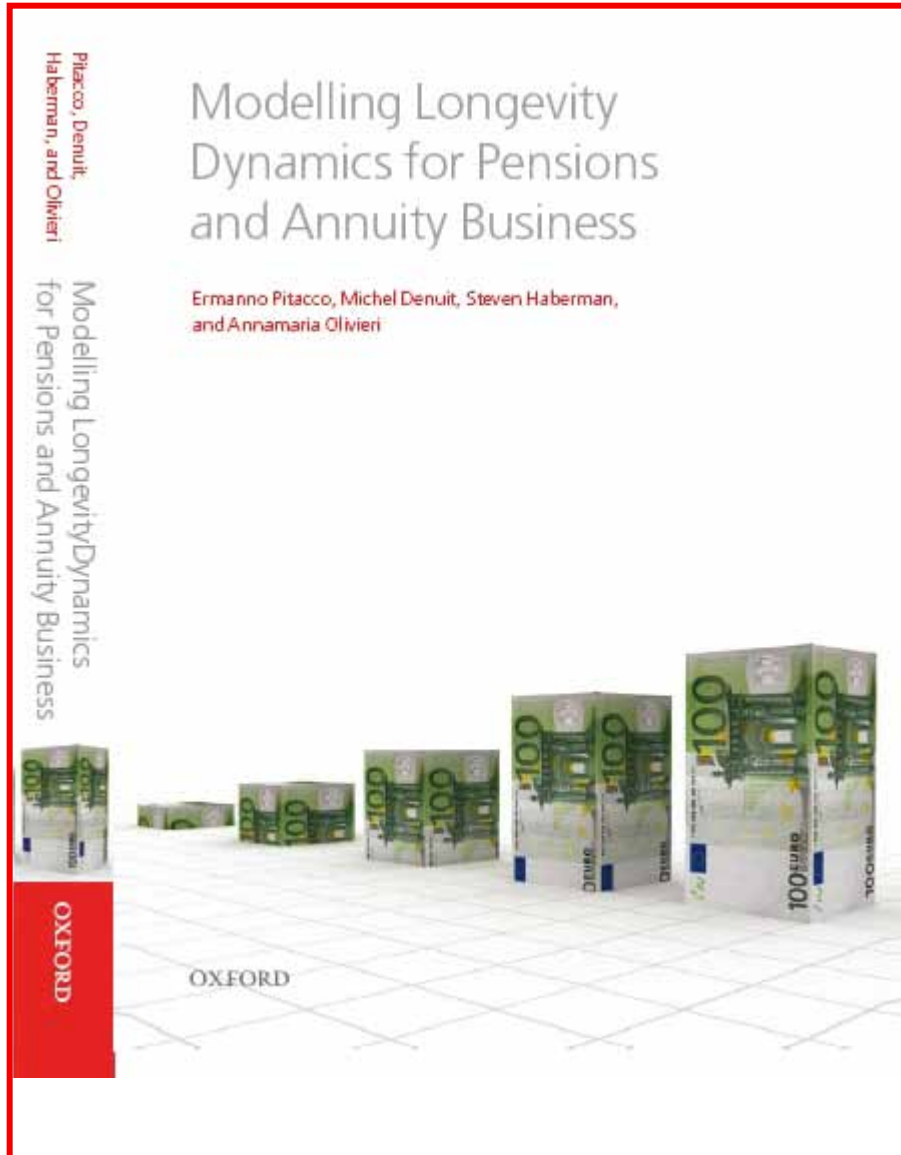
- Still a lack of contacts with firm management, corporate planning, etc
- This may result in actuarial teaching with contents rather far from needs of insurance and reinsurance companies
- Valuable opportunities for starting an innovation process currently provided by solvency regulation: a great emphasis on a RM approach

# Some references



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