Calculation of the actuarially balanced social security contribution rate: an application to the Brazilian case

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Objectives

Calculate the actuarially fair social security contribution rate for private sector workers in Brazil, for different combinations of gender, schooling years, number of years of contribution and type of benefit

Key findings:

Social security contribution rates (28%-31% are too high for length of contribution pension;

Social security contribution rates are too low for age retirement;

Actuarially fair rates vary according to gender and schooling.

The model:

Social security contributions

$$VPC = \sum_{j=1}^{T} \frac{cW_1(1+w)^{j-1}(1+i)^{T-j}}{(1+i)^T}$$

$$VPC = \frac{cW_1(1+i)^{T-1}}{(1+i)^T} + \frac{cW_1(1+w)(1+i)^{T-2}}{(1+i)^T} + \ldots + \frac{cW_1(1+w)^{T-1}(1+i)^{T-T}}{(1+i)^T} \qquad (2)$$

Sum of a GP: $a_1 = \frac{cW_1}{(1+i)}$, $q = \frac{(1+w)}{(1+i)}$ ***n = T***

$$VPC = \frac{cW_1}{(i-w)}\left[\frac{(1+i)^T - (1+w)^T}{(1+i)^T}\right] \qquad (3)$$

If *i* ® *w*, using L´Hospital's theorem

$$\lim_{i \to w} VPC = \frac{cW_1 T}{(1+w)}$$

Pension benefits

$$VPA = \sum_{j=T+1}^{T+N} \frac{A}{(1+i)^j}$$

$$VPA = \frac{A}{(1+i)^{T-1}} + \frac{A}{(1+i)^{T-2}} + \ldots + \frac{A}{(1+i)^{T+N}} \qquad (4)$$

$$VPA = \frac{A}{(1+i)^{T}} \frac{1}{i}\left(\frac{(1+i)^{N} - 1}{(1+i)^{N}}\right) \qquad (5)$$

Pension value *A* is given by the 80% highest earnings, times the *Fator Previdenciário f*

$$SPG = a_1 . \frac{1 - q^n}{1 - q} \qquad a_1 = W_1 (1+w)^{0,2T}$$

$$f = \frac{Tc \,.\, a}{Es} \left(1 + \frac{Id + Tc \,.\, a}{100} \right)$$

$$q = 1 + w \qquad n = 0,8T$$

$$A = f \frac{1}{0,8T} W_1 (1+w)^{0,2T} \left(\frac{1 - (1+w)^{0,8T}}{1 - (1+w)} \right)$$

$$A = f \frac{1}{0,8T} W_1 (1+w)^{0,2T} \left(\frac{1 - (1+w)^{0,8T}}{1 - (1+w)} \right) \qquad \textbf{(6)}$$

VPC = VPA, i.e. (3) = (5), using (6):

$$\frac{cW_1}{(i-w)}\left(\frac{(1+i)^T-(1+w)^T}{(1+i)^T}\right)=\frac{A}{(1+i)^T}\frac{1}{i}\left(\frac{(1+i)^N-1}{(1+i)^N}\right)$$

$$\frac{c}{(i-w)}\left((1+i)^T-(1+w)^T\right)=\frac{f(1+w)^{0,2T}}{0,8wT}\frac{1}{i}\left((1+w)^{0,8T}-1\right)\left(\frac{(1+i)^N-1}{(1+i)^N}\right) \quad \textbf{(7)}$$

$$c=\frac{f(1+w)^{0,2T}}{0,8wT}\frac{1}{i}\left((1+w)^{0,8T}-1\right)\left(\frac{(1+i)^N-1}{(1+i)^N}\right)\frac{(i-w)}{(1+i)^T-(1+w)^T} \quad \textbf{(8a)}$$

Obs.: 8a valid if $i \neq w$

Using L´Hospital's theorem if $w \to i$

$$c = \left[\frac{(1+i)^N - 1}{(1+i)^N} \frac{f}{0{,}8Ti}\right] \frac{(1+w)^{0,2T}\left((1+w)^{0,8T} - 1\right)}{w} \frac{(i-w)}{(1+i)^T - (1+w)^T}$$

$$c = \left[\frac{(1+i)^N - 1}{(1+i)^N} \frac{f}{0{,}8Ti}\right] \frac{f(w)}{g(w)}$$

$$\frac{f'(w)}{g'(w)} = \frac{(1+w)^{0,2T}\left(\left(0{,}2T(1+w)^{-1}\left((1+w)^{0,8T} - 1\right)(i-w)\right) + \left(0{,}8T(1+w)^{0,8T-1}(i-w) + \left(1 - (1+w)^{0,8T}\right)\right)\right)}{(1+i)^T - (1+w)^T - wT(1+w)^{T-1}}$$

$$c = \left[\frac{(1+i)^N - 1}{(1+i)^N} \frac{f}{0{,}8Ti}\right] \frac{(1+i)^T - (1+i)^{0,2T}}{iT(1+i)^{T-1}} \quad \text{, if } i = w \qquad \textbf{(8b)}$$

Data

- Brazilian National Household survey (PNAD) 2007
- Private sector workers, non-rural areas
- Age between 20 and 60

Econometric procedure

- Estimation using OLS
- Dependent variable: ln(income)

Table 2
Income growth rates (% per year)

Years of schooling	Women	Men
Whole sample	1,29	1,97
0 a 8	0,15	0,89
9 a 11	1,07	2,02
12+	1,92	2,81

Actuarially fair Contribution rates - Women

Group	Rate of growth of income *w* (% per year)	Number of years of contribution: 30 *Fator previdenciário*: 0.61		Number of years of contribution: 35 *Fator previdenciário*: 0.84	
		Interest rate *i*		Interest rate *i*	
		3.0	4.0	3.0	4.0
Whole sample	1.29	26.8	20.2	26.2	19.5
0 a 8	0.15	25.3	19.0	24.4	18.0
9 a 11	1.07	26.5	20.0	25.9	19.2
12+	1.92	27.6	20.9	27.2	20.4
All groups	2.00	27.7	21.0	27.3	20.5
	3.00	29.0	22.1	28.8	21.9
	4.00	30.2	23.2	30.3	23.2
	5.00	31.4	24.3	31.8	24.5

Actuarially fair Contribution rates - Men

Group	Rate of growth of income *w* (% per year)	Number of years of contribution: 35 *Fator previdenciário*: 0.73		Number of years of contribution: 30 *Fator previdenciário*: 1.01	
		Taxa de juros *i*		Taxa de juros *i*	
		3.0	4.0	3.0	4.0
Whole sample	1.97	23.7	17.8	23.0	17.1
0 a 8	0.89	22.2	16.5	21.3	15.6
9 a 11	2.02	23.8	17.8	23.1	17.2
12+	2.81	24.8	18.8	24.3	18.3
All groups	2.00	23.7	17.8	23.1	17.1
	3.00	25.1	19.0	24.7	18.5
	4.00	26.4	20.2	26.2	19.9
	5.00	27.6	21.3	27.7	21.3

Replacement rates - Women

Group	Rate of growth of income *w* (% per year)	Years of contribution: 30 *Fator*: 0.61	Years of contribution: 35 *Fator*: 0.84	Number of years of contribution: 40 *Fator*: 1.15
Whole sample	1.29	0.53	0.71	0.95
0 a 8	0.15	0.60	0.82	1.12
9 a 11	1.07	0.54	0.73	0.98
12+	1.92	0.49	0.66	0.87
All groups	2.00	0.49	0.65	0.86
	3.00	0.44	0.58	0.75
	4.00	0.40	0.52	0.67
	5.00	0.37	0.47	0.60

Replacement rates - Men

Group	Rate of growth of income *w* (% per year)	Number of years of contribution: 35 *Fator*: 0.73	Number of years of contribution: 40 *Fator*: 1.01	Number of years of contribution: 45 *Fator*: 1.42
Whole sample	1.97	0.57	0.76	1.03
0 a 8	0.89	0.65	0.88	1.22
9 a 11	2.02	0.56	0.75	1.02
12+	2.81	0.52	0.68	0.91
All groups	2.00	0.57	0.76	1.03
	3.00	0.50	0.66	0.89
	4.00	0.45	0.59	0.78
	5.00	0.41	0.52	0.69

An exercise (I)

For how many years would it be possible to pay the age benefit if the social security system was FF? (Women; c = 0,31; w = 0.0261%

Years of contribution	Real interest rate (%)	
	3,0	4,0
15	7	10
20	8	12

An exercise (II)

Actuarially fair contribution rates for differents ages of the beginning of contribution (c = 0,31; w = 0.02%

$$S = S_{-1}(1+i) + cW - A$$

Age started contributing	Men	Women
40	33.6	52.7
45	42.3	70.7
50	56.7	-

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