Calculation of the actuarially balanced social security contribution rate: An application to the Brazilian case

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Abstract
The aim of this paper is to calculate the contribution tax rate that equals expected present values of contributions and retirement benefits in a social security system. A theoretical exercise is developed to generate an analytical formula for the contribution rate, complemented by an empirical analysis, using Brazilian data. The tax rate is calculated in diverse situations, including differences of gender, education level and type of retirement. The main conclusion is that contribution rates around 31%, as observed in Brazil, are too high for retirement by contribution time, but insufficient for retirement by age, especially for women.

Keywords: social security; contribution rate; actuarial equilibrium; retirement, Brazil.

1. Introduction[1]

In Brazil The General Social Security Regime (*RGPS*) of the National Institute of Social Security Social (*INSS*) is responsible for collecting social security contributions and paying benefits to the private sector workers. RGPS can be classified as a pay-as-you-go (PAYG) regime. Even without funding, a PAYG regime can be guided by principles of actuarial equilibrium. The contribution tax rate can be calculated so that the expected present value of the contribution flow during a person's active life be equivalent to the expected present value of the flow of benefits received after retirement.

This article aims to discuss exactly that question: what should be a "correct" contribution tax rate in Brazilian Social Security?[2] In debates about the social security disequilibrium, even in a specialized public, the idea often comes up that a rate of 31%, prevalent in Brazil – 20% for the employer and 11% for the employee – would be more than enough to generate resources for retirement benefits if the regime was fully funded (FF). This article will discuss to what extent that idea is correct.

The theme fits into the larger debate about the Social Security financial status, which has been a source of concern for Brazilian researchers for years (Stephanes, 1998) and of two reforms in which great political capital was invested, during the Cardoso and Lula governments.[3] Both reforms revealed to be unsatisfactory, however, in view of the challenges posed by future demographic perspectives (Cechin, 2002; Giambiagi et al, 2004). The possibility of a third round of reform figures on the country's agenda for the next decade and, therefore, studying the adequacy of the current social security contribution rate is is fundamental to define the outlines of this reform.

The paper contains six sections, including this short introduction. In section two, a concise review of relevant literature is presented. In section three, a mathematical model is developed to calculate the actuarial social security contribution rate. Section four presents estimates and results for the actuarially balanced rates for Length of Contribution (*LOC*) retirement benefits[4] for different groups of workers, according to gender, education level and number of years of contribution. Based on the methodology described, three additional exercises are done. The first one is the calculation of the social security replacement rate for LOC retirement benefits. The other two addresses the age retirement benefits[5]. In section five, the relevance of results is discussed in the light of current parameters in Brazil. Finally, conclusions and policy recommendations are presented, based on the obtained results.

The contribution this text intends to offer in comparison with other past articles with similar concerns, like Giambiagi (1993) or Oliveira, Beltrão and Pasinato (1998) is to update the debate in view of important innovations associated with the *Fator Previdenciário* (social security factor law) in 1999. The Fator Previdenciário (hereafter referred as *f*) is given by $f = \frac{Tc \,.\, a}{Es}\left(1 + \frac{Id + Tc \,.\, a}{100}\right)$. In this expression, *Tc* is the contribution time; *Es* represents life expectancy upon retirement, obtained at age *Id*. And *a* is the contribution rate, for which 0.31 is adopted. It's used to calculate the value of LOC benefits (the formula is detailed in section three) Past calculations, made by other authors, calculated the value of LOC benefit using the the former formula, where thisvalue was based solely on the average of the last three years of contribution. By changing the law, increasing the contributory period and multiplying the average income by the *Fator Previdenciário* (that can be lower than one for cases of early retirement), the present value of benefits is lower. That is the case because both the average

[1] The authors thank Samuel de Abreu Pessôa and Paulo Tafner for reading the earlier version of this text. The authors are also grateful for the comments and suggestions of the two Brazilian Review of Economics (RBE) reviewers.

[2] The article specifically addresses the case of INSS contributors and not of civil servants, whose rules are partially different in Brazil.

[3] For explanation about the Cardoso reform, see Ornelas and Vieira (1999). For an analysis of the Lula reform, see Souza et al (2006).

[4] In Brazil private sector workers can retire after 35 years of contribution (men) or 30 years, in the case of women.

[5] In Brazil private workers can also retire by age at 65 (man) or 60 (women). For applying for this benefit, the worker must have made contributions for at least 15 years.

and the *Fator* are lower to those used as a reference before the legal changed introduced by the social security reform under the mandate of Fernando Henrique Cardoso.[6] Consequently, the conclusions of earlier articles were outdated because of the new rules. It is precisely that gap that justifies this article.

2. Literature review

Neither Brazilian nor international literature is very rich in articles that calculate the contribution tax rate necessary to obtain actuarial equilibrium in social security regimes, either PAYG or FF. Most articles have focused on calculating internal rates of return of social security systems (Brazilian case), or on analyzing marginal contribution rates for other countries, particularly the USA, such as Feldstein and Samwick (1992). Exceptions are the works by Feldstein (1995) and Geanakoplos, Mitchell and Zeldes (2000), Feldstein and Samwick (1996), Feldstein and Samwick (2000) and Feldstein (2005), the latter three of which analyze the impacts of changing from a simple capital allocation to a total or partially funded system.

Roach and Ackerman (2005: 22) assess that, to maintain the actuarial equilibrium of the SSA's Trust Fund (USA) for the next 75 years, the joint (employees and employers) contribution tax rate should be raised by 1.9 percentage point, if salaries rise at 1.1% per year. If salaries rise by a mere 0.6% per year, the rate should be raised by 2.5 percentage points. Another calculation, presented in the OASDI Trustees Report (SSA, 2007: 56), estimates that the rate should be raised by 2.0 percentage points. Or, alternatively, the same result could be obtained in case of a 13% reduction in the expected value of future benefits.

In the Czech Republic, government calculations (MLSA, 2001) show that, although a contribution rate of 26% was being charged (6.5% for employees and 19.5% for employers), the necessary rate was 29%. This conclusion is similar to Schmaehl (2002), who analyzed Germany's attempt to reform social security in 1997. Given the expected demographic changes for the coming years, the rate should rise from 19% at the time of study to about 24% in 2030. This result is supported by Börsch-Supan and Wilke (2003), who projected a necessary contribution rate of nearly 26% in 2042.

In the Brazilian case , Oliveira, Beltrão and Maniero (1997) developed important research in this sense. The authors calculate the contribution tax rates needed for the RGPS, using the age of retirement and entry into the job market as the main parameters. Their general conclusion is that, for most combinations among the variables, the necessary rates would be higher than those defined by law, given the rules in force at that time. One year later, McGreevey, Oliveira and Beltrão (1998) performed a similar study, this time analyzing the Civil Servants Social Security Regime (RPPS) of in the state of Rio Grande do Sul. In the specific case of women teachers, who started working at the age of 20, worked for 25 years, with a life expectancy of up to 75 years, the contribution tax rate should be around 60%. Oliveira, Beltrão and Pasinato (1998) propose a social security system based on principles of actuarial equilibrium. Based on the rules in force at that time (before the social security factor came into force), the authors calculate fair rates for some groups of workers. Their results show, for example, that at a 3% discount rate, the actuarially fair rate for a man who started working at the age of 20 and retired at 60 should be 20.1%. Using the same parameters, the fair rate for a woman would be 22.9%.

Fernandes and Gremaud (2004) focus on federal public servants, calculating equilibrium rates if the rules set by Constitutional Amendment 20 (EC20) had been in force since they entered the job market. Their results show that, on the average, for different government contexts, rates should exceed 73%. Fernandes and Narita (2005) make similar calculations for the INSS. Their main qualitative conclusions are that equilibrium rates are higher for workers with lower education levels and also for the poorest Brazilian regions. These results are quite similar to the ones obtained by Souza et al. (2006), according to whom INSS rates (for social security benefits) figure around 20%.

[6] Before this measure, retirement benefits were calculated as if, in practice, the Social Security Factor was equal to one.

3. Algebra of actuarial equilibrium

In this section, an algebric formula is developed to calculate the actuarial equilibrium contribution tax rate. The first step is calculating the present value of contributions. The second step is calculating the present value of retirement flows. The third, and final step, is to equate both flows and compute the necessary tax rate.[7]

3.1Present value of contribution flow

The starting point is the formula of the sum of a finite geometric progression (*SPG*) with *n* terms, first term a_1 and index *q*:

$$SPG = a_1 . \frac{1-q^n}{1-q} \qquad (1)$$

A social security system is actuarially balanced if the expected present value of the flow of contributions an individual makes during his active life (*VPC*), duly discounted at a real interest rate *i*, is strictly equal to the expected present value of the flow of retirement benefits (*VPA*) to be received after retirement.[8]

The term *VPC* can be equated to an *SPG* in which the first term is the individual's first contribution.[9] This corresponds to the result of multiplying a contribution tax rate *c* by income *W* for periods *1*, *2*, ... , *T*, in which *T* is the number years of contribution. All calculations, for contributions as well as retirement benefits, are made in terms of present value for period *0*, immediately before the start of work periood[10]. The calculation is made as if each contribution corresponded to a deposit, capitalized at rate *i* up to period *T* and brought to present value for period *0*. The contribution corresponding to income for the first period is then capitalized for *T-1* periods; the one for the second period for *T-2* periods and so on. Consequently, considering that income increases at a real rate *w* in relation to the previous year, it is given that:

$$VPC = \sum_{j=1}^{T} \frac{cW_1(1+w)^{j-1}(1+i)^{T-j}}{(1+i)^T}$$

$$VPC = \frac{cW_1(1+i)^{T-1}}{(1+i)^T} + \frac{cW_1(1+w)(1+i)^{T-2}}{(1+i)^T} + \ldots + \frac{cW_1(1+w)^{T-1}(1+i)^{T-T}}{(1+i)^T} \qquad (2)$$

Highlighting some terms and given that this is an *SPG*, by analogy with (1), with $\frac{cW_1}{(1+i)}$ as the first term and $\frac{(1+w)}{(1+i)}$ as the index, (2) equals:[11]

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[7] This section closely follows Giambiagi (1993), with some modifications.

[8] Calculations do not include non-programmable benefits like disease aid or disability retirement. Moreover, the hypothesis is adopted that the death of a retired person does not imply survirvor's benefit for the spouse orchildren.

[9] It is implicitly supposed that monetary flows of income and retirement are annual. The results can present slight variations as to the use of monthly variables, due to the capitalization of values during the year, but do not significantly change the conclusions. A similar reasoning applies to the influence of inflation.

[10] The interest rate of the first period is already charged on the present value of the variables. That is, the present value of the initial contribution is $\frac{cW_1}{(1+i)}$. This is the term corresponding to a_1 in (1) and which will be used in subsequent equations.

[11] Equation 3 a priori presents an indetermination when $i \to w$. This case corresponds to $q \to 1$ in equation 1. By the direct application of L'Hôpital's Rule, it is proved that, if $i \to w$, in (3), $\lim_{i \to w} VPC = \frac{cW_1 T}{(1+w)}$.

$$VPC = \frac{cW_1}{(i-w)}\left(\frac{(1+i)^T-(1+w)^T}{(1+i)^T}\right) \quad (3)$$

3.2 Present value of retirement benefit flow

The calculation of the present value of the flow of retirement benefits (*VPA*) to be received after retiring follows a similar reasoning to that explained for contributions[12]. The starting point is the same equation (1) of an *SPG*, calculating the present value of a payment flow of constant real value in similar way of a "Price table". These payments correspond to the value of retirement benefit *A*, ignoring the existence of inflation for the sake of calculations. Conceptually, the calculus is similar to the calculation that equals the value of a good to the present value of a set of fixed installments. The present value of the first term in equation (4) to calculate *VPA* equals the value of *A* discounted at interest rate *i*, already used in earlier equations, always at prices for period *0*. As retirement benefits start to be received one period after *T*, which is the last activity period, the discount factor *1+i* needs to b raised to *T+1* in order to calculate the present value of the first flow of retirement benefits *A* at prices for period *0*, that is, $\frac{A}{(1+i)^{T+1}}$. The number of terms is *N*, corresponding to the number of periods in which a person receives the benefit after leaving the labor market, that is, that person's survival period. The index of this geometrical progression is $\frac{1}{(1+i)}$.

The value of *VPA* at prices for period *0* is expressed by:

$$VPA = \sum_{j=T+1}^{T+N} \frac{A}{(1+i)^j}$$

$$VPA = \frac{A}{(1+i)^{T-1}} + \frac{A}{(1+i)^{T-2}} + \ldots + \frac{A}{(1+i)^{T+N}}, \quad (4)$$

which, by analogy with (1), can be written as

$$VPA = \frac{A}{(1+i)^T}\frac{1}{i}\left(\frac{(1+i)^N-1}{(1+i)^N}\right) \quad (5)$$

In the RGPS, the value of retirement benefit *A* is determined by multiplying the *Fator Previdenciário f* by the real average of the 80% highest contribution salaries. Following (1), this average can be defined as an *SPG* with index *1+w*, *0.8T* periods, divided by the number of contribution years (*80%* of *T*) considered in that average. Supposing a constant annual growth rate *w*, the first contribution salary in the calculus corresponds to the first year after the non used period of *20%* of *T*, given that the average is calculated based on *80%* of contributions. Hence, through (1), *A* is given by:

$$A = f\frac{1}{0{,}8T}W_1(1+w)^{0{,}2T}\left(\frac{1-(1+w)^{0{,}8T}}{1-(1+w)}\right) \quad (6)$$

3.3 Calculation of actuarial equilibrium rate

[12] The term "inactivity" is used based on the classical case in which a person works for a number of years and, upon retirement, stops working. In Brazil, however, it is known that many people retire and keep on working. Therefore, in this text, the expression "inactivity" is associated with the reception of a retirement benefit.

An actuarially balanced social security system requires that the present value of the contribution flow be equal to the present value of the flow of retirement benefits to be received, that is, that *VPC* = *VPA*. This implies equaling (3) and (5), i.e.:

$$\frac{cW_1}{(i-w)}\left(\frac{(1+i)^T-(1+w)^T}{(1+i)^T}\right)=\frac{A}{(1+i)^T}\frac{1}{i}\left(\frac{(1+i)^N-1}{(1+i)^N}\right),$$

which, when using the expression of A, given by (6), can be rewritten as:

$$\frac{c}{(i-w)}\left((1+i)^T-(1+w)^T\right)=\frac{f(1+w)^{0,2T}}{0,8wT}\frac{1}{i}\left((1+w)^{0,8T}-1\right)\left(\frac{(1+i)^N-1}{(1+i)^N}\right) \qquad (7)$$

Evidencing the contribution tax rate c in (7), one reaches the expression of the "fair" contribution rate that generates a situation of actuarial equilibrium in the system[13]:

$$c=\frac{f(1+w)^{0,2T}}{0,8wT}\frac{1}{i}\left((1+w)^{0,8T}-1\right)\left(\frac{(1+i)^N-1}{(1+i)^N}\right)\frac{(i-w)}{(1+i)^T-(1+w)^T}, \qquad (8a)$$

if $i \neq w$.

The same proviso made in equation 3 applies to equation 8[a], i.e. if $w \to i$, then there is an indetermination. In order to solve the problem is necessary to reorder the equation, so as to separate the terms inside the brackets, which do not depend on w:

$$c=\left[\left(\frac{(1+i)^N-1}{(1+i)^N}\right)\frac{f}{0,8Ti}\right]\frac{(1+w)^{0,2T}\left((1+w)^{0,8T}-1\right)}{w}\frac{(i-w)}{(1+i)^T-(1+w)^T}$$

$$c=\left[\left(\frac{(1+i)^N-1}{(1+i)^N}\right)\frac{f}{0,8Ti}\right]\frac{f(w)}{g(w)}$$

Next, to deal with the indetermination, L'Hôpital's rule is applied, deriving *f(w)* and *g(w)* in relation to w. After some manipulations, one reaches the following:

$$\frac{f'(w)}{g'(w)}=\frac{(1+w)^{0,2T}\left(\left(0,2T(1+w)^{-1}\left((1+w)^{0,8T}-1\right)(i-w)\right)+\left(0,8T(1+w)^{0,8T-1}(i-w)+\left(1-(1+w)^{0,8T}\right)\right)\right)}{(1+i)^T-(1+w)^T-wT(1+w)^{T-1}}$$

When inserting the above expression into equation (8a) and doing $w \to i$, one obtains (8b). This expression is used to calculate the further sections when the specific condition $i = w$ occurs.

$$c=\left[\left(\frac{(1+i)^N-1}{(1+i)^N}\right)\frac{f}{0,8Ti}\right]\frac{(1+i)^T-(1+i)^{0,2T}}{iT(1+i)^{T-1}}, \qquad (8b)$$

if $i = w$.

4. Empirical procedure

This section describers the econometric procedure adopted to estimate some parameters used in the model described in section 3. Next, the actuarial equilibrium rates are calculated. Based on these calculations, two short additional exercises on resource accumulation and tax rates are presented. Finally, in item 4.3, some theoretical considerations are made based on some of the results.

4.1. Econometric procedure

Through equations (8a) and (8b), one can calculate the actuarially balanced rate, which equates the expected present values of contributions and benefits. One fundamental variable in this kind of actuarial model is w, the income growth rate. Therefore, its value was empirically estimated

[13] The approach presented here, in discrete time, is similar to the formulation Iyer (2002, section 1.6) developed in continuous time.

through a log-linear regression of income by OLS. Microdata from the 2007 National Household Sample Survey (PNAD) were used, carried out by the Brazilian Institute of Geography and Statistics (IBGE). In the PNAD, as Silva, Pessoa and Lila (2002, sections 1 and 2) highlight, a stratified sampling plan is used with one, two or three selection stages, according to the stratum. Due to this sample design, PNAD data cannot be analyzed as if they were observations produced through simple random sampling, without replacement. In view of this relevant characteristic, the estimations made consider the complex sampling plan of the PNAD, according to the strata and primary sampling units. Further details on how this procedure was put in practice are also available in Osorio (2002).

Consistently with the model presented in section 3 and with the study goals, the regression includes only people working in the private sector and who declared social security contributions. People working in the rural context were excluded, as it would be hard for them to retire in the conditions supposed here. The sample includes all people with an occupation, who receive a positive income from their main occupation, are between 20 and up to and including 60 years of age[14]. Four regressions were estimated, whose results are presented in Table 1, with the same functional form and different samples. In the first, the entire sample is used; regression 2 includes only people with up to 8 years of education; in regression 3, people with between 9 and 11 years of study are included; finally, the sample for regression 4 only includes people with 12 or more years of study.

The explanatory variables are *exp*, which represents the number of years the person has been on the job market (given by the difference between that person's age in 2007 and the age of entry into the job market); *educa*, which is the number of schooling years; and the age at which the person started to work, *inictrab*. Next, there is a series of *dummy* variables. The first is *branco*, corresponding to 1 if the person declares himself white or yellow and zero if not. The variable *chefe* shows if the person is the family head. The variable *metrop* corresponds to 1 if the person lives in a metropolitan region. Job positions are given by *posocup*i variables. *Posocup1* corresponds to 1 for workers with an employment record card and zero for others. In the same line, *posocup2* refers to workers without an employment record card or who did not declare whether they have one; *posocup3* relates to household workers, *posocup4* to autonomous workers and, finally, *posocup5* to employers (the latter *dummy* was omitted in the regression). Similarly, geographical dummies represent, respectively, the Brazilian North, Northeast (omitted), Southeast, South and Central-West regions

The last two explanatory variables represent interactions. *Inictrabeduca* is an interaction between *inictrab* and *educa*. The variable *exphomem* represents the interaction between the dummies experience (*exp*) and gender (*homem*). The latter permits obtaining distinct income growth rates for men and women. For the full sample for example, women's income increases, according to estimates, at an annual rate *w* of 1.29% (coefficient of *exp* variable). The growth rate *w* of male income, on the other hand, is given by the sum of *exp* and *exphomem* coefficients (0.0129199 + 0.0067335), which results in a growth rate of 1.97% per year. The same reasoning applies to the other regressions. As all coefficients are significant at 1%, there is evidence of different income growth rates for men and women. The coefficients of *exp* are not only significant, but also increase with the income range. One possible explanation for this fact is that, along their career, people with a higher education level have a greater possibility of access to leading functions or with a management profile, which are associated with higher revenues. The results of these estimates are reported in Table 1.

Table 1
Linear regressions (OLS) – Dependent variable: ln (income)

	Regression 1	Regression 2	Regression 3	Regression 4
	Whole sample	**0 a 8 years of schooling**	**9 a 11 years of schooling**	**12+ years of schooling**
exp	0,01292	0,00154	0,01017	0,01918
	(32,86)**	(3,11)**	(18,90)**	(19,40)**

[14] Some people receive social security benefits but are still active in the job market. In this case, that person is considered a normal worker.

educa	0,037	0,043	0,072	0,031
	(13,28)**	(9,35)**	(3,42)**	(1,15)
inictrab	-0,009	0,009	-0,006	-0,095
	(5,40)**	(4,97)**	(0,41)	(4,30)**
branco	0,153	0,095	0,125	0,197
	(26,69)**	(13,53)**	(16,42)**	(12,16)**
homem	0,220	0,174	0,147	0,251
	(25,08)**	(12,82)**	(12,85)**	(10,60)**
chefe	0,083	0,051	0,049	0,165
	(7,92)**	(3,39)**	(4,03)**	(4,85)**
metrop	0,107	0,036	0,087	0,259
	(13,91)**	(4,61)**	(10,33)**	(14,20)**
posocup1	-0,753	-0,865	-0,852	-0,452
	(41,85)**	(25,01)**	(32,59)**	(16,15)**
posocup2	-0,784	-0,929	-0,857	-0,589
	(36,44)**	(23,12)**	(26,67)**	(16,80)**
posocup3	-0,852	-0,951	-1,071	-1,142
	(40,76)**	(26,10)**	(35,52)**	(10,32)**
posocup4	-0,517	-0,632	-0,577	-0,328
	(23,88)**	(15,85)**	(17,28)**	(9,10)**
regiao1	0,170	0,137	0,166	0,245
	(12,32)**	(9,49)**	(10,18)**	(7,65)**
regiao3	0,249	0,236	0,261	0,222
	(26,60)**	(22,15)**	(26,35)**	(8,63)**
regiao4	0,225	0,223	0,253	0,174
	(18,68)**	(17,70)**	(19,63)**	(5,58)**
regiao5	0,264	0,202	0,240	0,333
	(19,47)**	(14,91)**	(18,18)**	(9,94)**
inictrabeduca	0,003	-0,001	0,002	0,009
	(16,92)**	(2,89)**	(1,35)	(5,73)**
exphomem	0,007	0,007	0,009	0,009
	(16,07)**	(12,58)**	(13,85)**	(7,22)**
Constante	5,82298	6,40449	5,71711	5,54126
	(152,24)**	(129,93)**	(25,07)**	(14,31)**
Nº de obs.	64.637	23.785	27.666	13.186
R^2	0,45	0,33	0,37	0,39

t statistc in brackets

* statistically significant at 5%. ** statistically significant at 1%

Before continuing data analysis, some comment is needed on the adopted hypotheses and their possible influence on the results of this research. The first comment refers to the calculus of contribution time by the difference between age in 2007 and age at which the person started working. This implicitly includes the hypothesis of a 100% contribution density (people do not stop contributing at any time) and no unemployment during active life. Together, these suppositions increase the contribution volume throughout a person's active life. In addition, without unemployment, the worker is expected to manage to retire exactly in the minimum conditions defined by law. The second comment isabout the worker's income. In our calculations, only income from the main occupation was used and, consequently, the situation as a social security contributor in this main job. This does not consider the fact that a group of workers have more than one occupation and can also contribute to the social security system through both jobs. However, this group is quite small. In the sample used in regression 1, shown in Table 1, i.e. 64,637 persons, only 409 (0.63%) have two jobs and contribute to the INSS in this second occupation. And only 83 (0.13%) contribute in a third occupation. Thus, the extent of the error in calculating the contribution amount is small.

Further consideration is needed as to the functional form adopted in the estimated models. In general, income estimation models through Mincer equations include a squared term among the explanatory variables. In fact, a visual analysis of graphs 1 to 4 permits inferring a certain concaveness in income over time. This fact could justify the inclusion, besides the term related to the

job market experience (*exp*), of a squared term ($exp^2 = exp*exp$). However, the squared term was not included in the estimates for two reasons. The first refers to consistency between the model developed in section 3 and the econometric procedure in section 4. The model is based on a constant income growth rate *w*. The model permits an analytic solution for the contribution rate, given by equations 8a (general case) and 8b (specific case of $i \rightarrow w$). The same applies to the extensions developed in section 4. The inclusion of the squared term would cause analytic complications (particularly for deducting the expressions given in equations 8a and 8b), which would greatly exceed the gain in the model's explanatory power. The second motive is the explanatory power of the variable exp^2, which revealed to be quite low. In the same line, the increase in R^2 could virtually be ignored[15]. Hence, people's revenues can be represented by a linear equation, as adopted, with a constant growth coefficient *w* and an almost ignorable loss in the model's adherence to the empirically verified data.

Based on the estimated coefficients, Table 2 shows the income growth rates *w* for men and women, separated per study year. Next, Graphs 1 to 4 display the mean income per age for each gender, for the full sample and for each of the three education levels. There are clear remuneration (and growth rate) differences between men and women and also between education levels. The greatest variations identified in more advanced age should be attributed to the smaller number of people, due to the fact that many retirement benefits are granted as from the fifth decade of life.

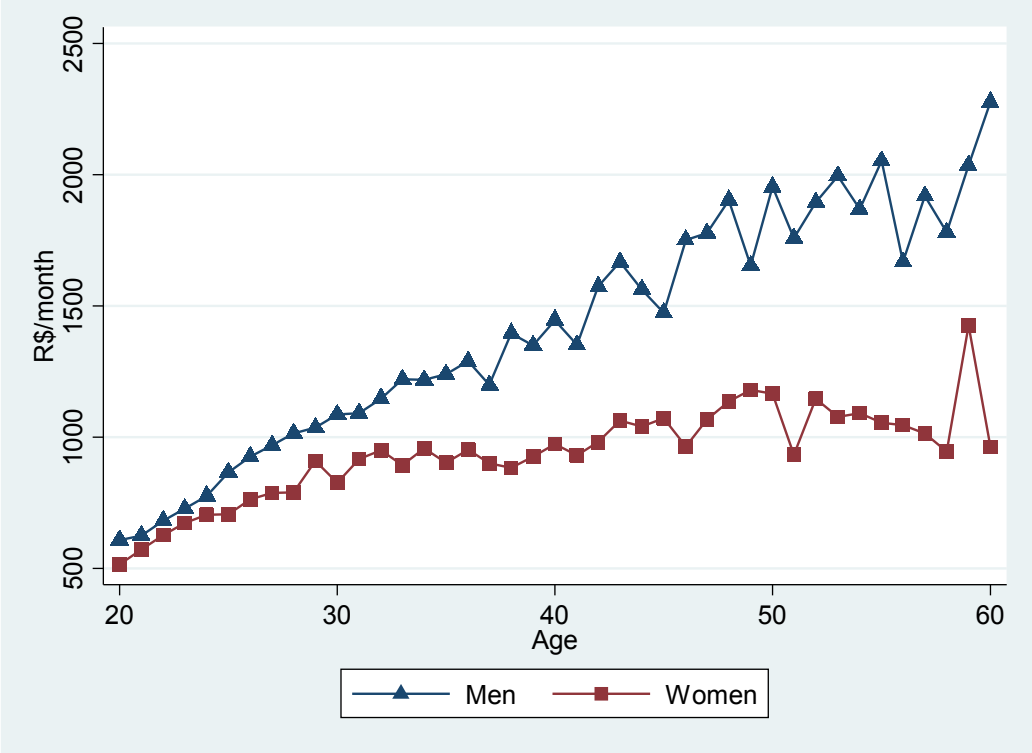
Table 2
Income growth rates (% per year)

Years of schooling	Women	Men
Whole sample	1,29	1,97
0 a 8	0,15	0,89
9 a 11	1,07	2,02
12+	1,92	2,81

Source: Authors' tabulations

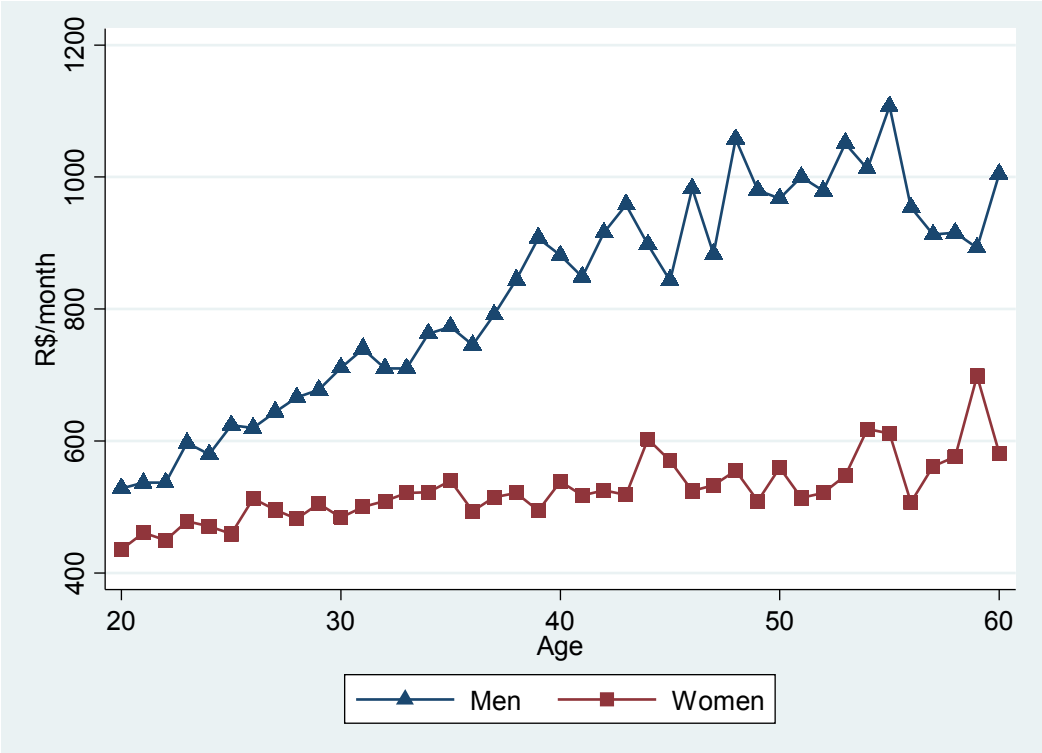
Graph 1
Mean monthly income – Men and women – Full sample
(Values in R$/month for September 2007)

[15] The coefficients of exp^2 were lower than 0.0006 in all regressions. The value of R^2 increased by a mere 0.01 in the 1st and 4th regressions. In the remainder, the increase was even larger. Due to limited space, the results of estimates including the squared term are not included.



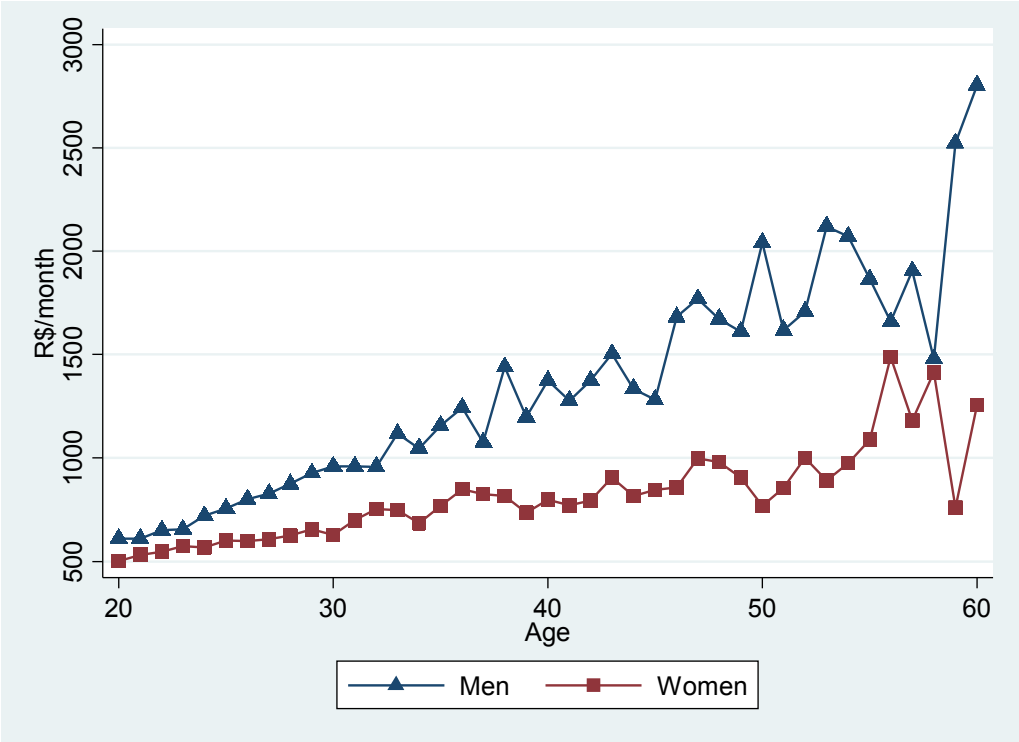
Source: PNAD 2007. Authors' tabulations.

Graph 2
Mean monthly income – Men and women – 0 to 8 years of schooling
(Values in R$/month for September 2007)



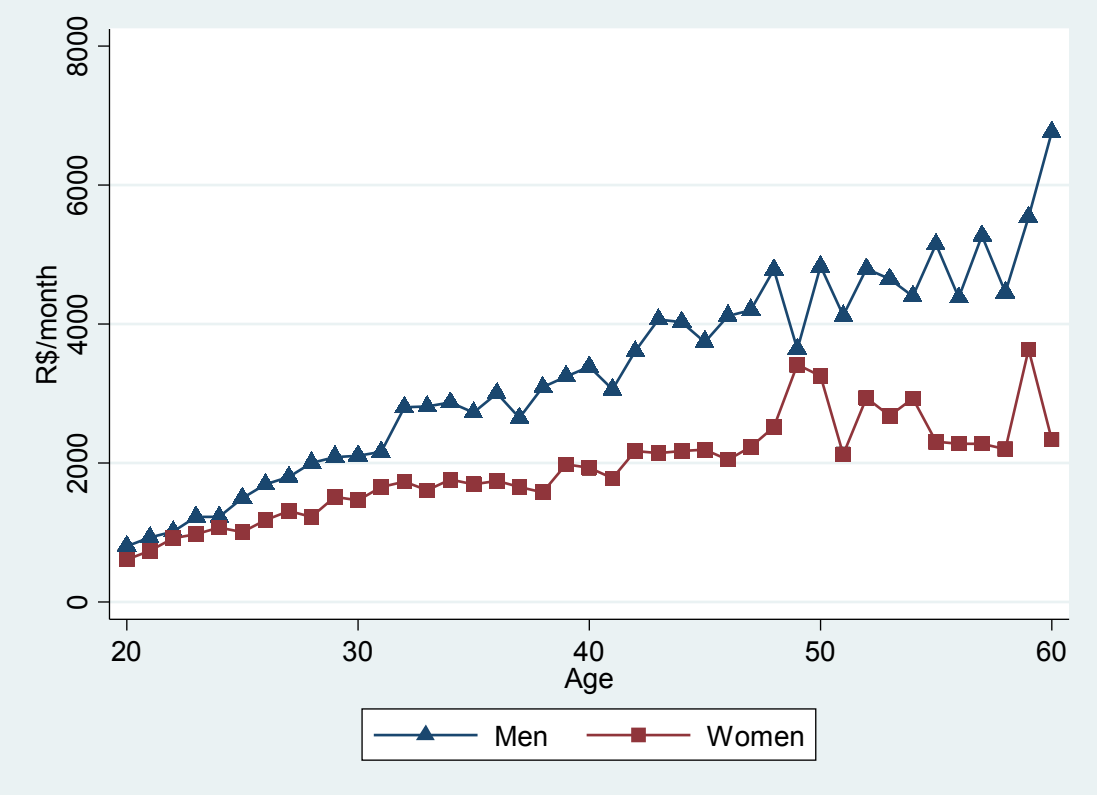
Source: PNAD 2007. Authors' tabulations.

Graph 3
Mean monthly income – Men and women – 9 to 11 years of schooling
(Values in R$/month for September 2007)



Source: PNAD 2007. Authors' tabulations.

Graph 4
Mean monthly income – Men and women – 12 or more years of schooling
(Values in R$/month for September 2007)



Source: PNAD 2007. Authors' tabulations.

4.2. Actuarially fair contribution tax rates

Based on equations 8a and 8b and on the parameters estimated in the linear regressions, the actuarially balanced rates were calculated for men and women, for the entire sample and also per education group. In Brazil, the retirement rule established in the Constitution for INSS contributors is that people can retire depending on their age: at 60 for women and 65 for men; or for contribution time, with 30 years for women and 35 for men. Calculated contribution rates will be presented for women and men, for the whole sample and for three study levels. For each gender, two possibilities are analyzed. For women, in the first possibility, retirement occurs after 30 years of contribution. In this case, the value of the social security *Fator Previdenciário* is 0.61. In the second possibility, retirement is obtained after 35 years of contribution and the *Fator* corresponds to 0.84. For men, the hypotheses are analyzed that retirement is obtained after 35 years of contribution (*Fator* 0.73) and after 40 years (*Fator* 1.01). The hypothesis is adopted that people start working (and contributing) at the age of 20 and die at 80. Hence, the sum of active and inactive periods is the same in all situations – 60 years. Discount rates of 3% and 4 % are used to present all results. The rates we calculated are lower than those practiced in Brazil for the last 10 years. However, they are consistent with similar exercises carried out in other countries, in which the interest rates used in actuarial exercises usually figure between 2% and 4%[16].

The parameters approximately reflect the case of the Brazilian middle class, which typically retires by age. In comparison with lower income classes, entry into the job market is delayed to conclude higher education and, at the time of retirement, life expectancy borders upon 80 years.[17] Following INSS rules, it is supposed that the value of the retirement benefit is given by the social security *Fator* multiplied by the average of the 80% highest contribution salaries and that, after the reception of the first benefit, its value continues, without variations in real terms.[18] Finally, the social security *Fator f* in (8a) and (8b) was obtained from Table A1 (see appendix), which corresponds to the factor actually in force in Brazil in function of laws approved during the second Cardoso government at the time this article is being concluded (2009). It should be reminded that, for women, the contribution time used to calculate the social security factor incorporates the five-year bonus they are entitled to, as defined by law.[19] The social security *Fator* of a woman who contributes to the INSS for 30 years and retires at 50 – however, as if it were 35 years for the sake of the factor – is 0.61 in Table A1, which means that, upon retirement, that woman receives a benefit equaling 61% of the average salaries considered for the contribution – in this case the 80% highest salaries in the 30-year period.

The results are reported in Tables 3 and 4. To assess the sensitivity of the calculated rate *c* for different *w* rates (and to incorporate occasional economic productivity gains each year, not captured in the estimates), in each table, the final lines show the rates calculated for growth rates *w* corresponding to 2%, 3%, 4% and 5%.

Table 3

[16] The choice of these rates is based on the real net profit of a public bond in Brazil. At the time this paper was finished, the *Selic* rate (treasury bonus rate) was expected to figure around 11% per year at the end of 2009. Considering a 20% income tax rate for a one-year application and an inflation rate of around 4.5% per year, the real values reached are close to the discount rates adopted here.

[17] Averaging both genders, life expectancy at the age of 50 in Brazil is 78.8 years (survival of 28.8 years); at 55, 79.8 years; and at 60, 81.1 years. Those are the retirement ages after 30, 35 and 40 years of work, respectively, for people who start working at the age of 20.

[18] The calculations presented here apply to situations in which the calculated retirement benefit lies between the minimum and maximum limits of the INSS benefit.

[19] This means that the time of 30 years, for example, is considered as 35 years of contribution to calculate the social security factor.

Actuarially fair contribution rates – Women

Group	Rate of growth of income *w* (% per year)	Number of years of contribution: 30		Number of years of contribution: 35	
		Fator previdenciário: 0.61		*Fator previdenciário*: 0.84	
		Interest rate *i*		Interest rate *i*	
		3.0	4.0	3.0	4.0
Whole sample	1.29	26.8	20.2	26.2	19.5
0 a 8	0.15	25.3	19.0	24.4	18.0
9 a 11	1.07	26.5	20.0	25.9	19.2
12+	1.92	27.6	20.9	27.2	20.4
All groups	2.00	27.7	21.0	27.3	20.5
	3.00	29.0	22.1	28.8	21.9
	4.00	30.2	23.2	30.3	23.2
	5.00	31.4	24.3	31.8	24.5

Source: Authors' tabulations.

Table 4
Actuarially fair contribution rates – Men

Group	Rate of growth of income *w* (% per year)	Number of years of contribution: 35		Number of years of contribution: 30	
		Fator previdenciário: 0.73		*Fator previdenciário*: 1.01	
		Taxa de juros *i*		Taxa de juros *i*	
		3.0	4.0	3.0	4.0
Whole sample	1.97	23.7	17.8	23.0	17.1
0 a 8	0.89	22.2	16.5	21.3	15.6
9 a 11	2.02	23.8	17.8	23.1	17.2
12+	2.81	24.8	18.8	24.3	18.3
All groups	2.00	23.7	17.8	23.1	17.1
	3.00	25.1	19.0	24.7	18.5
	4.00	26.4	20.2	26.2	19.9
	5.00	27.6	21.3	27.7	21.3

Source: Authors' tabulations.

Data in Tables 3 and 4 show that the equilibrium rates for women are always higher than for men. This is due to their shorter contribution period (and longer retirement benefit reception period) and, mainly to the formula used for calculating the social security factor, which grants them a five-year bonus. For example, for men with between 0 and 8 years of study (whose annual income growth rate is w = 0.89%), adopting a real interest rate of 3%, with a 35-year contribution period, a contribution tax rate of 22.2% is needed. This means that, if it were a capital redemption system, a rate like this, charged upon a person's income, would allow that person, after 35 years of work (and contribution), having started working at the age of 20, to have accumulated a certain capital. This capital, continuing to yield 3% each year, would permit withdraws in the next 25 years (from 55 to 80 years of age), so that it would be sufficient to cost the worker's retirement. For a woman with analogue conditions, the rate would be 25.3%. The most expressive differences in each gender are in the different columns, according to the adopted interest rates. The calculated rates are consistently higher for groups with the highest education level, although the differences are not as high in comparison with the other groups.

It is also observed that, for longer contribution periods, as expected, contribution tax rates are lower, but the difference is not as large as one would suppose in principle. The reason is that there are two effects working in opposite directions. On the one hand, a longer contribution period and a shorter benefit reception period imply a lower equilibrium rate. On the other, this extension of the contribution period makes people retire at a higher age and, therefore, with a higher social security factor, which raises the value of the retirement benefit. As the first effect is slightly stronger, the result is a slight reduction in the equilibrium rate.

As observed in Tables 3 and 4, in all situations under analysis (except for men who retire after 40 years of contribution), the social security *Fator* is smaller than one. As a result, the benefit is smaller than the average of the 80% highest revenues in the work period. In addition, it should be reminded that, in the calculations, people were expected to retire as soon as they finished the minimum contribution period. These facts arouse one question: how sensitive are the results found for the necessary contribution tax rates if workers decide to postpone their retirement for a few more years? This question can also be interpreted differently: what are the impacts of increasing the minimum contribution period required by law so that, for both genders, the social security factor exceeds the unit?[20] To answer this inquiry, actuarially fair rates are calculated for men and women, considering a five-year increase in the minimum contribution time. Hence, the results reported in Tables 5 and 6 refer, respectively, to the situation in which women retire after 40 years of contribution time (*Fator* equal to 1.15) and men after 45 years (*Fator* 1.42).

Table 5
Actuarially fair contribution rates – Women – Extended time

Group	Rate of growth of income *w* (% per year)	Number of years of contribution: 40	
		***Fator previdenciário*: 1.15**	
		Interest rate *i*	
		3.0	**4.0**
Whole sample	1.29	25.0	18.4
0 a 8	0.15	23.0	16.7
9 a 11	1.07	24.6	18.1
12+	1.92	26.1	19.4
All groups	2.00	26.3	19.5
	3.00	28.1	21.1
	4.00	29.8	22.7
	5.00	31.5	24.3

Source: Authors' tabulations.

Table 6
Actuarially fair contribution rates – Women – Extended time

Group	Rate of growth of income *w* (% per year)	Number of years of contribution: 45	
		***Fator previdenciário*: 1.42**	
		Interest rate *i*	
		3.0	**4.0**
Whole sample	1.97	21.7	16.0
0 a 8	0.89	19.8	14.3
9 a 11	2.02	21.8	16.0
12+	2.81	23.2	17.3
All groups	2.00	21.8	16.0
	3.00	23.5	17.6
	4.00	25.3	19.2
	5.00	26.9	20.7

Source: Authors' tabulations.

As expected, the calculated rates are lower in comparison with the corresponding cases in Tables 3 and 4. It should be noted that the variation in rates is not linear with the 5-year increases in the contribution period. For women, when moving from 30 to 35 years, the mean reduction in the

[20] In practice, this change corresponds to a minimum retirement age of 65 years for men and 60 years for women, closer to the ages most countries adopt.

calculated rate (at a 3% interest rate) is 1.25%.[21] When the extension goes from 35 to 40 years, on the average, the rate decreases by 3.51%. For men, analogue rates are 2.06% and 4.96%. In the second case, this more significant reduction is mainly associated with the variation in the social security factor, which is larger for men than for women. Another point that should be highlighted is that, the smaller the income growth rates, the larger the reduction in the calculated contribution tax rate.

4.3. Three extensions

The calculation method described in the previous section permits three extensions of the adopted model. The first is to calculate the *social security replacement rate,* given by the relation between the values of the first retirement benefit and the remuneration in the last work period, as presented in equation 9:

$$TR = \frac{A_{T+1}}{W_T} . \qquad (9)$$

The value of the benefit used to obtain the results in Tables 3 and 4 was calculated (in compliance with INSS rules) by multiplying the social security factor by the 80% highest contributions. In the model in question, as a monotonically growing revenue curve was supposed, this contribution period corresponds to the 80% last years in the person's work life. In case of a 35-year contribution time, this period is given by the interval between the 8th and the 35th years (included) of the person's time on the job market. Hence, the calculation does not consider periods 1 to 7, corresponding to the 20% lowest revenues. This exercise uses the actuarially fair rates for each group of workers, shown in Tables 3 to 6. In the calculations, it is supposed that the contributions are capitalized each period at an interest rate *i*. When the person retires, the accumulated resources are still capitalized at the same interest rate *i*, but now no more contributions are made. Instead, *A* retirement benefits are received until the accumulated amount is used up[22].

For example: for men whose number of schooling years lies between 0 and 8, gaining an initial remuneration of 100 monetary units, income upon retirement will correspond to 135.06 and mean revenue 120.17. Thus, at a social security fator of 0.73, the retirement benefit would amount to 87.73. According to equation (9), dividing 87.73 by 135.06, one reaches a replacement rate of 0.65. Table A2 in the Appendix shows the values used in this example.

Tables 7 and 8 present the replacement rates *TR* for all groups constituted by gender, education level and contribution period combinations, according to section 4.2. The data permit some conclusions. As expected, the longer the contribution period, the higher the replacement rate will be. For equal contribution periods, women's replacement rates are higher than for men. This is due to two motives. The first is the social security factor formula, in which 5 years are added to women's contribution time. The second motive is that women's income growth rates are higher than men's. Income growth rates demand higher actuarially balanced rates[23]. It can also be observed that, *ceteris paribus*, the lower the education level (and consequently income), the higher the replacement rate. Finally, it can also be mentioned that, as opposed to what common sense may usually assess, for contribution periods of 35 years for women and 40 for men, the replacement rate is high according to international standards, i.e. close to 0.73 for women and 0.78 for men.

Table 7
Replacement rates – Women

Group	Rate of growth of	Number of years of contribution: 30	Number of years of contribution: 35	Number of years of contribution: 40

[21] This value was obtained by calculating the average of percentage variations in rates presented in the 5th column (corresponding to the social security Fator 0.84) in Table 3 in relation to the values shown in the 3rd column (factor 0.61) in the same table. For the remaining values reported, calculations are analogue.

[22] As all values are capitalized (or discounted) at the same interest rate *i*, the choice of this rate does not affect the replacement rate *TR*.

[23] See section 4.4 on this point.

	income *w* (% per year)	*Fator*: 0.61	*Fator*: 0.84	*Fator*: 1.15
Whole sample	1.29	0.53	0.71	0.95
0 a 8	0.15	0.60	0.82	1.12
9 a 11	1.07	0.54	0.73	0.98
12+	1.92	0.49	0.66	0.87
All groups	2.00	0.49	0.65	0.86
	3.00	0.44	0.58	0.75
	4.00	0.40	0.52	0.67
	5.00	0.37	0.47	0.60

Source: Authors' tabulations.

Table 8
Replacement rates – Men

Group	Rate of growth of income *w* (% per year)	Number of years of contribution: 35 *Fator*: 0.73	Number of years of contribution: 40 *Fator*: 1.01	Number of years of contribution: 45 *Fator*: 1.42
Whole sample	1.97	0.57	0.76	1.03
0 a 8	0.89	0.65	0.88	1.22
9 a 11	2.02	0.56	0.75	1.02
12+	2.81	0.52	0.68	0.91
All groups	2.00	0.57	0.76	1.03
	3.00	0.50	0.66	0.89
	4.00	0.45	0.59	0.78
	5.00	0.41	0.52	0.69

Source: Authors' tabulations.

Now, a second extension will be made. It is calculated for how many years a fund constituted through the above described capital redemption procedure would permit ballasting a retirement benefit by age for a woman, at the contribution tax rate of 31% currently applied in Brazil. Differently from the previous case, the value of this benefit is given onlyby the average of the 80% highest salaries contributed (i. e. it's like the value of *fator* is equal to 1)[24]. It is supposed in these calculations that income grows by 0.15% per year. This rate refers to the income evolution of women with 0 to years of schooling. Calculations are made for contribution period of 15 to 20 years. The results are shown in Table 9 below. As observed, at an interest rate of 3% per year and a contribution period of 15 years, the fund is exhausted within 7 years.

Table 9
Number of years with funded retirement benefits – Different number of years of contribution (*c=0.31, w=0.15*% per year)

Years of contribution	Interest rate (%)	
	3.0	4.0
15	7	8
20	10	12

Source: Authors' calculations. Same methodology described in appendix.

Table 9 is important because it reveals the degree of distortion permitted by the figure of retirement by age which, in Brazil, is granted to women at the age of 60 and to men at 65. Despite the planned expansions, the contribution requisite is expected to be a mere 15 years and only in 2011, as it still corresponds to 14 years in 2009 because of current legislation, which establishes a six-month increase per year up to the above mentioned roof of 15 years. This means that, even at the end of this transition, in a fully funded system, a woman retired at the age of 60, after 15 years of contribution time would only have accumulated an amount of resources to receive retirement benefits for 7 years,

[24] Legislation establishes the facultative application of the social security factor to retirement benefits by age.

when in fact she is supposed to receive the benefit for 23 years, as the life expectancy of a women at the age of 60 is 83 years[25].

Finally, a third extension is made in addition to the first two. The starting point is equation 10:

$$S = S_{-1}(1+i) + cW - A\,, \qquad (10)$$

in which S is the balance of a hypothetic fully funded regime, i the real interest rate, c the contribution tax rate, W is the contribution income and A the value of the retirement benefit. For each time period, W and A are excluding, implying that, when contributions are still being made, the person does not receive retirement benefits and, when these start to be received, contribution salaries cease. Therefore, given an interest rate i and the retirement calculation formula, supposing perfect predictability of the date of decease, there is only one single c value that makes the final balance equal to zero. What is done here, hence, is to invert the reasoning of Table 9, which supposed a given contribution and calculated in which the accumulated balance was exhausted. Now, the contribution tax rate is calculated that preserves the positive balance up to the last moment, when it is zeroed.

In Table 10, the actuarially adequate contribution rate is presented, adopting an annual salary variation of 2% and a 3% capitalization rate i, supposing that the person lives up to the age of 80 years. The value of the retirement benefit by age equals 70% of the benefit salary, in addition to 1% for each set of 12 monthly contributions, up to the limit of 100% of the benefit salary. The benefit salary, in turn, is given by the average of the 80% highest contribution salaries, corrected by the IPCA consumer price index.[26]

Table 10
Actuarially fair contribution rates for differents ages of the beginning of contribution (*w=2.00%* per year*; i=0,03*) /a

Idade de início das contribuições	Men	Women
40	33.6	52.7
45	42.3	70.7
50	56.7	-

/a The woman retires at age of 60 and mana t age 65. We suppose the person dies at age of 80.

Source: Authors' tabulations.

In Table 10, a man who starts contributing at the age of 40 and contributes for 25 years can retire by age at 65. Considering the adopted hypothesis, he will live up to 80 years of age. A woman who starts to work at the same age, on the other hand, can retire at 60, contributing 5 years less and receiving retirement benefits for 20 instead of 15 years, which means that, in a pure capital redemption system, her contribution tax rate should be higher. It is observed that, in this case, the man's contribution rate should be 33.6%, while the woman's – who would contribute for 20 years and receive retirement for yet another 20 – should rise up to 52.7%. If contribution started at the age of 45 years, the contribution would be even higher.[27]

It should also be noted that, despite the relative importance of retirement by age in total retirement benefits, its average value is quite lower than the average retirement benefit by contribution time (LOC). According to the Brazilian Social Security Ministry, in December 2008, including only social security benefits, 14.453,455 retirement benefits were granted. This total comprises 7,500,092 (51.9%) benefits by age, 2,835,391 (19.6%) disability benefits and 4,117,972

[25] It should be observed that if, on the one hand, legislation is lenient about certain benefits, on the other, it is arbitrary because of "punishing" persons who contributed but left the job market. For example, a woman who worked and contributed for 12 years but then stopped contributing because she gave birth and left the job market will lose her contribution link and will not be entitled to any benefit.

[26] That is, for example, upon retirement, a person who contributed for 20 years receives 90% of the 80% highest contribution salaries.

[27] The example of a woman who starts to contribute at 50 was not taken into account as, when reaching the retirement age, she would not have complied with the eligibility requisite of 15 years of contribution time.

(28.5%) benefits per contribution time. The mean values of disability benefits and retirement benefits by age were R$ 552.01 and R$ 432.97, respectively. These data contrast with the mean value of ATC, which was R$ 1,058.93[28].

4.4. An additional look at the contribution tax rate

One should note, based on the results of section 4.2, for the parameters under analysis, that the necessary contribution rate *c* depends positively on the income growth rate *w*. This can be seen more clearly by rewriting 8a and making the derivation in relation to *w*.

$$\frac{dc}{dw} = \frac{d}{dw}\left(\frac{f(1+w)^{0,2T}}{0,8wT} \frac{1}{i}\left((1+w)^{0,8T} - 1\right)\left(\frac{(1+i)^N - 1}{(1+i)^N} \right) \frac{(i-w)}{(1+i)^T - (1+w)^T} \right) \qquad (11)$$

Rearranging the equation results in:

$$\frac{dc}{dw} = \frac{f}{0,8Ti}\left(\frac{(1+i)^N - 1}{(1+i)^N} \right) \frac{d}{dw}\left(\frac{(1+w)^T - (1+w)^{0,2T}}{w} \frac{(i-w)}{(1+i)^T - (1+w)^T} \right) \qquad (12)$$

And, finally, performing the derivation in relation to *w*:

$$\frac{dc}{dw} = \left[\frac{f}{0,8Ti}\left(\frac{(1+i)^N - 1}{(1+i)^N} \right) \right] \left[\left(\left(T(1+w)^{T-1} - 0,2T(1+w)^{0,2T-1} \right)(i-w) - \left((1+w)^T - (1+w)^{0,2T} \right) \right) \left(w\left((1+i)^T - (1+w)^T \right) \right) \right.$$
$$\left. - \left((1+w)^T - (1+w)^{0,2T} \right) \left((1+i)^T - (1+w)^T - wT(1+w)^{T-1} \right)(i-w) \right] / \left[w\left((1+i)^T - (1+w)^T \right) \right]^2 \qquad (13)$$

In equation 13, the expression within the first brackets, which does not depend on *w,* is positive. The term in the denominator (third brackets) is also positive. Thus, the sign of the derivate depends on the value of the expression within the second brackets. When $i > w$, the value of the expression is usually positive. If $i < w$, the negative signs of terms $(i-w)$ and $\left(w\left((1+i)^T - (1+w)^T\right)\right)$ cancel each other. Analogue reasoning applies to the final terms in the brackets. In principle, it may seem that a higher income growth rate *w* would imply a lower contribution rate. However, as the benefit is calculated based on the 80% highest contributions, if income grows at a higher rate, this implies a higher benefit, which can only be cost in an actuarially fair way when a higher contribution tax rate is applied.

5. What inferences do the results permit for Brazil?

Table A1 in the Appendix shows the social security *Fators* in force in Brazil at the start of 2009. This factor considers expected survival and is multiplied by the mean contribution salary – discarding the 20% lowest salaries – after June 1994 to achieve the value of the retirement benefit. As explained, women gain a 5-year bonus in contribution time counts. By the age of 55 – after 35 years of contribution that will count as 40 years, a woman who started contributing at the age of 20 and contributes on a constant fixed value will have a social security *fator* of 0.84. If delaying her retirement for 3 years, at the age of 58, after 38 years of contribution that will count as 43, the factor will already have exceeded the unit (1.02).

[28] At the time this paper was been finished, one Real (R$) – the Brazilian currency – was equivalent to US$ 0.55.

At 55/60 years, a man is expected to live up to the approximate age of 78/79 years. A woman, on the other hand, is expected to live up to 82/83 years (Table 11). Averaging both genders, according to IBGE mortality ages, life expectancy corresponds to 80 years at the age of 55 and 81 years at 60. By law, the expected survival used in the social security fator formula is the same for both genders. That is a distortion, as female life expectancy exceeds that of men. A second distortion is the 5-year contribution time bonus granted to women when the factor is calculated.

It should be reminded that, on the average, the retirement age of people retiring by contribution time in Brazil is 53 years, as shown in Table 12. In the case of women, this average age to receive the first benefits is even lower: 51 years[29]. Finally, it should also be reminded that retirement by age represents the largest part of retirement benefits (Table 13) and that, in the composition of this group, women represent 65% of urban and 61% of rural retirement benefits (AEPS, 2006).[30]

Table 11
Life expectancy, by age and gender (years) - Brazil

Age	Men	Women	Both
0	69	76	73
10	71	78	75
20	72	79	75
30	73	79	76
40	75	80	77
50	77	81	79
60	79	83	81
70	83	85	84
80+	89	90	89

Source: IBGE

Table 12
INSS – Mean age of retirement – LOC Benefit
Brazil - 2006

Group	Mean age (years)
Men	54
Women	51
Both	53

Source: Boletim Estatístico da Previdência Social. Vol. 12, Nº 02

Tabela 13
Number of Social Security benefits (RGPS/INSS) (%)/a
Brasil - 2008

Kind of benefit	%
Old-age	69.7
Age	36.2
Disability	13.7
Length of contribution (LOC)	19.9
Survivors	30.3
Total	100.0

/a Does not include sickness or other kinds of risk benefits.

[29] These figures are similar to Delgado et al (2006).
[30] It should be reminded that a considerable part of service retirements in the INSS come from the rural context, where limits to demand the benefit are 5 years shorter than for service retirement in the urban context.

Source: Boletim Estatístico da Previdência Social. Vol. 13, Nº 12.

What can one say, then, about the results obtained based on the data shown in the tables in this section? In the first place, one can conclude that the 31% rate, as opposed to what happened until the approval of social security factor legislation – which decreased the value of the retirement benefit when compared with earlier rules – seems to be relatively high[31]. This conclusion is supported by Rocha and Caetano (2008, section 4). The authors show that Brazil has one of the highest social security contribution tax rates, despite an elderly dependence index of around 8%, lower to the mean index in European and Latin American countries.

Considering the current social security *fator*, adopting a 3% discount rate and a 2.2% annual real income growth rate during a person's active life, for people who start working at 20 and are expected to live up to 80 years of age, the actuarially balanced rate should be 27.7% for women (who retire after 30 years of contribution) and 23.7% for men (who retire after 35 years of contribution)[32] (see Tables 3 and 4). In the second place, there is an important difference between the situation of men and women, as specified in the example mentioned in the previous paragraph. The role of the social security *fator* should also be highlighted. For lower retirement ages, corresponding to shorter contribution periods, the factor is considerably lower than 1 (see Table 3 for example, which brings down the value of the retirement benefit.

Finally, the figures suggest that the actuarial requisites partially covered in the social security *fator* should also be incorporated into the retirement benefit calculation for retirement by age. It is highlighted that, in Table 10, the hypothetically accumulated fund of people contributing for 15 years is exhausted within 7 years, while the retirement reception period in case of retirement by age approximately ranges between 15 and 20 years. No system can resist to a situation in which women contribute 31% of their salary for 15 years, retiring at the age of 60 and then gain retirement benefits practically on their full contribution salary (corresponding to 85% according to the rule) for yet another 20 year. It is observed in Table 10 that a woman who (within the adopted parameters) starts to contribute at the age of 40 with a view to retirement by age, should pay a contribution tax rate of 53%, much higher than the 31% actually paid.

6. Conclusions

The importance of a new social security reform to approach Brazil to the characteristics observed in other systems around the world in increasingly evident (Pinheiro, 2004; Tafner and Giambiagi, 2007). This paper attempted to support this debate by offering technical elements to verify whether the current 31% contribution tax rate is adequate or not, in view of what should be a situation of relative actuarial equilibrium between the present value of contributions and retirement benefits.

The calculations made permitted calculating the rates needed for a series of combinations among gender, education level, contribution time (and, hence, social security factor), discount rates and income growth rate of workers affiliated with the RGPS. In all calculations, the hypothesis was admitted that people start to work at the age of 20 and die at 80. For men who work for 35 years, using a 3% discount rate, contribution tax rates range between 22.2% and 24.8%, depending on the adopted parameters. For women who retire after 30 years of contribution, rates vary between 25.3%

[31] It should be reminded, however, that for a considerable part of the workforce, the contribution tax rate actually amounts to 28 or 29% instead of 31% as, for salaries of up to R$ 911.70, the rate is 8%, against 11% for salaries between R$ 911.71 and R$ 1,519.50. The employer's 20% should be added to these amounts.

[32] On the other hand, it should be reminded that, although the examples used indicate the feasibility of actuarial equilibrium with rates below 31%, these calculations do not consider the existence of non-programmable benefits, typically disease aid or disability retirement. In other words, the calculations presuppose that people contribute during a number of years as programmed and, after retirement, regularly withdraw benefits. However, among the almost 23 million benefits issued by the RGPS (data for December 2008), about 1.7 million (7.5%) are aids and 3.0 million (13.1%) disability retirements, whose cost the calculations do not include.

and 27.6%. It was also demonstrated that, in the case of retirement by age, current contribution requisites are insufficient to fully cost the benefit. In the most favorable hypothesis, the amount of contributions paid is equivalent to only 12 years of reception, quite below expectations. In the same line, actuarially necessary rates were also calculated for retirement by age. The rates range from 33.6% to 56.7% for men and from 52.7% to 70.7% for women.

Although these figures suggest that the current rate of 31% is reasonable or even excessive, five elements should be taken into account.

First, although the rates calculated here reveal to be sufficient to cover programmable benefits, particularly retirement by contribution time, they may be insufficient to cost, in addition, non-programmable risk benefits, such as disease aid or disability retirement. And those benefits represent a considerable part of INSS spending. Second, it should be reminded that the calculations made here do not cover pensions, which are another important component of social security spending. Third, the actuarially balanced rate of 31% this research reached for men is higher than the 20% paid by autonomous workers. Fourth, the 31% rate is blatantly insufficient from an actuarial viewpoint for cases of retirement by age, with a very short contribution period. Finally, for many workers, with the lowest incomes, the rate charged on their income is 28% instead of 31%.

The primary goal of this research was to present a set of actuarial calculations with a view to assessing to what extent the 31% rate practiced today is correct, and not to infer public policy prescriptions. However, we could not terminate without presenting some evident recommendations originating in the interpretation of the research results:

a) There is a need to move towards greater approximation between the retirement rules of men and women as, from an actuarial viewpoint, the benefit granted to women in the form of implicit subsidies paid by the rest of society reveals to be extremely significant. Even in the case of retirement by age, in which the incidence of the social security factor is optional, the actuarial rate necessary for women is quite higher than that for men;
b) Considering that the rate adopted today does not cover the sum of what would be a retirement benefit combined with the ballast for risk benefits, contributors should pay specifically for the right to use non-programmable benefits;
c) It is important to take a fundamental measure with a view to the system's equilibrium, increasing the contribution period for retirement by age, today limited to a mere 14 years, and which is expected to increase, but only up to 15 years in 2011;
d) One alternative would be to increase the contribution period (and, consequently, reduce the time during which the benefit is received). In the future, this policy could produce a drop in rates, besides implying a lower intergenerational transference burden between individuals, due to the decrease in distortions associated with higher rates.

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Appendix

Table A1
***Fator Previdenciário* – Men**
(Contribution rate = 0.31)

	Age of retirement																									
	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
ES	28.8	27.9	27.1	26.3	25.6	24.8	24.0	23.3	22.5	21.8	21.1	20.3	19.6	18.9	18.3	17.6	16.9	16.3	15.6	15.0	14.4	13.8	13.3	12.7	12.2	11.7

Number of years of contribution																										
25	0.42	0.44	0.46	0.47	0.49	0.51	0.53	0.55	0.57	0.59	0.62	0.64	0.67	0.70	0.73	0.76	0.80	0.83	0.87	0.91	0.96	1.00	1.05	1.10	1.15	1.21
26	0.44	0.46	0.48	0.49	0.51	0.53	0.55	0.57	0.59	0.62	0.64	0.67	0.70	0.73	0.76	0.79	0.83	0.87	0.91	0.95	1.00	1.05	1.09	1.15	1.20	1.26
27	0.46	0.48	0.50	0.51	0.53	0.55	0.57	0.59	0.62	0.64	0.67	0.70	0.73	0.76	0.79	0.82	0.86	0.90	0.95	0.99	1.04	1.09	1.14	1.20	1.25	1.31
28	0.48	0.50	0.51	0.53	0.55	0.57	0.60	0.62	0.64	0.67	0.69	0.73	0.76	0.79	0.82	0.86	0.90	0.94	0.98	1.03	1.08	1.13	1.18	1.24	1.30	1.36
29	0.50	0.52	0.53	0.55	0.57	0.59	0.62	0.64	0.67	0.69	0.72	0.75	0.78	0.82	0.85	0.89	0.93	0.97	1.02	1.07	1.12	1.17	1.22	1.29	1.35	1.41
30	0.51	0.53	0.55	0.57	0.59	0.62	0.64	0.66	0.69	0.72	0.75	0.78	0.81	0.85	0.88	0.92	0.96	1.01	1.06	1.11	1.16	1.22	1.27	1.33	1.40	1.46
31	0.53	0.55	0.57	0.59	0.61	0.64	0.66	0.69	0.72	0.74	0.77	0.81	0.84	0.88	0.91	0.95	1.00	1.04	1.09	1.14	1.20	1.26	1.31	1.38	1.45	1.52
32	0.55	0.57	0.59	0.61	0.64	0.66	0.69	0.71	0.74	0.77	0.80	0.84	0.87	0.91	0.94	0.99	1.03	1.08	1.13	1.18	1.24	1.30	1.36	1.43	1.50	1.57
33	0.57	0.59	0.61	0.63	0.66	0.68	0.71	0.73	0.76	0.79	0.83	0.86	0.90	0.94	0.97	1.02	1.07	1.11	1.17	1.22	1.28	1.34	1.40	1.48	1.54	1.62
34	0.59	0.61	0.63	0.66	0.68	0.70	0.73	0.76	0.79	0.82	0.85	0.89	0.93	0.97	1.01	1.05	1.10	1.15	1.21	1.26	1.32	1.39	1.45	1.52	1.59	1.67
35	0.61	0.63	0.65	0.68	0.70	0.73	0.75	0.78	0.81	0.85	0.88	0.92	0.96	1.00	1.04	1.08	1.14	1.18	1.24	1.30	1.36	1.43	1.49	1.57	1.64	1.72
36	0.62	0.65	0.67	0.70	0.72	0.75	0.78	0.81	0.84	0.87	0.91	0.95	0.99	1.03	1.07	1.12	1.17	1.22	1.28	1.34	1.40	1.47	1.54	1.62	1.69	1.78
37	0.64	0.67	0.69	0.72	0.74	0.77	0.80	0.83	0.86	0.90	0.93	0.97	1.02	1.06	1.10	1.15	1.20	1.26	1.32	1.38	1.45	1.52	1.58	1.67	1.74	1.83
38	0.66	0.69	0.71	0.74	0.76	0.79	0.82	0.85	0.89	0.92	0.96	1.00	1.04	1.09	1.13	1.18	1.24	1.29	1.36	1.42	1.49	1.56	1.63	1.71	1.79	1.88
39	0.68	0.71	0.73	0.76	0.78	0.81	0.85	0.88	0.91	0.95	0.99	1.03	1.07	1.12	1.16	1.22	1.27	1.33	1.40	1.46	1.53	1.60	1.67	1.76	1.84	1.93
40	0.70	0.73	0.75	0.78	0.81	0.84	0.87	0.90	0.94	0.97	1.01	1.06	1.10	1.15	1.20	1.25	1.31	1.36	1.43	1.50	1.57	1.65	1.72	1.81	1.89	1.99
41	0.72	0.75	0.77	0.80	0.83	0.86	0.89	0.93	0.96	1.00	1.04	1.09	1.13	1.18	1.23	1.28	1.34	1.40	1.47	1.54	1.61	1.69	1.77	1.86	1.95	2.04
42	0.74	0.77	0.79	0.82	0.85	0.88	0.92	0.95	0.99	1.03	1.07	1.12	1.16	1.21	1.26	1.32	1.38	1.44	1.51	1.58	1.65	1.74	1.81	1.91	2.00	2.09
43	0.76	0.79	0.81	0.84	0.87	0.90	0.94	0.97	1.02	1.05	1.10	1.14	1.19	1.24	1.29	1.35	1.41	1.47	1.55	1.62	1.70	1.78	1.86	1.96	2.05	2.15
44	0.78	0.80	0.83	0.86	0.89	0.93	0.96	1.00	1.04	1.08	1.12	1.17	1.22	1.27	1.32	1.38	1.45	1.51	1.59	1.66	1.74	1.82	1.90	2.00	2.10	2.20
45	0.79	0.82	0.85	0.89	0.92	0.95	0.99	1.02	1.07	1.11	1.15	1.20	1.25	1.31	1.36	1.42	1.49	1.55	1.63	1.70	1.78	1.87	1.95	2.05	2.15	2.25
46	0.81	0.84	0.87	0.91	0.94	0.97	1.01	1.05	1.09	1.13	1.18	1.23	1.28	1.34	1.39	1.45	1.52	1.59	1.67	1.74	1.82	1.91	2.00	2.10	2.20	2.31
47	0.83	0.86	0.90	0.93	0.96	1.00	1.04	1.07	1.12	1.16	1.21	1.26	1.31	1.37	1.42	1.49	1.56	1.62	1.71	1.78	1.87	1.96	2.04	2.15	2.25	2.36
48	0.85	0.88	0.92	0.95	0.98	1.02	1.06	1.10	1.14	1.19	1.23	1.29	1.34	1.40	1.45	1.52	1.59	1.66	1.74	1.82	1.91	2.00	2.09	2.20	2.30	2.41
49	0.87	0.90	0.94	0.97	1.00	1.04	1.08	1.12	1.17	1.21	1.26	1.32	1.37	1.43	1.49	1.56	1.63	1.70	1.78	1.87	1.95	2.05	2.14	2.25	2.36	2.47
50	0.89	0.93	0.96	0.99	1.03	1.07	1.11	1.15	1.20	1.24	1.29	1.35	1.40	1.46	1.52	1.59	1.66	1.74	1.82	1.91	2.00	2.09	2.19	2.30	2.41	2.52
51	0.91	0.95	0.98	1.01	1.05	1.09	1.13	1.17	1.22	1.27	1.32	1.38	1.43	1.50	1.55	1.62	1.70	1.77	1.86	1.95	2.04	2.14	2.23	2.35	2.46	2.58
52	0.93	0.97	1.00	1.04	1.07	1.11	1.16	1.20	1.25	1.29	1.35	1.41	1.46	1.53	1.59	1.66	1.74	1.81	1.90	1.99	2.08	2.19	2.28	2.40	2.51	2.63
53	0.95	0.99	1.02	1.06	1.09	1.14	1.18	1.22	1.27	1.32	1.37	1.44	1.50	1.56	1.62	1.69	1.77	1.85	1.94	2.03	2.13	2.23	2.33	2.45	2.56	2.69
54	0.97	1.01	1.04	1.08	1.12	1.16	1.20	1.25	1.30	1.35	1.40	1.47	1.53	1.59	1.65	1.73	1.81	1.89	1.98	2.07	2.17	2.28	2.38	2.50	2.62	2.74
55	0.99	1.03	1.06	1.10	1.14	1.18	1.23	1.27	1.33	1.38	1.43	1.50	1.56	1.62	1.69	1.76	1.85	1.93	2.02	2.11	2.21	2.32	2.42	2.55	2.67	2.80

Table A2
Resources accumulated (monetary units)
Men – 0 a 8 years of schooling. Discount rate: 3% per year.

Year	Wage	Resources accumulated	Year	Wage	Resources accumulated
1	100.00	22.23	31	130.37	1,247.14
2	100.89	45.33	32	131.53	1,313.80
3	101.78	69.32	35	132.69	1,382.71
4	102.69	94.23	34	133.87	1,453.96
5	103.60	120.08	35	135.06	1,527.60
6	104.52	146.92	36		1,485.70
7	105.45	174.77	37		1,442.55
8	106.38	203.67	38		1,398.10
9	107.33	233.64	39		1,352.31
10	108.28	264.72	40		1,305.15
11	109.24	296.95	41		1,256.58
12	110.21	330.36	42		1,206.55
13	111.19	364.99	43		1,155.02
14	112.18	400.88	44		1,101.95
15	113.17	438.07	45		1,047.28
16	114.18	476.59	46		990.97
17	115.19	516.50	47		932.97
18	116.22	557.83	48		873.23
19	117.25	600.63	49		811.70
20	118.29	644.95	50		748.33
21	119.34	690.83	51		683.05
22	120.40	738.32	52		615.82
23	121.47	787.48	53		546.56
24	122.55	838.35	54		475.23
25	123.63	890.98	55		401.76
26	124.73	945.44	56		326.09
27	125.84	1,001.78	57		248.15
28	126.96	1,060.06	58		167.86
29	128.08	1,120.34	59		85.17
30	129.22	1,182.68	60		0.00

Obs. Value of old-age benefit is equal to 87.73 monetary units. Benefit starts being paid at year # 36.
Source: Authors' tabulations.