# On ruin probability and climate change

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## Some few questions...

1) Is climate really changing?

2) If climate has been changing, what is the impact on natural disasters?

3) ...and the impact on insurance companies?

4) and how to integrate climate change in actuarial models?

## Definition of climate change

Climate change in IPCC (Intergovernmental Panel on Climate Change) usage refers to a change in the state of the climate that can be identified (e.g. using statistical tests) by changes in the mean and/or the variability of its properties, and that persists for an extended period, typically decades or longer.

It refers to any change in climate over time, whether due to natural variability or as a result of human activity.

...so, is climate changing or not?

# Some facts

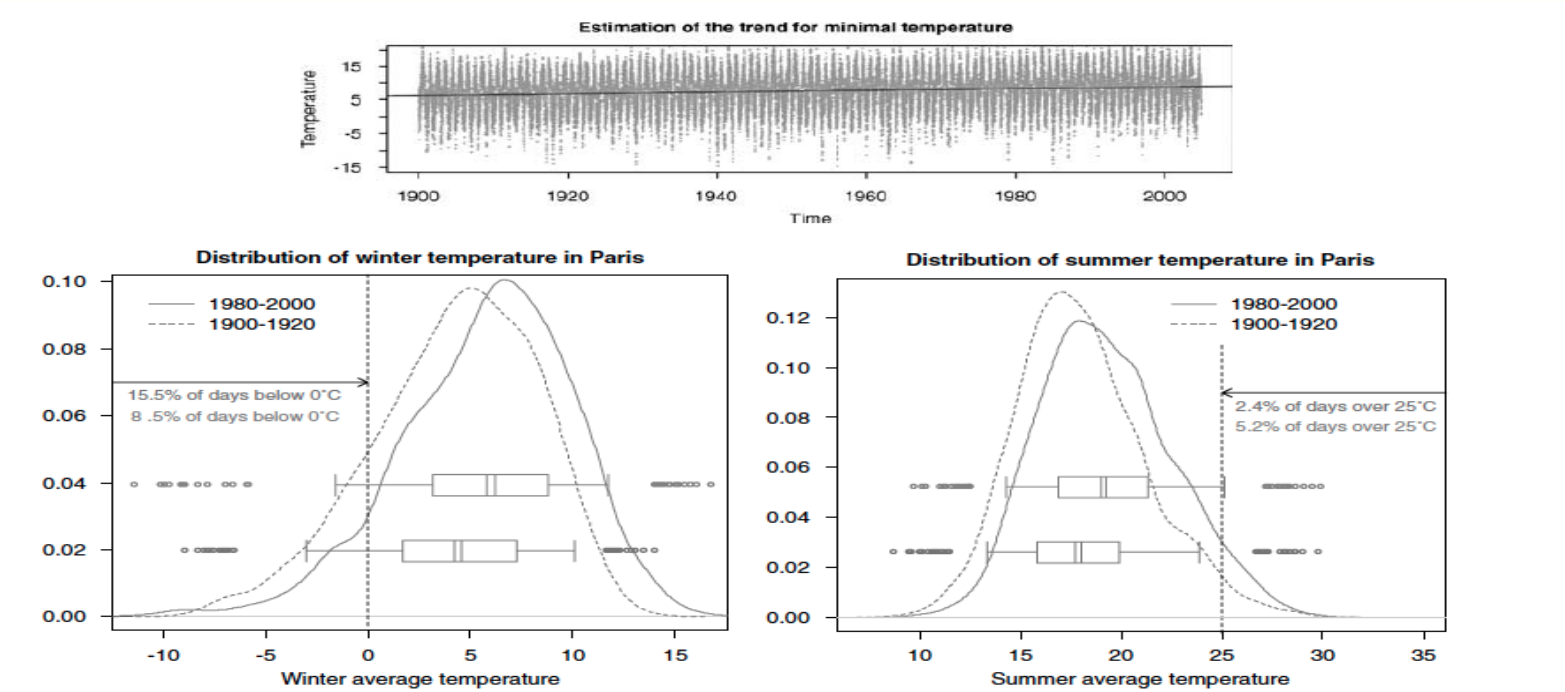



Figure 1: Temperature in Paris, over one century (+3 degrees), and distribution of winter and summer temperature for periods 1900-1920 and 1980-2000 (including boxplot with 95, 75, 50, 25 and 5 per cent quantiles)

## SRES scenarios

| Scenario | Population | Economy | Environment | Equity | Technology | Globalization |
|---|---|---|---|---|---|---|
| A1FI | ↗ | ↗ | → | → | ↗ | → |
| A1B | ↗ | ↗ | → | → | ↗ | → |
| A1T | ↗ | ↗ | ↗ | → | ↗ | → |
| B1 | ↗ | → | ↗ | → | ↗ | → |
| A2 | ↗ | → | → | → | → | → |
| B2 | → | → | ↗ | → | → | → |

Figure 2: scenarios described in the IPCC (Intergovernmental Panel on Climate Change) Special Report on Emissions Scenarios.

## More details

- A1 scenarios : rapid economic growth
  - A1FI: $+\ 4^\circ$ C (2,4 - 6,4): an emphasis on fossil-fuels;
  - A1B: $+\ 2,8^\circ$ C (1,7 - 4,4): a balanced emphasis on all energy sources;
  - A1T: $+\ 2,4^\circ$ C (1,4 - 3,8): emphasis on non-fossil energy sources;
- B1 scenario: $+\ 1,8^\circ$ C (1,1 - 2,9): a world more integrated, and more ecologically friendly, an emphasis on global solutions to economic, social and environmental stability;
- A2 scenario: $+\ 3.4^\circ$ C (2 - 5,4): a world of independently operating, self-reliant nations, continuously increasing population, regionally oriented economic development;
- B2 scenario: $+\ 2,4^\circ$ C (1,4 - 3,8): a world more divided, but more ecologically friendly, emphasis on local rather than global solutions to economic, social and environmental stability.

## Scenarios for emissions

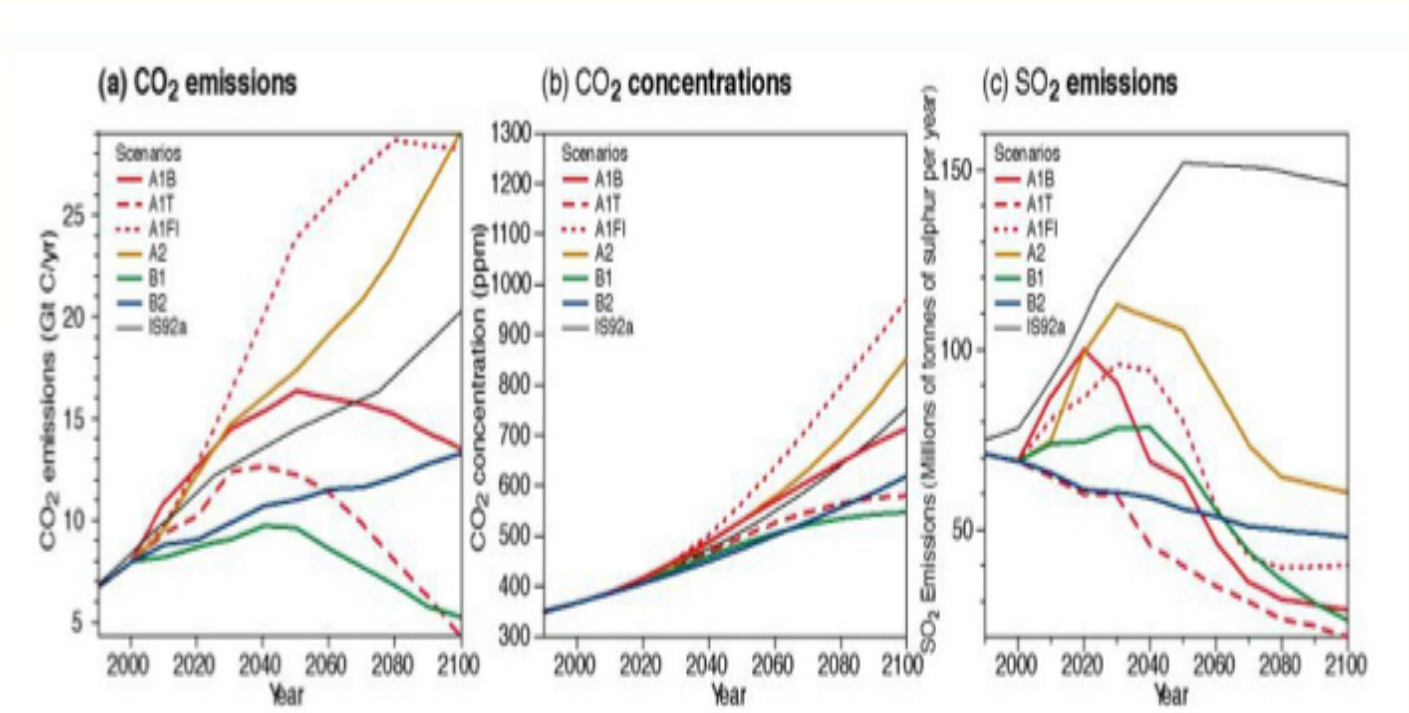



Figure 3: scenarios for $CO_2$ emissions, $CO_2$ concentrations and $S0_2$ emissions from 2000 to 2100.

## Surface warming

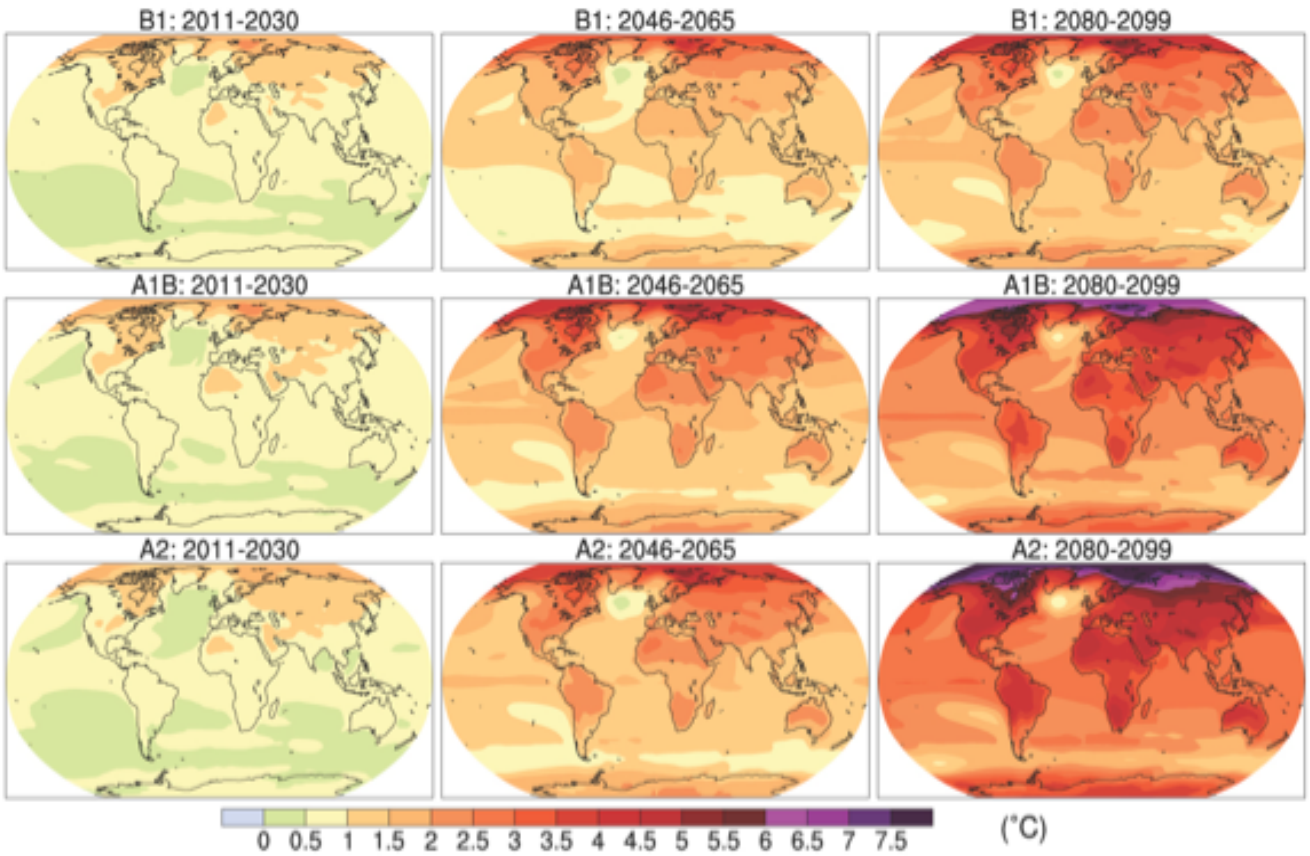



Figure 4: Projected surface temperature changes for the B1 (top), A1B (middle) and A2 (bottom) SRES scenarios averaged over the years 2011-2030 (left), 2046-2065 (middle) and 2080-2099 (right) compared to 1980-1999.

## Some facts

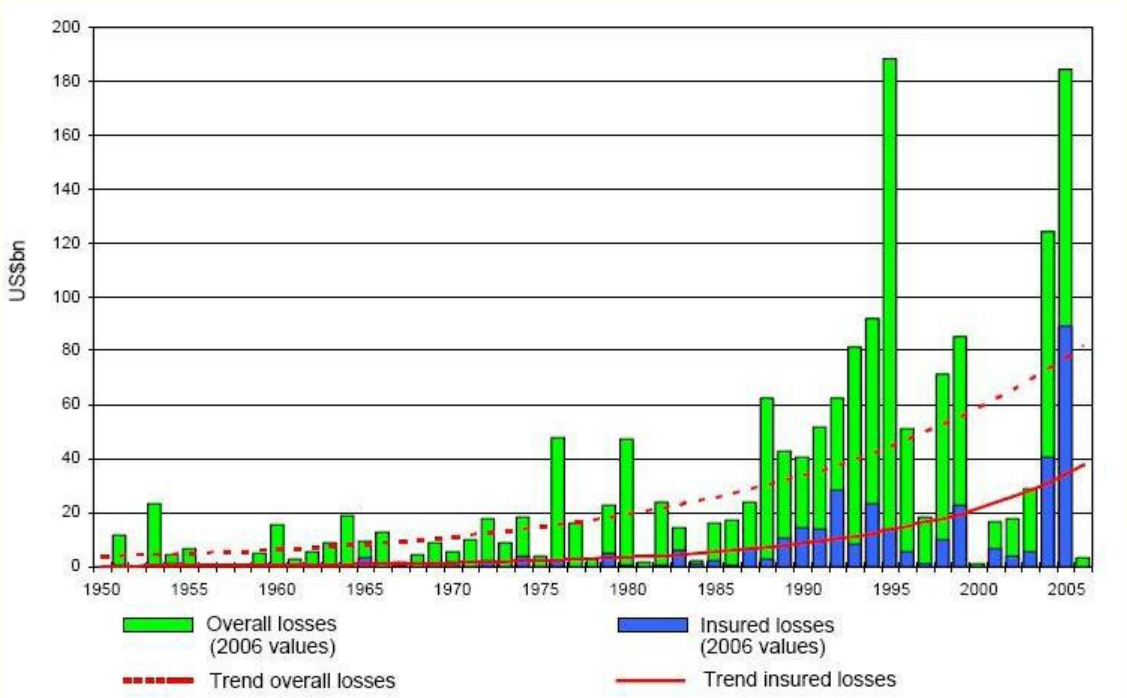



Figure 5: Major natural catastrophes on the period 1950-2005 (from Munich Re (2006).)

## Natural catastrophes

| Date | Loss event | Region | Overall losses | Insured losses | Fatalities |
|---|---|---|---|---|---|
| 25.8.2005 | Hurricane Katrina | USA | 125,000 | 61,000 | 1,322 |
| 23.8.1992 | Hurricane Andrew | USA | 26,500 | 17,000 | 62 |
| 17.1.1994 | Earthquake Northridge | USA | 44,000 | 15,300 | 61 |
| 21.9.2004 | Hurricane Ivan | USA, Caribbean | 23,000 | 13,000 | 125 |
| 19.10.2005 | Hurricane Wilma | Mexico, USA | 20,000 | 12,400 | 42 |
| 20.9.2005 | Hurricane Rita | USA | 16,000 | 12,000 | 10 |
| 11.8.2004 | Hurricane Charley | USA, Caribbean | 18,000 | 8,000 | 36 |
| 26.9.1991 | Typhoon Mireille | Japan | 10,000 | 7,000 | 62 |
| 9.9.2004 | Hurricane Frances | USA, Caribbean | 12,000 | 6,000 | 39 |
| 26.12.1999 | Winter storm Lothar | Europe | 11,500 | 5,900 | 110 |

Table 1: The 10 most expensive natural catastrophes, 1950-2005 (from Munich Re (2006)).

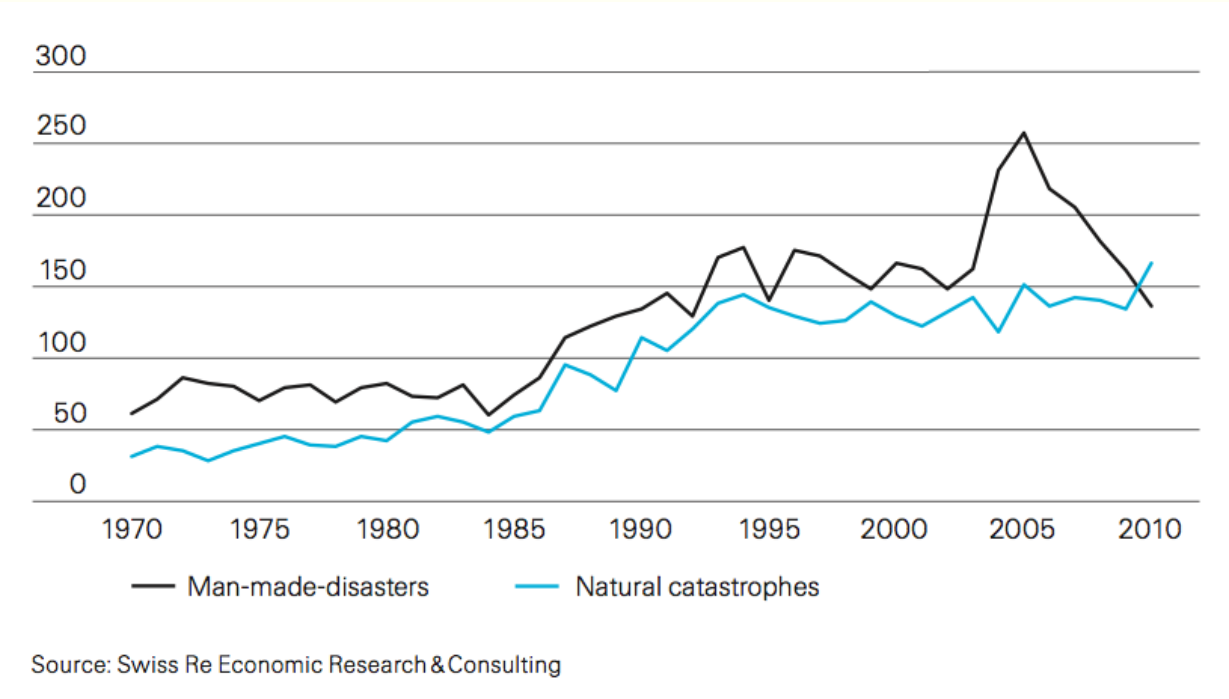



Figure 6: Number of events (man-made disasters and natural catastrophes) on the period 1970-2010.

## 2010: a year of devastating and costly events

| Region | Number | Victims | Insured loss (in USDm) | Economic losses (in USDm) | As a % of GDP |
|---|---|---|---|---|---|
| North America | 36 | 139 | 15348 | 20551 | 0.13% |
| Latin America and the Caribbean | 39 | 225784 | 8977 | 53378 | 1.10% |
| Oceania/Australia | 7 | 50 | 8860 | 13131 | 0.95% |
| Europe | 37 | 56490 | 6303 | 35204 | 0.19% |
| Asia | 139 | 17955 | 2240 | 74840 | 0.28% |
| Africa | 32 | 2'640 | 124 | 337 | 0.02% |
| Seas / Space | 14 | 515 | 1623 | 20623 | – |
| World total | 304 | 303573 | 43475 | 218064 | 0.31% |

Source: Swiss Re Economic Research & Consulting

Figure 7: Catastrophes in 2010 by region.

Gross domestic product (GDP) refers to the market value of all final goods and services produced within a country in a given period.

# 2010: the 20 most costly insurance losses

| Insured loss[3] (in USD m) | Victims[4] | Date (start) | Event | Country |
|---|---|---|---|---|
| 8000 | 562 | 27.02.2010 | Earthquake ($M_W$ 8.8) triggers tsunami, over 200 aftershocks | Chile |
| 4453 | 0 | 04.09.2010 | Earthquake ($M_W$ 7.0), over 300 aftershocks | New Zealand |
| 2754 | 64 | 27.02.2010 | Winter storm Xynthia, winds up to 160 km/h | France, Germany, Belgium et al |
| 2165 | – | 04.10.2010 | Thunderstorms, tornadoes, hail, floods | United States |
| 2050 | 1 | 23.12.2010 | Floods caused by heavy rains, tropical cyclone Tasha | Australia |
| 2000 | – | 12.05.2010 | Storms, winds up to 130 km/h, hail | United States |
| 1231 | – | 13.03.2010 | Storm, winds up to 120 km/h, heavy rain, floods | United States |
| 1079 | – | 22.03.2010 | Storm, winds up to 120 km/h, hail, rain, mudslides | Australia |
| 1070 | – | 06.03.2010 | Storms, hail, rain, floods | Australia |
| 1000 | 11 | 20.04.2010 | Explosion on Deepwater Horizon oil rig | Gulf of Mexico, United States |
| 820 | 33 | 30.04.2010 | Floods caused by heavy rain, storms | United States |
| 818 | 25 | 15.06.2010 | Floods, caused by heavy rain | France |
| 785 | 5 | 20.07.2010 | Storms, floods, hail, tornado | United States |
| 761 | 2490 | 29.05.2010 | Floods, mudslides caused by heavy monsoonal rain | China |
| 695 | 20 | 10.06.2010 | Storms, winds up to 97 km/h, hail, heavy rain, floods | United States |
| 620 | 20 | 17.06.2010 | Thunderstorms, heavy rains, floods | United States |
| 610 | 5 | 10.05.2010 | Tornadoes, hail | United States |
| 600 | – | 09.02.2010 | Winter storm, winds up to 80 km/h, snow | United States |
| 591 | 18 | 06.08.2010 | Floods caused by heavy rain | Germany, Czech Republic et al |
| > 500 | – | 12.07.2010 | Hailstorm, heavy rain; damage to buildings and cars | Canada |

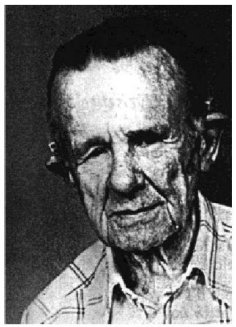
Source: Swiss Re Economic Research & Consulting

## Potential threats to the insurance industry

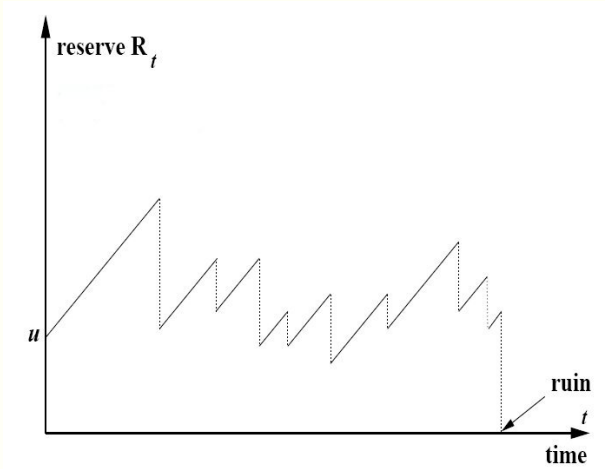
- Property and casualty insurance
  - Flooding - increased precipitation, rise in sea level etc.
  - Increase in frequency and severity of floods and storms
  - Pests - forestry and agriculture
- Life and health insurance
  - Change in tropical disease vectors (*new* diseases or resurgence of diseases that were supposed to have disappeared, e.g. dengue fever, malaria, cholera in North America)
  - Mortality rate changes (e.g. - increase of respiratory diseases and allergies)
- Potential financial areas of vulnerability?
  - Reserves (investments and surplus)
  - Ratings and solvency

## The classical model

F. Lundberg (1903)



H. Cramér




$$R_t = u + ct - \sum_{i=1}^{N_t} X_i \qquad t \geq 0$$

$u \geq 0$ initial capital

$c > 0$ premium intensity

$X_i$ the size f the $i$-th claim (i.i.d. r.v.s of d.f. $F$)

$N(t)$ homogeneous Poisson process ($\lambda$)

## Trying to update actuarial models...

Assume that $X_1, X_2, \ldots$ are i.i.d. regularly varying r.v.'s with index $\alpha > 1$ and d.f. $F$. Further consider the risk process

$$S_t = \sum_{i=1}^{N_t} \mu(\mathcal{T}_i) X_i - ct$$

where

- $N_t$ is a Poisson process with intensity function $\lambda(t)$
- $\mathcal{T}_i$ is the time of the $i$-th claim
- $c$ is the premium intensity

Further denote with $T_i = \mathcal{T}_i - \mathcal{T}_{i-1}$, $\mu_m = \min_{0 \le t \le T} \mu(t)$, $\mu_M = \max_{0 \le t \le T} \mu(t)$ and $\Lambda(t) = \int_0^t \lambda(x) \mathrm{d}x$.

## The finite time ruin probability

We are interested in the finite time ruin probability

$$\psi(u,T) = \mathbb{P}\left(\sup_{0\leq t\leq T} S_t > u\right)$$

It is easy to find that as $u \to \infty$

$$\psi(u,T) \sim \int_0^T \mathbb{P}(X_1 > u/\mu(x))\lambda(x)\mathrm{d}x \qquad (1)$$

Remark: this formula does not depend on the premium intensity $c$.

Our aim: to derive the error term for the asymptotic in (1).

## The finite time ruin probability (ctd)

Let $p_n := \mathbb{P}(N_T = n)$. We have that

$$\begin{aligned}\psi(u,T) &= \sum_{n=1}^{\infty} p_n \mathbb{P}\left(\sum_{i=1}^{n} \mu(\mathcal{T}_i) X_i > u + c\mathcal{T}_n\right) \\ &\quad + \sum_{n=2}^{\infty} p_n \sum_{k=1}^{n-1} \mathbb{P}\left(\sum_{i=1}^{k} \mu(\mathcal{T}_i) X_i > u + c\mathcal{T}_k, \right. \\ &\quad \left. \sup_{k<l\leq n} \sum_{i=1}^{l} \mu(\mathcal{T}_i) X_i - c\mathcal{T}_l \leq u\right)\end{aligned}$$

...and after some calculations...

## A second order approximation

Let $X_1, X_2, \ldots$ be i.i.d. r.v.'s with d.f. $F$ that is regularly varying with index $\alpha > 1$ and has regularly varying density $f$. Further let $\mu(t) \leq \mu_M$ a bounded function (on $[0, T]$) then

$$\begin{aligned}
\psi(u,T) &- \mathbb{E}[N_T] \int_0^T \mathbb{P}(X_1 > u/\mu(x)) \frac{\lambda(x)}{\Lambda(T)} \mathrm{d}x \\
&\sim f(u) \left( \mathbb{E}[X_1] \, \mathbb{E}\left[ \sum_{k=1}^{N_T} \mu(\mathcal{T}_k)^\alpha \sum_{i \neq k}^{N_T} \mu(\mathcal{T}_i) \right] \right. \\
&- c\mathbb{E}\left[ \mathcal{T}_{N_T} \sum_{k=1}^{N_T} \mu(\mathcal{T}_k)^\alpha \right] \\
&\left. + \mathbb{E}\left[ \sum_{k=1}^{N_T - 1} \sum_{l=1}^{k} \mu(\mathcal{T}_l)^\alpha \left( 0 \vee \min_{k < m \leq N_T} \left( c(\mathcal{T}_m - \mathcal{T}_k) - \sum_{i=k+1}^{m} \mu(\mathcal{T}_i) X_i \right) \right) \right] \right)
\end{aligned}$$

## Ruin probability with random $\mu$ and $\lambda$

Denote by $\psi(u, T, \lambda, \mu)$ the ruin probability given the processes $\mu(t)$ and $\lambda(t)$.

We have that $\psi(u, T) = \mathbb{E}\left[\psi(u, T, \lambda, \mu)\right]$ and

$$\psi(u, T, \lambda, \mu) = \int_0^T \lambda(t)\mathbb{E}_{\lambda,\mu}\left[\overline{F}\left(\frac{u + ct - \sum_{i=1}^{N_t} \mu(\mathcal{T}_t)X_i}{\mu(t)}\right) 1_{\left\{\inf_{0\leq s<t} u+cs-\sum_{i=1}^{N_s} \mu\mathcal{T}_s X_i>0\right\}}\right] \mathrm{d}t$$

...and after some calculations again...

## Upper and lower bounds

$$\begin{aligned}\psi(u,T) &- \mathbb{E}\left[\int_0^T \lambda(t)\overline{F}(u/\mu(t))\mathsf{d}t\right] \\ &\lesssim f(u)\mathbb{E}\left[\int_0^T \lambda(t)\left(\mu(t)\mathbb{E}\left[X_1\right]\int_0^t \mu(y)^\alpha\lambda(y)\mathsf{d}y\right.\right. \\ &\left.\left. + \mu(t)^\alpha\left(\mathbb{E}\left[X_1\right]\int_0^t \mu(x)\lambda(x)\mathsf{d}x - ct\right)\right)\mathsf{d}t\right]\end{aligned}$$

$$\begin{aligned}\psi(u,T) &- \mathbb{E}\left[\int_0^T \lambda(t)\overline{F}(u/\mu(t))\mathsf{d}t\right] \\ &\gtrsim f(u)\mathbb{E}\left[\int_0^T \lambda(t)\left(\mu(t)\mathbb{E}\left[X_1 1_{\{X_1>ct\mu(t)\}}\right]\int_0^t \mu(y)^\alpha\lambda(y)\mathsf{d}y\right.\right. \\ &\left.\left. + \mu(t)^\alpha\left(\mathbb{E}\left[X_1\right]\int_0^t \mu(x)\lambda(x)\mathsf{d}x - ct\right)\right)\mathsf{d}t\right]\end{aligned}$$

## An example

Let

$$\overline{F}(x) = (1+x)^{-\alpha}$$

with $\alpha = 2.5$. In case of climate change, we will have

$$\mu(t) = \lambda(t) = \frac{1}{4} + \frac{1}{10}\sin\left(2\pi\frac{t}{T}\right) + \frac{0.075}{120}\left(\frac{t}{T}\right)^2$$

If there is not climate change we will simply have

$$\mu^*(t) = \lambda^*(t) = \frac{1}{4} + \frac{1}{10}\sin\left(2\pi\frac{t}{T}\right)$$

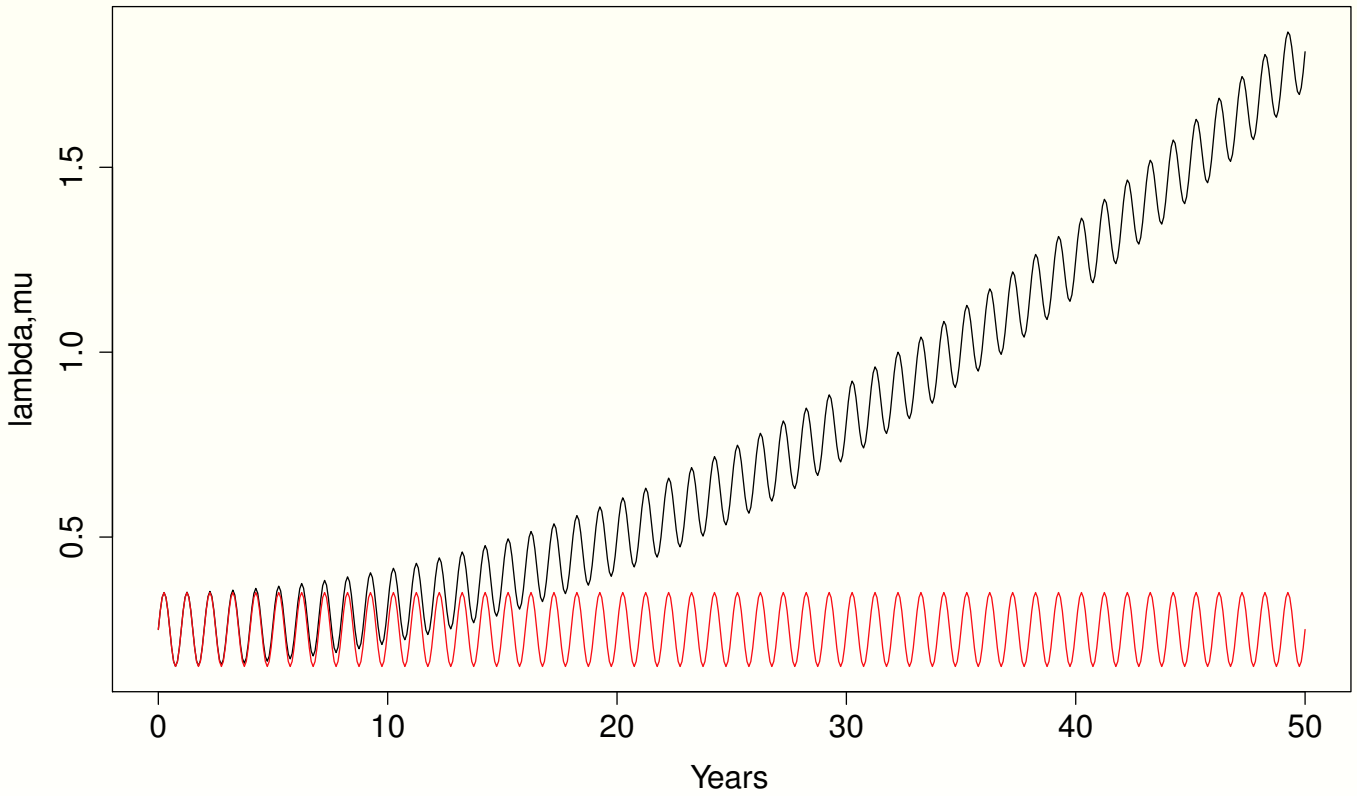



Figure 8: The intensity of the Poisson process, $\lambda(t)$, with (black) and without (red) climate change

## The expected claim amount

Let us consider $T = 12$ (12 months).

The expected claim amount in one year is given by

$$\mathbb{E}\left[\sum_{i=1}^{N_T} \mu(\mathcal{T}_i) X_i\right] = \mathbb{E}\left[X_1\right] \int_0^T \mu(t)\lambda(t)\mathrm{d}t$$

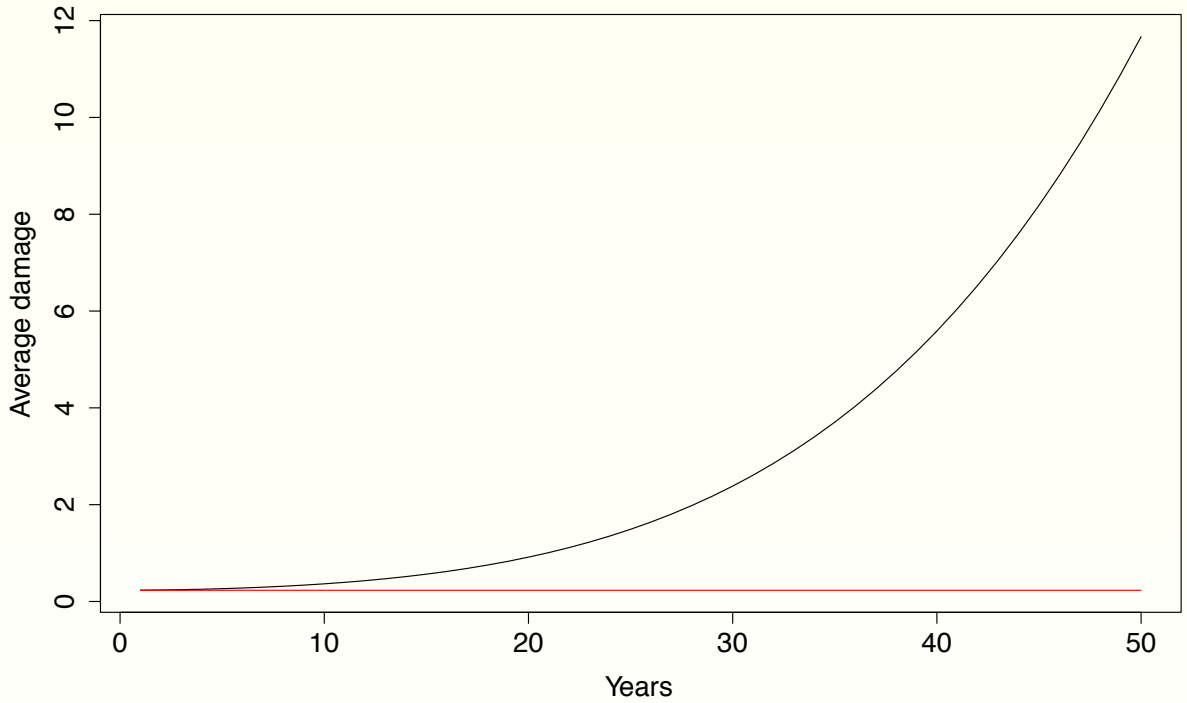



Figure 9: Average claim amount for the different years

## Solvency capital

We now want to collect premiums according to

$$c = 1.1 \times \frac{1}{T}\mathbb{E}\left[\sum_{i=1}^{N_t} \mu(\mathcal{T}_i)X_i\right]$$

Question: what is the value of $u$ such that $\psi(u, T) = 1/200$?

N.B. As an approximation for the ruin probability we will use the asymptotic upper bound.

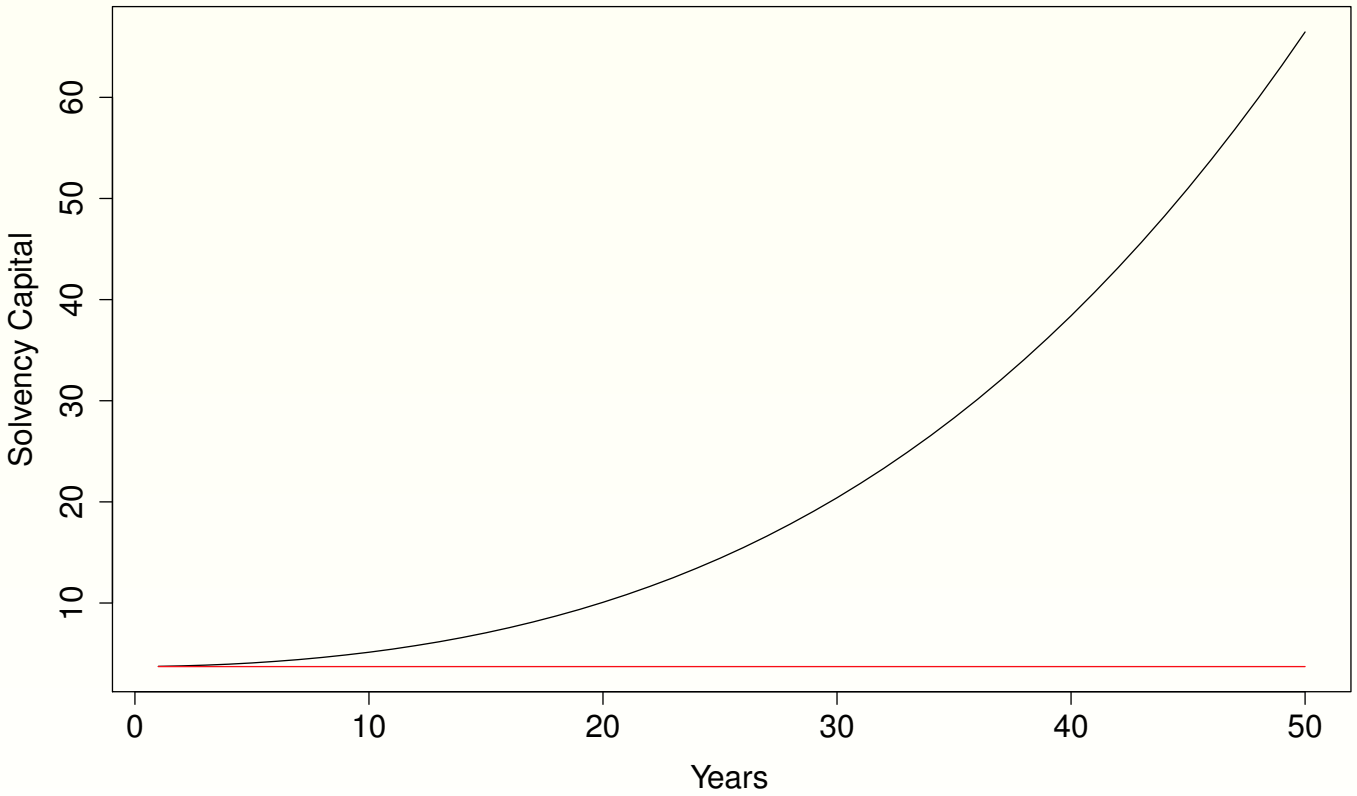



Figure 10: Solvency capital needed at the beginning of every year

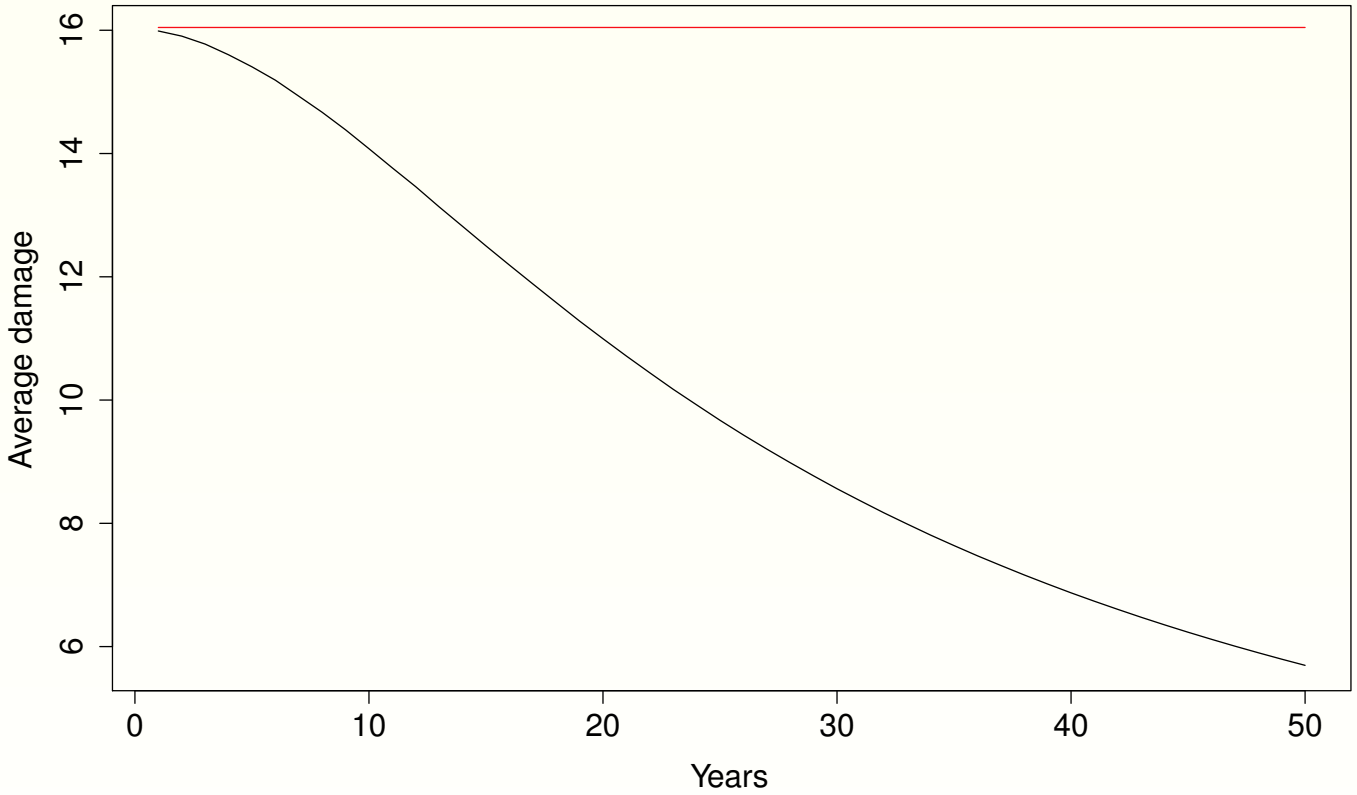



Figure 11: Solvency capital needed at the beginning of every year over the expected claim amount

## The cost of climate change

Question: what is the extra amount of money that an insurance company has to provide up to the year $k$ in a model with climate change?

The cost of climate change will be defined as

$$\begin{aligned} \text{Cost of CC} &= \text{Solvency Capital with CC} \\ &\quad - \text{Solvency Capital without CC} \\ &\quad - 0.1 \times \text{Collected Premiums} \end{aligned}$$

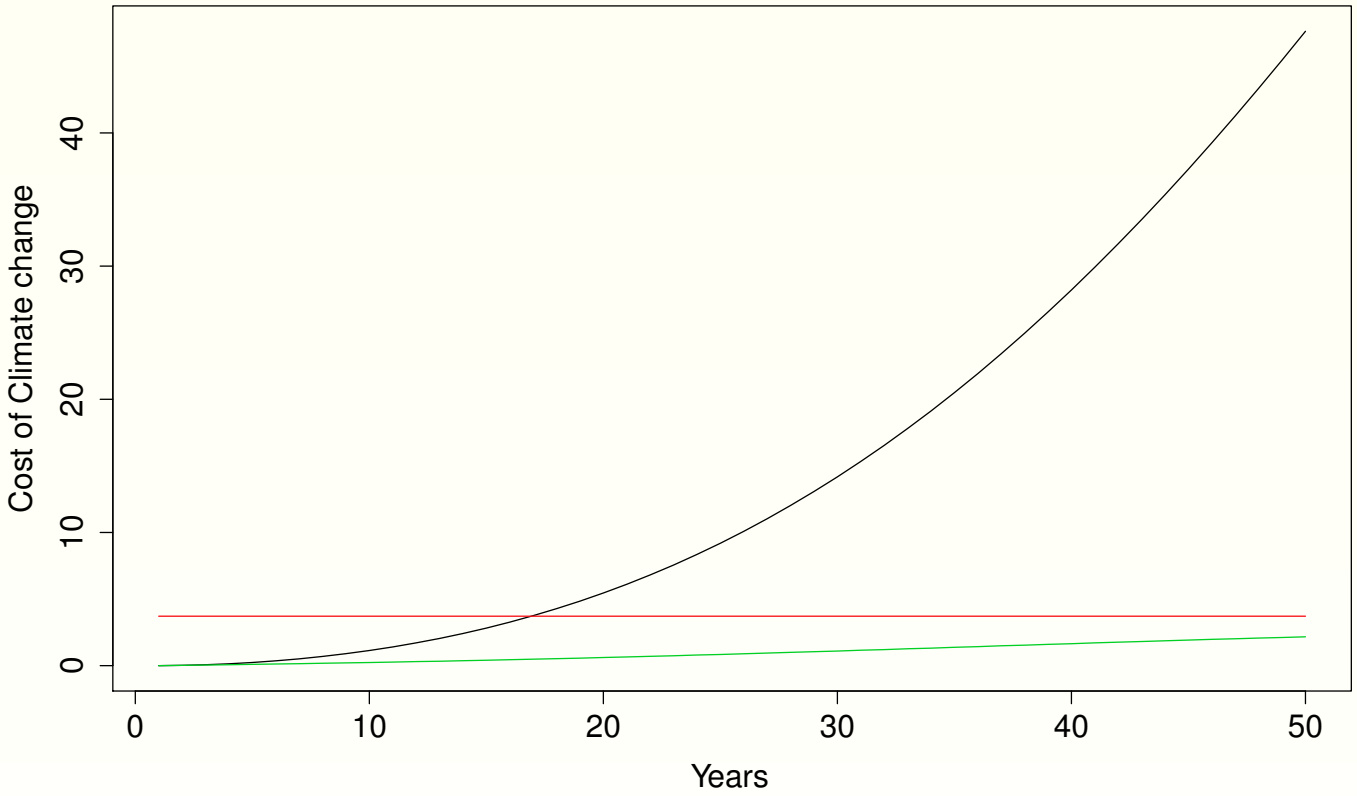



Figure 12: Cost of Climate change (black line) needed at the beginning of every year, together with the solvency capital without CC (red line) and the safety loading (green line)

## Conclusions

- Insurance company are particularly concerned by natural disaster and the impact of those disasters has been increasing because the climate has been changing!
- An attempt to integrate climate change in ruin probability calculations.
- To be continued...