Regression models for the joint development of individual payments and claim incurred

Łukasz Delong, SGH Warsaw School of Economics

Joint work with Mario V. Wüthrich, ETH Zürich

May 11th – May 15th 2020

About the speaker

Łukasz Delong

- Associate Professor at SGH Warsaw School of Economics
- Qualified actuary and Vice-President of the Polish Society of Actuaries
- PhD in Mathematics and Habilitation Degree in Economics
- Scientific research includes different areas of actuarial mathematics with focus on stochastic modelling of financial risks in insurance
- Author of numerous papers on optimal stochastic control applied to insurance and published a book on applications of backward stochastic differential equations with jumps in insurance
- Interested recently in neural networks and their applications in life and non-life insurance
- Webpage: www.lukaszdelong.pl

The goal of the research

- Our analysis focuses on the development of the so-called Reported But Not Settled (RBNS) claims,
- The development process of individual RBNS claims could be characterized with regression models since claims reported with different features and claim histories should generate different cash flows in time and amount,
- Regression models for individual claims should improve reserving methods and provide more detailed information about claim developments and ultimate losses in insurance portfolios,
- We jointly model the development of individual claim payments and claim incurred,
- We explore neural networks for this task because they seem to be particularly suited for our prediction problem.

Models for individual claims development

- Let $i \in \{1, 2, \dots\}$ denote the accident period of the occurrence date of an insurance claim, $j \in \{0, 1, 2, \dots\}$ denote the reporting delay after the claim occurrence date, $k \in \{0, 1, 2, \dots\}$ measure the development period of a reported claim, initialized to the respective reporting date $i + j$,
- Let $P_k^{i,j}$ denote the incremental payment in development period k, $I_k^{i,j}$ denote the claim incurred at the end of development period k, $R_k^{i,j}$ denote the case reserve at the end of development period k, for a claim from accident period i reported with delay j,
- At the reporting date of a claim, we observe the first payment and the first evaluation of the claim incurred, i.e. we have information $(P_0^{i,j}, I_0^{i,j})$. Next, we observe a process $(P_k^{i,j}, I_k^{i,j})_{k \geq 1}$,
- We define a filtration $(\mathcal{C}_k)_{k=0,1,\dots}$ which describes the history of payments and claim incurred on an individual claim:

$$\mathcal{C}_k^{i,j} = \sigma\left\{P_s^{i,j}, I_s^{i,j};\ 0 \leq s \leq k\right\}, \quad k = 0, 1, 2, \dots,$$

  To each individual claim, we also associate a vector of (static or dynamic) features, which we denote by $\boldsymbol{z}_k^{i,j}$. In our numerical example, $\boldsymbol{z}_k^{i,j}$ includes accident date, reporting delay, claim segment, claim type and claim origin,
- We aim at modeling the development $(P_k^{i,j}, I_k^{i,j})$ for each individual claim for all later time points $k = 1, 2, \dots$, given the individual claim history and the claim features.

Model 1: Occurrence of payments and changes in claim incurred

- We define the indicator process $(\mathcal{I}_k^{i,j}, \mathcal{P}_k^{i,j})_{k=1,2,\ldots}$:

$$\mathcal{I}_k^{i,j} = \mathbb{1}_{\{I_k^{i,j} - I_{k-1}^{i,j} \neq 0\}} \quad \text{and} \quad \mathcal{P}_k^{i,j} = \mathbb{1}_{\{P_k^{i,j} \neq 0\}},$$

and we introduce the stochastic process $(Y_k^{i,j})_{k=1,2,\ldots}$:

$$Y_k^{i,j} = 2\mathcal{I}_k^{i,j} + \mathcal{P}_k^{i,j} = \begin{cases} 0 & \text{if} \quad \mathcal{P}_k^{i,j} = 0 \text{ and } \mathcal{I}_k^{i,j} = 0, \\ 1 & \text{if} \quad \mathcal{P}_k^{i,j} = 1 \text{ and } \mathcal{I}_k^{i,j} = 0, \\ 2 & \text{if} \quad \mathcal{P}_k^{i,j} = 0 \text{ and } \mathcal{I}_k^{i,j} = 1, \\ 3 & \text{if} \quad \mathcal{P}_k^{i,j} = 1 \text{ and } \mathcal{I}_k^{i,j} = 1. \end{cases}$$

- We use a multinomial logistic regression to model the categorical conditional probabilities for the two-dimensional indicator process $(\mathcal{I}_k^{i,j}, \mathcal{P}_k^{i,j})$:

$$\log\left(\frac{\mathbb{P}\left(Y_k^{i,j} = y \,\middle|\, \mathcal{C}_{k-1}^{i,j}, \boldsymbol{z}_{k-1}^{i,j}\right)}{\mathbb{P}\left(Y_k^{i,j} = 0 \,\middle|\, \mathcal{C}_{k-1}^{i,j}, \boldsymbol{z}_{k-1}^{i,j}\right)}\right) = f_y(\mathcal{C}_{k-1}^{i,j}, \boldsymbol{z}_{k-1}^{i,j}), \quad y = 1, 2, 3, \quad k \geq 1,$$

where f_y denotes a regression function,
- If $Y_k^{i,j} = 0$, then we immediately know the values of the process $(P_k^{i,j}, I_k^{i,j})$ in the next development period k.

Models 2 and 3: Claim severities of incremental payments

- In practice, we observe both positive and negative incremental payments (salvages and subrogations). We use a spliced distribution to model non-zero incremental payments,
- We introduce the sequences of random variables $(P_k^{i,j,(+)}, P_k^{i,j,(-)})_{k=1,2,\ldots}$:

$$\begin{aligned} P_k^{i,j,(+)} &= P_k^{i,j} \Big| Y_k^{i,j} \in \{1,3\}, P_k^{i,j} > 0, \quad k = 1,2,\ldots, \\ P_k^{i,j,(-)} &= -P_k^{i,j} \Big| Y_k^{i,j} \in \{1,3\}, P_k^{i,j} < 0, \quad k = 1,2,\ldots, \end{aligned}$$

- We use a binomial logistic regression to model the conditional probabilities of a positive or a negative incremental payment:

$$\log\left(\frac{\mathbb{P}\left(P_k^{i,j} > 0 \Big| Y_k^{i,j} \in \{1,3\}, \mathcal{I}_k^{i,j}, \mathcal{C}_{k-1}^{i,j}, z_{k-1}^{i,j}\right)}{\mathbb{P}\left(P_k^{i,j} < 0 \Big| Y_k^{i,j} \in \{1,3\}, \mathcal{I}_k^{i,j}, \mathcal{C}_{k-1}^{i,j}, z_{k-1}^{i,j}\right)}\right) = f\left(\mathcal{I}_k^{i,j}, \mathcal{C}_{k-1}^{i,j}, z_{k-1}^{i,j}\right), \quad k \geq 1,$$

where f denotes a regression function.

Models 2 and 3: Claim severities of incremental payments

- We model claim severities with a double Gamma regression,
- We assume that $P_k^{i,j,(+)}$ and $P_k^{i,j,(-)}$ have Gamma distributions with the mean value:

$$\log\left(\mathbb{E}\left[P_k^{i,j,(+)} \middle| \mathcal{I}_k^{i,j}, \mathcal{C}_{k-1}^{i,j}, \boldsymbol{z}_{k-1}^{i,j}\right]\right) = f\left(\mathcal{I}_k^{i,j}, \mathcal{C}_{k-1}^{i,j}, \boldsymbol{z}_{k-1}^{i,j}\right), \quad k \geq 1,$$

for a regression function f, and the second moment given by

$$\begin{aligned}\mathrm{Var}\left[P_k^{i,j,(+)} \middle| \mathcal{I}_k^{i,j}, \mathcal{C}_{k-1}^{i,j}, \boldsymbol{z}_{k-1}^{i,j}\right] &= e^{\phi\left(\mathcal{I}_k^{i,j}, \mathcal{C}_{k-1}^{i,j}, \boldsymbol{z}_{k-1}^{i,j}\right)} \\ &\quad \cdot \left(\mathbb{E}\left[P_k^{i,j,(+)} \middle| \mathcal{I}_k^{i,j}, \mathcal{C}_{k-1}^{i,j}, \boldsymbol{z}_{k-1}^{i,j}\right]\right)^2, \quad k \geq 1,\end{aligned}$$

for another regression function ϕ,
- If $Y_k^{i,j} = 1$, we can derive the values of the process $(P_k^{i,j}, I_k^{i,j})$ in the next development period k. If $Y_k^{i,j} = 3$, we have to model the change in claim incurred at the end of the development period k. If $Y_k^{i,j} = 2$, the payment $P_k^{i,j} = 0$ is zero but we need to consider a change in claim incurred.

Model 4: Closing times

- We introduce the sequences of random variables $(\mathcal{R}_k^{i,j})_{k=1,2,\dots}$:

$$\mathcal{R}_k^{i,j} = R_k^{i,j} \Big| Y_k^{i,j} \in \{2,3\}, \quad k = 1,2,\dots.$$

  The event $\{\mathcal{R}_k^{i,j} = 0\}$ is interpreted as claim closing in development period k,
- We a binomial logistic regression to model the event that a claim is closed:

$$\log\left(\frac{\mathbb{P}\left(\mathcal{R}_k^{i,j} = 0 \Big| \mathcal{P}_k^{i,j}, P_k^{i,j}, \mathcal{C}_{k-1}^{i,j}, \boldsymbol{z}_{k-1}^{i,j}\right)}{\mathbb{P}\left(\mathcal{R}_k^{i,j} \neq 0 \Big| \mathcal{P}_k^{i,j}, P_k^{i,j}, \mathcal{C}_{k-1}^{i,j}, \boldsymbol{z}_{k-1}^{i,j}\right)}\right) = f(\mathcal{P}_k^{i,j}, P_k^{i,j}, \mathcal{C}_{k-1}^{i,j}, \boldsymbol{z}_{k-1}^{i,j}), \quad k \geq 1,$$

- It is reasonable to include an indicator for zero case reserves directly into all regression functions. This indicator is already included indirectly since the regression functions are assumed to depend on the whole history of paid and incurred claims from which the value of the case reserve can be derived. A direct inclusion should allow the neural network to learn the relevant structure more quickly,
- For claims that have zero case reserves at the end of development period k, we exactly know the change in claim incurred.

Model 5: Severities for claim incurred for open claims

- We define the sequence of random variables $(I_k^{i,j,(open)})_{k=1,2,\dots}$:

$$I_k^{i,j,(open)} = I_k^{i,j} | Y_k^{i,j} \in \{2,3\}, R_k^{i,j} \neq 0, \quad k = 1, 2, \dots,$$

- As for payments, we assume a double Gamma regression. The claim incurred $I_k^{i,j,(open)}$ has Gamma distribution with the mean value:

$$\log\left(\mathbb{E}\left[I_k^{i,j,(open)} \middle| \mathcal{P}_k^{i,j}, P_k^{i,j}, \mathcal{C}_{k-1}^{i,j}, \boldsymbol{z}_{k-1}^{i,j}\right]\right) = f(\mathcal{P}_k^{i,j}, P_k^{i,j}, \mathcal{C}_{k-1}^{i,j}, \boldsymbol{z}_{k-1}^{i,j}), \quad k \geq 1,$$

  for a regression function f, and choosing another regression function ϕ we specify the second moment of the distribution:

$$\begin{aligned}\operatorname{Var}\left[I_k^{i,j,(open)} \middle| \mathcal{P}_k^{i,j}, P_k^{i,j}, \mathcal{C}_{k-1}^{i,j}, \boldsymbol{z}_{k-1}^{i,j}\right] &= e^{\phi(\mathcal{P}_k^{i,j}, P_k^{i,j}, \mathcal{C}_{k-1}^{i,j}, \boldsymbol{z}_{k-1}^{i,j})} \\ &\cdot \left(\mathbb{E}\left[I_k^{i,j,(open)} \middle| \mathcal{P}_k^{i,j}, P_k^{i,j}, \mathcal{C}_{k-1}^{i,j}, \boldsymbol{z}_{k-1}^{i,j}\right]\right)^2, \quad k \geq 1.\end{aligned}$$

- We model claim incurred $I_k^{i,j} | Y_k^{i,j} \in \{2,3\}$ at the end of development period k and we can derive the value of the process $(P_k^{i,j}, I_k^{i,j})$ in the next development period k.

Large incremental payments and claim incurred in Models 3 and 5

- We model large incremental payments and large changes in claim incurred in each development period $k = 1, 2, \ldots,$
- For positive incremental payments $P_k^{i,j,(+)}$ in development period k, we can postulate a Pareto tail:

$$\mathbb{P}\Big(P_k^{i,j,(+)} - d_k > x \big| P_k^{i,j,(+)} > d_k, \mathcal{I}_k^{i,j}, \mathcal{C}_{k-1}^{i,j}, \boldsymbol{z}_{k-1}^{i,j}\Big)$$
$$= \left(\frac{\lambda(\mathcal{I}_k^{i,j}, \mathcal{C}_{k-1}^{i,j}, \boldsymbol{z}_{k-1}^{i,j})}{\lambda(\mathcal{I}_k^{i,j}, \mathcal{C}_{k-1}^{i,j}, \boldsymbol{z}_{k-1}^{i,j}) + x}\right)^{\gamma\left(\mathcal{I}_k^{i,j}, \mathcal{C}_{k-1}^{i,j}, \boldsymbol{z}_{k-1}^{i,j}\right)}, \quad x > 0, \quad k \geq 1,$$

- Regression models should be built for the probability of a large claim and the severity of the large claim (for λ and γ) since claims with particular features (e.g. body claims) are likely to have higher propensity to generate large claims,
- We choose a simpler approach. In each development period k, we set a fixed probability for the occurrence of a large claim, from which we deduce the threshold d_k, and estimate constant parameters (λ_k, γ_k) of the large claim distribution using EVT. Next, based on descriptive statistics and knowledge about business, we determine key features in $\boldsymbol{z}_{k-1}^{i,j}$ which have high propensity to generate large claims and allocate large claims in simulations to claims with these features in the first place.

Estimation approach

- We train a neural network with M hidden layers and $q_m \in \mathbb{N}$ hidden neurons in layers $m = 1, \ldots, M$,
- We define the network layers:

$$\begin{aligned}\theta^m(\boldsymbol{x}) &= \left(\theta_1^m(\boldsymbol{x}), \ldots, \theta_{q_m}^m(\boldsymbol{x})\right)' \in \mathbb{R}^{q_m}, \quad m = 1, \ldots, M, \\ \theta_r^m(\boldsymbol{x}) &= \varphi\left(c_r^m + \langle \boldsymbol{w}_r^m, \boldsymbol{x}\rangle\right), \quad r = 1, \ldots, q_m,\end{aligned}$$

  where φ denotes the (non-linear) activation function, c_r^m denotes the bias term (the constant), $\boldsymbol{w}_r$ denotes the network weights and $\boldsymbol{x}$ denotes a vector of predictors,
- For layers $m = 1, \ldots, M$, we use the hyperbolic tangent activation function for φ,
- The mapping

$$\boldsymbol{x} \mapsto c^{M+1} + \left\langle \boldsymbol{w}^{M+1}, \left(\theta^M \circ \cdots \circ \theta^1\right)(\boldsymbol{x})\right\rangle,$$

  gives us the prediction in the output layer $M+1$ with linear activation function and the output of dimension 1.

SECTIONS VIRTUAL COLLOQUIUM 2020

Estimation approach

- Let $\boldsymbol{x}_\ell$ denote a vector of predictors which characterizes an individual observation ℓ,
- We choose an initial regression model (GLM, GAM, tree) and derive the initial predictions $(\hat{p}^{\text{init}}_{\ell,a})_{a\in\mathcal{A}}$, respectively $\hat{\mu}^{\text{init}}_\ell$, for the probability that observation ℓ is in class $a \in \mathcal{A}$, and the expected value of the response for observation ℓ,
- For the Gamma regressions, we use the exponential activation function with an output of dimension 1 in layer $M+1$. We model the expected values of an individual case ℓ by

$$\begin{aligned}\mu_\ell &= e^{f(\boldsymbol{x}_\ell)}\\ f(\boldsymbol{x}_\ell) &= c^{M+1} + \alpha \log\left(\hat{\mu}^{\text{init}}_\ell\right) + \beta\Big\langle \boldsymbol{w}^{M+1}, \left(\theta^M \circ \cdots \circ \theta^1\right)(\boldsymbol{x}_\ell)\Big\rangle,\end{aligned}$$

- For the categorical regressions with $A = dim(\mathcal{A})$ classes, we use the softmax activation function with output of dimension A in layer $M+1$. We model the (softmax) probabilities for a single case ℓ as follows

$$\begin{aligned}p_{\ell,a} &= \frac{e^{f_a(\boldsymbol{x}_\ell)}}{\sum_{u\in\mathcal{A}} e^{f_u(\boldsymbol{x}_\ell)}}, \quad a\in\mathcal{A},\\ f_a(\boldsymbol{x}_\ell) &= c_a^{M+1} + \sum_{u\in\mathcal{A}} \alpha_u \log\left(\hat{p}^{\text{init}}_{\ell,u}/\hat{p}^{\text{init}}_{\ell,a^*}\right)\\ &\quad + \sum_{u\in\mathcal{A}} \beta_u \Big\langle \boldsymbol{w}_u^{M+1}, \left(\theta_u^M \circ \cdots \circ \theta_u^1\right)(\boldsymbol{x}_\ell)\Big\rangle, \quad a\in\mathcal{A},\end{aligned}$$

Estimation approach

- Neural networks are fitted by minimizing the categorical cross-entropy or the unscaled Gamma deviance loss of the regression model on a validation set,
- For fitting double Gamma regressions, we use a simplified approach: We first fit a neural network to the mean of the response assuming Gamma distribution with constant dispersion coefficient. Next, we calculate the Pearson residuals and we fit a second neural network for the dispersion coefficient with Pearson residuals as the response. We do not iterate the estimation process as it is done when we fit double GLMs/GAMs,
- We use drop-out probabilities as a regularization technique,
- We perform the bias regularization technique. For each categorical Model 1,2 and 4, we estimate a multinomial or binomial GLM with canonical link function (logit-link) where we use neurons from the last output layer estimated for the neural network as regressors in the GLM. For Gamma Models 3 and 5, we simply scale the predictions from the neural networks to obtain the correct sample mean of the response (this also includes the predictions for the dispersion coefficients).

Estimation approach

In initial regression models M_0:

- We use cumulative payments defined by $CP^{i,j}_{k-1} = \sum_{l=0}^{k-1} P^{i,j}_l$ and claim incurred $I^{i,j}_{k-1}$ as predictor variables,
- We choose regression trees for payments and claim incurred. We resign from M_0 for the categorical regressions.

In 0^{th} neural networks NN_0:

- We use all predictor variables which we choose for M_0,
- We include the indicators $\mathcal{I}^{i,j}_k, \mathcal{P}^{i,j}_k, \mathbb{1}_{\{R^{i,j}_k=0\}}$ and the incremental payment $P^{i,j}_k$ as regressors,
- As far as the vector of additional features $z^{i,j}_{k-1}$ is concerned, we include all available claims features such as accident quarter, reporting delay, claim segment, claim type and claim origin. We do not include accident year. When we fit the regression function f_{K+1} to all development periods latter than K, then $z^{i,j}_{k-1}$ also includes the development period k as a regressor,
- We use the prediction from the initial model M_0 as a regressor in the neural network (Combined Actuarial Neural Network),
- By this choice, we assume that there is a Markovian structure in the claims development process and only the most recent information about the claim is relevant for the next step predictions.

In main neural networks NN_1:

- We use all predictor variables which we choose for NN_0,
- We add all variables included in the individual claim history $\mathcal{C}^{i,j}_{k-1}$ as predictors. Hence, we relax the Markovian assumption postulated in neural network NN_0 and M_0. We keep the cumulative payments $CP^{i,j}_{k-1}$ in the regression functions.

Numerical example

- We have a data set consisting of $1,331,856$ individual claims. The data set describes the development processes of claims with accident dates and reporting dates both between January 2005 and December 2018,
- In order to anonymize the results, incremental payments and case reserves are scaled with a constant,
- We fit the neural networks on quarterly data. Hence, we deal with development periods $k = 1, 2, \ldots, 55$,
- We only model positive payments and resign from modeling negative payments (salvages and subrogations),
- We fit separate neural networks for each development period $k = 1, \ldots, 16$, and one neural network for all development periods $k = 17, \ldots, 55$, for Models 1, 3_positive, 4 and 5,
- Large incremental payments and large claim incurred are removed in each development period from the training set before Gamma regression models are fitted,
- Continuous predictors are transformed with logarithmic function and normalized with *MinMaxScaler* transformation,
- The responses for the Gamma regressions are scaled so that their empirical means are equal one.

Development period $k = 8$

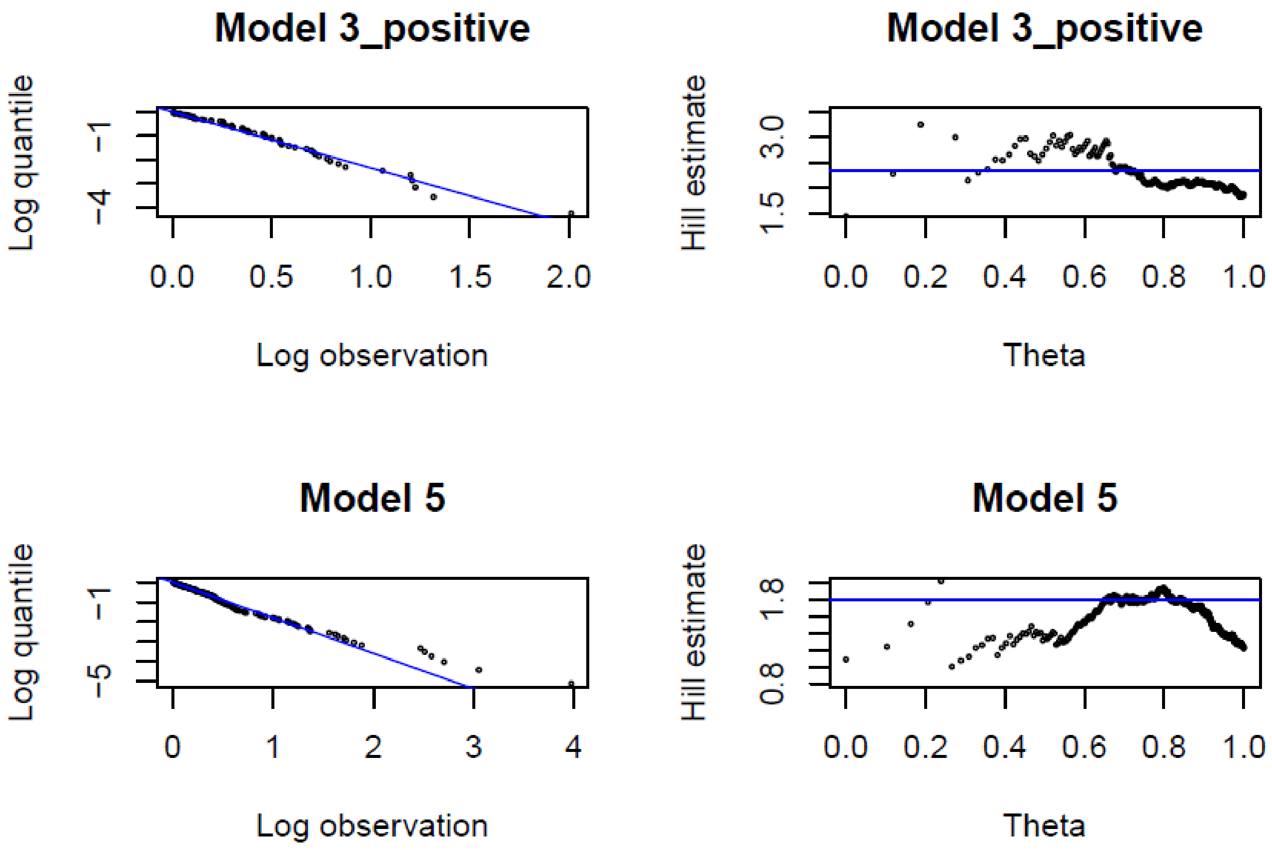

Figure: Pareto quantile plots and altHill plots for incremental payments and changes in claim incurred in $k = 8$.

Development period $k = 8$

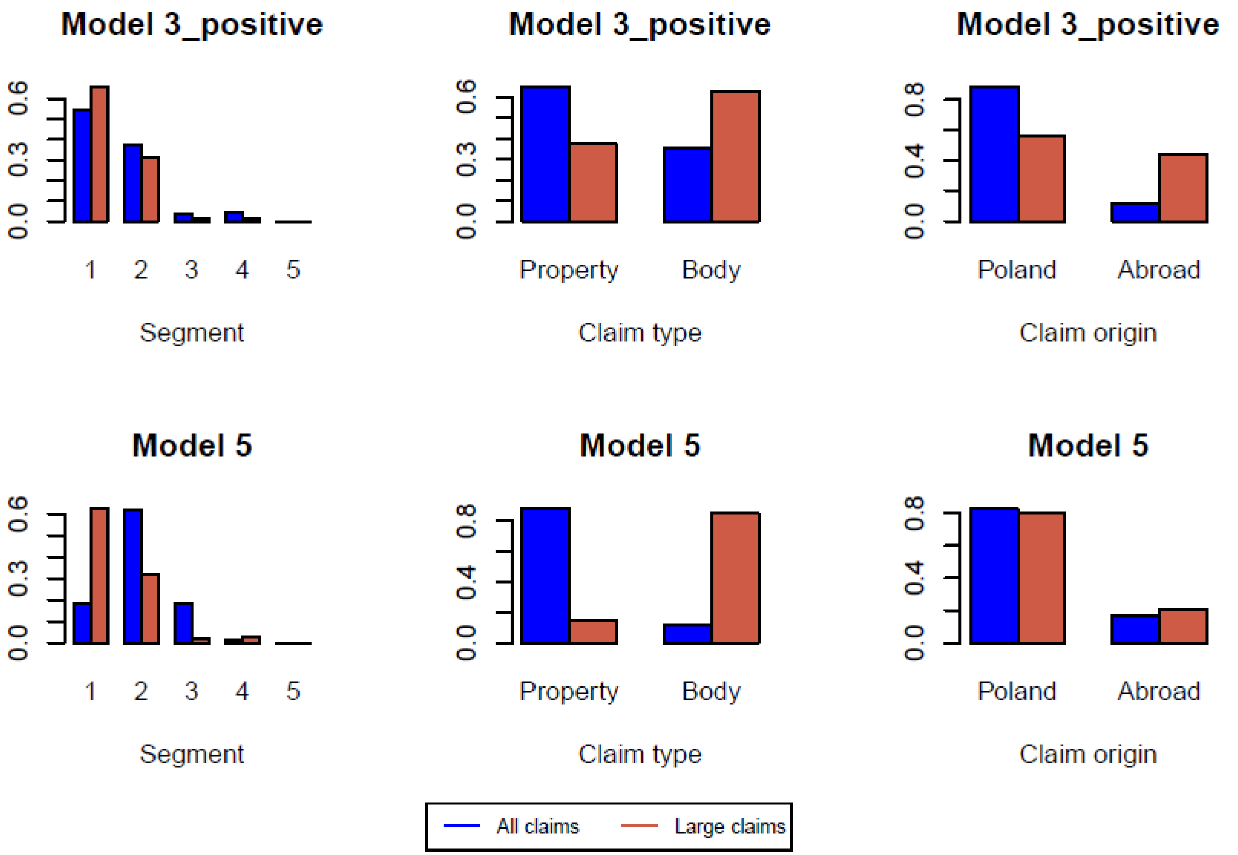

Figure: Proportions of claim features in the whole data set and in the data set of large claims in $k = 8$.

Development period $k = 8$

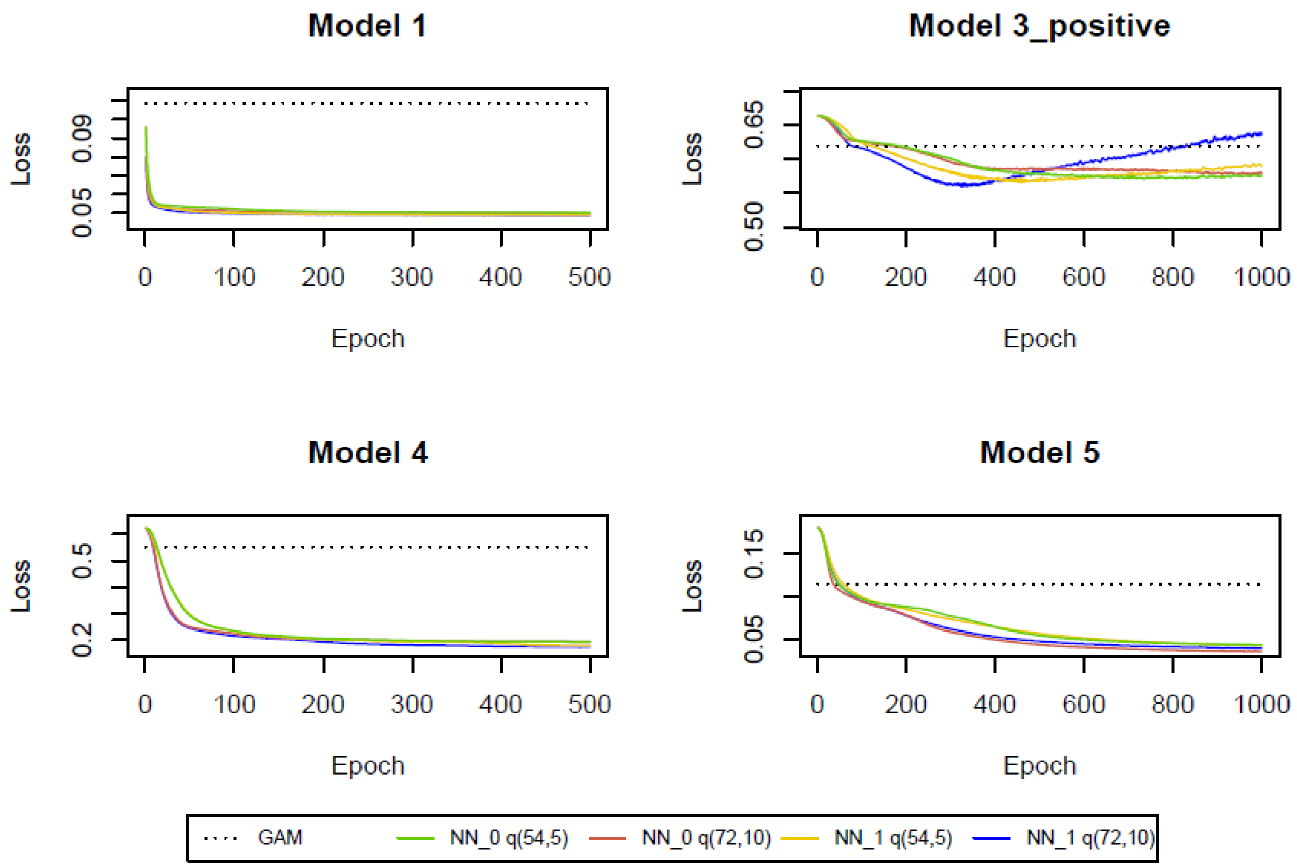

Figure: Cross-entropy and deviance loss functions on validation sets observed during the training of the neural networks in $k = 8$.

Development period $k = 8$

	D_{GAM}	D_{NN_0}	D_{NN_1}	$1 - \frac{D_{NN_0}}{D_{\text{GAM}}}$	$1 - \frac{D_{NN_1}}{D_{\text{GAM}}}$
Model 1: $q = (72, 10)$	0.1086	0.0499	0.0487	54.03%	**55.17%**
Model 1: $q = (54, 5)$	0.1086	0.0503	0.0488	53.73%	55.03%
Model 3_positive: $q = (72, 10)$	0.6190	0.5763	0.5593	6.90%	**9.64%**
Model 3_positive: $q = (54, 5)$	0.6190	0.5707	0.5657	7.80%	8.61%
Model 4: $q = (72, 10)$	0.5533	0.1941	0.1742	64.92%	**68.52%**
Model 4: $q = (54, 5)$	0.5533	0.1939	0.1792	64.96%	67.61%
Model 5: $q = (72, 10)$	0.1152	0.0349	0.0390	**69.71%**	66.17%
Model 5: $q = (54, 5)$	0.1152	0.0431	0.0424	62.61%	63.19%

Table: Minimal cross-entropy and deviance loss functions on validation sets observed during the training of the neural networks in $k = 8$.

Development period $k = 8$

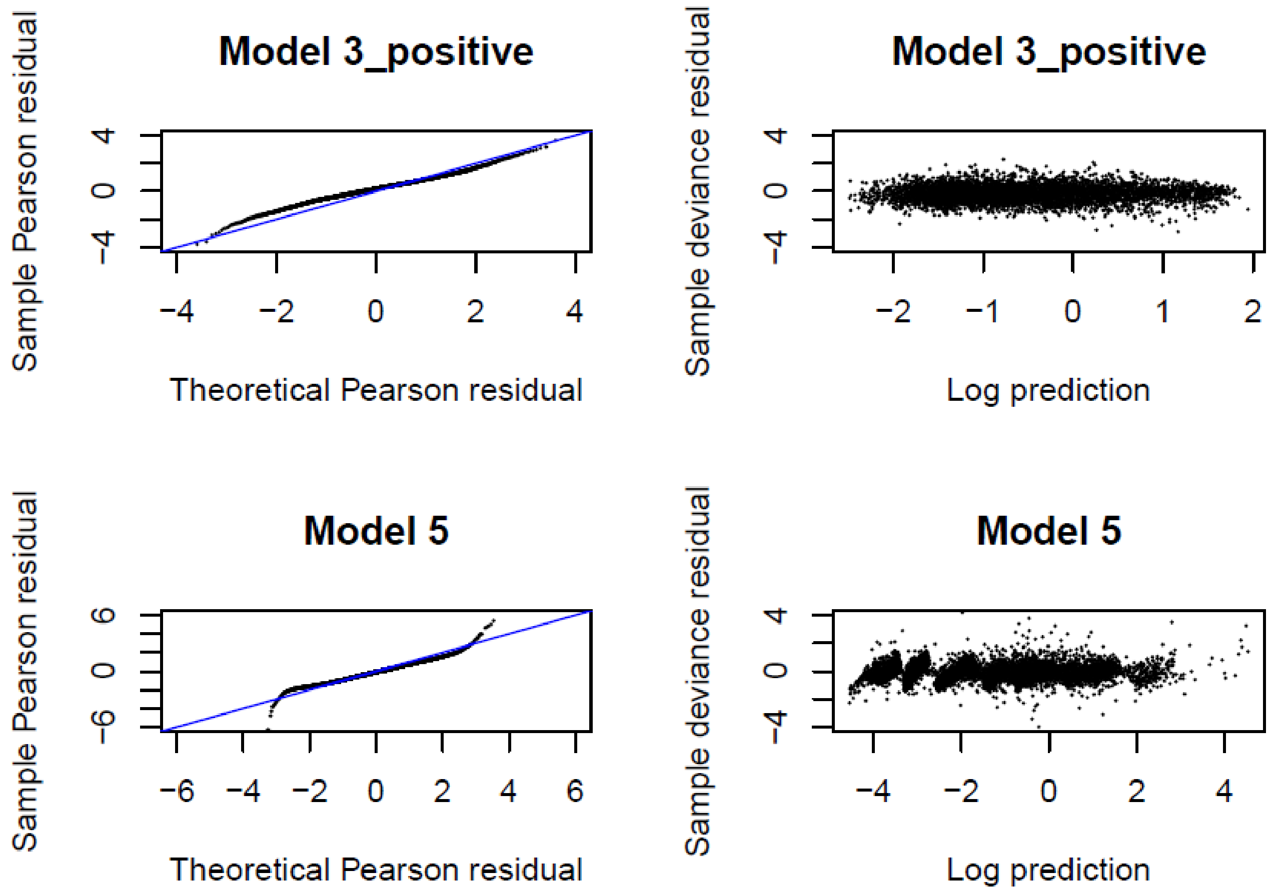

Figure: QQ normal plots and Tukey-Anscombe plots in Models 3_positive and 5 fitted with neural networks NN_1 in $k = 8$

Development period $k = 16$

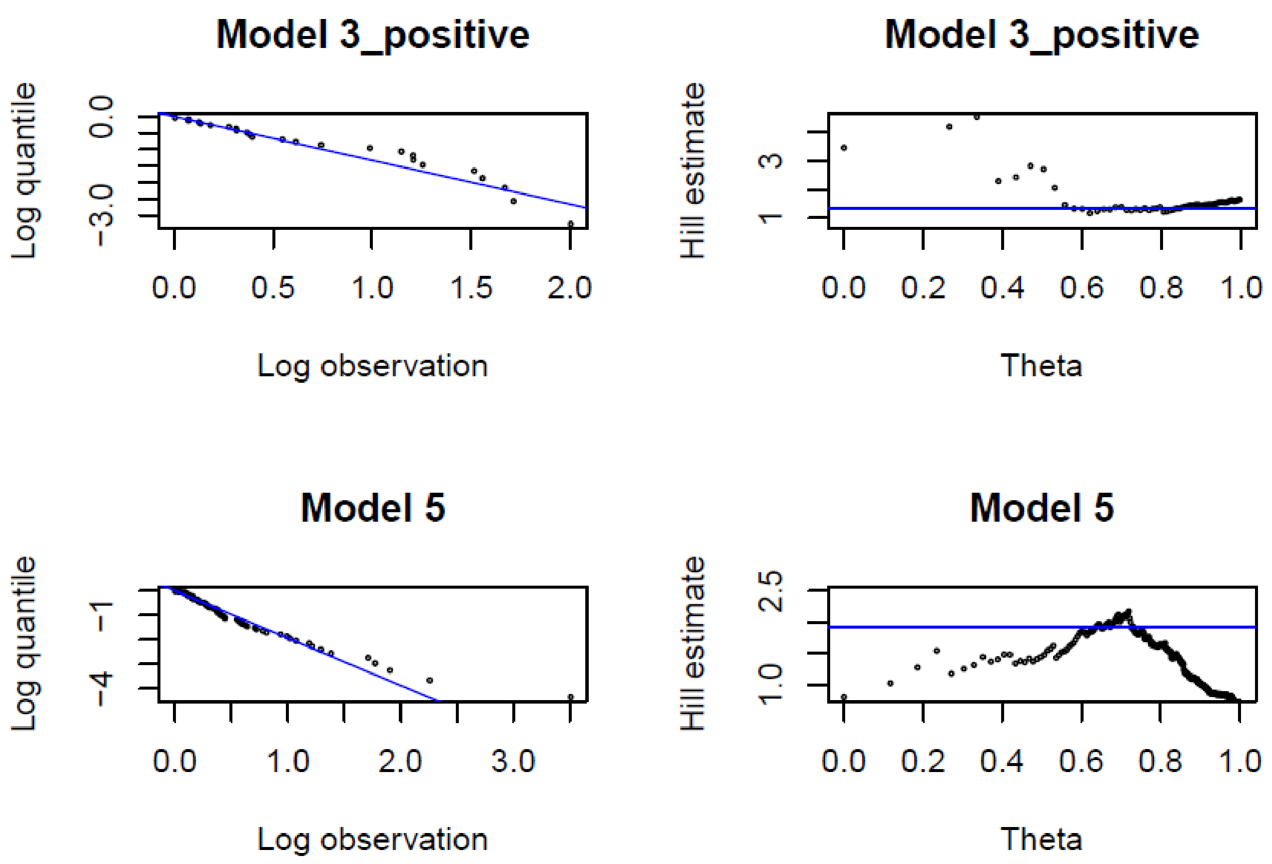

Figure: Pareto quantile plots and altHill plots for incremental payments and changes in claim incurred in $k = 16$.

Development period $k = 16$

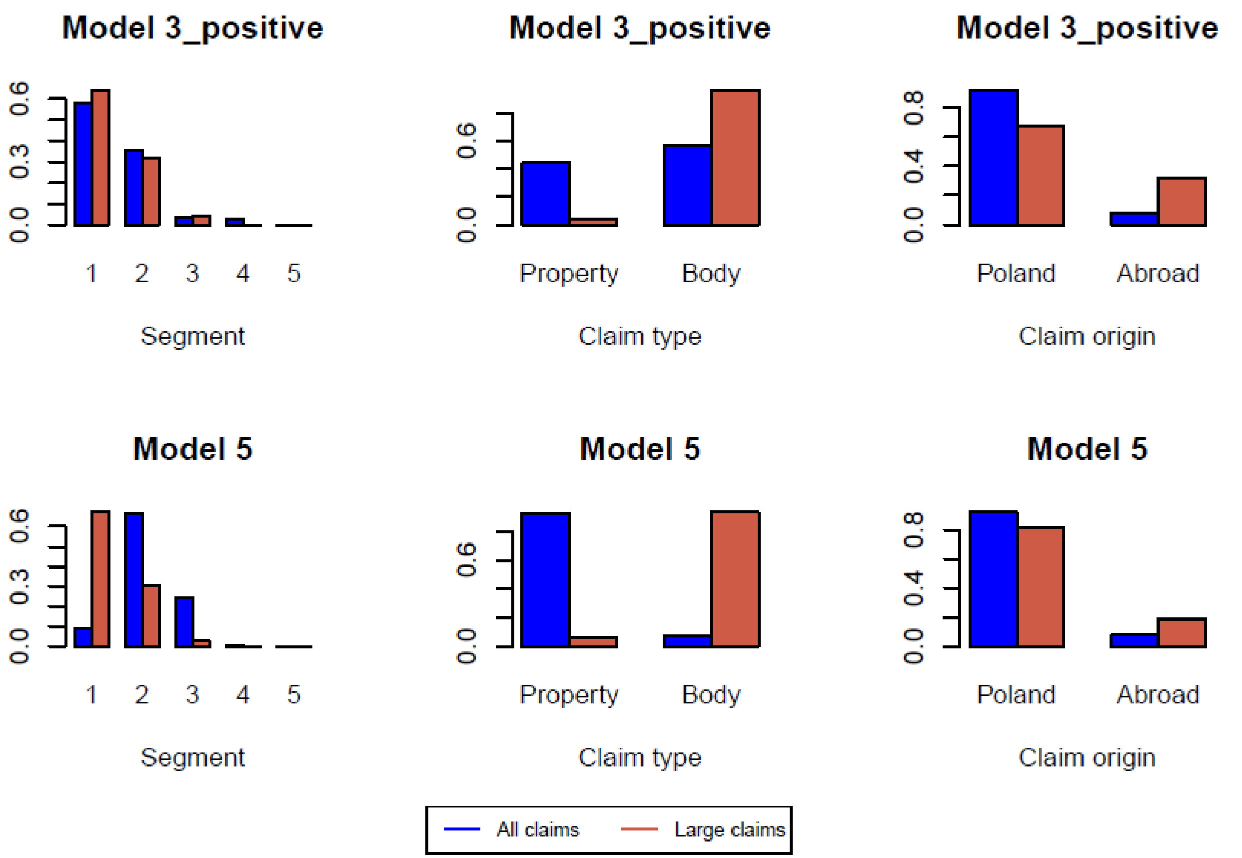

Figure: Proportions of claim features in the whole data set and in the data set of large claims in $k = 16$.

Development period $k = 16$

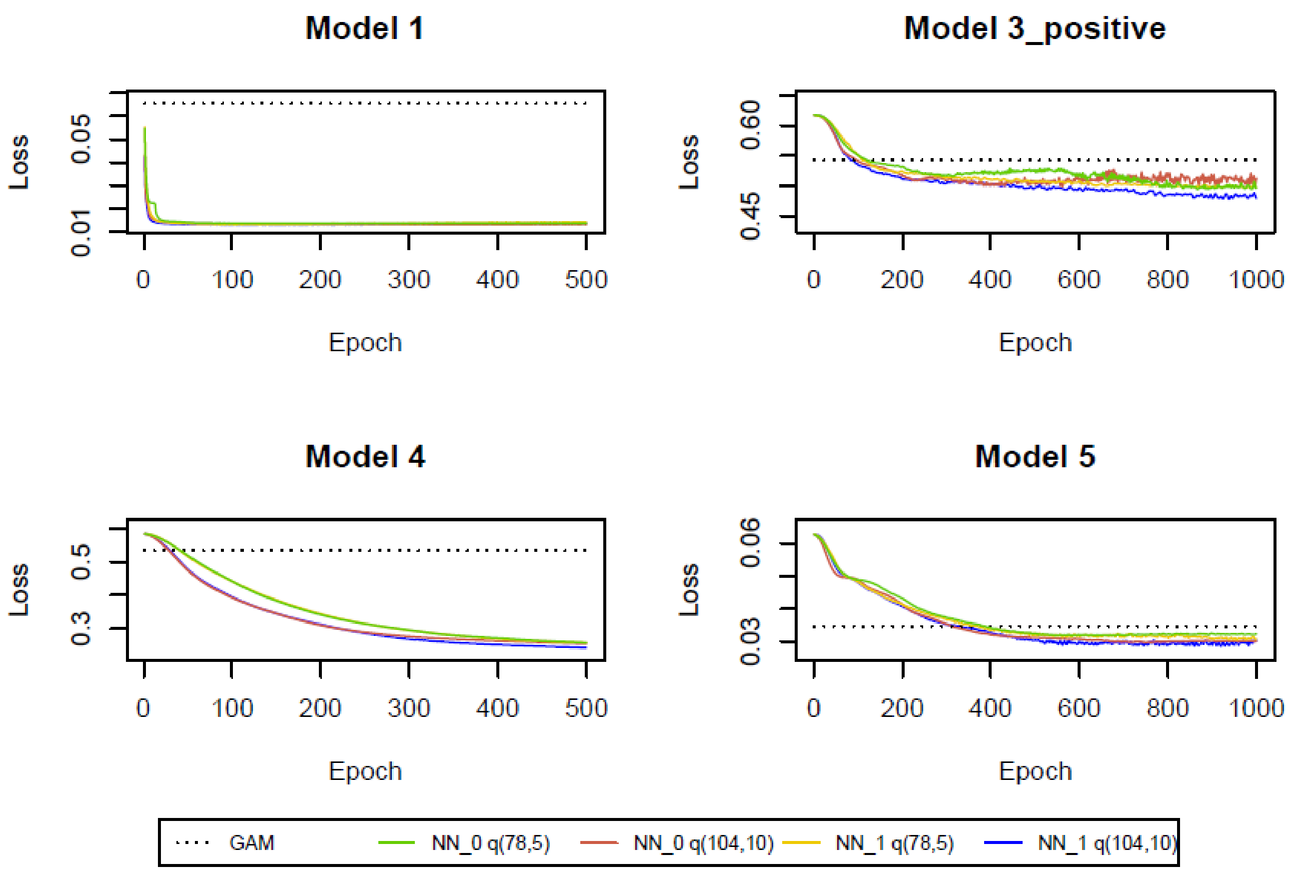

Figure: Cross-entropy and deviance loss functions on validation sets observed during the training of the neural networks in $k = 16$.

SECTIONS VIRTUAL COLLOQUIUM 2020

Development period $k = 16$

	D_{GAM}	D_{NN_0}	D_{NN_1}	$1 - \frac{D_{NN_0}}{D_{\text{GAM}}}$	$1 - \frac{D_{NN_1}}{D_{\text{GAM}}}$
Model 1: $q = (104, 10)$	0.0653	0.0132	0.0130	79.84%	**80.10%**
Model 1: $q = (78, 5)$	0.0653	0.0136	0.0132	79.15%	79.85%
Model 3_positive: $q = (104, 10)$	0.5425	0.4988	0.4781	8.05%	**11.87%**
Model 3_positive: $q = (78, 5)$	0.5425	0.4972	0.4943	8.35%	8.88%
Model 4: $q = (104, 10)$	0.5350	0.2540	0.2417	52.51%	**54.82%**
Model 4: $q = (78, 5)$	0.5350	0.2569	0.2568	51.98%	51.99%
Model 5: $q = (104, 10)$	0.0347	0.0297	0.0285	14.41%	**17.96%**
Model 5: $q = (78, 5)$	0.0347	0.0319	0.0305	8.23%	12.22%

Table: Minimal cross-entropy and deviance loss functions on validation sets observed during the training of the neural networks in $k = 16$.

Development period $k = 16$

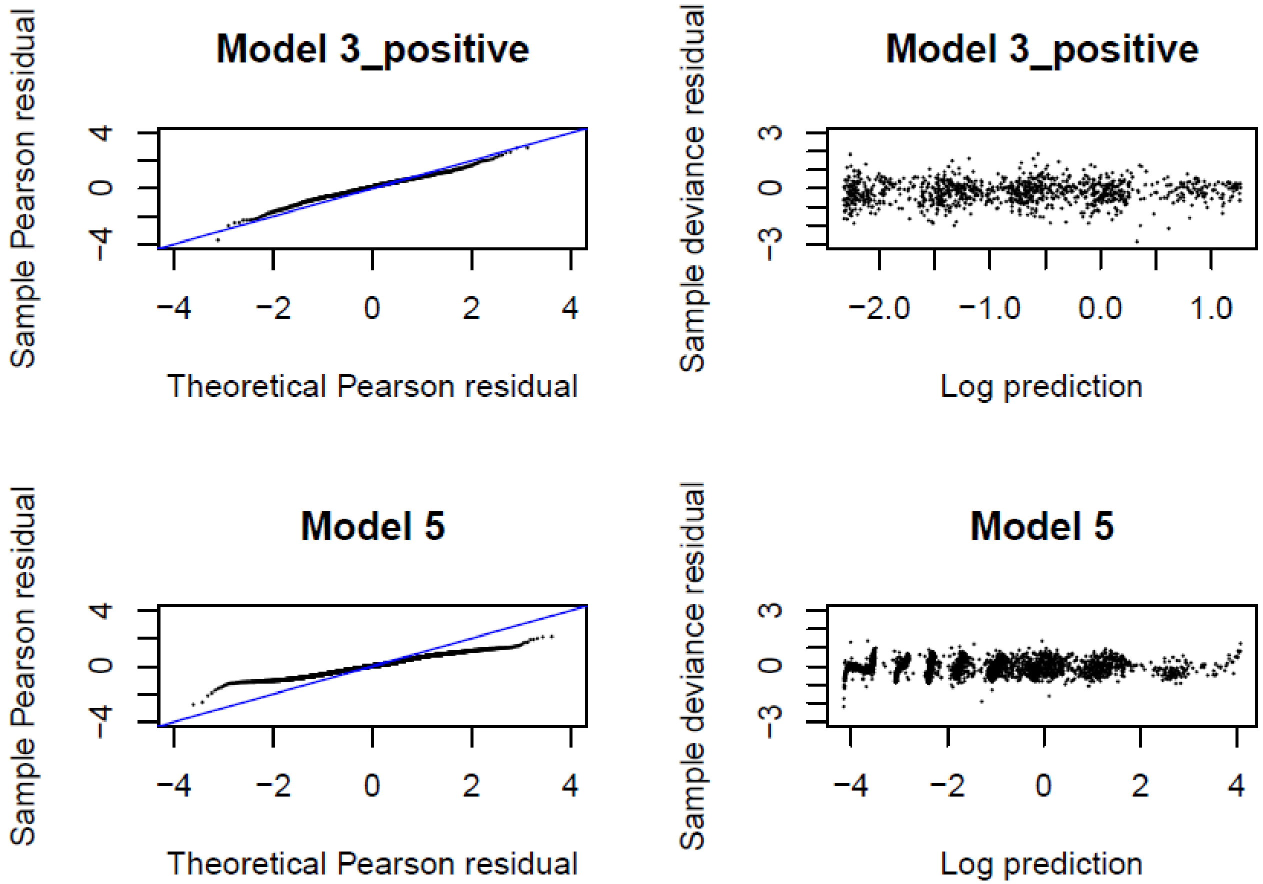

Figure: QQ normal plots and Tukey-Anscombe plots in Models 3_positive and 5 fitted with neural networks NN_1 in $k = 16$

Individual claims simulations

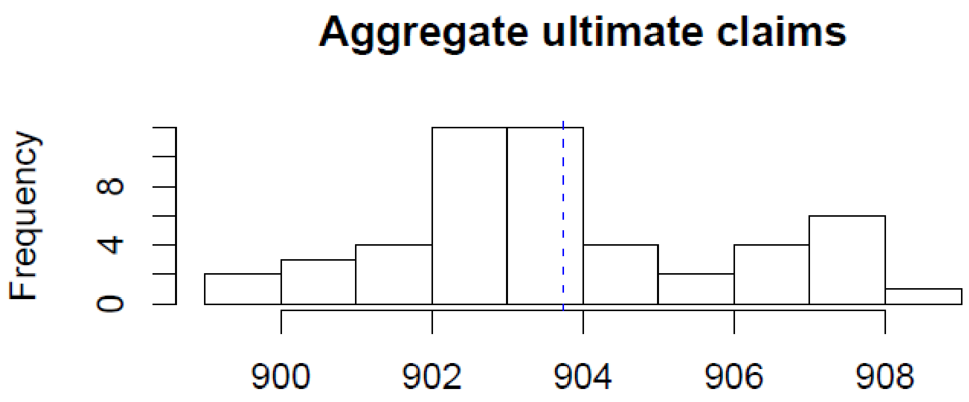

Figure: Histogram of the aggregate ultimate payments from the RBNS claims (in MM). The dashed line indicates the mean value from the simulations.

Individual claims simulations

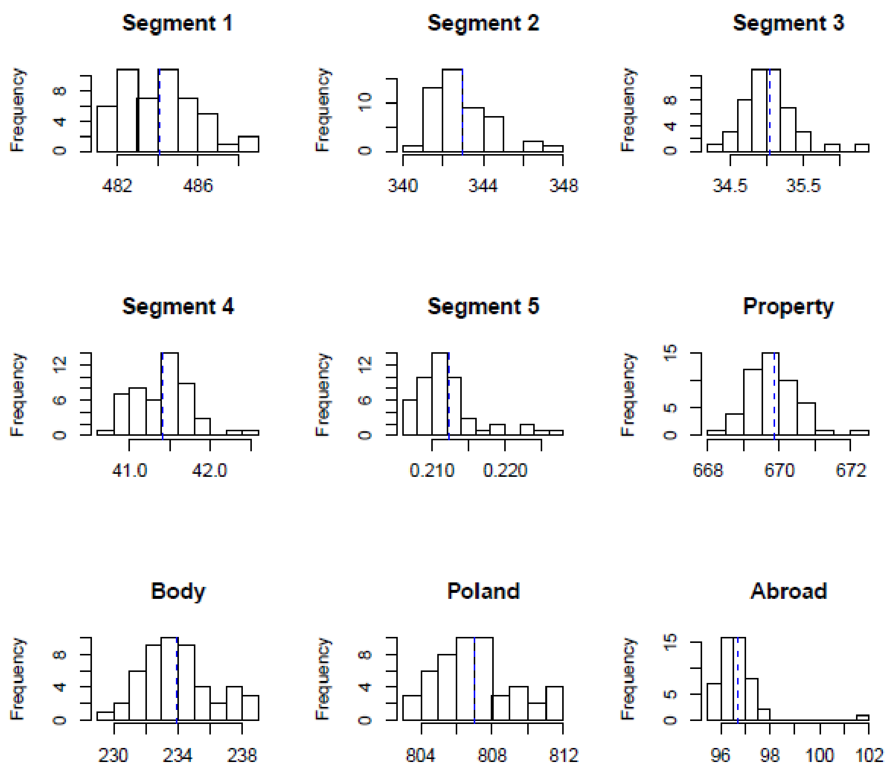

Figure: Histograms of the aggregate ultimate payments from the RBNS claims (in MM). The dashed lines indicate the mean value from the simulations.

Individual claims simulations

Accident year	RBNS reserve				Case reserve
	Mean	25th quantile	75th quantile	Chain-Ladder	
2009	1.26	1.13	1.39	1.35	0.65
2010	1.51	1.32	1.68	1.97	1.56
2011	2.29	2.03	2.39	2.66	3.14
2012	3.65	3.26	3.78	3.85	3.63
2013	5.10	4.69	5.18	5.11	5.07
2014	5.99	5.43	6.35	6.61	7.00
2015	8.27	7.57	8.36	9.36	6.90
2016	11.91	11.48	12.27	13.43	10.81
2017	17.20	16.67	17.80	19.62	11.65
2018	35.09	34.64	35.59	39.77	28.02
All	92.27	88.21	94.78	103.74	78.43

Table: Simulations results and Chain-Ladder (CL) estimates (in MM).

Individual claims simulations

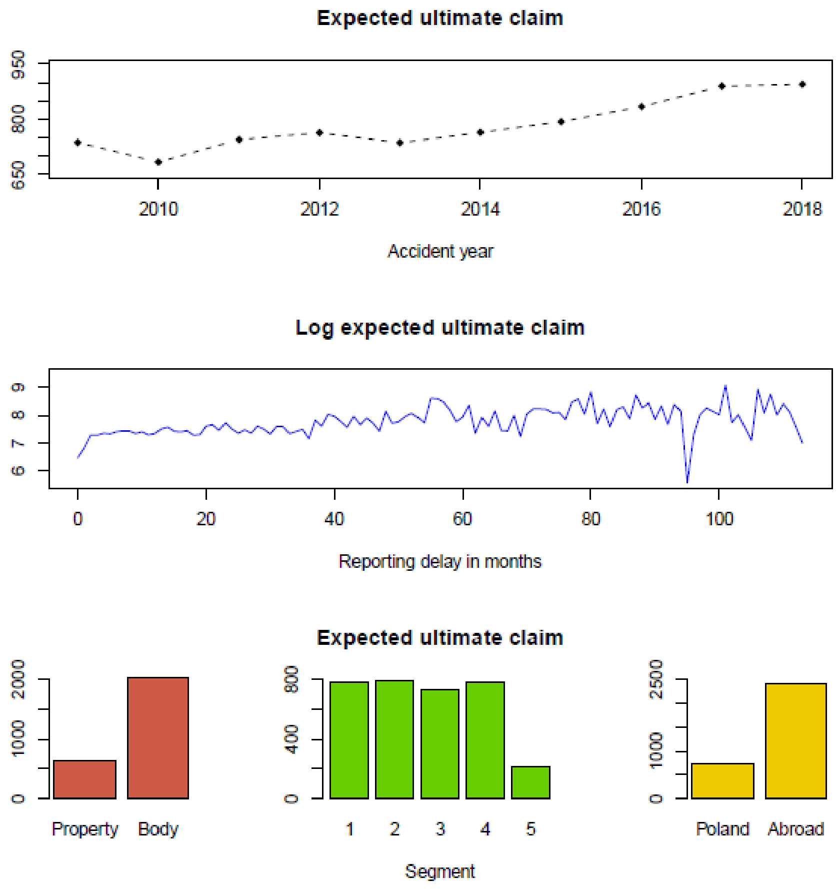

Figure: Expected aggregate ultimate payments from an individual claim.

Individual claims simulations

Accident year	RBNS reserve Mean	25th quantile	75th quantile	Chain-Ladder	Case reserve
2009	0.94	0.79	1.08	1.35	0.65
2010	1.31	1.18	1.36	1.97	1.56
2011	1.94	1.74	2.09	2.66	3.14
2012	3.11	2.94	3.32	3.85	3.63
2013	4.51	4.29	4.71	5.11	5.07
2014	5.49	5.17	5.78	6.61	7.00
2015	7.78	7.43	8.02	9.36	6.90
2016	11.40	10.84	11.79	13.43	10.81
2017	17.16	16.83	17.44	19.62	11.65
2018	36.17	35.64	36.68	39.77	28.02
All	89.80	86.87	92.28	103.74	78.43

Table: Simulations results and Chain-Ladder (CL) estimates (in MM) - a new re-calibration of NNs and a new simulation run.

References

- Ł. Delong, M.V. Wüthrich (2020) *Regression models for the joint development of individual payments and claim incurred*, Risks, available at SSRN,
- Ł. Delong, M.M. Lindholm, M.V. Wüthrich (2020) *Collective reserving using individual claims data*, preprint, available at SSRN.

Thank you for your attention

IAA AAI
INSTITUT DES ACTUAIRES
SECTIONS VIRTUAL COLLOQUIUM | 2020

Contact details :

Łukasz Delong

SGH Warsaw School of Economics,
Institute of Econometrics, Department of Probabilistic Methods

lukasz.delong@sgh.waw.pl

https://www.actuarialcolloquium2020.com/

Disclaimer:

The views or opinions expressed in this presentation are those of the authors and do not necessarily reflect official policies or positions of the Institut des Actuaires (IA), the International Actuarial Association (IAA) and its Sections.